Section 1. $\mathbb{R}^\ell$ – as a commodity -price space

1.1. $\mathbb{R}^\ell$ as the commodity space

1.2. $\mathbb{R}^\ell$ – as a space of prices

1.1 $\mathbb{R}^\ell$ as the commodity space

Definition 1.1 A **commodity** is a physical good or service (non-physical) for which there is demand and it satisfies human wants or needs.

Each commodity is completely specified by:

- its physical characteristics,
- its location,
- date at which it is available (availability location).

Mathematical structure

Definition 1.2. The **Cartesian product (the cross product)** of two sets A and B (read: "A cross B") is defined as the set of all ordered pairs having the first member from set A and the second member from set B: $A \times B = \{(a,b) : a \in A \ \wedge \ b \in B\}$.

The set $\mathbb{R}^\ell = \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R} = \{x = (x_1, x_2, \ldots, x_\ell) : x_k \in \mathbb{R} \text{ for } k \in \{1, \ldots, \ell\}, \ \ell \in \mathbb{N}\}$ is ℓ −dimensional real (Euclidean/vector) space (a set of ℓ − elementary sequences/vectors).

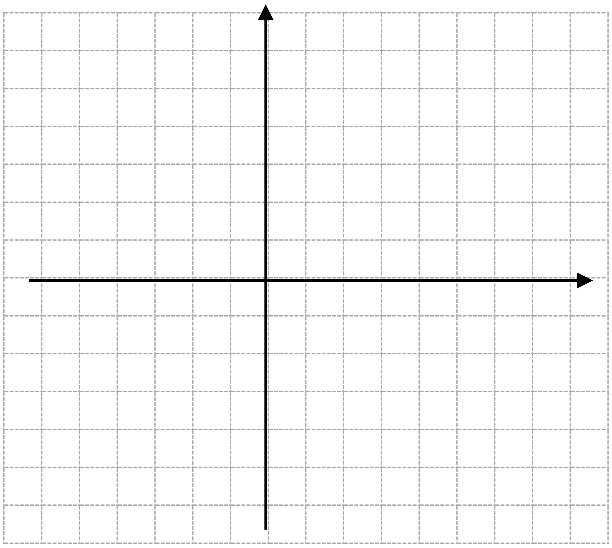
$$\forall x, y \in \mathbb{R}^\ell \quad x + y = (x_1, x_2, \ldots, x_\ell) + (y_1, y_2, \ldots, y_\ell) = (x_1 + y_1, x_2 + y_2, \ldots, x_\ell + y_\ell)$$

$$\forall x \in \mathbb{R}^\ell \quad \forall a \in \mathbb{R} \quad a \cdot x = a \cdot (x_1, x_2, \ldots, x_\ell) = (ax_1, ax_2, \ldots, ax_\ell)$$

$$\forall x, y \in \mathbb{R}^\ell \quad x \circ y = (x_1, x_2, \ldots, x_\ell) \circ (y_1, y_2, \ldots, y_\ell) = x_1y_1 + x_2y_2 + \cdots + x_\ell y_\ell$$

Example 1.1 Find (and make a graph if possible) $x + y, a \cdot x, x \circ y$ where:

a) $x = (4,1), y = (2,5), a = 2, \ a = -1$



b) $x = (-2,3,5,0), y = (1,0,-2,3), a = 3$

Assumption: there exists a finite number ℓ of commodities (markets).

A commodity bundle, i.e. a list of real numbers $x_k \in \mathbb{R}$ indicating the quantity of commodity k for $k = 1, 2, \dots, \ell$, can be described therefore as an ℓ -dimensional vector $(x_1, x_2, \dots, x_\ell)$, where $x_k \in \mathbb{R}$ and as a point in ℓ -dimensional Euclidean (vector) space $\mathbb{R}^\ell$, the commodity space:

$$\mathbb{R}^\ell = \{(x_1, x_2, \dots, x_\ell) : x_k \in \mathbb{R} \quad for \quad k = 1, 2, \dots, \ell\,\}$$

$x \in \mathbb{R}^\ell \Leftrightarrow x = (x_1, x_2, \dots, x_\ell)$ consists of x_1 units of the first good, x_2 units of the second good, up to x_ℓ units of the ℓ -th good and is called:

- commodity bundle or
- plan of action of economic agent (consumption plan or production plan),

Properties (conventions) of commodities:

1) a unit of each commodity can be defined (physical units or pieces), thus $x_k \in \mathbb{R}$ indicates the quantity of given commodity k (goods are measurable),
2) „perfect" or infinite divisibility of commodities (any real number is possible as a quantity for each commodity),
3) quantities of each commodity are measured by real numbers positive or negative, to justify negative real numbers, the following convention is introduced. Each commodity bundle is assumed to be an input-output vector in $\mathbb{R}^\ell$, where what is made available to an economic agent is called an input for him, and what is made available by an economic agent is called an output for him.

