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FIXED INCOME

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Fixed Income

LEARNING MODULE

1

The Term Structure and Interest Rate Dynamics

by Thomas S.Y. Ho, PhD, Sang Bin Lee, PhD, and Stephen E. Wilcox, PhD, CFA.

Thomas S.Y. Ho, PhD, is at Thomas Ho Company Ltd (USA). Sang Bin Lee, PhD, is at Hanyang University (South Korea). Stephen E. Wilcox, PhD, CFA, is at Minnesota State University, Mankato (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve
☐	describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping
☐	describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management
☐	describe the strategy of rolling down the yield curve
☐	explain the swap rate curve and why and how market participants use it in valuation
☐	calculate and interpret the swap spread for a given maturity
☐	describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk
☐	explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve
☐	explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks
☐	explain the maturity structure of yield volatilities and their effect on price volatility
☐	explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes

1 SPOT RATES, FORWARD RATES, AND THE FORWARD RATE MODEL

- ☐ describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve
- ☐ describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping

Interest rates are both a barometer of the economy and an instrument for its control. The term structure of interest rates—market interest rates at various maturities—is a vital input into the valuation of many financial products. The quantification of interest rate risk is of critical importance to risk managers. Understanding the determinants of interest rates, and thus the drivers of bond returns, is imperative for fixed-income market participants. Here, we explore the tools necessary to understand the term structure and interest rate dynamics—that is, the process by which bond yields and prices evolve over time.

Section 1 explains how spot (or current) rates and forward rates, which are set today for a period starting in the future, are related, as well as how their relationship influences yield curve shape. Section 2 builds upon this foundation to show how forward rates impact the yield-to-maturity and expected bond returns. Section 3 explains how these concepts are put into practice by active fixed-income portfolio managers.

The swap curve is the term structure of interest rates derived from a periodic exchange of payments based on fixed rates versus short-term market reference rates rather than default-risk-free government bonds. Sections 4 and 5 describe the swap curve and its relationship to government yields, known as the swap spread, and explains their use in valuation.

Section 6 describes traditional theories of the term structure of interest rates. These theories outline several qualitative perspectives on economic forces that may affect the shape of the term structure.

Section 7 describes yield curve factor models. The focus is a popular three-factor term structure model in which the yield curve changes are described in terms of three independent movements: level, steepness, and curvature. These factors can be extracted from the variance–covariance matrix of historical interest rate movements.

Section 8 builds on the factor model and describes how to manage the risk of changing rates over different maturities. Section 9 concludes with a discussion of key variables known to influence interest rates, the development of interest rate views based on forecasts of those variables, and common trades tailored to capitalize on an interest rate view. A summary of key points concludes the reading.

Spot Rates and Forward Rates

We first explain the relationships among spot rates, forward rates, yield-to-maturity, expected and realized returns on bonds, and the shape of the yield curve. We then discuss the assumptions made about forward rates in active bond portfolio management.

The price of a risk-free single-unit payment (e.g., $1, €1, or £1) after N periods is called the **discount factor** with maturity N, denoted by PV_N. The yield-to-maturity of the payment is called a **spot rate**, denoted by Z_N. That is,

$$DF_N = \frac{1}{(1 + Z_N)^N} \quad (1)$$

The N-period discount factor, DF_N, and the N-period spot rate, Z_N, for a range of maturities in years $N > 0$ are called the **discount function** and the **spot yield curve** (or, more simply, **spot curve**), respectively. This spot curve represents the term structure of interest rates. Note that the discount function completely identifies the spot curve and vice versa, because both contain the same set of information about the time value of money.

The spot curve shows, for various maturities, the annualized return on an option-free and default-risk-free **zero-coupon bond** (**zero** for short) with a single payment at maturity. For this reason, spot rates are also referred to as zero-coupon yields or zero rates. The spot rate as a yield concept avoids the need for a reinvestment rate assumption for coupon-paying securities.

As Equation 1 suggests, the spot curve is a benchmark for the time value of money received on a future date as determined by the market supply and demand for funds. It is viewed as the most basic term structure of interest rates because no reinvestment risk is involved; the stated yield equals the actual realized return if the zero is held to maturity. Thus, the yield on a zero-coupon bond maturing in year T is regarded as the most accurate representation of the T-year interest rate.

A **forward rate** is an interest rate determined today for a loan that will be initiated in a future period. The set of forward rates for loans of different maturities with the same future start date is called the **forward curve**. Forward rates and forward curves can be mathematically derived from the current spot curve.

Denote the forward rate of a loan initiated A periods from today with tenor (further maturity) of B periods by $f_{A,B-A}$. Consider a forward contract in which one party, the buyer, commits to pay another party, the seller, a forward contract price $f_{A,B-A}$ at time A for a zero-coupon bond with maturity $B - A$ and unit principal. Because this is an agreement to do something in the future, no money is exchanged at contract initiation. At A, the buyer will pay the seller the contracted forward price and will receive from the seller at time B a payment defined here as a single currency unit.

The **forward pricing model** describes the valuation of forward contracts. The no-arbitrage principle, which simply states that tradable securities with identical cash flow payments must have the same price, may be used to derive the model as shown in Equation 2:

$$DF_B = DF_A \times F_{A,B-A} \tag{2}$$

The discount factors DF_A and DF_B represent the respective prices for period A and a longer period B needed to derive the forward price, $F_{A,B-A}$, a contract which starts in the future at time A and ends at time B.To understand the reasoning behind Equation 2, consider two alternative investments: (1) buying a two-year zero-coupon bond at a cost of $DF_2 = 0.93$ and (2) entering into a one-year forward contract to purchase a one-year zero-coupon bond for $DF_1 = 0.95$. Because the payoffs in two years are the same and the initial costs of the investments must be equal, the no-arbitrage forward price $F_{1,1}$ must equal 0.93/0.95, or 0.9789. Otherwise, any trader could sell the overvalued investment and buy the undervalued investment with the proceeds to generate risk-free profits with zero net investment.

Example 1 should help confirm your understanding of discount factors and forward prices. Please note that the solutions in the examples that follow may be rounded to two or four decimal places.

EXAMPLE 1

Spot and Forward Prices and Rates (1)

Consider a two-year loan beginning in one year ($A = 1$, $B = 3$). The one-year spot rate is $z_1 = z_A = 7\% = 0.07$. The three-year spot rate is $z_3 = z_B = 9\% = 0.09$.

1. Calculate the one-year discount factor: $DF_A = DF_1$.

Solution:

Using Equation 1,

$$DF_1 = \frac{1}{(1 + 0.07)^1} = 0.9346$$

2. Calculate the three-year discount factor: $DF_B = DF_3$.

Solution:

$$DF_3 = \frac{1}{(1 + 0.09)^3} = 0.7722$$

3. Calculate the forward price of a two-year bond to be issued in one year: $F_{A,B-A} = F_{1,2}$.

Solution:

Using Equation 2,

$$0.7722 = 0.9346 \times F_{1,2}.$$

$$F_{1,2} = 0.7722 \div 0.9346 = 0.8262.$$

4. Interpret your answer to Problem 3.

Solution:

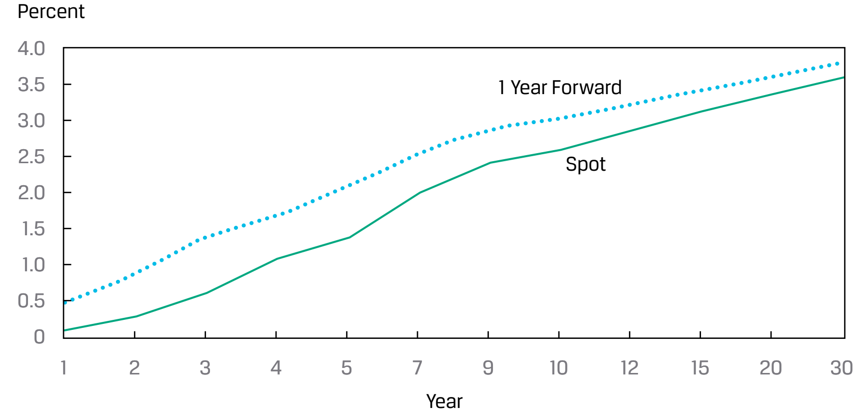
The forward contract price of $F_{1,2}$ = 0.8262 is the price agreed on today, to be paid one year from today for a bond with a two-year maturity and a risk-free unit-principal payment (e.g., $1, €1, or £1) at maturity in three years. As shown in the solution to 3, it is calculated as the three-year discount factor, DF_3 = 0.7722, divided by the one-year discount factor, DF_1 = 0.9346.

The Forward Rate Model

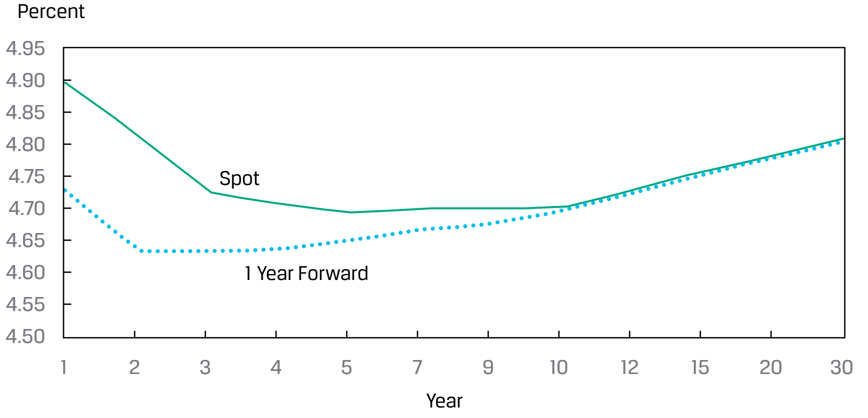
This section uses the forward rate model to establish that forward rates are above spot rates when the spot curve is upward sloping and below spot rates when the spot curve slopes downward. Exhibit 1 shows these spot versus forward relationships for the US Treasury yield curve in July 2013 versus December 2006, respectively. As we illustrate later, the relationship between spot and forward rates is important for future rate expectations as well as valuing fixed-income instruments.

Exhibit 1: Spot and Forward Curves

A. Spot vs. Forward US Treasury Yields, July 2013



B. Spot vs. Forward US Treasury Yields, December 2006



In contrast to the forward price $F_{A,B-A,}$ the forward *rate*$f_{A,B\text{-}A}$ is the discount rate for a risk-free unit-principal payment (e.g., \$1, €1, or £1) B periods from today, valued at time A, such that the present value equals the forward contract price, $DF_{A,B\text{-}A}$. Then, by definition,

$$DF_{A,B-A} = \frac{1}{\left(1 + F_{A,B-A}\right)^{B-A}} \tag{3}$$

By substituting Equation 1 and Equation 3 into Equation 2, the forward pricing model can be expressed in terms of rates as noted by Equation 4, which is the **forward rate model**:

$$\left(1 + z_B\right)^B = \left(1 + z_A\right)^A \left(1 + f_{A,B-A}\right)^{B-A} \tag{4}$$

Thus, the spot rate for B periods, which is z_B, and the spot rate for A periods, which is z_A, imply a value for the (B-A)-period forward rate at A, $f_{A,B\text{-}A}$. Equation 4 is important because it shows how forward rates may be extrapolated from spot rates—that is, they are implicit in the spot rates at any given point in time.

Equation 4 suggests two ways to interpret forward rates. For example, suppose $f_{7,1}$, the rate agreed on today for a one-year loan to be made seven years from today, is 3%. Then 3% is the

- reinvestment rate that would make an investor indifferent between buying an eight-year zero-coupon bond or investing in a seven-year zero-coupon bond and at maturity reinvesting the proceeds for one year. In this sense, the forward rate can be viewed as a type of breakeven interest rate.
- one-year rate that can be locked in today by buying an eight-year zero-coupon bond rather than investing in a seven-year zero-coupon bond and, when it matures, reinvesting the proceeds in a zero-coupon instrument that matures in one year. In this sense, the forward rate can be viewed as a rate that can be locked in by extending maturity by one year.

Example 2 addresses forward rates and the relationship between spot and forward rates.

EXAMPLE 2

Spot and Forward Prices and Rates (2)

The spot rates for three hypothetical zero-coupon bonds (zeros) with maturities of one, two, and three years are given in the following table.

Maturity (*T*)	**1**	**2**	**3**
Spot rates	z_1 = 9%	z_2 = 10%	z_3 = 11%

1. Calculate the forward rate for a one-year zero issued one year from today, $f_{1,1}$.

Solution:

$f_{1,1}$ is calculated as follows (using Equation 4):

$$(1 + z_2)^2 = (1 + z_1)^1 (1 + f_{1,1})^1$$
$$(1 + 0.10)^2 = (1 + 0.09)^1 (1 + f_{1,1})^1$$
$$f_{1,1} = \frac{(1.10)^2}{1.09} - 1 = 11.01\%$$

2. Calculate the forward rate for a one-year zero issued two years from today, $f_{2,1}$.

Solution:

$f(2,1)$ is calculated as follows:

$$(1 + z_3)^3 = (1 + z_2)^2 (1 + f_{2,1})^1$$
$$(1 + 0.11)^3 = (1 + 0.10)^2 (1 + f_{2,1})^1$$
$$f_{2,1} = \frac{(1.11)^3}{(1.10)^2} - 1 = 13.03\%$$

3. Calculate the forward rate for a two-year zero issued one year from today, $f_{1,2}$.

Solution:

$f_{1,2}$ is calculated as follows:

$$\left(1+z_3\right)^3 = \left(1+z_1\right)^1\left(1+f_{1,2}\right)^2$$

$$\left(1+0.11\right)^3 = \left(1+0.09\right)^1\left(1+f_{1,2}\right)^2$$

$$f_{1,2} = \sqrt[2]{\frac{(1.11)^3}{(1.09)^1}}-1 = 12.01\%$$

4. Based on your answers to 1 and 2, describe the relationship between the spot rates and the implied one-year forward rates.

Solution:

The upward-sloping zero-coupon yield curve is associated with an upward-sloping forward curve (a series of increasing one-year forward rates because 13.03% is greater than 11.01%). This dynamic is explained further in the following discussion.

The relationship between spot rates and one-period forward rates may be demonstrated using the forward rate model and successive substitution, resulting in Equation 5 and Equation 6:

$$\left(1+z_T\right)^T = \left(1+z_1\right)\left(1+f_{1,1}\right)\left(1+f_{2,1}\right)\left(1+f_{3,1}\right)\ldots\left(1+f_{T-1,1}\right) \quad (5)$$

$$z_T = \left\{\left(1+z_1\right)\left(1+f_{1,1}\right)\left(1+f_{2,1}\right)\left(1+f_{3,1}\right)\ldots\left(1+f_{T-1,1}\right)\right\}^{\frac{1}{T}}-1 \quad (6)$$

Equation 6 shows that the spot rate for a security with a maturity of $T > 1$ can be expressed as a geometric mean of the spot rate for a security with a maturity of $T = 1$ and a series of $T - 1$ forward rates.

Equation 6 is critical for active fixed-income portfolio managers. Although the question of whether forward rates are unbiased estimators of market consensus expectations remains open to debate, implied forward rates are generally the best available and most accessible proxy for market expectations of future spot rates. If an active trader can identify a series of short-term bonds whose actual returns exceed today's quoted forward rates, then the total return over her investment horizon would exceed the return on a maturity-matching, buy-and-hold strategy if the yield curve were to remain relatively stable. Later, we will apply this concept to dynamic hedging strategies and the local expectations theory.

Example 3 and Example 4 explore the relationship between spot and forward rates.

EXAMPLE 3

Spot and Forward Prices and Rates (3)

1. Given the data and conclusions for z_1, $f_{1,1}$, and $f_{2,1}$ from Example 2:

$z_1 = 9\%$

$f_{1,1} = 11.01\%$

$f_{2,1}$ = 13.03%

Show that the two-year spot rate of z_2 = 10% and the three-year spot rate of z_3 = 11% are geometric averages of the one-year spot rate and the forward rates.

Solution:

Using Equation 5,

$$\left(1+z_2\right)^2 = \left(1+z_1\right)\left(1+f_{1,1}\right)$$
$$z_2 = \sqrt[2]{(1+0.09)(1+0.1101)} - 1 \approx 10\%$$

$$\left(1+z_3\right)^3 = \left(1+z_1\right)\left(1+f_{1,1}\right)\left(1+f_{2,1}\right)$$
$$z_3 = \sqrt[3]{(1+0.09)(1+0.1101)(1+0.1303)} - 1 \approx 11\%$$

We can now consolidate our knowledge of spot and forward rates to explain important relationships between the spot and forward rate curves. The forward rate model (Equation 4) can also be expressed as Equation 7.

$$\left\{\frac{1+z_B}{1+z_A}\right\}^{\frac{A}{B-A}}\left(1+z_B\right) = 1+f_{A,B-A} \tag{7}$$

To illustrate, suppose A = 1, B = 5, z_1 = 2%, and z_5 = 3%; the left-hand side of Equation 7 is

$$\left(\frac{1.03}{1.02}\right)^{\frac{1}{4}}(1.03) = (1.0024)(1.03) = 1.0325,$$

so $f_{1,4}$ = 3.25%. Given that the yield curve is upward sloping—so, $z_B > z_A$—Equation 7 implies that the forward rate from A to B is greater than the long-term spot rate: $f_{A,B-A} > z_B$. This is the case in our example, because 3.25% > 3.00%. Conversely, when the yield curve is downward sloping, then $z_B < z_A$ and the forward rate from A to B is lower than the long-term spot rate: $f_{A,B-A} < z_B$. Equation 7 also shows that if the spot curve is flat, all one-period forward rates equal the spot rate. For an upward-sloping yield curve— $z_B > z_A$ —the forward rate rises as time periods increase. For a downward-sloping yield curve— $z_B < z_A$ —the forward rate declines as time periods increase.

EXAMPLE 4

Spot and Forward Prices and Rates (4)

Given the spot rates z_1 = 9%, z_2 = 10%, and z_3 = 11%, as in Example 2 and Example 3:

1. Determine whether the forward rate $f_{1,2}$ is greater than or less than the long-term rate, z_3.

Solution:

The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$, or in general, $z_B > z_A$. Thus, the forward rate will be greater than the long-term rate, or $f_{A,B-A} > z_B$. Note from Example 2 that $f_{1,2}$ = 12.01% > z_3 = 11%.

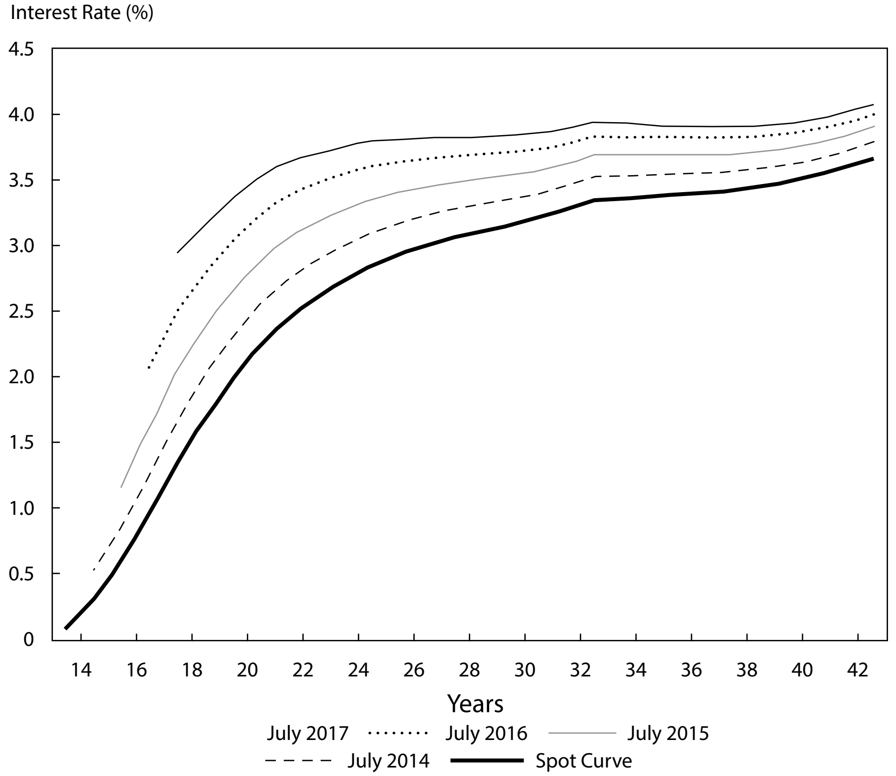
2. Determine whether forward rates rise or fall as the initiation date, A, for the forward rate is later.

Solution:

The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$. Thus, the forward rates will rise with increasing A. This relationship was shown in Example 2, in which $f_{1,1}$ = 11.01% and $f_{2,1}$ = 13.03%.

These relationships are illustrated in Exhibit 2 and Exhibit 3 as an extension of Exhibit 1. The spot rates for US Treasuries as of 31 July 2013 constructed using interpolation are the lowest, as shown in the table following the exhibit. Note that the spot curve is upward sloping. The forward curves for the end of July 2014, 2015, 2016, and 2017 are also presented in Exhibit 2. Because the yield curve is upward sloping, these forward curves are all above the spot curve and become successively higher and steeper as the forward period increases, the highest of which is that for July 2017.

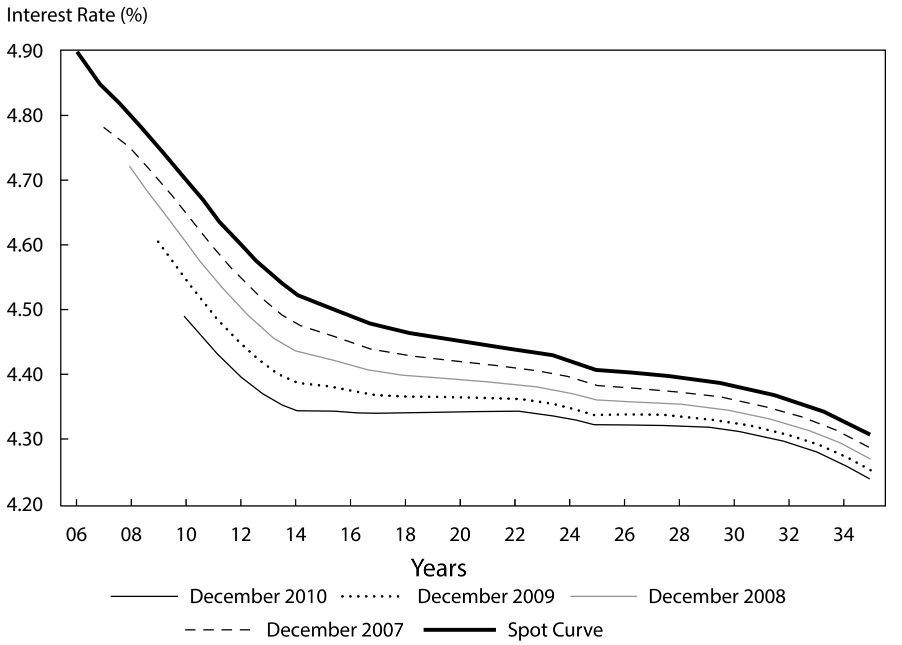
Exhibit 2: Historical Example: Upward-Sloping Spot Curve vs. Forward Curves, 31 July 2013



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	0.11	0.33	0.61	1.37	2.00	2.61	3.35	3.66

Exhibit 3 shows the opposite case of a downward sloping spot curve based on US Treasury rates as of 31 December 2006. This data also uses interpolation and is somewhat modified to make the yield curve more downward sloping for illustrative purposes. The spot curve and forward curves for the end of December 2007, 2008, 2009, and 2010 are presented in Exhibit 3.

Exhibit 3: Historical Example: Downward-Sloping Spot Curve vs. Forward Curves, 31 December 2006 (modified for illustrative purposes)



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	4.90	4.82	4.74	4.70	4.60	4.51	4.41	4.31

The highest curve is the spot yield curve, and it is downward sloping. The forward curves are below the spot curve, with longer forward periods associated with lower forward curves, the lowest of which is dated December 2010.

An important point that can be inferred from Exhibit 2 and Exhibit 3 is that forward rates do not extend beyond the longest maturity on today's yield curve. For example, if yields reach a 30-year maturity on today's yield curve, then a three-year forward model will extend just 27 years. Similarly, four years hence, the longest-maturity forward rate would be $f_{4,26}$.

In summary, when the spot curve slopes upward, the forward curve will lie above the spot curve. Conversely, when the spot curve slopes downward, the forward curve will lie below the spot curve. This dynamic reflects the basic mathematical truth that when an average is rising (falling), the marginal data point must be above (below) the average. In this case, the spot curve represents an average over an entire period and the forward rates represent the marginal changes between future periods.

We have thus far discussed the spot curve and the forward curve. Another curve important in practice is the government par curve. The **par curve** represents the yields to maturity on coupon-paying government bonds, priced at par, over a range of maturities. In practice, recently issued ("on the run") bonds are most often used to create the par curve, because these securities are most liquid and typically priced at or close to par.

The par curve is important for valuation in that it can be used to construct a zero-coupon yield curve. The process considers a coupon-paying bond as a portfolio of zero-coupon bonds. The zero-coupon rates are determined by using the par yields and solving for the zero-coupon rates one by one, from the shortest to longest maturities using a forward substitution process known as **bootstrapping**.

WHAT IS BOOTSTRAPPING?

Because the practical details of deriving the zero-coupon yield are beyond the scope of this reading, the concept of bootstrapping may be best shown using a numerical illustration. Suppose the following yields are observed for annual coupon sovereign debt:

Par Rates:

One-year par rate = 5%, two-year par rate = 5.97%, three-year par rate = 6.91%, four-year par rate = 7.81%. From these data, we can bootstrap zero-coupon rates.

Zero-Coupon Rates:

Given annual coupons, the one-year zero-coupon rate equals the one-year par rate because it has one cash flow, whereas two-year and longer maturity bonds have coupon payments prior to maturity.

The derivation of zero-coupon rates begins with the two-year maturity. The two-year zero-coupon rate is determined by using z_1 = 5% and solving for z_2 in the following equation for of one monetary unit of current market value:

$$1 = \frac{0.0597}{(1.05)} + \frac{1 + 0.0597}{\left(1 + z_2\right)^2}$$

In the equation, 0.0597 and 1.0597 represent payments from interest and principal and interest, respectively, per unit of principal value. The equation implies that z_2 = 6%. We have bootstrapped the two-year spot rate. Continuing with forward substitution, the three-year zero-coupon rate can be bootstrapped by solving for z_3 using the known values of the one-year and two-year spot rates of 5% and 6%:

$$1 = \frac{0.0691}{(1.05)} + \frac{0.0691}{(1.06)^2} + \frac{1 + 0.0691}{\left(1 + z_3\right)^3}$$

Thus, z_3 = 7%. Finally, we solve for the four-year zero-coupon rate, z_4:

$$1 = \frac{0.0781}{(1.05)} + \frac{0.0781}{(1.06)^2} + \frac{0.0781}{(1.07)^3} + \frac{1 + 0.0781}{\left(1 + z_4\right)^4}$$

In summary, z_1 = 5%, z_2 = 6%, z_3 = 7%, and z_4 = 8%.

In the preceding discussion, we considered an upward-sloping (spot) yield curve (Exhibit 2) and an inverted or downward-sloping (spot) yield curve (Exhibit 3). In developed markets, yield curves are most commonly upward sloping with diminishing marginal increases in yield for identical changes in maturity; that is, the yield curve "flattens" at longer maturities. Because nominal yields incorporate a premium for expected inflation, an upward-sloping yield curve is generally interpreted as reflecting a market expectation of rising or at least stable future inflation (associated with relatively strong economic growth). The existence of risk premiums (e.g., for the greater interest rate risk of longer-maturity bonds) also contributes to a positive slope.

An inverted yield curve (Exhibit 3) is less common. Such a term structure may reflect a market expectation of declining future inflation rates (because a nominal yield incorporates a premium for expected inflation) from a relatively high current level. Expectations of an economic slowdown may be one reason to anticipate a decline in inflation, and a downward-sloping yield curve is frequently observed before recessions. A flat yield curve typically occurs briefly in the transition from an upward-sloping to a downward-sloping yield curve, or vice versa. A humped yield curve, which is relatively rare, occurs when intermediate-term interest rates are higher than short- and long-term rates.

2 YTM IN RELATION TO SPOT AND FORWARD RATES

☐ describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management

Yield-to-maturity (YTM) is perhaps the most familiar pricing concept in bond markets. In this section, we clarify how it is related to spot rates and a bond's expected and realized returns.

How is the yield-to-maturity related to spot rates? In bond markets, most bonds outstanding have coupon payments and many have various options, such as a call provision. The YTM of these bonds with maturity T would not be the same as the spot rate at T but should be mathematically related to the spot curve. Because the principle of no arbitrage shows that a bond's value is the sum of the present values of payments discounted by their corresponding spot rates, the YTM of the bond should be some weighted average of spot rates used in the valuation of the bond.

Example 5 addresses the relationship between spot rates and YTM.

EXAMPLE 5

Spot Rate and Yield-to-Maturity

Recall from earlier examples the spot rates z_1= 9%, z_2 = 10%, and z_3 = 11%. Let y_T be the YTM.

1. Calculate the price of a two-year annual coupon bond using the spot rates. Assume the coupon rate is 6% and the face value is $1,000. Next, state the formula for determining the price of the bond in terms of its YTM. Is z_2 greater than or less than y_2? Why?

Solution:

Using the spot rates,

$$\text{Price} = \frac{\$60}{(1 + 0.09)^1} + \frac{\$1,060}{(1 + 0.10)^2} = \$931.08$$

Using the YTM,

$$Price = \frac{\$60}{(1 + y_2)} + \frac{\$1,060}{(1 + y_2)^2} = \$931.08$$

Note that y_2 is used to discount both the first- and second-year cash flows. Because the bond can have only one price, it follows that $z_1 < y_2 < z_2$ because y_2 is a weighted average of z_1 and z_2 and the yield curve is upward sloping. Using a calculator, one can calculate the YTM as y_2 = 9.97%, which is less than z_2 = 10% and greater than z_1 = 9%, just as we would expect. Note that y_2 is much closer to z_2 than to z_1 because the bond's largest cash flow occurs in Year 2, thereby giving z_2 a greater weight than z_1 in the determination of y_2.

2. Calculate the price of a three-year annual coupon-paying bond using the spot rates. Assume the coupon rate is 5% and the face value is £100. Next, write a formula for determining the price of the bond using the YTM. Is z_3 greater or less than y_3? Why?

Solution:

Using the spot rates,

$$\text{Price} = \frac{£5}{(1+0.09)^1} + \frac{£5}{(1+0.10)^2} + \frac{£105}{(1+0.11)^3} = £85.49$$

Using the yield-to-maturity,

$$\text{Price} = \frac{£5}{(1+y_3)} + \frac{£5}{(1+y_3)^2} + \frac{£105}{(1+y_3)^3} = £85.49.$$

Note that y_3 is used to discount all three cash flows. Because the bond can have only one price, y_3 must be a weighted average of z_1, z_2, and z_3. Given that the yield curve is upward sloping in this example, $y_3 < z_3$. Using a calculator to compute YTM, y_3 = 10.93%, which is less than z_3 = 11% and greater than z_1 = 9%—just as we would expect, because the weighted YTM must lie between the highest and lowest spot rates. Note that y_3 is much closer to z_3 than it is to z_2 or z_1 because the bond's largest cash flow occurs in Year 3, thereby giving z_3 a greater weight than z_1 and z_2 in the determination of y_3.

Investors can expect to earn the yield-to-maturity on a bond only under extremely restrictive assumptions. The YTM is the expected rate of return for a bond held to maturity, assuming that all promised coupon and principal payments are made in full when due and that coupons are reinvested at the original YTM. As interest rates change, the reinvestment of coupons at the original YTM is unlikely. The YTM can provide a poor estimate of expected return if (1) interest rates are volatile, (2) the yield curve is sloped either upward or downward, (3) there is significant risk of default, or (4) the bond has one or more embedded options (e.g., put, call, or conversion). If either (1) or (2) is the case, reinvestment of coupons would not be expected to be at the assumed rate (YTM). Case 3 implies that actual cash flows may differ from those assumed in the YTM calculation, and in Case 4, the exercise of an embedded option would result in a holding period shorter than the bond's original maturity.

The realized return is the actual bond return during an investor's holding period. It is based on actual reinvestment rates and the yield curve at the end of the holding period. If we had perfect foresight, the expected bond return would equal the realized bond return.

To illustrate these concepts, assume that z_1 = 5%, z_2 = 6%, z_3 = 7%, z_4 = 8%, and z_5 = 9%. Consider a five-year annual coupon bond with a coupon rate of 10%. The forward rates extrapolated from the spot rates are $f_{1,1}$ = 7.0%, $f_{2,1}$ = 9.0%, $f_{3,1}$ = 11.1%, and $f_{4,1}$ = 13.1%. The price, determined as a percentage of par, is 105.43.

The yield-to-maturity of 8.62% can be determined by solving

$$105.43 = \frac{10}{(1+y_5)} + \frac{10}{(1+y_5)^2} + \frac{10}{(1+y_5)^3} + \frac{10}{(1+y_5)^4} + \frac{110}{(1+y_5)^5}$$

The yield-to-maturity of 8.62% is the bond's expected return assuming no default, a holding period of five years, and a reinvestment rate of 8.62%. But what if the forward rates are assumed to be the future spot rates?

Using the forward rates as the expected reinvestment rates results in the following expected cash flow at the end of Year 5:

$$10(1+0.07)(1+0.09)(1+0.111)(1+0.131) + 10(1+0.09)(1+0.011)(1+0.131) + 10(1+0.111)(1+0.131) + 10(1+0.131) + 110 \approx 162.22$$

Therefore, the expected bond return is (162.22 – 105.43)/105.43 = 53.87% and the expected annualized rate of return is 9.00% [solve $(1 + x)^5 = 1 + 0.5387$].

From this example, we can see that the expected rate of return is not equal to the YTM even if we make the generally unrealistic assumption that the forward rates are the future spot rates. The YTM is generally a realistic estimate of expected return only if the yield curve is flat. Note that in the foregoing formula, all cash flows were discounted at 8.62% regardless of maturity.

Example 6 will reinforce your understanding of various yield and return concepts.

EXAMPLE 6

Yield and Return Concepts

1. When the spot curve is upward sloping, the forward curve:

   A. lies above the spot curve.
   B. lies below the spot curve.
   C. is coincident with the spot curve.

Solution:

A is correct. Points on a spot curve can be viewed as an average of single-period rates over given maturities, whereas forward rates reflect the marginal changes between future periods.

2. Which of the following statements concerning the YTM of a default-risk-free bond is *most* accurate? The YTM of such a bond:

   A. equals the expected return on the bond if the bond is held to maturity.
   B. can be viewed as a weighted average of the spot rates applying to its cash flows.
   C. will be closer to the realized return if the spot curve is upward sloping rather than flat through the life of the bond.

Solution:

B is correct. The YTM is the discount rate that, when applied to a bond's promised cash flows, equates those cash flows to the bond's market price and the fact that the market price should reflect discounting promised cash flows at appropriate spot rates.

3. When the spot curve is downward sloping, a later initiation date results in a forward curve that is:

   A. closer to the spot curve.
   B. a greater distance above the spot curve.
   C. a greater distance below the spot curve.

Solution:

C is correct. This answer follows from the forward rate model as expressed in Equation 6. If the spot curve is downward sloping (upward sloping), a later initiation date will result in a forward curve that is a greater distance below (above) the spot curve. See Exhibit 2 and Exhibit 3.

Yield Curve Movement and the Forward Curve

This section establishes several important results concerning forward prices and the spot yield curve to demonstrate the relevance of the forward curve to active bond investors.

The forward contract price remains unchanged as long as future spot rates evolve as predicted by today's forward curve. If a trader expects the future spot rate to be below what is predicted by the prevailing forward rate, the forward contract value is expected to increase and the trader would buy the forward contract. Conversely, if the trader expects the future spot rate to be above that predicted by the existing forward rate, then the forward contract value is expected to decrease and the trader would sell the forward contract.

Using the forward pricing model defined by Equation 2, we can determine the forward contract price that delivers a (B – A)-period-maturity bond at time A, $F_{A,B-A}$, using Equation 8 (which is Equation 2 solved for the forward price):

$$F_{A,B-A} = \frac{DF_B}{DF_A} \tag{8}$$

Now suppose that after t periods, the new discount function for some maturity time T period, denoted as DF_T^{new}, is the same as the forward discount function implied by today's discount function, as shown by Equation 9.

$$DF_T^{new} = \frac{DF_{t+T}}{DF_t} \tag{9}$$

Next, after a lapse of t periods, the time to expiration of the contract is $A - t$, and the forward contract price at time t is $F_{A-t,B-A}^{new}$. Equation 8 can be rewritten as Equation 10:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} \tag{10}$$

Substituting Equation 9 into Equation 10 and adjusting for the lapse of time t results in Equation 11:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} = \frac{\frac{DF_B}{DF_t}}{\frac{DF_A}{DF_t}} = \frac{DF_B}{DF_A} = F_{A,B-A} \tag{11}$$

Equation 11 shows that the forward contract price remains unchanged as long as future spot rates are equal to what is predicted by today's forward curve. Therefore, a change in the forward price is the result of a deviation of the spot curve from what is predicted by today's forward curve.

To make these observations concrete, consider a flat yield curve for which the interest rate is 4%. Using Equation 1, the discount factors for the one-year, two-year, and three-year terms are, to four decimal places, as follows:

$$DF_1 = \frac{1}{(1+0.04)} = 0.9615$$

$$DF_2 = \frac{1}{(1+0.04)^2} = 0.9246$$

$$DF_3 = \frac{1}{(1+0.04)^3} = 0.8890$$

Therefore, using Equation 8, the forward contract price that delivers a one-year bond at Year 2 is

$$F_{2,1} = \frac{DF_3}{DF_2} = \frac{0.8890}{0.9246} = 0.9615.$$

Suppose the future discount function at Year 1 is the same as the forward discount function implied by the Year 0 spot curve. The lapse of time is t = 1. Using Equation 9, the discount factors for the one-year and two-year terms one year from today are as follows:

$$DF_1^{new} = \frac{DF_2}{DF_1} = \frac{0.9246}{0.9615} = 0.9615$$

$$DF_2^{new} = \frac{DF_3}{DF_1} = \frac{0.8890}{0.9615} = 0.9246$$

Using Equation 10, the price of the forward contract one year from today is

$$F_{2,1}^{new} = \frac{DF_2^{new}}{DF_1^{new}} = \frac{0.9246}{0.9615} = 0.9615.$$

The price of the forward contract is unchanged. This will be the case as long as future discount functions are the same as those based on today's forward curve.

From this numerical example, we can see that if the spot rate curve is unchanged, then each bond "rolls down" the curve and earns the current one-period spot rate and subsequent forward rates. Specifically, when one year passes, a three-year bond will return (0.9246 − 0.8890)/0.8890 = 4%, which is equal to the spot rate. Furthermore, if another year passes, the bond will return (0.9615 − 0.9246)/0.9246 = 4%, which is equal to the implied forward rate for a one-year security one year from today.

3 ACTIVE BOND PORTFOLIO MANAGEMENT

☐ describe the strategy of rolling down the yield curve

One way that active bond portfolio managers attempt to outperform the bond market's return is by anticipating changes in interest rates relative to the projected evolution of spot rates reflected in today's forward curves.

The forward rate model (Equation 4) provides insight into these issues. By rearranging terms in Equation 4 and setting the time horizon to one period, A = 1, we obtain

$$\frac{(1 + z_B)^B}{(1 + f_{A,B-A})^{B-A}} = (1 + z_A)^A. \tag{12}$$

The numerator of the left-hand side of Equation 12 is for a bond with an initial maturity of B periods and a remaining maturity of $B - A$ periods after A periods pass. Suppose the prevailing spot yield curve after one period (A = 1) is the current forward curve; then, Equation 12 shows that the total return on the bond is the one-period risk-free rate. The following sidebar shows that returns on bonds of varying tenor over a one-year period always equal the one-year rate (the risk-free rate over the one-year period) if the spot rates evolve as implied by the current forward curve at the end of the first year.

WHEN SPOT RATES EVOLVE AS IMPLIED BY THE CURRENT FORWARD CURVE

As in earlier examples, assume the following:

z_1 = 9%

z_2 = 10%

$z_3 = 11\%$

$f_{1,1} = 11.01\%$

$f_{1,2} = 12.01\%$

If the spot curve one year from today reflects the current forward curve, the return on a zero-coupon bond for the one-year holding period is 9%, regardless of the bond's maturity. The following computations assume a par amount of 100 and represent the percentage change in price. Given the rounding of price and the forward rates to the nearest hundredth, the returns all approximate 9%. With no rounding, however, all answers would be precisely 9%.

The return of the one-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 91.74 and is worth the par amount of 100 at maturity.

$$\left(100 \div \frac{100}{1 + z_1}\right) - 1 = \left(100 \div \frac{100}{1 + 0.09}\right) - 1 = \frac{100}{91.74} - 1 = 9\%.$$

The return of the two-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 82.64. One year from today, the two-year bond has a remaining maturity of one year. Its price one year from today is 90.08, determined as the par amount divided by 1 plus the forward rate for a one-year bond issued one year from today.

$$\left(\frac{100}{\left(1 + f_{1,1}\right)} \div \frac{100}{\left(1 + z_2\right)^2}\right) - 1 = \left(\frac{100}{(1 + 0.1101)} \div \frac{100}{(1 + 0.10)^2}\right) - 1 = \frac{90.08}{82.64} - 1$$
$$= 9\%$$

The return of the three-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 73.12. One year from today, the three-year bond has a remaining maturity of two years. Its price one year from today of 79.71 reflects the forward rate for a two-year bond issued one year from today.

$$\left(\frac{100}{\left(1 + f_{1,2}\right)^2} \div \frac{100}{\left(1 + z_3\right)^3}\right) - 1 = \left(\frac{100}{(1 + 0.1201)^2} \div \frac{100}{(1 + 0.11)^3}\right) - 1$$
$$= \frac{79.71}{73.12} - 1 \approx 9\%$$

This numerical example shows that the return of a bond over a one-year period is always the one-year rate (the risk-free rate over the one period) if the spot rates evolve as implied by the current forward curve.

But if the spot curve one year from today differs from today's forward curve, the returns on each bond for the one-year holding period will not all be 9%. To show that the returns on the two-year and three-year bonds over the one-year holding period are not 9%, we assume that the spot rate curve at Year 1 is flat with yields of 10% for all maturities.

The return on a one-year zero-coupon bond over the one-year holding period is

$$\left(100 \div \frac{100}{1 + 0.09}\right) - 1 = 9\%.$$

The return on a two-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{1+0.10} \div \frac{100}{(1+0.10)^2}\right) - 1 = 10\%.$$

The return on a three-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{(1+0.10)^2} \div \frac{100}{(1+0.11)^3}\right) - 1 = 13.03\%.$$

The bond returns are 9%, 10%, and 13.03%. The returns on the two-year and three-year bonds differ from the one-year risk-free interest rate of 9%.

Equation 12 provides a total return investor with a means to evaluate the cheapness or expensiveness of a bond of a certain maturity. If any of the investor's expected future spot rates is below a quoted forward rate for the same maturity, then (all else being equal) the investor would perceive the bond to be undervalued, in the sense that the market is effectively discounting the bond's payments at a higher rate than the investor and the bond's market price is below the intrinsic value perceived by the investor.

Another example will reinforce the point that if a portfolio manager's projected spot curve is above (below) the forward curve and his expectation turns out to be true, the return will be less (more) than the one-period risk-free interest rate.

For the sake of simplicity, assume a flat yield curve of 8% and that a trader holds a three-year bond paying an 8% annual coupon. Assuming a par value of 100, the current market price is also 100. If today's forward curve turns out to be the spot curve one year from today, the trader will earn an 8% return.

If the trader projects that the spot curve one year from today is above today's forward curve—for example, a flat yield curve of 9%—the trader's expected rate of return is 6.24%, which is less than 8%:

$$\frac{8 + \frac{8}{1+0.09} + \frac{108}{(1+0.09)^2}}{100} - 1 = 6.24\%$$

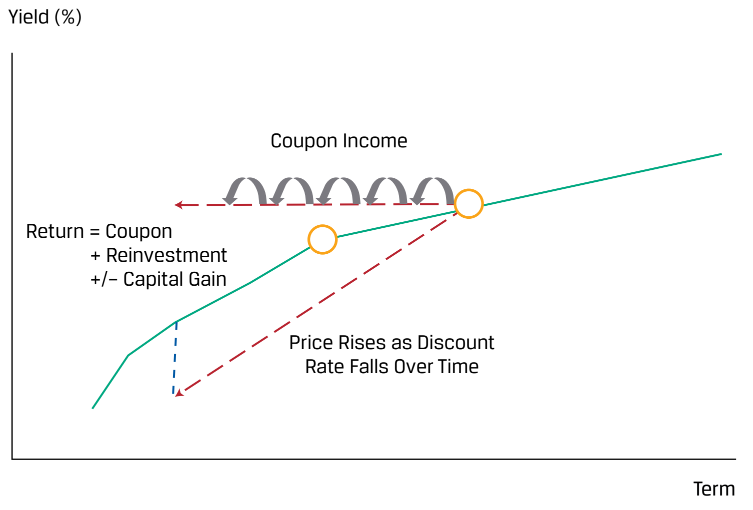
If the trader predicts a flat yield curve of 7%, the trader's expected return is 9.81%, which is greater than 8%:

$$\frac{8 + \frac{8}{1+0.07} + \frac{108}{(1+0.07)^2}}{100} - 1 = 9.81\%$$

As the gap between the projected future spot rate and the forward rate widens, so too will the difference between the trader's expected return and the original YTM of 8%.

This logic is the basis for a popular yield curve trade called **rolling down the yield curve**, also referred to as riding the yield curve. As we have noted, when a yield curve is upward sloping, the forward curve is always above the current spot curve. If the trader expects the yield curve to remain static over an investment horizon, then buying bonds with a maturity longer than the investment horizon would provide a total return greater than the return on a maturity-matching strategy. The bond's total return will depend on the spread between the forward rate and the spot rate as well as the maturity of the bond. The longer the bond's maturity, the more sensitive its total return is to the spread. This strategy is shown in Exhibit 4.

Exhibit 4: Rolling Down the Yield Curve



The return on a yield curve rolldown strategy may be demonstrated using a simple example. As stated earlier, the investment return on a fixed-rate (non-defaulted and non-callable) bond return may be defined as follows:

Bond return = Receipt of promised coupons (and principal)

+ Reinvestment of coupon payments (12)

+/– Capital gain/Loss on sale prior to maturity

Say we observe one-, three-, four-, five- and six-year spot rates on annual coupon bonds trading at par of 2%, 4%, 5%, 6%, and 7%, respectively. An investor with a five-year maturity target decides to forgo a matched-maturity 6% five-year bond in favor of the 7%, six-year bond given her expectation of an unchanged yield curve over the next two years. We can compare the annualized return over two years for both bonds, assuming unchanged yields, as follows.

The 6% five-year bond purchased for 100 returns 117.67 in two years [(6 × 1.02) + 6 + 105.55], which consists of the first year's coupon reinvested at the one-year rate, the second annual coupon, and the capital gain on the sale of the 6% bond with three years to maturity at an unchanged three-year yield of 4% [$105.55 = 6/1.04 + 6/(1.04)^2 + 106/(1.04)^3$]. The annualized rate of return is 8.476% [solve for r, where $(117.67/100) = (1 + r)^2$].

The 7% six-year bond purchased at par returns 121.23 in two years [(7 × 1.02) + 7 + 107.09] with an annualized return of 1 bond 10.10%. The excess return of nearly 2% results from both higher coupon income than the five-year matched maturity bond as well as a larger capital gain on the sale of the 7% bond with four years to maturity at an unchanged four-year yield of 5% [$107.09 = 7/1.05 + 7/(1.05)^2 + + 7/(1.05)^3 + 107/(1.05)^4$].

In the years following the 2008 financial crisis, many central banks acted to keep short-term interest rates very low. As a result, yield curves subsequently had a steep upward slope (see Exhibit 2). For active fixed-income managers, this situation provided an incentive to access short-term funding and invest in long-term bonds. This is just one form of a carry trade, referred to as a maturity spread carry trade, and is subject to significant interest rate risk, such as an unexpected increase in future spot rates (e.g., as a result of a spike in inflation). The maturity spread carry trade, in which the trader borrows short term and lends long term in the same currency, is common in an upward-sloping yield curve environment.

In summary, when the yield curve slopes upward, as a bond approaches maturity or "rolls down the yield curve," it is valued at successively lower yields and higher prices. Using this strategy, a bond can be held for a period of time as it appreciates in price and then sold before maturity to realize a higher return. As long as interest rates remain stable and the yield curve retains an upward slope, this strategy can continuously add to the total return of a bond portfolio.

Example 7 addresses how the preceding analysis relates to active bond portfolio management.

EXAMPLE 7

Active Bond Portfolio Management

1. The "rolling down the yield curve" strategy is executed by buying bonds whose maturities are:

   A. equal to the investor's investment horizon.

   B. longer than the investor's investment horizon.

   C. shorter than the investor's investment horizon.

Solution:

B is correct. A bond with a longer maturity than the investor's investment horizon is purchased but then sold prior to maturity at the end of the investment horizon. If the yield curve is upward sloping and yields do not change, the bond will be valued at successively lower yields and higher prices over time. The bond's total return will exceed that of a bond whose maturity is equal to the investment horizon.