Input-output interpretation:

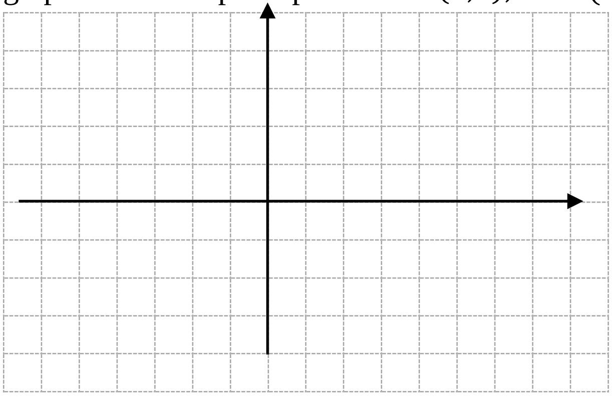
- **inputs** – those commodities which are made available to economic agents (for producer- land, machines, for consumer – consumption goods),
- **outputs** - those commodities which are made available by economic agents to others (for a producer –final products, for a consumer – kinds of labour).

Scheme:

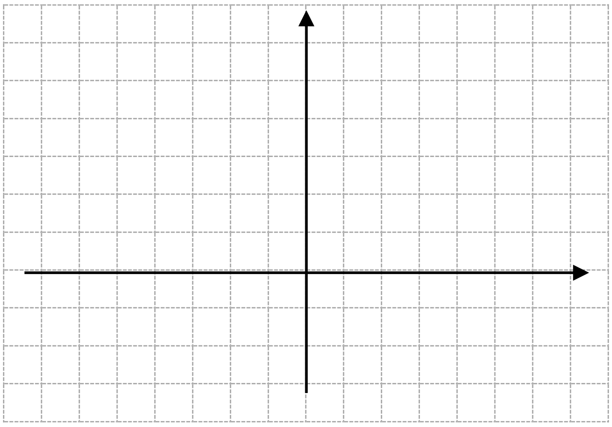
	producers	consumers
INPUTS		
OUTPUTS		

Example 1.2

a) Consumer viewpoint. The two-commodities world: wine and bread. Units: a bottle, a loaf. Make a graph of consumption plans: $x = (3,1)$, $\bar{x} = (0,2)$



b) Producer viewpoint. The two-commodities world: flour and bread. Units: kg, a loaf. Make a graph of a production plans: $y = (-1,2)$, $\bar{y} = (-2,0)$



Example 1.3 Construct a plan of production and a plan of consumption in a space of commodities.

Commodity:									
Unit:									
Price (per unit):									
Production plan:									
Consumption plan:									

1. 2. $\mathbb{R}^\ell$ – as a space of prices

Definition 1.2 A price is a real number which is the amount paid in exchange for one unit of the commodity.

Remarks:

- price of the given commodity is determined by technologies, costs of production, relative scarcity, resources, tastes,
- it allows estimating actions of economic agents (to calculate the profit of producer and cost of realisation of consumption plan),
- price can be any real number (rational or irrational) positive or negative.

Given the good or service in the space $\mathbb{R}^\ell$ the number $p_k \in \mathbb{R}$ for $k = 1,2,\dots,\ell$ indicates the price of a unit of commodity k (p_k is the amount paid in exchange for one unit of the commodity k). A price system or a price vector can thus be represented by a point in Euclidean space $\mathbb{R}^\ell$:

$$p = (p_1, p_2, \dots, p_\ell) \in \mathbb{R}^\ell,$$

Assumptions:

- $p_k > 0$ for scarce commodity (a price is a positive number),
- $p_k = 0$ for „free” commodity,
- $p_k < 0$ for harmful commodity (a price is a negative number).

Thus $\mathbb{R}^\ell$ is interpreted as a commodity-price space.

The value of a commodity bundle $x \in \mathbb{R}^\ell$ given a price vector $p \in \mathbb{R}^\ell$ (i.e. the profit of a producer, the expenditure of a consumer) is :

$$p \circ x = (p_1, p_2, \dots, p_\ell) \circ (x_1, x_2, \dots, x_\ell) = p_1x_1 + p_2x_2 + \dots + p_\ell x_\ell = \sum_{k=1}^{\ell} p_k x_k \in \mathbb{R}$$

Example 1.4 Let prices of commodities from Example 1.3 be given.

Calculate the profit of the producer and the expenditure of the consumer.