2. A bond will be overvalued if the expected spot rate is:

   A. equal to the current forward rate.

   B. lower than the current forward rate.

   C. higher than the current forward rate.

Solution:

C is correct. If the expected discount rate is higher than the forward rate, then the bond will be overvalued. The expected price of the bond is lower than the price obtained from discounting using the forward rate.

3. Assume a flat yield curve of 6%. A three-year £100 bond is issued at par paying an annual coupon of 6%. What is the portfolio manager's expected return if he predicts that the yield curve one year from today will be a flat 7%?

   A. 4.19%

   B. 6.00%

   C. 8.83%

Solution:

A is correct. Expected return will be less than the current YTM of 6% if yields increase to 7%. The expected return of 4.19% is computed as follows:

$$\frac{6 + \frac{6}{1 + 0.07} + \frac{106}{(1 + 0.07)^2}}{100} - 1 \approx 4.19\%$$

4. A forward contract price will increase if:

   A. future spot rates evolve as predicted by current forward rates.

   B. future spot rates are lower than what is predicted by current forward rates.

   C. future spot rates are higher than what is predicted by current forward rates.

Solution:

B is correct. The forward rate model can be used to show that a change in the forward contract price requires a deviation of the spot curve from that predicted by today's forward curve. If the future spot rate is lower than what is predicted by the prevailing forward rate, the forward contract price will increase because it is discounted at an interest rate that is lower than the originally anticipated rate.

THE SWAP RATE CURVE 4

☐ explain the swap rate curve and why and how market participants use it in valuation

Earlier, we described the spot rate curve of default-risk-free bonds as a measure of the time value of money. The swap rate curve, or swap curve for short, is another important representation of the time value of money used in fixed-income markets. Here we will discuss how the swap curve is used in valuation, where the spread of swap rates over government benchmark rates is a proxy for perceived credit risk relative to risk-free debt.

Swap Rate Curve

Interest rate swaps are an integral part of the fixed-income market. These derivative contracts usually involve the net exchange, or swap, of fixed-rate for floating-rate interest payments, and these contracts are an essential tool for investors who use them to hedge, speculate on, or otherwise modify risk. The fixed and floating payments are determined by multiplying the respective rate by a principal (or notional) amount for each interest period over the swap maturity. The rate for the fixed leg of an interest rate swap is known as the **swap rate**. The swap rate is analogous to the YTM on a government bond, which as we saw earlier may be derived from zero rates using bootstrapping. The key difference between the swap rate and the government bond rate is that the swap rate is derived using short-term lending rates rather than default-risk-free rates. Swap floating rates historically referenced short-term survey-based interest rates, such as three- or six-month US dollar Libor (London Interbank Offered Rate) and are slated to transition to transaction-based market reference rates (MRR) based on secured overnight funding transactions. The yield curve of swap rates is called the **swap rate curve** or, more simply, the **swap curve**. Because it is based on so-called **par swaps**, in which the fixed rate is set so that no money is exchanged at contract initiation—the present values of the fixed-rate and benchmark floating-rate legs being equal— the swap curve is a type of par curve. When we refer to the "par curve" here, however, the reference is to the government par yield curve.

The swap market is a highly liquid market for two reasons. First, unlike bonds, a swap does not have multiple borrowers or lenders, only counterparties who exchange cash flows. Such arrangements offer significant flexibility and customization in the swap contract's design. Second, swaps provide one of the most efficient ways to hedge interest rate risk. The Bank for International Settlements (BIS) estimates that the notional amount outstanding on interest rate swaps was nearly $350 trillion as of June 2020.

Many countries do not have a liquid government bond market with maturities longer than one year. The swap curve is a necessary market benchmark for interest rates in these countries. In countries where the private sector is much bigger than the public sector, the swap curve is a far more relevant measure of the time value of money than is the government's cost of borrowing.

Swaps are frequently used as a benchmark in Europe, whereas in Asia, the swap markets and the government bond markets have developed in parallel, and both are used in valuation in credit and loan markets.

Why Do Market Participants Use Swap Rates When Valuing Bonds?

Government spot curves and swap rate curves are the chief reference curves in fixed-income valuation. The choice between them can depend on multiple factors, including the relative liquidity of these two markets. In the United States, where there is both an active Treasury security market and a swap market, the choice of a benchmark for the time value of money often depends on the interest rate exposure profile of the institution using the benchmark. On one hand, wholesale banks frequently use the swap curve to value assets and liabilities because they hedge their balance sheet with swaps. On the other hand, retail banks with little exposure to the swap market are more likely to use the government spot curve as their benchmark.

Let us illustrate how a financial institution uses the swap market for its internal operations. Consider the case of a bank raising funds using a certificate of deposit (CD). Assume the bank can borrow $10 million in the form of a CD that bears interest of 1.5% for a two-year term. Another $10 million CD offers 1.70% for a three-year term. The bank can arrange two swaps: (1) The bank receives 1.50% fixed and pays MRR minus 10 bps with a two-year term and a notional amount of $10 million, and (2) the bank receives 1.70% fixed and pays MRR minus 15 bps with a three-year term and a notional amount of $10 million. After issuing the two CDs and committing to the two swaps, the bank has raised $20 million with an annual funding cost for the first two years of MRR minus 12.5 bps applied to the total notional amount of $20 million. The fixed interest payments received from the counterparty to the swap are paid to the CD investors; in effect, fixed-rate liabilities have been converted to floating-rate liabilities. The margins on the floating rates become the standard by which value is measured in assessing the bank's total funding cost.

By using the swap curve as a benchmark for the time value of money, the investor can adjust the swap spread so that the swap will be fairly priced given the spread. Conversely, given a swap spread, the investor can determine a fair price for the bond. We will use the swap spread in the following section to determine the value of a bond.

How Do Market Participants Use the Swap Curve in Valuation?

Although benchmark swap rates are quoted for specific maturities, swap contracts may be customized by two parties in the over-the-counter market. The fixed payment can be specified by an amortization schedule or involve a coupon with non-standard payment dates. In this section, we will focus on zero-coupon bonds. The yields on these bonds determine the swap curve, which, in turn, can be used to determine bond values.

Each forward date has an associated discount factor that represents the value today of a unit payment that one would hypothetically receive on the forward date expressed as a decimal fraction. For example, if we expect to receive ₩10,000 (10,000 South Korean won) in one year and the current price of the security is ₩9,259.30, then the discount factor for one year will be 0.92593 (= ₩9,259.30/₩10,000). Note that the rate associated with this discount factor is 1/0.92593 − 1 ≈ 8.00%.

To price a swap using current market rates, as mentioned we must solve for a constant fixed rate that sets the present value of fixed-leg payments equal to the present value of floating-leg payments over the life of the swap. Once established, the fixed cash flows are specified by the coupon rate set at the time of the original agreement. Pricing the floating leg is more complex than pricing the fixed leg because, by definition, its cash flows change with future changes in interest rates. The forward rate for each floating payment date is calculated by using the forward curves.

Let s_T stand for the T-period swap rate. Because the value of a swap at origination is set to zero, the swap rates must satisfy Equation 12. Note that the swap rates can be determined from the spot rates and the spot rates can be determined from the swap rates.

$$\sum_{t=1}^{T} \frac{s_T}{(1+z_t)^t} + \frac{1}{(1+z_T)^T} = 1 \qquad (13)$$

The right-hand side of Equation 13 is the value of the floating leg, which is always 1 at origination. The swap rate is determined by equating the value of the fixed leg, on the left-hand side, to the value of the floating leg.

Example 8 addresses the relationship between the swap rate curve and spot curve.

EXAMPLE 8

Determining the Swap Rate Curve

Suppose a government spot curve implies the following discount factors:

$DF_1 = 0.9524$

$DF_2 = 0.8900$

$DF_3 = 0.8163$

$DF_4 = 0.7350$

1. Given this information, determine the swap rate curve.

Solution:

Recall from Equation 1 that $DF_N = \frac{1}{(1+Z_N)^N}$. Therefore,

$$z_N = \left(\frac{1}{DF_N}\right)^{1/N} - 1$$

$$z_1 = \left(\frac{1}{0.9524}\right)^{1/1} - 1 = 5.00\%$$

$$z_2 = \left(\frac{1}{0.8900}\right)^{1/2} - 1 = 6.00\%$$

$$z_3 = \left(\frac{1}{0.8163}\right)^{1/3} - 1 = 7.00\%$$

$$z_4 = \left(\frac{1}{0.7350}\right)^{1/4} - 1 = 8.00\%$$

Using Equation 12, for $N = 1$,

$$\frac{s_1}{(1+z_1)} + \frac{1}{(1+z_1)} = \frac{s_1 + 1}{(1 + 0.05)} = 1$$

Therefore, $s_1 = 5\%$.

For $T = 2$,

$$\frac{s_2}{(1+z_1)} + \frac{s_2}{(1+z_2)^2} + \frac{1}{(1+z_2)^2} = \frac{s_2}{(1 + 0.05)} + \frac{s_2 + 1}{(1 + 0.06)^2} = 1$$

Therefore, $s_2 = 5.97\%$.

For $T = 3$,

$$\frac{s_3}{(1+z_1)} + \frac{s_3}{(1+z_2)^2} + \frac{s_3}{(1+z_3)^3} + \frac{1}{(1+z_3)^3}$$

$$= \frac{s_3}{(1 + 0.05)} + \frac{s_3}{(1 + 0.06)^2} + \frac{s_3}{(1 + 0.07)^3} + \frac{1}{(1 + 0.07)^3} = 1$$

Therefore, $s_3 = 6.91\%$.

For $T = 4$,

$$\frac{s_4}{(1+z_1)} + \frac{s_4}{(1+z_2)^2} + \frac{s_4}{(1+z_3)^3} + \frac{s_4}{(1+z_4)^4} + \frac{1}{(1+z_4)^4}$$

$$= \frac{s_4}{(1 + 0.05)} + \frac{s_4}{(1 + 0.06)^2} + \frac{s_4}{(1 + 0.07)^3} + \frac{s_4}{(1 + 0.08)^4} + \frac{1}{(1 + 0.08)^4} = 1$$

Therefore, $s_4 = 7.81\%$.

Note that the swap rates, spot rates, and discount factors are all mathematically linked together. Having access to data for one of the series allows you to calculate the other two.

5 THE SWAP SPREAD AND SPREADS AS A PRICE QUOTATION CONVENTION

- ☐ calculate and interpret the swap spread for a given maturity
- ☐ describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk

The swap spread is a popular way to indicate credit spreads in a market. The **swap spread** is defined as the spread paid by the fixed-rate payer of an interest rate swap over the rate of the "on-the-run" (most recently issued) government security with the same maturity as the swap. The spread captures the yield premium required for credit relative to the benchmark government bond. Because swap rates are built from market rates for short-term risky debt, this spread is a barometer of the market's perceived credit risk relative to default-risk-free rates. This spread typically widens countercyclically, exhibiting greater values during recessions and lower values during economic expansions.

The term “swap spread” is sometimes also used as a reference to a bond’s basis point spread over the interest rate swap curve and is a measure of the credit and/or liquidity risk of a bond. Here, a swap spread is an excess yield of swap rates over the yields on government bonds, and we use the terms I-spread, ISPRD, or interpolated spread to refer to bond yields net of the swap rates of the same maturities. In its simplest form, the I-spread can be measured as the difference between the yield-to-maturity of the bond and the swap rate given by a straight-line interpolation of the swap curve.

Often, fixed-income prices will be quoted as a swap rate plus (or minus) a spread, for which the yield is simply the yield on an equal-maturity government bond plus the swap spread. For example, if the fixed rate of a five-year fixed-for-float MRR swap is 2.00% and the five-year Treasury is yielding 1.70%, the swap spread is 2.00% − 1.70% = 0.30%, or 30 bps.

For euro-denominated swaps, the government yield used as a benchmark is most frequently Bunds (German government bonds) with the same maturity. Gilts (UK government bonds) are used as a benchmark in the United Kingdom.

Although the Libor swap curve is being phased out, it has historically been considered to reflect the default risk of A1/A+ rated commercial banks. The transition from Libor to MRR based on secured overnight funding rates will increase the influence of demand and supply conditions in government debt markets on swap rates. Another reason for the popularity of the swap market is that it is led by major financial institutions rather than controlled by governments, so swap rates are more comparable across different countries. The swap market also has more maturities with which to construct a yield curve than do government bond markets. Historically, cash or deposit rates such as Libor have been used for short-maturity yields; interest rate futures such as Eurodollar futures contracts have maturities of up to a year; and swap rates extend to maturities of up to 50 years in US dollars or euro. As the market transitions from Libor, the concept of this spread will be consistent with whichever market-based alternative to Libor emerges.

HISTORY OF THE US SWAP SPREAD SINCE 2008

The fact that governments generally pay less than private entities do in order to borrow suggests that swap spreads should always be positive. However, the 30-year Treasury swap spread turned negative following the collapse of Lehman Brothers Holdings Inc. in September 2008. Strong demand for duration combined with tighter liquidity and greater counterparty risk were widely cited as reasons for this phenomenon. For the period shown, the 30-year Treasury swap spread hit a record low (–62 bps intramonth) during November 2008. The 30-year Treasury swap spread was at or above zero for more than a year before becoming negative once again (see Exhibit 5). A recent study by the Federal Reserve Bank of New York (Boyarchenko, Gupta, Steele, and Yen, 2018) suggests that negative swap spreads have persisted because of increased regulatory capital requirements among swap dealers following the financial crisis.

Exhibit 5: US Swap Spread, January 2008–May 2020 (monthly data)

To illustrate the use of the swap spread in fixed-income pricing, consider a US$1 million investment in GE Capital (GECC) notes with a coupon rate of 1 5/8% (1.625%) that matures on 2 July 2024. Coupons are paid semiannually. The evaluation date is 12 July 2021, so the remaining maturity is 2.97 years [= 2 + (350/360)]. The Treasury rates for two-year and three-year maturities are 0.525% and 0.588%, respectively. By simple interpolation between these two rates, the US Treasury rate for 2.97 years is 0.586% [= 0.525% + (350/360)(0.588% − 0.525%)]. If the swap spread for the same maturity is 0.918%, then the yield-to-maturity on the bond (assuming the *i*-spread is zero for this case) is 1.504% (= 0.918% + 0.586%). Given the yield-to-maturity, the invoice price (price including accrued interest) for US$1 million face value is as follows:

$$\frac{1,000,000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(1-\frac{10}{180}\right)}}+\frac{1,000,000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(2-\frac{10}{180}\right)}}+\cdots+$$

$$\frac{1,000,000\left(\frac{0.01625}{2}\right)}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}}+\frac{1,000,000}{\left(1+\frac{0.01504}{2}\right)^{\left(6-\frac{10}{180}\right)}} = \text{US\$1,003,954.12}$$

The left-hand side sums the present values of the semiannual coupon payments and the final principal payment of US$1,000,000. The accrued interest rate amount is US$451.39 [= 1,000,000 × (0.01625/2)(10/180)]. Therefore, the clean price (price not including accrued interest) is US$1,003,502.73 (= 1,003,954.12 – 451.39).

The swap spread helps an investor to identify the time value, credit, and liquidity components of a bond's YTM. If the bond is default free, then the swap spread could provide an indication of the bond's liquidity, or it could provide evidence of market mispricing. The higher the swap spread, the higher the return that investors require for credit and/or liquidity risks. Another approach introduced in an earlier reading is to calculate a constant yield spread over a government (or interest rate swap) spot curve instead. This spread is known as the zero volatility spread (Z-spread) of a bond over the benchmark rate.

Spreads as a Price Quotation Convention

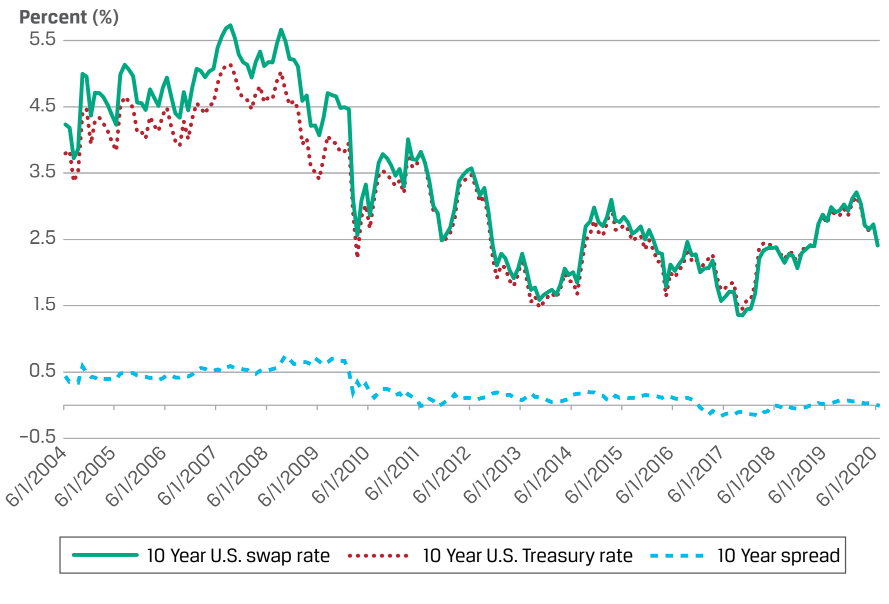
Treasury curves and swap curves represent different benchmarks for fixed-income valuation. It is therefore important to distinguish between a bond price quote that uses the bond yield net of a benchmark Treasury yield and one that uses a swap rate.

The Treasury rate can differ from the swap rate for the same term for several reasons. Unlike the cash flows from US Treasury bonds, the cash flows from swaps are subject to greater default risk. Market liquidity for specific maturities may differ. For example, some parts of the term structure of interest rates may be more actively traded with swaps than with Treasury bonds. Finally, arbitrage between these two markets cannot be perfectly executed.

Swap spreads to the Treasury rate (as opposed to **I-spreads**, which are bond rates net of the swap rates of the same maturities) are simply the differences between swap rates and government bond yields of a particular maturity. One problem in defining swap spreads is that, for example, a 10-year swap matures in exactly 10 years, whereas this condition is true for a 10-year government bond only at the time of issuance. By convention, therefore, the 10-year swap spread is defined as the difference between the 10-year swap rate and the 10-year on-the-run government bond. Swap spreads of other maturities are defined similarly.

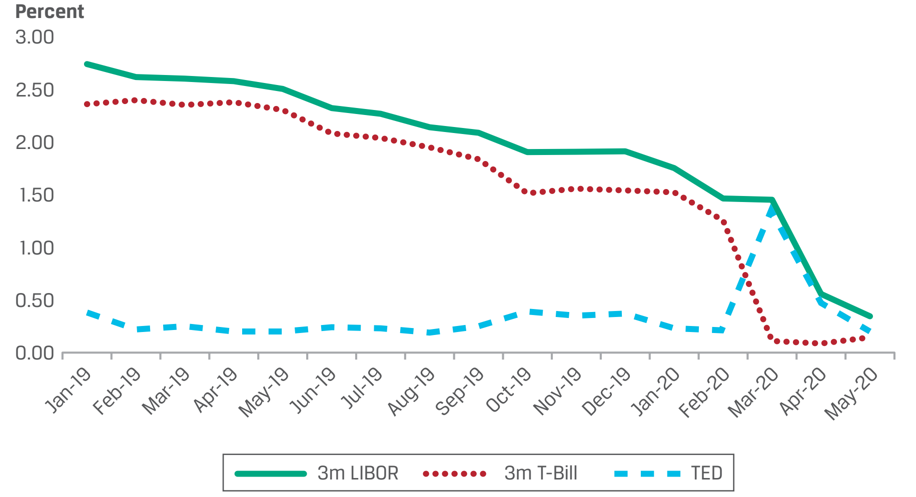
The curves in Exhibit 6 show the relationship between 10-year Treasury notes and 10-year swap rates. The 10-year swap spread is the 10-year swap rate less the 10-year Treasury note yield. Although positive swap spreads reflecting the difference between MRR-based rates and default-risk-free US government yields were historically the norm, these spreads have narrowed to zero or negative levels since the 2008 financial crisis because of higher swap dealer capital requirements and leverage constraints.

Exhibit 6: 10-Year US Swap Rate vs. 10-Year US Treasury Rate



Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk. For example, the difference between MRR and the yield on a Treasury bill of the same maturity, or **TED spread**, has historically been a key indicator of perceived credit and liquidity risk. TED is an acronym formed from an abbreviation for the US T-bill (T) and the ticker symbol for the MRR-based Eurodollar futures contract (ED). Exhibit 7 shows the historical TED spread. An increase in the TED spread signals greater perceived credit and liquidity risk, as occurred in early 2020 amid market turmoil related to the COVID-19 pandemic.

Exhibit 7: TED Spread, January 2019–May 2020 (end-of-month data)



Another popular measure of such risk is the MRR–OIS spread, formerly the **Libor-OIS spread**, which is the difference between MRR and the **overnight indexed swap (OIS) rate**. An OIS is an interest rate swap in which the periodic floating rate of the swap equals the geometric average of a daily unsecured overnight rate (or overnight index rate). The index rate is typically the rate for overnight unsecured lending between banks, such as the federal funds rate for US dollars or Eonia (Euro OverNight Index Average) for euros. As market participants transition away from survey-based Libor to alternative benchmarks based on actual transaction data, the **secured overnight financing rate (SOFR)**, or overnight cash borrowing rate collateralized by US Treasuries, has gained prominence and is expected to replace Libor in the future. A barometer of the US Treasury repurchase (or repo) market, SOFR is a daily volume-weighted index of all qualified repo market transactions and is influenced by supply and demand conditions in secured funding markets. The shift to overnight secured funding benchmarks extends globally—for example, the secured European Short-Term Rate (ESTR) has been recommended to replace Eonia, and the Canadian Overnight Repo Rate Average (CORRA) is proposed to replace the survey-based unsecured Canadian Dollar Offered Rate (CDOR).

6 TRADITIONAL THEORIES OF THE TERM STRUCTURE OF INTEREST RATES

☐ explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve

This section presents four traditional theories of the underlying economic factors that affect the shape of the yield curve.

Expectations Theory

One branch of traditional term structure theory focuses on interpreting term structure shape in terms of investors' expectations. Historically, the first such theory is known as the **unbiased expectations theory**, also called **pure expectations theory**. It says that the forward rate is an unbiased predictor of the future spot rate; its broadest interpretation is that bonds of any maturity are perfect substitutes for one another. For example, buying a bond with a maturity of five years and holding it for three years has the same expected return as buying a three-year bond or buying a series of three one-year bonds.

The predictions of the unbiased expectations theory are consistent with the assumption of risk neutrality. In a risk-neutral world, investors are unaffected by uncertainty and risk premiums do not exist. Every security is risk free and yields the risk-free rate for that particular maturity. Although such an assumption leads to interesting results, it clearly is in conflict with the large body of evidence showing that investors are risk averse.

A theory that is similar but more rigorous than the unbiased expectations theory is the **local expectations theory**. Rather than asserting that every maturity strategy has the same expected return over a given investment horizon, this theory instead contends that the expected return for every bond over short periods is the risk-free rate. This conclusion results from an assumed no-arbitrage condition in which bond pricing does not allow for traders to earn arbitrage profits.

The primary way that the local expectations theory differs from the unbiased expectations theory is that it can be extended to a world characterized by risk. Although the theory requires that risk premiums be nonexistent for very short holding periods, no such restrictions are placed on longer-term investments. Thus, the theory is applicable to both risk-free as well as risky bonds.

Although the local expectations theory is economically appealing, it is often observed that short-holding-period returns on long-dated bonds in fact exceed those on short-dated bonds. The need for liquidity and the ability to hedge risk essentially ensure that the demand for short-term securities will exceed that for long-term securities. Thus, both the yields and the actual returns for short-dated securities are typically lower than those for long-dated securities.

Liquidity Preference Theory

Whereas expectations theories leave no room for risk aversion, liquidity preference theory attempts to account for it. **Liquidity preference theory** asserts that **liquidity premiums** exist to compensate investors for the added interest rate risk they face when lending long term and that these premiums increase with maturity. Thus, given an expectation of unchanging short-term spot rates, liquidity preference theory predicts an upward-sloping yield curve. The forward rate provides an estimate of the expected spot rate that is biased upward by the amount of the liquidity premium, which invalidates the unbiased expectations theory. The liquidity premium for each consecutive future period should be no smaller than that for the prior period.

For example, the US Treasury offers bonds that mature in 30 years. Most investors, however, have shorter investment horizons than 30 years. For investors to hold these bonds, they would demand a higher return for taking the risk that the yield curve changes and that they must sell the bond prior to maturity at an uncertain price. That incrementally higher return is the liquidity premium. Note that this premium is not to be confused with a yield premium for the lack of liquidity that thinly traded bonds may bear. Rather, it is a premium applying to all long-term bonds, including those with deep markets.

Liquidity preference theory fails to offer a complete explanation of the term structure. Rather, it simply argues for the existence of liquidity premiums. For example, a downward-sloping yield curve could still be consistent with the existence of liquidity premiums if one of the factors underlying the shape of the curve is an expectation of deflation (i.e., a negative rate of inflation resulting from monetary or fiscal policy actions). Expectations of sharply declining spot rates may also result in a downward-sloping yield curve if the expected decline in interest rates is severe enough to offset the effect of the liquidity premiums.

In summary, liquidity preference theory claims that lenders require a liquidity premium as an incentive to lend long term. Thus, forward rates derived from the current yield curve provide an upwardly biased estimate of expected future spot rates. Although downward-sloping or hump-shaped yield curves may sometimes occur, the existence of liquidity premiums implies that the yield curve will typically be upward sloping.

Segmented Markets Theory

Unlike expectations theory and liquidity preference theory, **segmented markets theory** allows for lender and borrower preferences to influence the shape of the yield curve. The result is that yields are not a reflection of expected spot rates or liquidity premiums. Rather, they are solely a function of the supply and demand for funds of a particular maturity. That is, each maturity sector can be thought of as a segmented market in which yield is determined independently from the yields that prevail in other maturity segments.

The theory is consistent with a world in which asset/liability management constraints exist, either regulatory or self-imposed. In such a world, investors might restrict their investment activity to a maturity sector that provides the best match for the maturity of their liabilities. Doing so avoids the risks associated with an asset/liability mismatch.

For example, because life insurers sell long-term liabilities against themselves in the form of life insurance contracts, they tend to be most active as buyers in the long end of the bond market. Similarly, because the liabilities of pension plans are long term, they typically invest in long-term securities. Why would they invest short term given that those returns might decline while the cost of their liabilities stays fixed? In contrast, money market funds would be limited to investing in debt with maturity of one year or less, in general.

In summary, the segmented markets theory assumes that market participants are either unwilling or unable to invest in anything other than securities of their preferred maturity. It follows that the yield of securities of a particular maturity is determined entirely by the supply and demand for funds of that particular maturity.

Preferred Habitat Theory

The **preferred habitat theory** is similar to the segmented markets theory in proposing that many borrowers and lenders have strong preferences for particular maturities, but it does not assert that yields at different maturities are determined independently of each other.

The theory contends, however, that if the expected additional returns to be gained become large enough, institutions will be willing to deviate from their preferred maturities or habitats. For example, if the expected returns on longer-term securities exceed those on short-term securities by a large enough margin, an intermediate-term bond fund might lengthen the maturities of their assets. And if the excess returns expected from buying short-term securities become large enough, life insurance companies might stop limiting themselves to long-term securities and place a larger part of their portfolios in shorter-term investments.

The preferred habitat theory is based on the realistic notion that agents and institutions will accept additional risk in return for additional expected returns. In accepting elements of both the segmented markets theory and the unbiased expectations theory, yet rejecting their extreme polar positions, the preferred habitat theory moves closer to explaining real-world phenomena. In this theory, both market expectations and the institutional factors emphasized in the segmented markets theory influence the term structure of interest rates.

PREFERRED HABITAT AND QE

The term "quantitative easing" (QE) refers to an unconventional monetary policy used by central banks to increase the supply of money in an economy when central bank and/or interbank interest rates are already close to zero. The first of several QE efforts by the US Federal Reserve began in late 2008, following the establishment of a near-zero target range for the federal funds rate. Since then, the Federal Reserve has greatly expanded its holdings of long-term securities via a series of asset purchase programs, with the goal of putting downward pressure on long-term interest rates and thereby making financial conditions even more accommodative. Exhibit 8 presents information regarding the securities held by the Federal Reserve on 20 September 2007 (when all securities held by the Fed were US Treasury issuance) and on 29 October 2014 (when the Federal Reserve ended its third round of QE).

Exhibit 8: Securities Held by the US Federal Reserve

(US$ billions)	20 Sep 2007	29 Oct 2014
Securities held outright	780	4,219
US Treasury	780	2,462
Bills	267	0
Notes and bonds, nominal	472	2,347
Notes and bonds, inflation indexed	36	115
Inflation compensation	5	16
Federal agency	0	40
Mortgage-backed securities	0	1,718

As Exhibit 8 shows, the Federal Reserve's security holdings on 20 September 2007 consisted entirely of US Treasury securities, and about 34% of those holdings were short term in the form of T-bills. On 29 October 2014, only about 58% of the Federal Reserve's security holdings were Treasury securities, and none were T-bills. Furthermore, the Federal Reserve held well over US$1.7 trillion of mortgage-backed securities (MBS), which accounted for 41% of all securities held.

Prior to the QE efforts, the yield on MBS was typically in the 5%–6% range. It declined to less than 2% by the end of 2012. Concepts related to preferred habitat theory could possibly help explain that drop in yield.

The purchase of MBS by the Federal Reserve reduced the supply of these securities that was available for private purchase. Assuming that many MBS investors are either unwilling or unable to withdraw from the MBS market because of their comparative experience and expertise in managing interest rate and repayment risks of MBS versus option-free bonds, MBS investing institutions would have a "preferred habitat" in the MBS market. If they were unable to meet investor demand without bidding more aggressively, these buyers would drive down yields on MBS.

The Federal Reserve's purchase of MBS also resulted in a reduction in MBS yields. If a homeowner prepays on a mortgage, the payment is sent to MBS investors on a pro rata basis. Although investors are uncertain about when such a prepayment will be received, prepayment is more likely in a declining interest rate environment.

Use Example 9 to test your understanding of traditional term structure theories.

EXAMPLE 9

Traditional Term Structure Theories

1. Many fixed-income portfolio managers are limited in or prohibited from high-yield bond investments. When a bond is downgraded from an investment-grade to a high-yield (junk) rating, it is referred to as a *fallen angel*. Because of restrictions, many pension funds sell fallen angels when they are downgraded from investment grade to high yield (junk). This coordinated selling action often results in depressed prices and attractive yields for the fallen angels. Which of the following reasons best explains why fallen angel yields often exceed otherwise identical bonds?

   A. The preferred habitat theory

   B. The segmented markets theory

   C. The local expectations theory

Solution:

B is correct. Market segmentation in this example results from the requirement that some fixed-income fund managers are prohibited or limited in their capacity to hold high-yield bonds. The segmentation results in selling pressure on fallen angels that depresses their prices.

2. The term structure theory in which investors can be induced by relatively attractive yields to hold debt securities whose maturities do not match their investment horizon is *best* described as the:

   A. preferred habitat theory.

   B. segmented markets theory.

   C. unbiased expectations theory.

Solution:

A is correct. Preferred habitat theory asserts that investors are willing to deviate from their preferred maturities if yield differentials encourage the switch. Segmented markets theory is more rigid than preferred habitat in that asset/liability management constraints force investors to buy securities whose horizons match those of their liabilities. The unbiased expectations theory makes no assumptions about maturity preferences. Rather, it contends that forward rates are unbiased predictors of future spot rates.

3. The unbiased expectations theory assumes investors are:

   A. risk averse.

B. risk neutral.

C. risk seeking.

Solution:

B is correct. The unbiased expectations theory asserts that different maturity strategies, such as rollover, maturity matching, and riding the yield curve, have the same expected return. By definition, a risk-neutral party is indifferent about choices with equal expected payoffs, even if one choice is riskier. Thus, the predictions of the theory are consistent with the existence of risk-neutral investors.

4. Market evidence shows that forward rates are:

A. unbiased predictors of future spot rates.

B. upwardly biased predictors of future spot rates.

C. downwardly biased predictors of future spot rates.

Solution:

B is correct. The existence of a liquidity premium ensures that the forward rate is an upwardly biased estimate of the future spot rate. Market evidence clearly shows that liquidity premiums exist, and this evidence effectively refutes the predictions of the unbiased expectations theory.

5. Market evidence shows that short holding-period returns on short-maturity bonds *most* often are:

A. less than those on long-maturity bonds.

B. about equal to those on long-maturity bonds.

C. greater than those on long-maturity bonds.

Solution:

A is correct. Although the local expectations theory predicts that the short-run return for all bonds will equal the risk-free rate, most of the evidence refutes that claim. Returns from long-dated bonds are generally higher than those from short-dated bonds, even over relatively short investment horizons. This market evidence is consistent with the risk–expected return trade-off that is central to finance and the uncertainty surrounding future spot rates.

YIELD CURVE FACTOR MODELS 7

☐ explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks

The effect of yield volatilities on price is an important consideration in fixed-income investment, particularly for risk management and portfolio evaluation. In this section, we describe measuring and managing the interest rate risk of bonds.

A Bond's Exposure to Yield Curve Movement

Shaping risk is defined as the sensitivity of a bond's price to the changing shape of the yield curve. The yield curve's shape changes continually, and yield curve shifts are rarely parallel. For active bond management, a bond investor may want to base trades on a forecasted yield curve shape or may want to hedge the yield curve risk on a bond portfolio using swaps. Shaping risk also affects the value of many options, which is very important because many fixed-income instruments have embedded options.

Exhibit 9 and Exhibit 10 show historical yield curve movements for US and European swap rates from March 2006 until March 2020. The exhibits show the considerable swap yield curve changes over time. In both cases, the pre-financial-crisis March 2006 yield curves represent the highest swap yields and those from March 2020 (amid the COVID-19 pandemic-related market turmoil) the lowest. In the United States, however, the end of quantitative easing and tighter monetary policy resulted in a rebound in swap yields prior to 2020, whereas in Europe, yields remained low or negative because of continued accommodative monetary policy. Note that the vertical axis values of the three exhibits differ, and the horizontal axis is not to scale.

Exhibit 9: Historical US Swap Yield Curve Movements

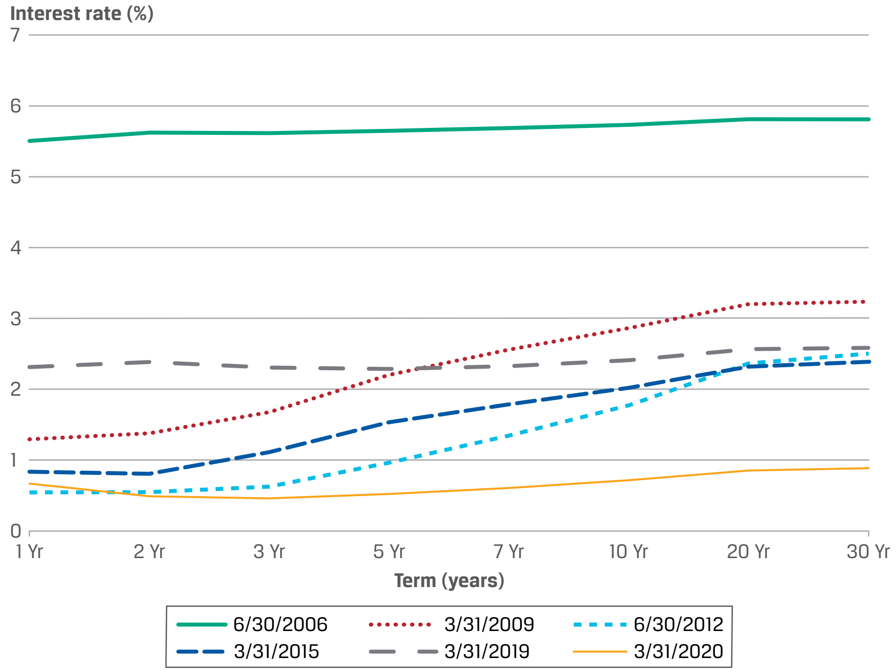
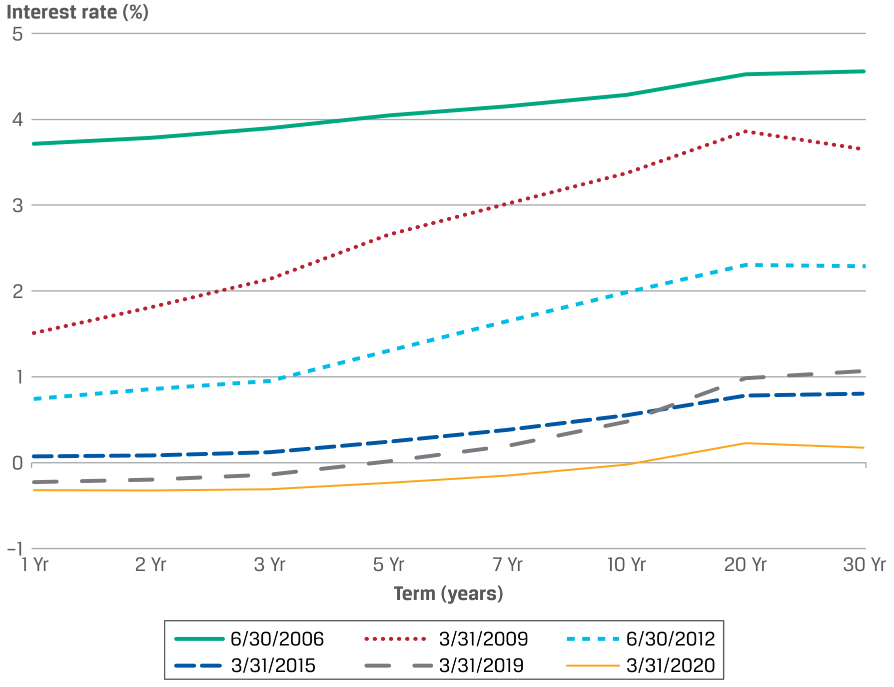


Exhibit 10: Historical European Swap Yield Curve Movements

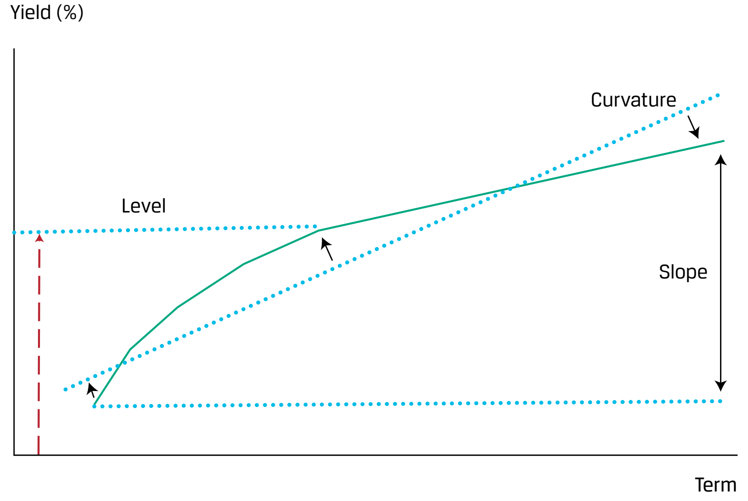


Factors Affecting the Shape of the Yield Curve

The previous section showed that the yield curve can take nearly any shape. The challenge for a fixed-income manager is to implement a process to manage the yield curve shape risk in her portfolio. One approach is to find a model that reduces most of the possible yield curve movements to a probabilistic combination of a few standardized yield curve movements. This section presents one of the best-known yield curve factor models.

A **yield curve factor model** is defined as a model or a description of yield curve movements that can be considered realistic when compared with historical data. Research has led to models that can describe these movements with some accuracy. One specific yield curve factor model is the three-factor model of Litterman and Scheinkman (1991), who found that yield curve movements are historically well described by a combination of three independent movements, which they interpreted as **level**, **steepness**, and **curvature**. The level movement refers to an upward or downward shift in the yield curve. The steepness movement refers to a non-parallel shift in the yield curve when either short-term rates change more than long-term rates or long-term rates change more than short-term rates. The curvature movement is a reference to movement in three segments of the yield curve: The short-term and long-term segments rise while the middle-term segment falls, or vice versa. Exhibit 11 illustrates these factors.

Exhibit 11: Primary Yield Curve Factors: Level, Slope, and Curvature



In practice, the level movement factor explains most of the total changes in swap and bond market yields. This factor may be interpreted as a reflection of parallel yield curve moves in which rates move in the same direction and by a similar order of magnitude. The steepness factor addresses the shape of the curve, with short-term yields typically moving more than long-term yields. These changes take place over time and therefore explain less of the total variance in rates than the level factor. Finally, the third factor, curvature, tends to have a negative impact on intermediate yields and a positive impact on short- and long-term yields. This variable explaining the "twist" in the yield curve has the smallest impact of the three.

8 THE MATURITY STRUCTURE OF YIELD CURVE VOLATILITIES

☐ explain the maturity structure of yield volatilities and their effect on price volatility

Yield Volatility

Quantifying interest rate volatilities is important for fixed income managers for at least two reasons. First, most fixed-income instruments and derivatives have embedded options. Option values, and hence the values of the fixed-income instrument, crucially depend on the level of interest rate volatilities. Second, fixed-income interest rate risk management is clearly an important part of any management process, and such risk management includes controlling the impact of interest rate volatilities on the instrument's price volatility.

The term structure of interest rate volatilities is a representation of the yield volatility of a zero-coupon bond for every maturity of security. This volatility curve (or "vol") or volatility term structure measures yield curve risk.

Interest rate volatility is not the same for all interest rates along the yield curve. On the basis of the typical assumption of a lognormal model, the uncertainty of an interest rate is measured by the annualized standard deviation of the proportional change in a bond yield over a specified interval. For example, if the interval is a one-month period, then the specified interval equals 1/12 years. This measure, called interest rate volatility, is denoted σ(*t*,*T*), which is the volatility of the rate for a security with maturity *T* at time *t*. The term structure of volatilities is given by Equation 14:

$$\sigma(t,T) = \frac{\sigma[\Delta r(t,T)/r(t,T)]}{\sqrt{\Delta t}} \quad (14)$$

In Exhibit 12, to illustrate a term structure of volatility, the data series is deliberately chosen to end before the 2008 financial crisis, which was associated with some unusual volatility magnitudes.

Exhibit 12: Historical Example: US Treasuries, August 2005–December 2007

Maturity (years)	0.25	0.50	1	2	3	5	7	10	20	30
σ(*t*,*T*)	0.3515	0.3173	0.2964	0.2713	0.2577	0.2154	0.1885	0.1621	0.1332	0.1169

For example, the 35.15% standard deviation for the three-month T-bill in Exhibit 12 is based on a monthly standard deviation of 0.1015 = 10.15%, which annualizes as

$$0.1015 \div \sqrt{\frac{1}{12}} = 0.3515 = 35.15\%.$$

The volatility term structure typically shows that short-term rates are more volatile than long-term rates. That said, long-term bond *prices* tend to vary more than short-term bond prices given the impact of duration. Research indicates that short-term volatility is most strongly linked to uncertainty regarding monetary policy, whereas long-term volatility is most strongly linked to uncertainty regarding the real economy and inflation. Furthermore, most of the co-movement between short-term and long-term volatilities appears to depend on the ever-changing correlations among these three determinants (monetary policy, the real economy, and inflation). During the period of August 2005–December 2007, long-term volatility was lower than short-term volatility, falling from 35.15% for the 0.25-year rate to 11.69% for the 30-year rate.

Managing Yield Curve Risks Using Key Rate Duration

Yield curve risk—the risk to portfolio value arising from unanticipated changes in the yield curve—can be managed on the basis of several measures of sensitivity to yield curve movements. Management of yield curve risk involves changing the identified exposures to desired values by trades in security or derivative markets (the details fall under the rubric of fixed-income portfolio management and thus are outside the scope of this reading).

One available measure of yield curve sensitivity is effective duration, which measures the sensitivity of a bond's price to a small parallel shift in a benchmark yield curve. Another is based on **key rate duration**, which measures a bond's sensitivity to a small change in a benchmark yield curve at a specific maturity segment. Using one of these last two measures allows identification and management of "shaping risk"—that is, sensitivity to changes in the shape of the benchmark yield curve—in addition to the risk associated with parallel yield curve changes, which is addressed adequately by effective duration.

To make the discussion more concrete, consider a portfolio of 1-year, 5-year, and 10-year zero-coupon bonds with \$100 value in each position; total portfolio value is therefore \$300. Also consider the hypothetical set of factor movements shown in the following table:

Year	1	5	10
Parallel	1	1	1
Steepness	−1	0	1
Curvature	1	0	1

In the table, a parallel movement or shift means that all the rates shift by an equal amount—in this case, by a unit of 1. A steepness movement means that the yield curve steepens with the long rate shifting up by one unit and the short rate shifting down by one unit. A curvature movement means that both the short rate and the long rate shift up by one unit, whereas the medium-term rate remains unchanged. These movements need to be defined, as they are here, such that none of the movements can be a linear combination of the other two movements. Next, we address the calculation of the various yield curve sensitivity measures.

Because the bonds are zero-coupon bonds, each bond's effective duration is the same as its maturity. The portfolio's effective duration is the weighted sum of the effective duration of each bond position; for this equally weighted portfolio, effective duration is 0.333(1 + 5 + 10) = 5.333.

To calculate key rate durations, consider various yield curve movements. First, suppose that the one-year rate changes by 100 bps while the other rates remain the same; the sensitivity of the portfolio to that shift is 1/[(300)(0.01)] = 0.3333. We conclude that the key rate duration of the portfolio ($KeyDur_{Full}$) to the one-year rate, denoted $KeyDur_1$, is 0.3333. Likewise, the key rate durations of the portfolio to the 5-year rate, $KeyDur_5$, and the 10-year rate, $KeyDur_{10}$, are 1.6667 and 3.3333, respectively. Note that the sum of the key rate durations is 5.333, which is the same as the effective duration of the portfolio. This fact can be explained intuitively. Key rate duration measures the portfolio risk exposure to each key rate. If all the key rates move by the same amount, then the yield curve has made a parallel shift, and as a result, the proportional change in value has to be consistent with effective duration. The related model for yield curve risk based on key rate durations ($KeyDur$) is as follows:

$$KeyDur_{Full} = \%\Delta P = \left(\frac{\Delta P}{P}\right) \approx -KeyDur_1 \Delta z_1 - KeyDur_5 \Delta z_5 - KeyDur_{10} \Delta z_{10}$$
$$= -0.3333 \Delta z_1 - 1.667 \Delta z_5 - 3.3333 \Delta z_{10} \quad (15)$$

Next, we can calculate a measure based on the decomposition of yield curve movements into parallel, steepness, and curvature movements, as described earlier. Define D_L, D_S, and D_C as the sensitivities of portfolio value to small changes in the level, steepness, and curvature factors, respectively. Based on this factor model, Equation 16 shows the proportional change in portfolio value that would result from a small change in the level factor (Δx_L), the steepness factor (Δx_S), and the curvature factor (Δx_C).

$$KeyDur_{Full} = \%\Delta P$$
$$= \left(\frac{\Delta P}{P}\right) \approx -KeyDur_L \Delta x_L - KeyDur_S \Delta x_S - KeyDur_C \Delta x_C \quad (16)$$

Because $KeyDur_L$ is by definition sensitivity to a parallel shift, the proportional change in the portfolio value per unit shift (the line for a parallel movement in the table) is 5.3333 = (1 + 5 + 10)/[(300)(0.01)]. The sensitivity for steepness movement can be calculated as follows (see the line for steepness movement in the table). When the steepness makes an upward shift of 100 bps, it would result in a downward shift of 100 bps for the 1-year rate, resulting in a gain of \$1, and an upward shift for the 10-year rate, resulting in a loss of \$10. The change in value is therefore (1 – 10). $KeyDur_S$ is the negative of the proportional change in price per unit change in this movement and in this case is 3.0 = −(1 – 10)/[(300)(0.01)]. Considering the line for curvature movement in the table, $KeyDur_C$ = 3.6667 = (1 + 10)/[(300)(0.01)]. Thus, for our hypothetical bond portfolio, we can analyze the portfolio's yield curve risk using the following equation:

$$KeyDur_{Full} = \%\Delta P = \left(\frac{\Delta P}{P}\right) \approx -5.3333\Delta x_L - 3.0\Delta x_S - 3.6667\Delta x_C$$

For example, if $\Delta x_L = -0.0050$, $\Delta x_S = 0.002$, and $\Delta x_C = 0.001$, the predicted change in portfolio value would be +1.7%. It can be shown that key rate durations are directly related to level, steepness, and curvature in this example and that one set of sensitivities can be derived from the other. One can use the numerical example to verify that relation by decomposing changes in the term structure into level, slope, and curvature factors:

$$KeyDur_L = KeyDur_1 + KeyDur_5 + KeyDur_{10}$$

$$KeyDur_S = -KeyDur_1 + KeyDur_{10}$$

$$KeyDur_C = KeyDur_1 + KeyDur_{10}$$

Example 10 reviews concepts from this section and the preceding sections.

EXAMPLE 10

Term Structure Dynamics

1. The most important factor in explaining changes in the yield curve has been found to be:

   A. level.

   B. curvature.

   C. steepness.

Solution:

A is correct. Research shows that upward and downward shifts in the yield curve explain more than 75% of the total change in the yield curve.

2. A movement of the yield curve in which the short rate decreases by 150 bps and the long rate decreases by 50 bps would *best* be described as a:

   A. flattening of the yield curve resulting from changes in level and steepness.

   B. steepening of the yield curve resulting from changes in level and steepness.

   C. steepening of the yield curve resulting from changes in steepness and curvature.

Solution:

B is correct. Both the short-term and long-term rates have declined, indicating a change in the level of the yield curve. Short-term rates have declined more than long-term rates, indicating a change in the steepness of the yield curve.

3. The yield curve starts off flat, and then intermediate-maturity yields decrease by 10 bps while short- and long-maturity yields remain constant. This movement is *best* described as involving a change in:

   A. level only.

   B. curvature only.

   C. level and curvature.

Solution:

B is correct. The curve starts off flat, with identical short, intermediate, and long rates. Both the short-term and long-term rates remained constant, indicating no change in the level of the yield curve. Intermediate rates decreased, however, resulting in curvature.

4. Typically, short-term interest rates:

   A. are less volatile than long-term interest rates.

   B. are more volatile than long-term interest rates.

   C. have about the same volatility as long-term interest rates.

Solution:

B is correct. A possible explanation is that expectations for long-term inflation and real economic activity affecting longer-term interest rates are slower to change than those related to shorter-term interest rates.

5. Suppose for a given portfolio that key rate changes are considered to be changes in the yield on 1-year, 5-year, and 10-year securities. Estimated key rate durations are $KeyDur_1$ = 0.50, $KeyDur_2$ = 0.70, and $KeyDur_3$ = 0.90. What is the percentage change in the value of the portfolio if a parallel shift in the yield curve results in all yields declining by 50 bps?

   A. −1.05%.

   B. +1.05%.

   C. +2.10%.

Solution:

B is correct. A decline in interest rates would lead to an increase in bond portfolio value: −0.50(−0.005) − 0.70(−0.005) − 0.90(−0.005) = 0.0105 = 1.05%.

9 DEVELOPING INTEREST RATE VIEWS USING MACROECONOMIC VARIABLES

☐ explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes

Interest rate dynamics such as changes in spot versus forward rates and the level, steepness, and curvature of the yield curve are influenced by key economic variables and market events. Implied forward rates serve as market-neutral reference points for fixed income traders. As we illustrated earlier, if today's forward rates are realized in the future, then bond values will simply roll down the yield curve. In practice, active fixed-income market participants establish their own views on future interest rate developments and then position their portfolios in order to capitalize on differences between their own rate view and the market consensus. If their forecast is accurate, the portfolio generates greater returns than it would have otherwise.

This section reviews the key drivers of interest rates before moving on to establishing views and positioning fixed-income portfolios to capitalize on a specific interest rate view.

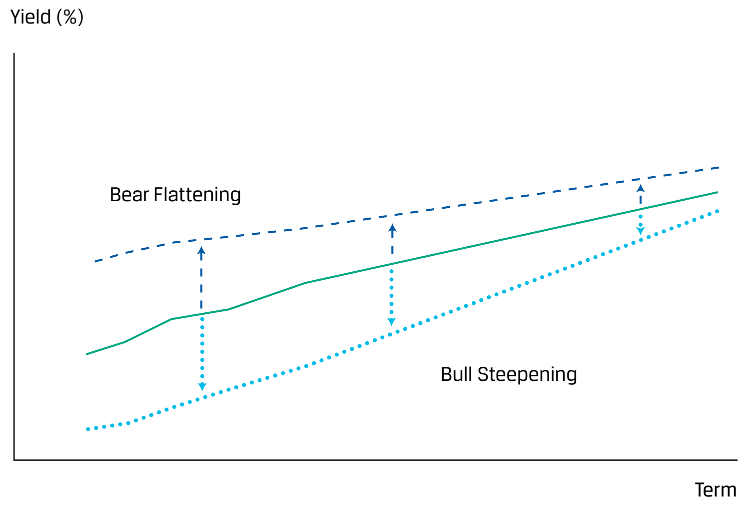
The term **bond risk premium** refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate. This premium is also referred to as the term (or duration) premium, and it is usually measured using government bonds to capture uncertainty of default-free rates, whereas credit, liquidity, and other risks may increase the overall risk premium for a specific bond. Unlike *ex post* observed historical returns, the bond risk premium is a forward-looking expectation and must be estimated.

Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.

Research shows that although inflation, GDP, and monetary policy explain most of the variance of bond yields, short- and intermediate-term bond yields are driven mostly by monetary policy, whereas other factors such as inflation are key drivers of long-term yields. Monetary policy explains about two-thirds of short- and intermediate-term bond yield variation, with the remaining third roughly equally attributable to economic growth and factors including inflation. In contrast, inflation explains nearly two-thirds of long-term yield variation, and the remaining third is largely attributable to monetary policy.

Monetary policy impacts the bond risk premium. Central banks such as the European Central Bank control the money supply and influence interest rates through policy tools in order to achieve stable prices and sustainable economic growth. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is often consistent with **bearish flattening**, or short-term bond yields rising more than long-term bond yields, resulting in a flatter yield curve. During economic recessions or anticipated recessions, the monetary authority cuts benchmark rates to help stimulate economic activity. The lowering of interest rates is associated with **bullish steepening**, in which short-term rates fall by more than long-term yields, resulting in a steeper term structure. These monetary policy actions lead to procyclical short-term interest rate changes. Exhibit 13 shows these two yield curve changes.

Exhibit 13: Examples of Yield Curve Flattening and Steepening



In recent years, central banks have increasingly used their balance sheets for large-sale asset purchases. For example, the Federal Reserve has bought large quantities of US Treasury bonds and mortgage-backed securities. The intended purpose is to stimulate economic activity by increasing the money supply through benchmark bond purchases and driving down the bond risk premium, encouraging capital allocation to incrementally higher-risk assets. Asset purchases impact the term structure by raising demand in a range of maturity segments.

Other factors that influence bond prices, yields, and the bond risk premium include fiscal policy, the maturity structure of debt, and investor demand.

Benchmark government bonds are the means by which nations fund their cumulative (current and past) budget deficits. Greater deficits require more borrowing, which influences both bond supply and required yield. Thus, fiscal supply-side effects affect bond prices and yields by increasing (decreasing) yields when budget deficits rise (fall). In the late 1990s, market participants believed the US government would run fiscal surpluses, leading to a reduction in government bond supply as the Treasury stopped issuing new 30-year bonds for four years. The expected reduction in supply drove long-maturity Treasury yields lower.

Longer government debt maturity structures predict greater excess bond returns. This is effectively a segmented market factor, wherein the greater supply of bonds of long-term maturity increases the yield in that market segment.

Domestic investor demand is a key driver of bond prices, especially among pension funds and insurance companies that use long-dated government bonds to match expected future liabilities. Greater domestic investor demand increases prices and reduces the bond risk premium.

Non-domestic investor demand influences government bond prices and may result either from holding reserves or from actions associated with currency exchange rate management. Non-domestic flows significantly influence bond prices because inflows (outflows) bid up (down) bond prices, lowering (raising) the bond risk premium.

During highly uncertain market periods, investors flock to government bonds in what is termed a **flight to quality**. This term refers to investors' selling off higher-risk asset classes such as stocks and commodities in favor of default-risk-free government bonds. A flight to quality is often associated with **bullish flattening**, in which the yield curve flattens as long-term rates fall by more than short-term rates.

Fixed-income trades based on interest rate forecasts can take a variety of forms, often using bond futures contracts to avoid significant portfolio turnover. Remember that any interest rate view must be evaluated relative to the current short rate and forward curve, because they reflect returns earned by investors rolling down the curve under the current set of implied forward rates.

Investors expecting interest rates to fall will generally extend portfolio duration relative to a benchmark to take advantage of bond price increases from falling rates, whereas investors expecting higher rates will shorten portfolio duration to reduce exposure to falling bond prices.

To capitalize on a steeper curve under which long-term rates rise relative to short-term rates, traders will short long-term bonds and purchase short-term bonds. If on the other hand a trader forecasts curve flattening, whereby short-term rates rise relative to long-term rates, she may capitalize on this trend by purchasing long-term bonds and selling short-term bonds short. In both the expected steepening and flattening trades, the position may be designed as duration neutral in order to insulate from changes in the level of the term structure. Fixed-income investors with long-only investment mandates may alternate between portfolios concentrated in a single maturity, known as a **bullet portfolio**, and those with similar duration that combine short and long maturities, known as a **barbell portfolio**. For example, an investor may seek to capitalize on an expected bullish flattening of the yield curve by shifting from a bullet to a barbell position.

EXAMPLE 11

Building a Rate View Based On Economic Forecasts and Monetary Policy

Morgan Salaz is a fixed income analyst responsible for advising fixed income clients about bond trading opportunities. In the current recessionary environment, the level of government bond yields is low and the term structure is nearly flat. Salaz's firm forecasts that after a brief recession, economic growth will return quickly during the coming 12 months.

1. Which of the following changes to the yield curve is consistent with Salaz's expectation of increasing economic growth over the coming year?

   A. Decrease in the level

   B. Decrease in the term spread of long-term rates over short-term rates

   C. Increase in the term spread of long-term rates over short-term rates

   Solution:

   Answer: C is correct. Economic growth forecasts impact long-term rates. The view that economic growth will return to robust levels is consistent with a shift to a positively sloped term structure.

2. Salaz also expects the Federal Reserve to decrease asset purchases of long-term bonds as the economic recovery continues. Which of the following scenarios is consistent with this view? The reduced asset purchases will likely:

   A. amplify the effect of increased economic activity on the term spread.

   B. dampen the effect of increased economic activity on the term spread.

C. have no effect on the term spread.

Solution:

Answer: A. Reduced asset purchases constitute a negative shift in demand for longer-term bonds, which raises their yields. The reduced asset purchases of long-maturity bonds would add to the effect of greater economic activity, both of which will increase the term spread.

SUMMARY

- The spot rate for a given maturity can be expressed as a geometric average of the short-term rate and a series of forward rates.
- Forward rates are above (below) spot rates when the spot curve is upward (downward) sloping, whereas forward rates are equal to spot rates when the spot curve is flat.
- If forward rates are realized, then all bonds, regardless of maturity, will have the same one-period realized return, which is the first-period spot rate.
- If the spot rate curve is upward sloping and is unchanged, then each bond "rolls down" the curve and earns the forward rate that rolls out of its pricing (i.e., an N-period zero-coupon bond earns the N-period forward rate as it rolls down to be a $N - 1$ period security). This dynamic implies an expected return in excess of short-maturity bonds (i.e., a **term premium**) for longer-maturity bonds if the yield curve is upward sloping.
- Active bond portfolio management is consistent with the expectation that today's forward curve does not accurately reflect future spot rates.
- The swap curve provides another measure of the time value of money.
- Swaps are an essential tool frequently used by investors to hedge, take a position in, or otherwise modify interest rate risk.
- Bond quote conventions often use measures of spreads. Those quoted spreads can be used to determine a bond's price.
- Swap curves and Treasury curves can differ because of differences in their credit exposures, liquidity, and other supply/demand factors.
- Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk.
- The local expectations theory, liquidity preference theory, segmented markets theory, and preferred habitat theory provide traditional explanations for the shape of the yield curve.
- Historical yield curve movements suggest that they can be explained by a linear combination of three principal movements: level, steepness, and curvature.
- The volatility term structure can be measured using historical data and depicts yield curve risk.
- The sensitivity of a bond value to yield curve changes may make use of effective duration, key rate durations, or sensitivities to parallel, steepness, and curvature movements. Using key rate durations or sensitivities to parallel, steepness, and curvature movements allows one to measure and manage shaping risk.

- The term bond risk premium refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate
- Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.
- During highly uncertain market periods, investors flock to government bonds in a flight to quality that is often associated with bullish flattening, in which long-term rates fall by more than short-term rates.
- Investors expecting rates to fall will generally extend (shorten) portfolio duration to take advantage of expected bond price increases (decreases)
- When investors expect a steeper (flatter) curve under which long-term rates rise (fall) relative to short-term rates, they will sell (buy) long-term bonds and purchase (sell) short-term bonds.

REFERENCES

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PRACTICE PROBLEMS

1. Given spot rates for one-, two-, and three-year zero coupon bonds, how many forward rates can be calculated?

2. Give two interpretations for the following forward rate: The two-year forward rate one year from now is 2%.

3. Describe the relationship between forward rates and spot rates if the yield curve is flat.

4. Which forward rate cannot be computed from the one-, two-, three-, and four-year spot rates? The rate for a:

   A. one-year loan beginning in two years

   B. two-year loan beginning in two years

   C. three-year loan beginning in two years

5. Consider spot rates for three zero-coupon bonds: z(1) = 3%, z(2) = 4%, and z(3) = 5%. Which statement is correct? The forward rate for a one-year loan beginning in one year will be:

   A. less than the forward rate for a one-year loan beginning in two years.

   B. greater than the forward rate for a two-year loan beginning in one year.

   C. greater than the forward rate for a one-year loan beginning in two years.

6. If one-period forward rates are decreasing with maturity, the yield curve is *most likely*:

   A. flat.

   B. upward sloping.

   C. downward sloping.

The following information relates to questions 7-17

A one-year zero-coupon bond yields 4.0%. The two- and three-year zero-coupon bonds yield 5.0% and 6.0%, respectively.

7. The rate for a one-year loan beginning in one year is *closest* to:

   A. 4.5%.

   B. 5.0%.

   C. 6.0%.

8. The forward rate for a two-year loan beginning in one year is *closest* to:

   A. 5.0%.

B. 6.0%.

C. 7.0%.

9. The forward rate for a one-year loan beginning in two years is *closest* to:

A. 6.0%.

B. 7.0%.

C. 8.0%.

10. The five-year spot rate is not provided here; however, the forward price for a two-year zero-coupon bond beginning in three years is known to be 0.8479. The price today of a five-year zero-coupon bond is *closest* to:

A. 0.7119.

B. 0.7835.

C. 0.9524.

11. The one-year spot rate z_1 is 4%, the forward rate for a one-year loan beginning in one year is 6%, and the forward rate for a one-year loan beginning in two years is 8%. Which of the following rates is *closest* to the three-year spot rate?

A. 4.0%

B. 6.0%

C. 8.0%

12. The one-year spot rate z_1 is 5%, and the forward price for a one-year zero-coupon bond beginning in one year is 0.9346. The spot price of a two-year zero-coupon bond is *closest* to:

A. 0.87.

B. 0.89.

C. 0.93.

13. In a typical interest rate swap contract, the swap rate is *best* describedas the interest rate for the:

A. fixed-rate leg of the swap.

B. floating-rate leg of the swap.

C. difference between the fixed and floating legs of the swap.

14. A two-year fixed-for-floating MRR swap is 1.00%, and the two-year US Treasury bond is yielding 0.63%. The swap spread is *closest* to:

A. 37 bps.

B. 100 bps.

C. 163 bps.

15. The swap spread is quoted as 50 bps. If the five-year US Treasury bond is yielding

2%, the rate paid by the fixed payer in a five-year interest rate swap is *closest* to:

A. 0.50%.

B. 1.50%.

C. 2.50%.

16. If the three-month T-bill rate drops and MRR remains the same, the relevant TED spread:

A. increases.

B. decreases.

C. does not change.

17. Given the yield curve for US Treasury zero-coupon bonds, which spread is *most* helpful pricing a corporate bond? The:

A. Z-spread.

B. TED spread.

C. MRR–OIS spread, formerly the Libor–OIS spread.

The following information relates to questions 18-24

Jane Nguyen is a senior bond trader for an investment bank, and Chris Alexander is a junior bond trader at the bank. Nguyen is responsible for her own trading activities and also for providing assignments to Alexander that will develop his skills and create profitable trade ideas. Exhibit 1 presents the current par and spot rates.

Exhibit 1: Current Par and Spot Rates

Maturity	Par Rate	Spot Rate
One year	2.50%	2.50%
Two years	2.99%	3.00%
Three years	3.48%	3.50%
Four years	3.95%	4.00%
Five years	4.37%	

Note: Par and spot rates are based on annual-coupon sovereign bonds.

Nguyen gives Alexander two assignments that involve researching various questions:

Assignment 1 What is the yield-to-maturity of the option-free, default-risk-free bond presented in Exhibit 2? Assume that the bond is held to maturity, and use the rates shown in Exhibit 1.

Exhibit 2: Selected Data for $1,000 Par Bond

Bond Name	Maturity (*T*)	Coupon
Bond Z	Three years	6.00%

Note: Terms are today for a *T*-year loan.

Assignment 2 Assuming that the projected spot curve two years from today will be below the current forward curve, is Bond Z fairly valued, undervalued, or overvalued?

After completing his assignments, Alexander asks about Nguyen's current trading activities. Nguyen states that she has a two-year investment horizon and will purchase Bond Z as part of a strategy to ride the yield curve. Exhibit 1 shows Nguyen's yield curve assumptions implied by the spot rates.

18. Based on Exhibit 1, the five-year spot rate is *closest to*:

A. 4.40%.

B. 4.45%.

C. 4.50%.

19. Based on Exhibit 1, the market is *most likely* expecting:

A. deflation.

B. inflation.

C. no risk premiums.

20. Based on Exhibit 1, the forward rate of a one-year loan beginning in three years is *closest to*:

A. 4.17%.

B. 4.50%.

C. 5.51%.

21. Based on Exhibit 1, which of the following forward rates can be computed?

A. A one-year loan beginning in five years

B. A three-year loan beginning in three years

C. A four-year loan beginning in one year

22. For Assignment 1, the yield-to-maturity for Bond Z is *closest* to the:

A. one-year spot rate.

B. two-year spot rate.

C. three-year spot rate.

23. For Assignment 2, Alexander should conclude that Bond Z is currently:

A. undervalued.

B. fairly valued.

C. overvalued.

24. By choosing to buy Bond Z, Nguyen is *most likely* making which of the following assumptions?

A. Bond Z will be held to maturity.

B. The three-year forward curve is above the spot curve.

C. Future spot rates do not accurately reflect future inflation.

The following information relates to questions 25-29

Laura Mathews recently hired Robert Smith, an investment adviser at Shire Gate Advisers, to assist her in investing. Mathews states that her investment time horizon is short, approximately two years or less. Smith gathers information on spot rates for on-the-run annual-coupon government securities and swap spreads, as presented in Exhibit 1. Shire Gate Advisers recently published a report for its clients stating its belief that, based on the weakness in the financial markets, interest rates will remain stable, the yield curve will not change its level or shape for the next two years, and swap spreads will also remain unchanged.

Exhibit 1: Government Spot Rates and Swap Spreads

	Maturity (years)			
	1	**2**	**3**	**4**
Government spot rate	2.25%	2.70%	3.30%	4.05%
Swap spread	0.25%	0.30%	0.45%	0.70%

Smith decides to examine the following three investment options for Mathews:

Investment 1:	Buy a government security that would have an annualized return that is nearly risk free. Smith is considering two possible implementations: a two-year investment or a combination of two one-year investments.
Investment 2:	Buy a four-year, zero-coupon corporate bond and then sell it after two years. Smith illustrates the returns from this strategy using the swap rate as a proxy for corporate yields.
Investment 3:	Buy a lower-quality, two-year corporate bond with a coupon rate of 4.15% and a Z-spread of 65 bps.

When Smith meets with Mathews to present these choices, Mathews tells him that she is somewhat confused by the various spread measures. She is curious to know whether there is one spread measure that could be used as a good indicator of the risk and liquidity of money market securities during the recent past.

25. In his presentation of Investment 1, Smith could show that under the no-arbitrage principle, the forward price of a one-year government bond to be

issued in one year is *closest* to:

A. 0.9662.

B. 0.9694.

C. 0.9780.

26. In presenting Investment 1, using Shire Gate Advisers' interest rate outlook, Smith could show that riding the yield curve provides a total return that is *most likely*:

A. lower than the return on a maturity-matching strategy.

B. equal to the return on a maturity-matching strategy.

C. higher than the return on a maturity-matching strategy.

27. In presenting Investment 2, Smith should show an annual return *closest* to:

A. 4.31%.

B. 5.42%.

C. 6.53%.

28. The bond in Investment 3 is *most likely* trading at a price of:

A. 100.97.

B. 101.54.

C. 104.09.

29. The *most* appropriate response to Mathews question regarding a spread measure is the:

A. Z-spread.

B. TED spread.

C. MRR–OIS spread, formerly the Libor–OIS spread.

The following information relates to questions 30-40

Liz Tyo is a fund manager for an actively managed global fixed-income fund that buys bonds issued in Countries A, B, and C. She and her assistant are preparing the quarterly markets update. Tyo begins the meeting by distributing the daily rates sheet, which includes the current government spot rates for Countries A, B, and C as shown in Exhibit 1.

Exhibit 1: Today's Government Spot Rates

Maturity	Country A	Country B	Country C
One year	0.40%	–0.22%	14.00%
Two years	0.70	–0.20	12.40
Three years	1.00	–0.12	11.80
Four years	1.30	–0.02	11.00
Five years	1.50	0.13	10.70

Tyo asks her assistant how these spot rates were obtained. The assistant replies, "Spot rates are determined through the process of bootstrapping. It entails backward substitution using par yields to solve for zero-coupon rates one by one, in order from latest to earliest maturities."

Tyo then provides a review of the fund's performance during the last year and comments, "The choice of an appropriate benchmark depends on the country's characteristics. For example, although Countries A and B have both an active government bond market and a swap market, Country C's private sector is much bigger than its public sector, and its government bond market lacks liquidity."

Tyo further points out, "The fund's results were mixed; returns did not benefit from taking on additional risk. We are especially monitoring the riskiness of the corporate bond holdings. For example, our largest holdings consist of three four-year corporate bonds (Bonds 1, 2, and 3) with identical maturities, coupon rates, and other contract terms. These bonds have Z-spreads of 0.55%, 1.52%, and 1.76%, respectively."

Tyo continues, "We also look at risk in terms of the swap spread. We considered historical three-year swap spreads for Country B, which reflect that market's credit and liquidity risks, at three different points in time." Tyo provides the information in Exhibit 2.

Exhibit 2: Selected Historical Three-Year Rates for Country B

Period	Government Bond Yield (%)	Fixed-for-Floating MRR Swap (%)
1 month ago	–0.10	0.16
6 months ago	–0.08	0.01
12 months ago	–0.07	0.71

Tyo then suggests that the firm was able to add return by riding the yield curve. The fund plans to continue to use this strategy but only in markets with an attractive yield curve for this strategy.

She moves on to present her market views on the respective yield curves for a five-year investment horizon.

> **Country A:** "The government yield curve has changed little in terms of its level and shape during the last few years, and I expect this trend to continue. We assume that future spot rates reflect the current forward curve for all maturities."
>
> **Country B:** "Because of recent economic trends, I expect a reversal in the slope of the current yield curve. We assume that future spot rates will be higher than current forward rates for all maturities."

Country C: "To improve liquidity,Country C's central bank is expected to intervene, leading to a reversal in the slope of the existing yield curve. We assume that future spot rates will be lower than today's forward rates for all maturities."

Tyo's assistant asks, "Assuming investors require liquidity premiums, how can a yield curve slope downward? What does this imply about forward rates?"

Tyo answers, "Even if investors require compensation for holding longer-term bonds, the yield curve can slope downward—for example, if there is an expectation of severe deflation. Regarding forward rates, it can be helpful to understand yield curve dynamics by calculating implied forward rates. To see what I mean, we can use Exhibit 1 to calculate the forward rate for a two-year Country C loan beginning in three years."

30. Did Tyo's assistant accurately describe the process of bootstrapping?

A. Yes

B. No, with respect to par yields

C. No, with respect to backward substitution

31. The swap curve is a better benchmark than the government spot curve for:

A. Country A.

B. Country B.

C. Country C.

32. Based on Exhibit 2, the implied credit and liquidity risks as indicated by the historical three-year swap spreads for Country B were the lowest:

A. 1 month ago.

B. 6 months ago.

C. 12 months ago.

33. Based on Exhibit 1 and Tyo's expectations, which country's term structure is currently best for traders seeking to ride the yield curve?

A. Country A

B. Country B

C. Country C

34. Based on Exhibit 1 and assuming Tyo's market views on yield curve changes are realized, the forward curve of which country will lie below its spot curve?

A. Country A

B. Country B

C. Country C

35. Based on Exhibit 1 and Tyo's expectations for the yield curves, Tyo *most likely* perceives the bonds of which country to be fairly valued?

A. Country A

B. Country B

C. Country C

36. With respect to their discussion of yield curves, Tyo and her assistant are *most likely* discussing which term structure theory?

A. Pure expectations theory

B. Local expectations theory

C. Liquidity preference theory

37. Tyo's assistant should calculate a forward rate *closest* to:

A. 9.07%.

B. 9.58%.

C. 9.97%.

38. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is *most* often consistent with:

A. bearish flattening.

B. bullish steepening.

C. bearish steepening.

39. When government budget deficits fall, fiscal supply-side effects are *most* likely to result in:

A. higher bond yields.

B. a steeper yield curve.

C. lower bond yields.

40. A flight to quality is most often associated with:

A. a general rise in the level of interest rates.

B. bullish flattening.

C. bearish flattening.

The following information relates to questions 41-42

41. Define the yield-to-maturity for a coupon bond.

42. Is it possible for a coupon bond to earn less than the yield-to-maturity if held to maturity?

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43. If a bond trader believes that current forward rates overstate future spot rates, how might she profit from that conclusion?

44. Explain the strategy of rolling down the yield curve.

45. What are the advantages of using the swap curve as a benchmark of interest rates relative to a government bond yield curve?

46. What is the TED spread, and what type of risk does it measure?

47. What is the SOFR rate, and which market conditions does it reflect?

The following information relates to questions 48-51

Rowan Madison is a junior analyst at Cardinal Capital. Sage Winter, a senior portfolio manager and Madison's supervisor, meets with Madison to discuss interest rates and review two bond positions in the firm's fixed-income portfolio.

Winter begins the meeting by asking Madison to state her views on the term structure of interest rates. Madison responds:

"Yields are a reflection of expected spot rates and risk premiums. Investors demand risk premiums for holding long-term bonds, and these risk premiums increase with maturity."

Winter tells Madison that, based on recent changes in spreads, she is concerned about a perceived increase in counterparty risk in the economy and its effect on the portfolio. Madison asks Winter:

"Which spread measure should we use to assess changes in counterparty risk in the economy?"

Winter is also worried about the effect of yield volatility on the portfolio. She asks Madison to identify the economic factors that affect short-term and long-term rate volatility. Madison responds:

"Short-term rate volatility is mostly linked to uncertainty regarding monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation."

Finally, Winter asks Madison to analyze the interest rate risk portfolio positions in a 5-year and a 20-year bond. Winter requests that the analysis be based on level, slope, and curvature as term structure factors. Madison presents her analysis in Exhibit 1.

Exhibit 1: Three-Factor Model of Term Structure

	Time to Maturity (years)	
Factor	**5**	**20**
Level	–0.4352%	–0.5128%
Steepness	–0.0515%	–0.3015%
Curvature	0.3963%	0.5227%

Note: Entries indicate how yields would change for a one standard deviation increase in a factor.

Winter asks Madison to perform two analyses:

Analysis 1:	Calculate the expected change in yield on the 20-year bond resulting from a two-standard-deviation increase in the steepness factor.
Analysis 2:	Calculate the expected change in yield on the five-year bond resulting from a one-standard-deviation decrease in the level factor and a one-standard-deviation decrease in the curvature factor.

48. Madison's views on the term structure of interest rates are *most* consistent with the:

A. local expectations theory.

B. segmented markets theory.

C. liquidity preference theory.

49. Is Madison's response regarding the factors that affect short-term and long-term rate volatility correct?

A. Yes

B. No, she is incorrect regarding factors linked to long-term rate volatility

C. No, she is incorrect regarding factors linked to short-term rate volatility

50. Based on Exhibit 1, the results of Analysis 1 should show the yield on the 20-year bond decreasing by:

A. 0.3015%.

B. 0.6030%.

C. 0.8946%.

51. Based on Exhibit 1, the results of Analysis 2 should show the yield on the five-year bond:

A. decreasing by 0.8315%.

B. decreasing by 0.0389%.

C. increasing by 0.0389%.

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52. According to the local expectations theory, what would be the difference in the one-month total return if an investor purchased a five-year zero-coupon bond versus a two-year zero-coupon bond?

53. Compare the segmented market and the preferred habitat term structure theories.

The following information relates to questions 54-56

54. List the three factors that have empirically been observed to affect Treasury security returns and explain how each of these factors affects returns on Treasury securities.

55. What has been observed to be the most important factor in affecting Treasury returns?

56. Which measures of yield curve risk can measure shaping risk?

SOLUTIONS

1. Three forward rates can be calculated from the one-, two- and three-year spot rates. The rate on a one-year loan that begins at the end of Year 1 can be calculated using the one- and two-year spot rates; in the following equation, one would solve for $f_{1,1}$:

   $[1 + z_2]^2 = [1 + z_1]^1[1 + f_{1,1}]^1$

   The rate on a one-year loan that starts at the end of Year 2 can be calculated from the two- and three-year spot rates. In the following equation, one would solve for $f_{2,1}$:

   $[1 + z_3]^3 = [1 + z_2]^2[1 + f_{2,1}]^1$

   Additionally, the rate on a two-year loan that begins at the end of Year 1 can be computed from the one- and three-year spot rates. In the following equation, one would solve for $f_{1,2}$:

   $[1 + z_3]^3 = [1 + z_1]^1[1 + f_{1,2}]^2$

2. For the two-year forward rate one year from now of 2%, the two interpretations are as follows:

   - 2% is the rate that will make an investor indifferent between buying a three-year zero-coupon bond or investing in a one-year zero-coupon bond and, when it matures, reinvesting in a zero-coupon bond that matures in two years.
   - 2% is the rate that can be locked in today by buying a three-year zero-coupon bond rather than investing in a one-year zero-coupon bond and, when it matures, reinvesting in a zero-coupon bond that matures in two years.

3. A flat yield curve implies that all spot interest rates are the same. When the spot rate is the same for every maturity, successive applications of the forward rate model will show that all the forward rates will also be the same and equal to the spot rate.

4. C is correct. There is no spot rate information to provide rates for a loan that terminates in five years. That is $f_{2,3}$ is calculated as follows:

   $$f_{2,3} = \sqrt[3]{\frac{[1 + z_5]^5}{[1 + z_2]^2}} - 1$$

   This equation indicates that in order to calculate the rate for a three-year loan beginning at the end of two years, one needs the five-year spot rate, z_5, and the two-year spot rate, z_2. However, z_5 is not provided.

5. A is correct. The forward rate for a one-year loan beginning in one year, $f_{1,1}$, is $1.04^2/1.03 - 1 = 5\%$. The rate for a one-year loan beginning in two years, $f_{2,1}$, is $1.05^3/1.04^2 - 1 = 7\%$. This confirms that an upward-sloping yield curve is consistent with an upward-sloping forward curve.

6. C is correct. If one-period forward rates are decreasing with maturity, then the forward curve is downward sloping. This turn implies a downward-sloping yield curve where longer-term spot rates z_{B-A} are less than shorter-term spot rates z_A.

7. C is correct. From the forward rate model, we have

   $[1 + z_2]^2 = [1 + z_1]^1[1 + f_{1,1}]^1$

   Using the one- and two-year spot rates, we have

   $(1 + 0.05)^2 = (1 + 0.04)^1[1 + f_{1,1}]^1$, so $\frac{(1 + 0.05)^2}{(1 + 0.04)^1} - 1 = f_{1,1} = 6.010\%$.

8. C is correct. From the forward rate model,

   $[1 + z_3]^3 = [1 + z_1]^1[1 + f_{1,2}]^2$

   Using the one- and three-year spot rates, we find

   $(1 + 0.06)^3 = (1 + 0.04)^1[1 + f_{1,2}]^2$, so $\sqrt{\frac{(1 + 0.06)^3}{(1 + 0.04)^1}} - 1 = f_{1,2} = 7.014\%$.

9. C is correct. From the forward rate model,

   $[1 + z_3]^3 = [1 + z_2]^2[1 + f_{2,1}]^1$

   Using the two- and three-year spot rates, we find

   $(1 + 0.06)^3 = (1 + 0.05)^2[1 + f_{2,1}]^1$, so $\frac{(1 + 0.06)^3}{(1 + 0.05)^2} - 1 = f_{2,1} = 8.029\%$.

10. A is correct. We can convert spot rates to spot prices to find $DF_3 = \frac{1}{(1.06)^3} =$ 0.8396.

    The forward pricing model can be used to find the price of the five-year zero as $DF_B = DF_A \times F_{A,B-A}$, so $DF_5 = DF_3F_{3,2} = 0.8396 \times 0.8479 = 0.7119$.

11. B is correct. Applying the forward rate model, we find

    $[1 + z_3]^3 = [1 + z_1]^1[1 + f_{1,1}]^1[1 + f_{2,1}]^1$

    So $[1 + z_3]^3 = (1 + 0.04)^1(1 + 0.06)^1(1 + 0.08)^1$,

    $\sqrt[3]{1.1906} - 1$

    $= z_3 = 5.987\%$.

12. B is correct. We can convert spot rates to spot prices and use the forward pricing model, so we have $DF_1 = \frac{1}{(1.05)^1} = 0.9524$.

    The forward pricing model is

    $DF_B = DF_A \times F_{A,B-A}$, so $DF_2 = DF_1F_{1,1} = 0.9524 \times 0.9346 = 0.8901$.

13. A is correct. The swap rate is the interest rate for the fixed-rate leg of an interest rate swap.

14. A is correct. The swap spread = 1.00% – 0.63% = 0.37%, or 37 bps.

15. C is correct. The fixed leg of the five-year fixed-for-floating swap will be equal to the five-year Treasury rate plus the swap spread: 2.0% + 0.5% = 2.5%.

16. A is correct. The TED spread is the difference between the three-month MRR and the three-month Treasury bill rate. If the T-bill rate falls and MRR does not change, the TED spread will increase.

17. A is correct. The Z spread is the single rate that, when added to the rates of the spot yield curve, will provide the correct discount rates to price a particular risky

bond.

18. B is correct. The five-year spot rate is determined by using forward substitution and using the known values of the one-year, two-year, three-year, and four-year spot rates, as follows:

$$1 = \frac{0.0437}{1.025} + \frac{0.0437}{(1.03)^2} + \frac{0.0437}{(1.035)^3} + \frac{0.0437}{(1.04)^4} + \frac{1 + 0.0437}{(1 + z_5)^5}$$

$$z_5 = \sqrt[5]{\frac{1.0437}{0.8394}} - 1 = 4.453\%$$

19. B is correct. The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$. Because nominal yields incorporate a premium for expected inflation, an upward-sloping yield curve is generally interpreted as reflecting a market expectation of increasing, or at least level, future inflation (associated with relatively strong economic growth).

20. C is correct. A one-year loan beginning in three years, or $f_{3,1}$, is calculated as follows:

$$[1 + z_{3+1}]^{3+1} = [1 + z_3]^3 [1 + f_{3,1}]^1$$

$$[1.04]^4 = [1.035]^3 [1 + f_{3,1}]$$

$$f_{3,1} = \frac{(1.04)^4}{(1.035)^3} - 1 = 5.514\%$$

21. C is correct. Exhibit 1 provides five years of par rates, from which the spot rates for z_1, z_2, z_3, z_4, and z_5 can be derived. Thus the forward rate $f_{1,4}$ can be calculated as follows:

$$f_{1,4} = \sqrt[4]{\frac{[1 + z_5]^5}{[1 + z_1]^1}} - 1$$

22. C is correct. The yield-to-maturity, y_3, of Bond Z should be a weighted average of the spot rates used in the valuation of the bond. Because the bond's largest cash flow occurs in Year 3, z_3 will have a greater weight than z_1 and z_2 in determining y_3.
Using the spot rates:

$$\text{Price} = \frac{\$60}{(1.025)^1} + \frac{\$60}{(1.030)^2} + \frac{\$1,060}{(1.035)^3} = \$1,071.16$$

Using the yield-to-maturity:

$$\text{Price} = \frac{\$60}{[1 + y(3)]^1} + \frac{\$60}{[1 + y(3)]^2} + \frac{\$1,060}{[1 + y(3)]^3} = \$1,071.16$$

The computed result is $y_3 = 3.46\%$, which is closest to the three-year spot rate of 3.50%.

23. A is correct. Alexander projects that the spot curve two years from today will be below the current forward curve, which implies that her expected future spot rates beyond two years will be lower than the quoted forward rates. Alexander would perceive Bond Z to be undervalued in the sense that the market is effectively discounting the bond's payments at a higher rate than she would, and the bond's market price is below her estimate of intrinsic value.

24. B is correct. Nguyen's strategy is to ride the yield curve, which is appropriate when the yield curve is upward sloping. The yield curve implied by Exhibit 1 is upward sloping, which implies that the three-year forward curve is above the current spot curve. When the yield curve slopes upward, as a bond approaches maturity or "rolls down the yield curve," the bond is valued at successively lower yields and higher prices.

25. B is correct. The forward pricing model is based on the no-arbitrage principle and is used to calculate a bond's forward price based on the spot yield curve. The spot curve is constructed by using annualized rates from option-free and default-risk-free zero-coupon bonds.

    Equation 2: $DF_B = DF_A \times F_{A,B\text{-}A}$; we need to solve for $F_{1,1}$.

    $DF_1 = 1/(1 + 0.0225)^1$ and $DF_2 = 1/(1 + 0.0270)^2$,

    $F_{1,1} = DF_2/DF_1 = 0.9481/0.9780 = 0.9694$.

26. C is correct. When the spot curve is upward sloping and its level and shape are expected to remain constant over an investment horizon (Shire Gate Advisers' view), buying bonds with a maturity longer than the investment horizon (i.e., riding the yield curve) will provide a total return greater than the return on a maturity-matching strategy.

27. C is correct. The swap spread is a common way to indicate credit spreads in a market. The four-year swap rate (fixed leg of an interest rate swap) can be used as an indication of the four-year corporate yield. Riding the yield curve by purchasing a four-year zero-coupon bond with a yield of 4.75% {i.e., 4.05% + 0.70%, [$P_4 = 100/(1 + 0.0475)^4 = 83.058$]} and then selling it when it becomes a two-year zero-coupon bond with a yield of 3.00% {i.e., 2.70% + 0.30%, [$P_2 = 100/(1 + 0.0300)^2 = 94.260$]} produces an annual return of 6.53%: $(94.260/83.058)^{0.5} - 1.0 = 0.0653$.

28. B is correct. The Z-spread is the constant basis point spread that is added to the default-free spot curve to price a risky bond. A Z-spread of 65 bps for a particular bond would imply adding a fixed spread of 65 bps to maturities along the spot curve to correctly price the bond. Therefore, for the two-year bond, z_1 = 2.90% (i.e., 2.25% + 0.65%), z_2 = 3.35% (i.e., 2.70% + 0.65%), and the price of the bond with an annual coupon of 4.15% is as follows:

    $P = 4.15/(1 +0.\ 029)^1 + 4.15/(1 + 0.0335)^2 + 100/(1 + 0.0335)^2$,

    $P = 101.54$.

29. C is correct. The MRR–OIS spread is considered an indicator of the risk and liquidity of money market securities. This spread measures the difference between MRR and the OIS rate.

30. C is correct. The assistant states that bootstrapping entails *backward* substitution using par yields to solve for zero-coupon rates one by one, in order from latest to earliest maturities. Bootstrapping entails *forward* substitution, however, using par yields to solve for zero-coupon rates one by one, in order from earliest to latest maturities.

31. C is correct. Country C's private sector is much bigger than the public sector, and the government bond market in Country C currently lacks liquidity. Under such circumstances, the swap curve is a more relevant benchmark for interest rates.

32. B is correct. The historical three-year swap spread for Country B was the lowest six months ago. Swap spread is defined as the spread paid by the fixed-rate payer of an interest rate swap over the rate of the "on the run" (most recently issued) government bond security with the same maturity as the swap. The lower (higher) the swap spread, the lower (higher) the return that investors require for credit and/or liquidity risks.

    The fixed rate of the three-year fixed-for-floating Libor swap was 0.01% six months ago, and the three-year government bond yield was –0.08% six months ago. Thus the swap spread six months ago was 0.01% – (–0.08%) = 0.09%.

    One month ago, the fixed rate of the three-year fixed-for-floating Libor swap was 0.16%, and the three-year government bond yield was –0.10%. Thus the swap spread one month ago was 0.16% – (–0.10%) = 0.26%.

    Twelve months ago, the fixed rate of the three-year fixed-for-floating Libor swap was 0.71%, and the three-year government bond yield was –0.07%. Thus, the swap spread 12 months ago was 0.71% – (–0.07%) = 0.78%.

33. A is correct. Country A's yield curve is upward sloping—a condition for the strategy—and more so than Country B's.

34. B is correct. The yield curve for Country B is currently upward sloping, but Tyo expects a reversal in the slope of the current yield curve. This means she expects the resulting yield curve for Country B to slope downward, which implies that the resulting forward curve would lie below the spot yield curve. The forward curve lies below the spot curve in scenarios in which the spot curve is downward sloping; the forward curve lies above the spot curve in scenarios in which the spot curve is upward sloping.

    A is incorrect because the yield curve for Country A is currently upward sloping and Tyo expects that the yield curve will maintain its shape and level. That expectation implies that the resulting forward curve would be above the spot yield curve.

    C is incorrect because the yield curve for Country C is currently downward sloping and Tyo expects a reversal in the slope of the current yield curve. She thus expects the resulting yield curve for Country C to slope upward, which implies that the resulting forward curve would be above the spot yield curve.

35. A is correct. Tyo's projected spot curve assumes that future spot rates reflect, or will be equal to, the current forward rates for all respective maturities. This assumption implies that the bonds for Country A are fairly valued because the market is effectively discounting the bond's payments at spot rates that match those projected by Tyo.

    B and C are incorrect because Tyo's projected spot curves for the two countries do not match the current forward rates for all respective maturities. In the case of Country B, she expects future spot rates to be higher (than the current forward rates that the market is using to discount the bond's payments). For Country C, she expects future spot rates to be lower (than the current forward rates). Hence, she perceives the Country B bond to be currently overvalued and the Country C bond to be undervalued.

36. C is correct. Liquidity preference theory suggests that liquidity premiums exist to compensate investors for the added interest rate risk that they face when lending long term and that these premiums increase with maturity. Tyo and her assistant are assuming that liquidity premiums exist.

37. A is correct. From the forward rate model, $f_{3,2}$, is found as follows:

$[1 + z_5]^5 = [1 + z_3]^3[1 + f_{3,2}]^2$

Using the three-year and five-year spot rates, we find

$(1 + 0.107)^5 = (1 + 0.118)^3[1 + f_{3,2}]^2$, so

$$\sqrt{\frac{(1 + 0.107)^5}{(1 + 0.118)^3}} - 1 = f_{3,2} = 9.07\%$$

38. A is correct. This action is most often consistent with bearish flattening, or short-term bond yields rising more than long-term bond yields resulting in a flatter yield curve.

39. C is correct. When government budget deficits fall, fiscal supply-side effects are most likely to result in lower bond yields.

40. B is correct. A flight to quality is most often associated with bullish flattening, in which the yield curve flattens as long term rates fall by more than short-term rates.

41. The yield-to-maturity of a coupon bond is the expected rate of return on a bond if the bond is held to maturity, there is no default, and the bond and all coupons are reinvested at the original yield-to-maturity.

42. Yes, it is possible. For example, if reinvestment rates for the future coupons are lower than the initial yield-to-maturity, a bondholder may experience lower realized returns.

43. If forward rates are higher than expected future spot rates, the market price of the bond will be lower than the intrinsic value. This dynamic occurs because, everything else held constant, the market is currently discounting the bonds cash flows at a higher rate than the investor's expected future spot rates. The investor can capitalize on this scenario by purchasing the undervalued bond. If expected future spot rates are realized, then bond prices should rise, thus generating gains for the investor.

44. The strategy of rolling down the yield curve is one in which a bond trader attempts to generate a total return over a given investment horizon that exceeds the return to bond with maturity matched to the horizon. The strategy involves buying a bond with maturity more distant than the investment horizon. Assuming an upward-sloping yield curve, if the yield curve does not change level or shape, as the bond approaches maturity (or rolls down the yield curve) it will be priced at successively lower yields. So as long as the bond is held for a period less than maturity, it should generate higher returns because of price gains.

45. Some countries do not have active government bond markets with trading at all maturities. For those countries without a liquid government bond market but with an active swap market, there are typically more points available to construct a swap curve than a government bond yield curve. For those markets, the swap curve may be a superior benchmark.

46. The TED spread is the difference between MRR and the US T-bill rate of matching maturity. It is an indicator of perceived credit and liquidity risk. In particular, because sovereign debt instruments are typically the benchmark for the lowest default risk instruments in a given market, and loans between banks (often at MRR) have some counterparty risk, the TED spread is considered to at least in part reflect default (or counterparty) risk in the banking sector.

47. The secured overnight financing rate (SOFR), or overnight cash borrowing rate collateralized by US Treasuries, is a barometer of the US Treasury repurchase (or repo) market. SOFR is a volume-weighted index of all qualified repo market transactions on a given day and is influenced by supply and demand conditions in secured funding markets.

48. C is correct. Liquidity preference theory asserts that investors demand a risk premium, in the form of a liquidity premium, to compensate them for the added interest rate risk they face when buying long-maturity bonds. The theory also states that the liquidity premium increases with maturity.

49. A is correct. Madison's response is correct; research indicates that short-term rate volatility is mostly linked to uncertainty regarding monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation.

50. B is correct. Because the factors in Exhibit 1 have been standardized to have unit standard deviations, a two-standard-deviation increase in the steepness factor will lead to the yield on the 20-year bond decreasing by 0.6030%, calculated as follows:

    Change in 20-year bond yield = –0.3015% × 2 = –0.6030%

51. C is correct. Because the factors in Exhibit 1 have been standardized to have unit standard deviations, a one-standard-deviation decrease in both the level factor and the curvature factor will lead to the yield on the five-year bond increasing by 0.0389%, calculated as follows:

    Change in five-year bond yield = 0.4352% – 0.3963% = 0.0389%

52. The local expectations theory asserts that the total return over a one-month horizon for a five-year zero-coupon bond would be the same as for a two-year zero-coupon bond.

53. Both theories attempt to explain the shape of any yield curve in terms of supply and demand for bonds. In segmented market theory, bond market participants are limited to purchase of maturities that match the timing of their liabilities. In the preferred habitat theory, participants have a preferred maturity for asset purchases, but they may deviate from it if they feel returns in other maturities offer sufficient compensation for leaving their preferred maturity segment.

54. Studies have shown that three factors affect Treasury returns: (1) changes in the level of the yield curve, (2) changes in the slope of the yield curve, and (3) changes in the curvature of the yield curve. Changes in the level refer to upward or downward shifts in the yield curve. For example, an upward shift in the yield curve is likely to result in lower returns across all maturities. Changes in the slope of the yield curve relate to the steepness of the yield curve. Thus, if the yield curve steepens, higher returns for short-maturity bonds and lower returns for long-maturity bonds will likely occur. An example of a change in the curvature of the yield curve is a situation where rates fall at the short and long end of the yield curve while rising for intermediate maturities. In this situation, returns on short and long maturities are likely to rise while declining for intermediate-maturity bonds.

55. Empirically, the most important factor is the change in the level of interest rates.

56. Key rate durations and a measure based on sensitivities to level, slope, and curvature movements can address shaping risk, but effective duration cannot.

LEARNING MODULE

2

The Arbitrage-Free Valuation Framework

by Steven V. Mann, PhD.

Steven V. Mann, PhD, is at the University of South Carolina (USA).

LEARNING OUTCOMES

Mastery	The candidate should be able to:
☐	explain what is meant by arbitrage-free valuation of a fixed-income instrument
☐	calculate the arbitrage-free value of an option-free, fixed-rate coupon bond
☐	describe a binomial interest rate tree framework
☐	describe the process of calibrating a binomial interest rate tree to match a specific term structure
☐	describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node
☐	compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice
☐	describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path
☐	describe a Monte Carlo forward-rate simulation and its application
☐	describe term structure models and how they are used

INTRODUCTION 1

☐	explain what is meant by arbitrage-free valuation of a fixed-income instrument

The presentation of the binomial trees in this reading was revised to conform with other readings in 2018 by Donald J. Smith, PhD, Boston University (USA).

The idea that market prices adjust until there are no arbitrage opportunities forms the basis for valuing fixed-income securities, derivatives, and other financial assets. If both the net proceeds (e.g., buying and selling the same value of an asset) and the risk of an investment are zero, the return on that investment should also be zero.

This reading is designed to equip candidates with a set of bond valuation tools that are consistent with this idea. The remainder of Section 1 further defines the concept of no arbitrage, and Section 2 provides a framework for an arbitrage-free valuation of fixed-income securities. Section 3 introduces the binomial interest rate tree framework based on a lognormal random walk, which is used to value an option-free bond. The binomial tree model is calibrated to the current yield curve in Section 4. This step ensures that the interest rate tree is consistent with pricing using the zero-coupon (i.e., spot) curve as illustrated in Section 5. The reading next turns to an introduction of pathwise valuation, in Section 6. Section 7 describes a Monte Carlo forward-rate simulation and its application. Section 8 goes beyond the lognormal random walk approach to introduce common term structure models. Building on principles established earlier in the reading, these models incorporate assumptions about changes in interest rates and volatility to capture term structure dynamics and are used by practitioners to price and hedge fixed-income securities and derivatives.

The Meaning of Arbitrage-Free Valuation

Arbitrage-free valuation refers to an approach to security valuation that determines security values that are consistent with the absence of an **arbitrage opportunity**, which is an opportunity for trades that earn riskless profits without any net investment of money. In well-functioning markets, prices adjust until there are no arbitrage opportunities, which is the **principle of no arbitrage** that underlies the practical validity of arbitrage-free valuation. This principle itself can be thought of as an implication of the idea that identical assets should sell at the same price.

These concepts will be explained in greater detail shortly, but to indicate how they arise in bond valuation, consider first an imaginary world in which financial assets are free of risk and the benchmark yield curve is flat. In this reading, the terms yield, interest rate, and discount rate will be used interchangeably. A flat yield curve implies that the relevant yield is the same for all cash flows regardless of when the cash flows are delivered in time. Accordingly, the value of a bond is the present value of its certain future cash flows. In discounting those cash flows—determining their present value—investors would use the risk-free interest rate because the cash flows are certain; because the yield curve is assumed to be flat, one risk-free rate would exist and apply to all future cash flows. This is the simplest case of bond valuation one can envision. When we exit this imaginary world and enter more realistic environs, bonds' cash flows are risky (i.e., there is some chance the borrower will default) and the benchmark yield curve is not flat. How would our approach change?

A fundamental principle of valuation is that the value of any financial asset is equal to the present value of its expected future cash flows. This principle holds for any financial asset, from zero-coupon bonds to interest rate swaps. Thus, the valuation of a financial asset involves the following three steps:

Step 1 Estimate the future cash flows.

Step 2 Determine the appropriate discount rate or discount rates that should be used to discount the cash flows.

Step 3 Calculate the present value of the expected future cash flows found in Step 1 by applying the appropriate discount rate or rates determined in Step 2.

The traditional approach to valuing bonds is to discount all cash flows with the same discount rate as if the yield curve were flat. However, a bond is properly thought of as a package or portfolio of zero-coupon bonds, also referred to as zeros or discount instruments. Each zero-coupon bond in such a package can be valued separately at a discount rate that depends on the shape of the yield curve and when its single cash flow is delivered in time. The term structure of these discount rates is referred to as the spot curve. Bond values derived by summing the present values of the individual zeros (cash flows) determined by such a procedure can be shown to be arbitrage free. Ignoring transaction costs for the moment, if the bond's value were much less than the sum of the values of its cash flows individually, a trader would perceive an arbitrage opportunity and buy the bond while selling claims to the individual cash flows and pocketing the excess value. Although the details bear further discussion, the valuation of a bond as a portfolio of zeros based on using the spot curve is an example of arbitrage-free valuation. Regardless of the complexity of the bond, each component must have an arbitrage-free value. A bond with embedded options can be valued in parts as the sum of the arbitrage-free bond without options (that is, a bond with no embedded options) and the arbitrage-free value of each of the options.

The Law of One Price

The central idea of financial economics is that market prices will adjust until there are no opportunities for arbitrage. We will define shortly what is meant by an arbitrage opportunity, but for now think of it as "free money." Prices will adjust until there is no free money to be acquired. Arbitrage opportunities arise from violations of the **law of one price**. The law of one price states that two goods that are perfect substitutes must sell for the same current price in the absence of transaction costs. Two goods that are identical, trading side by side, are priced the same. Otherwise, if it were costless to trade, one would simultaneously buy at the lower price and sell at the higher price. The riskless profit is the difference in the prices. An individual would repeat this transaction without limit until the two prices converge. An implication of these market forces is deceptively straightforward and basic. If you do not put up any of your own money and take no risk, your expected return should be zero.

Arbitrage Opportunity

With this background, let us define arbitrage opportunity more precisely. An arbitrage opportunity is a transaction that involves no cash outlay that results in a riskless profit. There are two types of arbitrage opportunities. The first type of arbitrage opportunity is often called **value additivity**; put simply, the value of the whole equals the sum of the values of the parts. Consider two risk-free investments with payoffs one year from today and the prices today provided in Exhibit 1. Asset A is a simple risk-free zero-coupon bond that pays off one dollar and is priced today at 0.952381 (= 1/1.05). Asset B is a portfolio of 105 units of Asset A that pays off 105 one year from today and is priced today at 97. The portfolio does not equal the sum of the parts. The portfolio (Asset B) is cheaper than buying 105 units of Asset A at a price of 100 and then combining. An astute investor would sell 105 units of Asset A for 105 × 0.952381 = 100 while simultaneously buying the portfolio, Asset B, for 97. This position generates a certain 3 today (100 – 97) and generates net 0 one year from today because cash inflow for Asset B matches the amount for the 105 units of Asset A sold. An investor would repeat this trade until the prices are equal.

The second type of arbitrage opportunity is often called **dominance**. A financial asset with a risk-free payoff in the future must have a positive price today. Consider two assets, C and D, that are risk-free zero-coupon bonds. Payoffs in one year and prices today are displayed in Exhibit 1. On careful review, it appears that Asset D is cheap relative to Asset C. If both assets are risk-free, they should have the same discount rate. To make money, sell two units of Asset C at a price of 200 and use the proceeds to purchase one unit of Asset D for 200. The construction of the portfolio involves no net cash outlay today. Although it requires zero dollars to construct today, the portfolio generates 10 one year from today. Asset D will generate a 220 cash inflow, whereas the two units of Asset C sold will produce a cash outflow of 210.

Exhibit 1: Price Today and Payoffs in One Year for Sample Assets

Asset	Price Today	Payoff in One Year
A	0.952381	1
B	97	105
C	100	105
D	200	220

This existence of both types of arbitrage opportunity is transitory. Investors aware of this mispricing will demand the securities in question in unlimited quantities. Something must change to restore stability. Prices will adjust until there are no arbitrage opportunities.

EXAMPLE 1

Arbitrage Opportunities

1. Which of the following investment alternatives includes an arbitrage opportunity?

   A. Bond A: The yield for a 3% annual coupon 10-year bond is 2.5% in New York City. The same bond sells for \$104.376 per \$100 face value in Chicago.

   B. Bond B: The yield for a 3% annual coupon 10-year bond is 3.2% in Hong Kong SAR. The same bond sells for RMB97.220 per RMB100 face value in Shanghai.

Solution:

Bond B is correct. Bond B's arbitrage-free price may be solved for using a financial calculator or Microsoft Excel as $3/1.032 + 3/1.032^2 + \ldots + 103/1.032^{10} = 98.311$, which is higher than the price in Shanghai. Therefore, an arbitrage opportunity exists. Buy bonds in Shanghai for RMB97.220 and sell them in Hong Kong SAR for RMB98.311. You make RMB1.091 per RMB100 of bonds traded.

Bond A's arbitrage-free price is $3/1.025 + 3/1.025^2 + \ldots + 103/1.025^{10} = 104.376$, which matches the price in Chicago. Therefore, no arbitrage opportunity exists in this market.

Implications of Arbitrage-Free Valuation for Fixed-Income Securities

Using the arbitrage-free approach, any fixed-income security should be thought of as a package or portfolio of zero-coupon bonds. Thus, a five-year 2% coupon Treasury issue should be viewed as a package of 11 zero-coupon instruments (10 semiannual coupon payments, 1 of which is made at maturity, and 1 principal value payment at maturity).

The market mechanism for US Treasuries that enables this approach is the dealer's ability to separate the bond's individual cash flows and trade them as zero-coupon securities. This process is called **stripping**. In addition, dealers can recombine the appropriate individual zero-coupon securities and reproduce the underlying coupon Treasury. This process is called **reconstitution**. Dealers in sovereign debt markets around the globe are free to engage in the same process.

Arbitrage profits are possible when value additivity does not hold. The arbitrage-free valuation approach does not allow a market participant to realize an arbitrage profit through stripping and reconstitution. By viewing any security as a package of zero-coupon securities, a consistent and coherent valuation framework can be developed. Viewing a security as a package of zero-coupon bonds means that two bonds with the same maturity and different coupon rates are viewed as different packages of zero-coupon bonds and valued accordingly. Moreover, two cash flows with identical risks delivered at the same time will be valued using the same discount rate even though they are attached to two different bonds.

ARBITRAGE-FREE VALUATION FOR AN OPTION-FREE BOND 2

☐ calculate the arbitrage-free value of an option-free, fixed-rate coupon bond

The goal of this section is to develop a method to produce an arbitrage-free value for an option-free bond and to provide a framework—based on interest rate trees—that is rich enough to be applied to the valuation of bonds with embedded options.

For bonds that are option-free, the simplest approach to arbitrage-free valuation involves determining the arbitrage-free value as the sum of the present values of expected future values using the benchmark spot rates. Benchmark securities are liquid, safe securities whose yields serve as building blocks for other interest rates in a country or currency. Sovereign debt is the benchmark in many countries. For example, on-the-run Treasuries serve as benchmark securities in the United States. Par rates derived from the Treasury yield curve can be used to obtain spot rates by means of bootstrapping. Gilts are the benchmark in the United Kingdom, while German bunds serve as the benchmark for euro-denominated bonds. In markets where the sovereign debt market is not sufficiently liquid, the swap curve is a viable alternative.

In this reading, benchmark bonds are assumed to be correctly priced by the market. The valuation model we develop will be constructed to reproduce exactly the prices of the benchmark bonds.

EXAMPLE 2

The Arbitrage-Free Value of an Option-Free Bond

1. The yield-to-maturity ("par rate") for a benchmark one-year annual coupon bond is 2%, for a benchmark two-year annual coupon bond is 3%, and for a benchmark three-year annual coupon bond is 4%. A three-year, 5% annual coupon bond with the same risk and liquidity as the benchmarks is selling

for 102.7751 today (t = 0) to yield 4%. Is this value correct for the bond given the current term structure?

Solution:

The first step in the solution is to find the correct spot rate (zero-coupon rates) for each year's cash flow. The spot rates may be determined using bootstrapping, which is an iterative process. Using the bond valuation equation below, one can solve iteratively for the spot rates, z_t (rate on a zero-coupon bond of maturity t), given the periodic payment, PMT, on the relevant benchmark bond.

$$100 = \frac{PMT}{(1+z_1)^1} + \frac{PMT}{(1+z_2)^2} + \cdots + \frac{PMT + 100}{(1+z_N)^N}.$$

A revised equation, which uses the par rate rather than PMT, may also be used to calculate the spot rates. The revised equation is

$$1 = \frac{\text{Par rate}}{(1+z_1)} + \frac{\text{Par rate}}{(1+z_2)^2} + \ldots + \frac{\text{Par rate} + 1}{(1+z_N)^N},$$

where par rate is PMT divided by 100 and represents the par rate on the benchmark bond and z_t is the t-period zero-coupon rate.
In this example, the one-year spot rate, z_1, is 2%, which is the same as the one-year par rate. To solve for z_2,

$$1 = \frac{0.03}{(1+z_1)} + \frac{0.03+1}{(1+z_2)^2} = \frac{0.03}{(1+0.02)} + \frac{0.03+1}{(1+z_2)^2}.$$
$$z_2 = 3.015\%.$$

To solve for z_3,

$$1 = \frac{0.04}{(1+z_1)} + \frac{0.04}{(1+z_2)^2} + \frac{0.04+1}{(1+z_3)^3} = \frac{0.04}{(1+0.02)} + \frac{0.04}{(1+0.03015)^2} + \frac{0.04+1}{(1+z_3)^3}.$$
$$z_3 = 4.055\%$$

The spot rates are 2%, 3.015%, and 4.055%. The correct arbitrage-free price for the bond, then, is

$$P_0 = 5/1.02 + 5/1.03015^2 + 105/1.04055^3 = 102.8102.$$

To be arbitrage free, each cash flow of a bond must be discounted by the spot rate for zero-coupon bonds maturing on the same date as the cash flow. Discounting early coupons by the bond's yield-to-maturity gives too much discounting with an upward sloping yield curve and too little discounting for a downward sloping yield curve. The bond is mispriced by 0.0351 per 100 of par value.

For option-free bonds, performing valuation discounting with spot rates produces an arbitrage-free valuation. For bonds that have embedded options, we need a different approach. The challenge one faces when developing a framework for valuing bonds with embedded options is that their expected future cash flows are interest rate dependent. If the bonds are option-free, changes in interest rates have no impact on the size and timing of the bond's cash flows. For bonds with options attached, changes in future interest rates impact the likelihood the option will be exercised and in so doing impact the cash flows. Therefore, to develop a framework that values bonds both without and with embedded options, we must allow interest rates to take on different potential values in the future based on some assumed level of volatility. The vehicle to portray this information is an interest rate "tree" representing possible future interest rates consistent with the assumed volatility. Because the interest rate tree resembles a lattice, these models are often called "lattice models." The interest rate tree performs two functions in the valuation process: (1) Generate the cash flows that are interest rate dependent, and (2) supply the interest rates used to determine the present value of the cash flows. This approach will be used in later readings when considering learning outcome statements involving callable bonds.

An interest rate model seeks to identify the elements or *factors* that are believed to explain the dynamics of interest rates. These factors are random or *stochastic* in nature, so we cannot predict the path of any factor. An interest rate model must, therefore, specify a statistical process that describes the stochastic property of these factors to arrive at a reasonably accurate representation of the behavior of interest rates. What is important to understand is that the interest rate models commonly used are based on how short-term interest rates can evolve (i.e., change) over time. Consequently, these interest rate models are referred to as one-factor models because only one interest rate is being modeled over time. More complex models consider how more than one interest rate changes over time (e.g., the short rate and the long rate) and are referred to as two-factor models.

Our task at hand is to describe the binomial interest rate tree framework. The valuation model we are attempting to build is the binomial lattice model. It is so named because the short interest rate can take on one of two possible values consistent with the volatility assumption and an interest rate model. As we will soon discover, the two possible interest rates next period will be consistent with the following three conditions: (1) an interest rate model that governs the random process of interest rates, (2) the assumed level of interest rate volatility, and (3) the current benchmark yield curve. We take the prices of the benchmark bonds as given so that our model recovers the market values for each benchmark bond. In this way, we tie the model to the current yield curve that reflects the underlying economic reality.

The Binomial Interest Rate Tree

The first step for demonstrating the binomial valuation method is to present the benchmark par curve by using bonds of a country or currency. For simplicity in our illustration, we will use US dollars. The same principles hold with equal force regardless of the country or currency. The benchmark par curve is presented in Exhibit 2. For simplicity, we assume that all bonds have annual coupon payments. Benchmark bonds are conveniently priced at par so the yields-to-maturity and the coupon rates on the bonds are the same. From these par rates, we use the bootstrapping methodology to uncover the underlying spot rates shown in Exhibit 3. Because the par curve is upward sloping, it comes as no surprise that after Year 1 the spot rates are higher than the par rates. In Exhibit 4 we present the one-year implied forward rates derived from the spot curve using no arbitrage. Because the par, spot, and forward curves reflect the same information about interest rates, if one of the three curves is known, it is possible to generate the other two curves. The three curves are identical only if the yield curve is flat.

Exhibit 2: Benchmark Par Curve

Maturity (Years)	Par Rate	Bond Price
1	1.00%	100
2	1.20%	100
3	1.25%	100

Maturity (Years)	Par Rate	Bond Price
4	1.40%	100
5	1.80%	100

Exhibit 3: Underlying One-Year Spot Rates of Par Rates

Maturity (Years)	One-Year Spot Rate
1	1.0000%
2	1.2012%
3	1.2515%
4	1.4045%
5	1.8194%

Exhibit 4: One-Year Implied Forward Rates

Maturity (Years)	Forward Rate
Current one-year rate	1.0000%
One-year rate, one year forward	1.4028%
One-year rate, two years forward	1.3521%
One-year rate, three years forward	1.8647%
One-year rate, four years forward	3.4965%

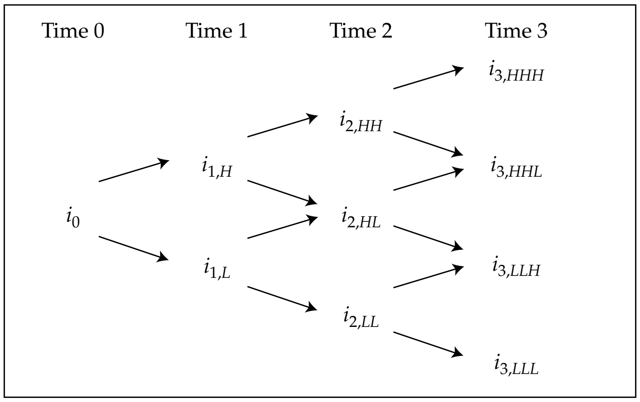
Recall from our earlier discussion that if we value the benchmark bonds using rates derived from these curves, we will recover the market price of par for all five bonds in Exhibit 2. Specifically, par rates represent the single interest applied to all the cash flows that will produce the market prices. Discounting each cash flow separately with the set of spot rates will also give the same answer. Finally, forward rates are the discount rates of a single cash flow over a single period. If we discount each cash flow with the appropriate discount rate for each period, the computed values will match the observed prices.

When we approach the valuation of bonds with cash flows that are interest rate dependent, we must explicitly allow interest rates to change. We accomplish this task by introducing interest rate volatility and generating an interest rate tree later in this reading. An interest rate tree is simply a visual representation of the possible values of interest rates based on an interest rate model and an assumption about interest rate volatility.

A binomial interest rate tree is presented in Exhibit 5. Our goal is to learn how to populate this structure with interest rates. Notice the *i*'s, which represent different potential values the one-year interest rates may take over time. As we move from left to right on the tree, the number of possible interest rates increases. The first is the current time (in years), or formally, Time 0. The interest rate displayed at Time 0 is the discount rate that converts Time 1 payments to Time 0 present values. At the bottom of the graph, time is the unit of measurement. Notice that there is one year between possible interest rates. This is called the "time step," and in our illustration, it matches the frequency of the annual cash flows. The *i*'s in Exhibit 5 are called nodes.

The first node is called the root of the tree and is simply the current one-year rate at Time 0. Each node thereafter is represented by a both time element and a rate change component.

Exhibit 5: Binomial Interest Rate Tree



We now turn to the question of how to obtain the two possible values for the one-year interest rate one year from today. Two assumptions are required: an interest rate model and a volatility of interest rates. Recall an interest rate model puts structure on the randomness. We are going to use the lognormal random walk, and the resulting tree structure is often referred to as a lognormal tree. A lognormal model of interest rates insures two appealing properties: (1) non-negativity of interest rates and (2) higher volatility at higher interest rates. At each node, there are two possible rates one year forward at Time 1. We will assume for the time being that each has an equal probability of occurring. The two possible rates we will calculate are going to be higher and lower than the one-year forward rate at Time 1 one year from now.

We denote i_L to be the rate lower than the implied forward rate and i_H to be the higher forward rate. The lognormal random walk posits the following relationship between $i_{1,L}$ and $i_{1,H}$:

$$i_{1,H} = i_{1,L}e^{2\sigma},$$

where σ is the standard deviation and *e* is Euler's number, the base of natural logarithms, which is a constant 2.7183. The random possibilities each period are (nearly) centered on the forward rates calculated from the benchmark curve. The intuition of this relationship is deceptively quick and simple. Think of the one-year forward implied interest rate from the yield curve as the average of possible values for the one-year rate at Time 1. The lower of the two rates, i_L, is one standard deviation below the mean (one-year implied forward rate), and i_H is one standard deviation above the mean. Thus, the higher and lower values (i_L and i_H) are multiples of each other, and the multiplier is $e^{2\sigma}$. Note that as the standard deviation (i.e., volatility) increases, the multiplier increases, and the two rates will grow farther apart but will still be (nearly) centered on the implied forward rate derived from the spot curve. We will demonstrate this soon.

We use the following notation to describe the tree at Time 1. Let

σ = assumed volatility of the one-year rate,

$i_{1,L}$ = the lower one-year forward rate one year from now at Time 1, and

$i_{1,H}$ = the higher one-year forward rate one year from now at Time 1.

For example, suppose that $i_{1,L}$ is 1.194% and σ is 15% per year; then $i_{1,H}$ = 1.194%($e^{2\times0.15}$) = 1.612%.

At Time 2, there are three possible values for the one-year rate, which we will denote as follows:

$i_{2,LL}$ = one-year forward rate at Time 2 assuming the lower rate at Time 1 and the lower rate at Time 2.

$i_{2,HH}$ = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the higher rate at Time 2.

$i_{2,HL}$ = one-year forward rate at Time 2 assuming the higher rate at Time 1 and the lower rate at Time 2, or equivalently, the lower rate at Time 1 and the higher rate at Time 2.

The middle rate will be close to the implied one-year forward rate two years from now derived from the spot curve, whereas the other two rates are two standard deviations above and below this value. (Recall that the multiplier for adjacent rates on the tree differs by a multiple of *e* raised to the 2σ.) This type of tree is called a recombining tree because there are two paths to get to the middle rate. This feature of the model results in faster computation because the number of possible outcomes each period grows linearly rather than exponentially.

The relationship between $i_{2,LL}$ and the other two one-year rates is as follows:

$i_{2,HH} = i_{2,LL}(e^{4\sigma})$, and $i_{2,HL} = i_{2,LL}(e^{2\sigma})$.

In a given period, adjacent possible outcomes in the tree are two standard deviations apart. So, for example, if $i_{2,LL}$ is 0.980%, and assuming once again that σ is 15%, we calculate

$i_{2,HH}$ = 0.980%($e^{4\times0.15}$) = 1.786%

and

$i_{2,HL}$ = 0.980%($e^{2\times0.15}$) = 1.323%.

There are four possible values for the one-year forward rate at Time 3. These are represented as follows: $i_{3,HHH}$, $i_{3,HHL}$, $i_{3,LLH}$ and $i_{3,LLL}$. Once again, all the forward rates in the tree are multiples of the lowest possible rates each year. The lowest possible forward rate at Time 3 is $i_{3,LLL}$ and is related to the other three as given below:

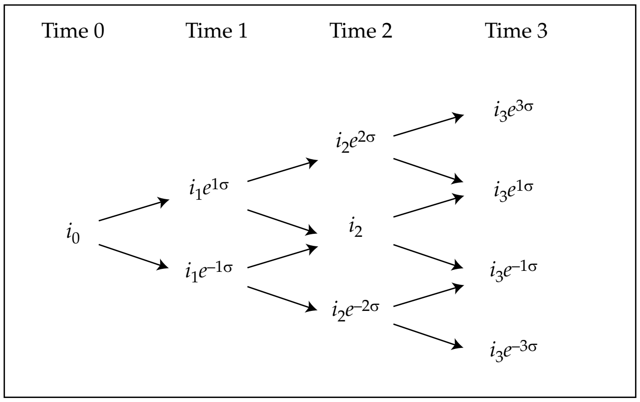
$i_{3,HHH} = (e^{6\sigma})i_{3,LLL}$.

$i_{3,HHL} = (e^{4\sigma})i_{3,LLL}$.

$i_{3,LLH} = (e^{2\sigma})i_{3,LLL}$.

Exhibit 6 shows the notation for a four-year binomial interest rate tree. We can simplify the notation by centering the one-year rates on the tree on implied forward rates on the benchmark yield curve, so i_t is the one-year rate *t* years from now and the centering rate. The subscripts indicate the rates at the end of the year, so in the second year, it is the rate at the end of Time 2 to the end of Time 3. Exhibit 6 uses this uniform notation. Note that adjacent forward rates in the tree are two standard deviations (σ) apart.

Exhibit 6: Four-Year Binomial Tree



Before we attempt to build an interest rate tree, two additional tools are needed. These tools are introduced in the next two sections.

CREATING A BINOMIAL INTEREST RATE TREE

3

☐ describe a binomial interest rate tree framework

Recall that variance is a measure of dispersion of a probability distribution. The standard deviation is the square root of the variance and is measured in the same units as the mean. With a simple lognormal distribution, the changes in interest rates are proportional to the level of the one-period interest rates each period. Volatility is measured relative to the current level of rates. It can be shown that for a lognormal distribution the standard deviation of the one-year rate is equal to $i_0\sigma$. For example, if σ is 10% and the one-year rate (i_0) is 2%, then the standard deviation of the one-year rate is 2% × 10% = 0.2%, or 20 bps. As a result, interest rate moves are larger when interest rates are high and are smaller when interest rates are low. One of the characteristics of a lognormal distribution is that negative interest rates are not possible, since as rates approach zero, the absolute change in interest rates becomes smaller and smaller.

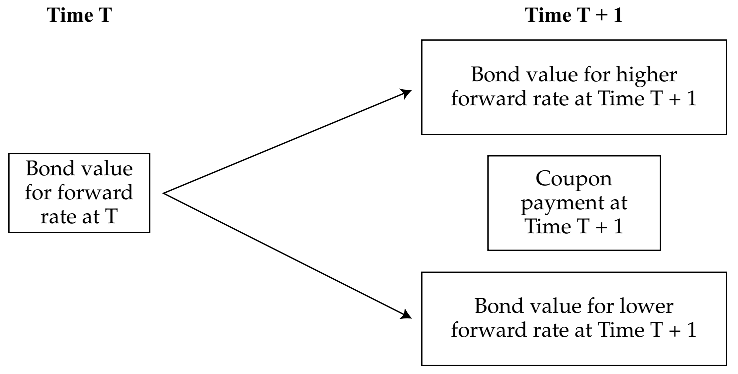
There are two methods commonly used to estimate interest rate volatility. The first method uses historical interest rate volatility based on data from the recent past, which is assumed to be indicative of the future. A second method to estimate interest rate volatility is that derived from observed market prices of interest rate derivatives (e.g., swaptions, caps, floors) known as implied volatility.

Determining the Value of a Bond at a Node

To find the value of the bond at a node, we use the backward induction valuation methodology. Barring default, we know that at maturity the bonds will be valued at par. So, we start at maturity, fill in those values, and work back from right to left to find the bond's value at the desired node. Suppose we want to determine the bond's value at the lowest node at Time 1. To find this value, we must first calculate the bond's value at the two nodes to the right of the node we selected. The bond's value at the two nodes immediately to the right must be available.

A bond's value at any node will depend on the future coupon payment, *C*, and the expected future value for the bond. This expected value is the average of the value for the forward rate being higher, to be denoted below by *VH*, and the value for the forward rate being lower, *VL*. It is a simple average because in the lognormal model the probabilities for the rate going up or down are equal. This is illustrated in Exhibit 7. Notice that the coupon payment due at the end of the period, at Time T + 1, is placed directly to the right of the node for Time T. The arrows point to the two possible future bond values, one for the forward rate going up at Time T + 1 and the other for the rate going down.

Exhibit 7: Finding a Bond's Value at Any Node



The next step is to determine the present value of the coupon payment and the expected future bond value. The relevant discount rate is the one-year forward rate prevailing at the beginning of the time period, *i*, at Time T. The bond's value at any node is determined by the following expression:

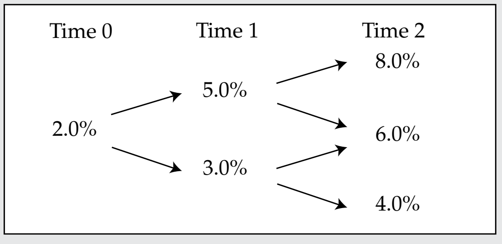
$$\text{Bond value at a node} = \frac{C + (0.5 \times VH + 0.5 \times VL)}{1 + i}.$$

EXAMPLE 3

Pricing a Bond Using a Binomial Tree

1. Using the interest rate tree in Exhibit 8, find the correct price for a three-year, annual pay bond with a coupon rate of 5%.

Exhibit 8: Three-Year Binomial Interest Rate Tree



Solution:

Exhibit 9 shows the binomial tree to value the three-year, 5% bond. We start with Time 3. The cash flow is 105, the redemption of par value (100) plus the final coupon payment (5), regardless of the level of the forward rate at Time 2. Using backward induction, we next calculate the present value of the bond as of Time 2 for the three possible forward rates:

105/1.08 = 97.2222.

105/1.06= 99.0566.

105/1.04 = 100.9615.

Working back to Time 1 requires the use of the general expression above for the value at any node. If the forward rate is 5.0% at Time 1, the bond value is 98.2280:

$$\frac{5 + (0.5 \times 97.2222 + 0.5 \times 99.0566)}{1.05} = 98.2280.$$

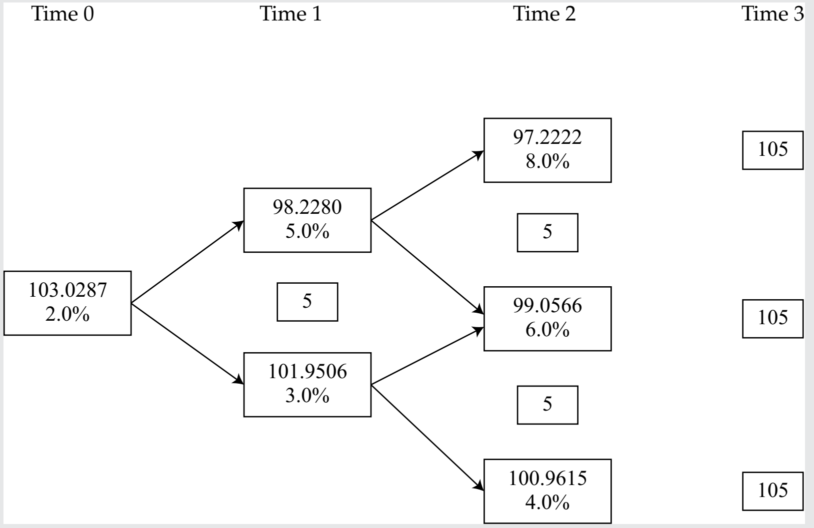
If the forward rate instead is 3.0%, the bond value is 101.9506:

$$\frac{5 + (0.5 \times 99.0566 + 0.5 \times 100.9615)}{1.03} = 101.9506.$$

Finally, the value of the bond at Time 0 is 103.0287:

$$\frac{5 + (0.5 \times 98.2280 + 0.5 \times 101.9506)}{1.02} = 103.0287.$$

Exhibit 9: Three-Year Binomial Tree



CALIBRATING THE BINOMIAL INTEREST RATE TREE TO THE TERM STRUCTURE

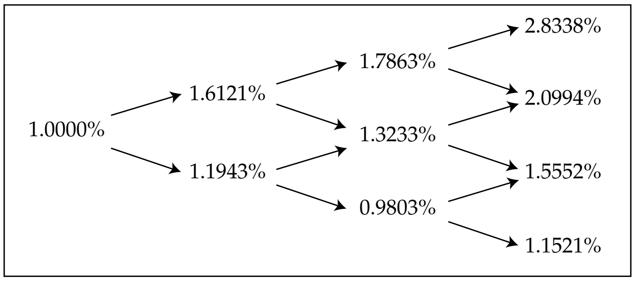
4

☐ describe the process of calibrating a binomial interest rate tree to match a specific term structure

The construction of a binomial interest rate tree requires multiple steps, but keep in mind what we are trying to accomplish. We assume a process that generates interest rates and volatility. The first step is to describe the calibration of a binomial interest rate tree to match a specific term structure. We do this to ensure that the model is arbitrage free. We fit the interest rate tree to the current yield curve by choosing interest rates such that the model produces the benchmark bond values reported earlier. By doing this, we tie the model to the underlying economic reality.

Recall from Exhibits 2, 3, and 4 the benchmark bond price information and the relevant par, spot, and forward curves. We will assume that volatility, σ, is 15% and construct a four-year tree starting with the two-year bond that carries a coupon rate of 1.20%. A complete four-year binomial interest rate tree is presented in Exhibit 10. We will demonstrate how these rates are determined. The current one-year rate is 1%, i_0.

Exhibit 10: Four-Year Binomial Interest Rate Tree



Finding the rates in the tree is an iterative process, and the interest rates are found numerically. There are two possible rates at Time 1—the higher rate and the lower rate. We observe these rates one year from today. These two rates must be consistent with the volatility assumption, the interest rate model, and the observed market value of the benchmark bond. Assume that the interest rate volatility is 15%. From our discussion earlier, we know that at Time 1 the lower one-year rate is lower than the implied one-year forward rate and the higher rate is a multiple of the lower rate. We iterate to a solution with constraints in mind. Once we select these rates, how will we know the rates are correct? The answer is when we discount the cash flows using the tree and produce a value that matches the price of the two-year benchmark bond. If the model does not produce the correct price with this result, we need to select another forward rate and repeat the process. The process of calibrating a binomial interest rate tree to match a specific term structure is illustrated in the following paragraphs.

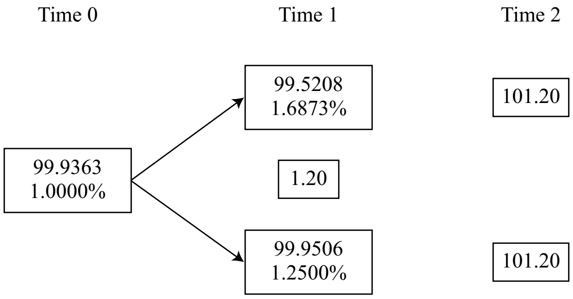
The procedure starts with the selection of a trial rate for one of the Time 1 forward rates—for instance, $i_{1,L}$. This rate should be lower than the implied forward rate from Exhibit 4 of 1.4028%. Suppose that we select 1.2500%. The other forward rate will be 1.6873% [= 1.2500% × ($e^{2\times0.15}$)]. Exhibit 11 shows that the Time 0 value for the 1.20%, two-year bond is 99.9363. The redemption of principal and the final interest payment are placed across from the two nodes for the forward rates. At Time 1, the interest payment due is placed across from the initial rate for Time 0. These are the calculations:

$$101.20/1.016873 = 99.5208.$$

$$101.20/1.012500 = 99.9506.$$

$$\frac{1.20 + (0.5 \times 99.5208 + 0.5 \times 99.9506)}{1.01} = 99.9363.$$

Exhibit 11: Calibrating the Two-Year Binomial Tree



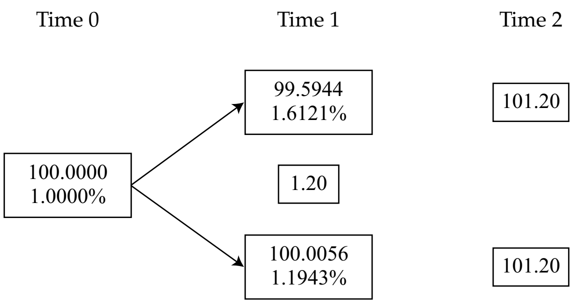
These two trial rates are clearly too high. They need to be lowered somewhat to raise the bond value to attain a Time 0 price for the bond of 100.0000. We could proceed with further trial-and-error search or use an analytic tool, such as Solver in Excel, to carry out this calculation. Essentially, we need to set the cell for the Time 0 bond price to a value of 100.0000 by changing the cell containing the initial lower forward rate for Time 1.

This procedure eventually obtains a value for $i_{1,L}$ of 1.1943%. This is the lower one-year rate. The higher one-year rate is 1.6121% [= 1.1943% × ($e^{2\times0.15}$)]. Notice that the average of these two forward rates is 1.4032% [= (1.6121% + 1.1943%)/2], slightly above the implied forward rate of 1.4028% from Exhibit 4. The binomial tree spreads out around the forward rate curve. The average is slightly higher than the implied forward rate because of the assumption of lognormality.

Recall from the information on the benchmark bonds that the two-year bond will pay its maturity value of 100 at Time 2 and an annual coupon payment of 1.20. The bond's value at Time 2 is 101.20. The present value of the coupon payment plus the bond's maturity value if the higher one-year rate is realized, *VH*, is 99.5944 (= 101.20/1.016121). Alternatively, the present value of the coupon payment plus the bond's maturity value if the lower one-year rate is realized, *VL*, is 100.0056 (= 101.20/1.011943). These two calculations determine the bond's value one year forward. Effectively, the forward rates move the bond's value from Time 2 to Time 1. Exhibit 12 demonstrates that the arbitrage-free forward rates for Time 1 are 1.6121% and 1.1943%. The value for the bond at Time 0 is 100.0000, confirming the calibration:

$$\frac{1.20 + (0.5 \times 99.5944 + 0.5 \times 100.0056)}{1.010000} = 100.0000.$$

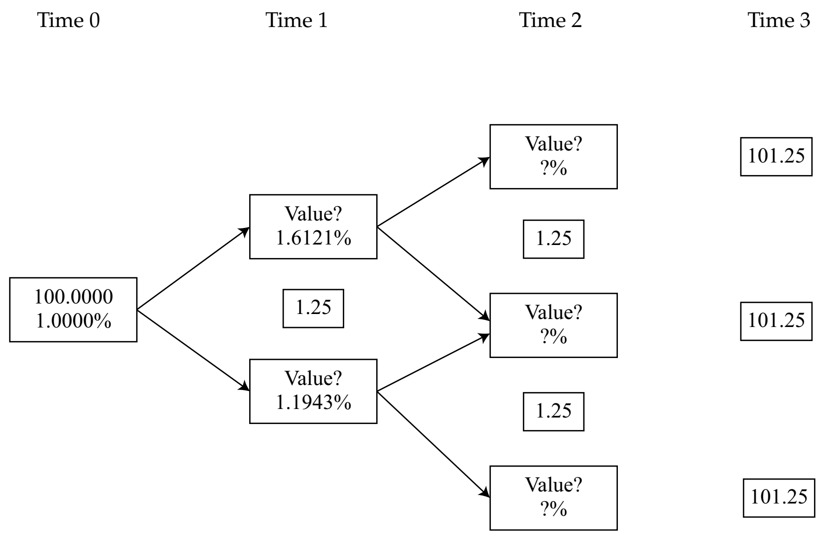
Exhibit 12: Building the Two-Year Binomial Tree



To build out the tree one more year, we repeat the same process, this time using a three-year benchmark bond with a coupon rate of 1.25%. Now, we are looking for three forward rates that are consistent with (1) the interest rate model assumed, (2) the assumed volatility of 15%, (3) a current one-year rate of 1.0%, and (4) the two possible forward rates one year from now (at Time 1) of 1.1943% (the lower rate) and 1.6121% (the higher rate).

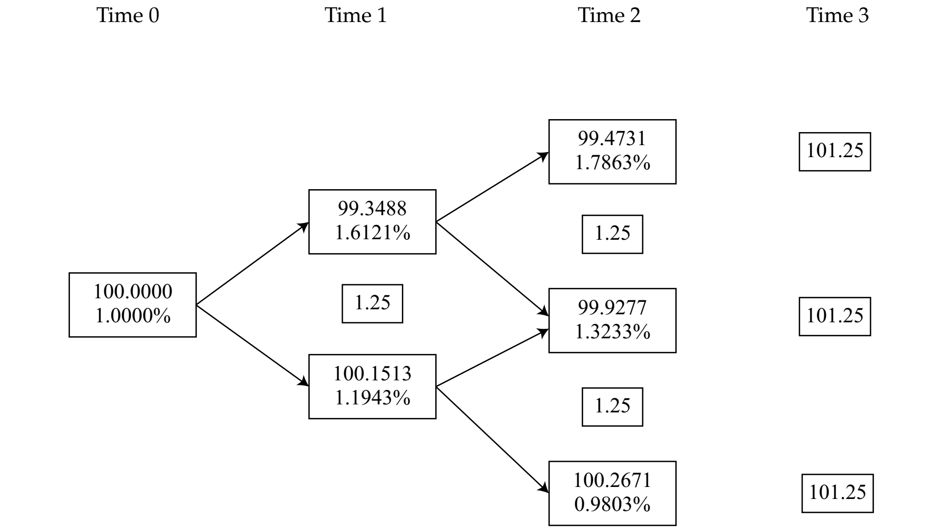
At Time 3, we receive the final coupon payment and maturity value of 101.25. In Exhibit 13, we see the known coupon payments of 1.25 for Times 1 and 2. Also entered are the Time 1 forward rates and the target price of par value for the three-year bond. The unknown items to determine are the Time 1 and Time 2 bond values (Value?) and the Time 2 forward rates (?%).

Exhibit 13: Finding the Time 2 Forward Rates



We need to select a trial value for the middle rate, $i_{2,HL}$. A good choice is the implied forward rate of 1.3521%. The trial value for the upper rate, $i_{2,HH}$, would need to be 1.3521% × ($e^{2\times0.15}$), and the trial value for the lower rate, $i_{2,LL}$, would need to be 1.3521%/($e^{2\times0.15}$). The middle rate is then changed, changing the others as well, until the value for the 1.25% three-year bond is 100.0000. It turns out that the three forward rates are 1.7863%, 1.3233%, and 0.9803%. To demonstrate that these are the correct values, we simply work backward from the cash flows at Time 3 of the tree in Exhibit 13. The same procedure is used to obtain the values at the other nodes. The completed tree is shown in Exhibit 14.

Exhibit 14: Completed Binomial Tree with Calculated Forward Rates



Let us focus on the impact of volatility on the possible forward rates in the tree. If we were to use a higher estimate of volatility—say, 20%—the possible forward rates should spread farther out around the forward curve. If we were to use a lower estimate of volatility—say, 0.01%—the rates should collapse to the implied forward rates from the current yield curve. Exhibit 15 and Exhibit 16 depict the interest rate trees for the volatilities of 20% and 0.01%, respectively, and confirm the expected outcome. Notice that in Exhibit 16 for 0.01% volatility, the Time 1 forward rates are very close to the implied forward rate of 1.4028% shown in Exhibit 4. Likewise, the Time 2 and Time 3 rates are a small range around the forward rates of 1.3521% and 1.8647%, respectively. In fact, if $\sigma = 0$, the binomial tree is simply the implied forward curve.

Exhibit 15: Completed Tree with σ = 20%

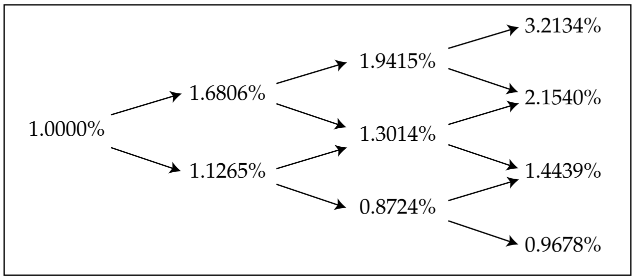
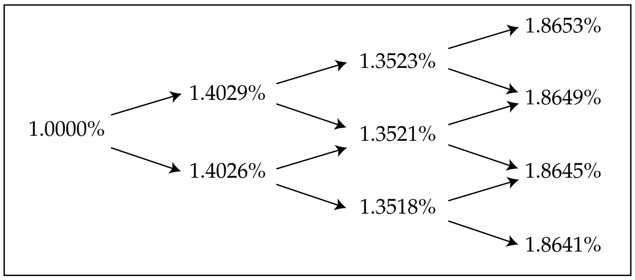


Exhibit 16: Completed Tree with σ = 0.01%



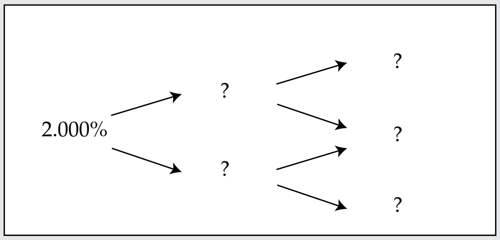
EXAMPLE 4

Calibrating a Binomial Tree to Match a Specific Term Structure

1. As in Example 2, the one-year par rate is 2.000%, the two-year par rate is 3.000%, and the three-year par rate is 4.000%. Consequently, the spot rates are S_0 = 2.000%, S_1 = 3.015%, and S_2 = 4.055%. The forward rates are F_0 = 2.000%, F_1 = 4.040%, and F_2 = 6.166%. Interest volatility is 15% for all years.

   Calibrate the binomial tree in Exhibit 17.

Exhibit 17: Binomial Tree to Calibrate



Solution:

Time 0

The par, spot, and forward rates are all the same for the first period in a binomial tree. Consequently, $Y_0 = S_0 = F_0$ = 2.000%.

Time 1

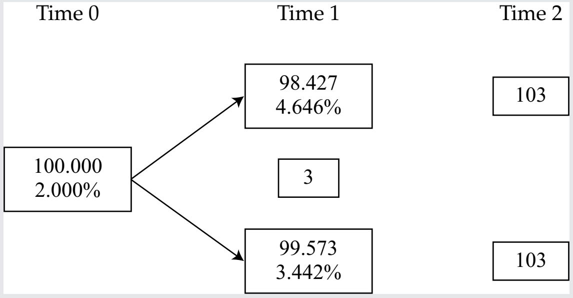
We need to use trial-and-error search (or Solver in Excel) to find the two forward rates that produce a value of 100.000 for the 3%, two-year bond. The lower trial rate needs to be lower than the implied forward rate of 4.040%—for instance, 3.500%. The higher trial rate would be 3.500% × ($e^{2\times0.15}$) = 4.725%. These lead to a Time 0 value for the bond of 99.936. Therefore, the next stage in the procedure lowers the trial rates. Finally, the calibrated forward rates are 4.646% and 3.442%. Exhibit 18 shows that these are the correct rates because the value of the bond at Time 0 is 100.000. These are the calculations:

103/1.04646 = 98.427.

$103/1.03442 = 99.573.$

$$\frac{3 + (0.5 \times 98.427 + 0.5 \times 99.573)}{1.02} = 100.0000.$$

Exhibit 18: Calibration of Time 1 Forward Rates



Time 2

The initial trial rate for the middle node for Time 2 is the implied forward rate of 6.166%. The rate for the upper node is 8.323% [= 6.166% × ($e^{2\times0.15}$)], and the rate for the lower node is 4.568% [= 6.166%/($e^{2\times0.15}$)]. Exhibit 19 shows that these rates for Time 2 and the already calibrated rates for Time 1 lead to a value of 99.898 for the 4% three-year bond as of Time 0. These are not the arbitrage-free rates: The Time 2 rates need to be lowered slightly to get the price up to 100.000.

Exhibit 19: Calibration of Time 2 Forward Rates

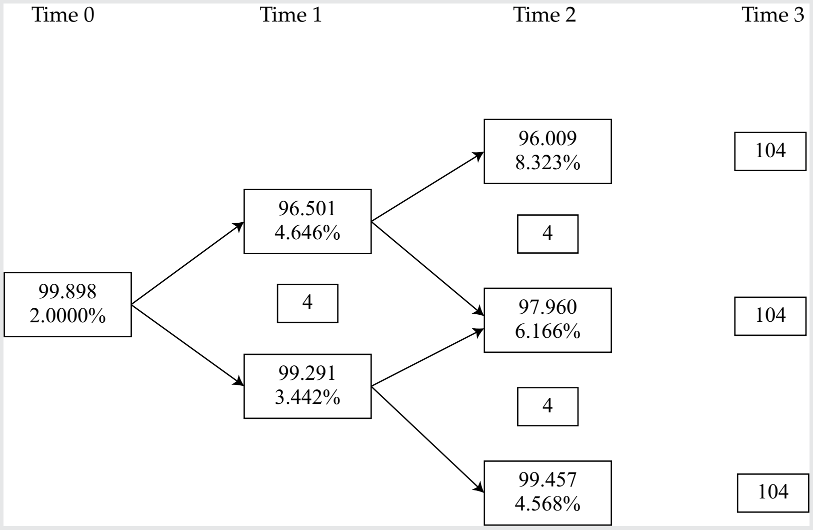


Exhibit 20 displays the completed binomial tree. The calibrated forward rates for Time 2 are 8.167%, 6.050%, and 4.482%. These are the calculations:

$104/1.08167 = 96.148.$

$104/1.06050 = 98.067.$

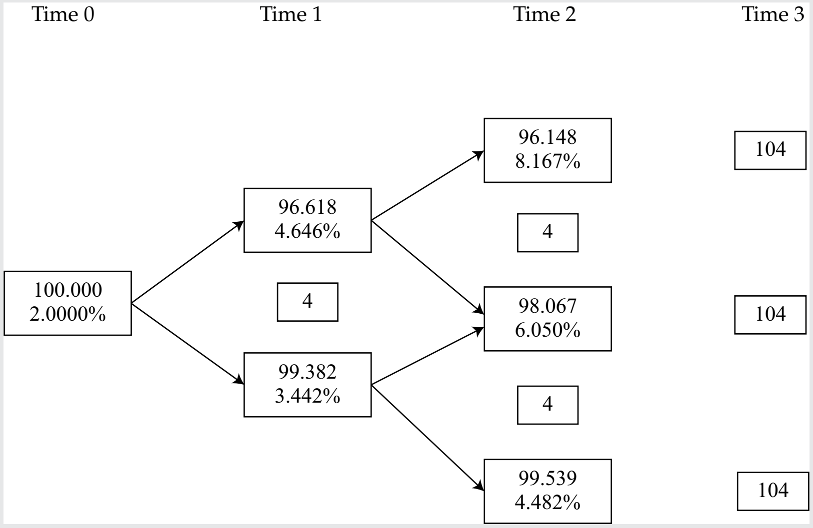
$104/1.04482 = 99.538.$

$$\frac{4 + (0.5 \times 96.148 + 0.5 \times 98.067)}{1.04646} = 96.618.$$

$$\frac{4 + (0.5 \times 98.067 + 0.5 \times 99.539)}{1.03442} = 99.382.$$

$$\frac{4 + (0.5 \times 96.618 + 0.5 \times 99.382)}{1.02000} = 100.000.$$

Exhibit 20: Completed Binomial Tree



Now that our tree gives the correct prices for the underlying par bonds maturing in one, two, and three years, we say that our tree is calibrated to be arbitrage free. It will price option-free bonds correctly, including prices for the zero-coupon bonds used to find the spot rates, and to the extent that we have chosen an appropriate interest rate process and interest rate volatility, it will provide insights into the value of bonds with embedded options and their risk parameters.

5 VALUING AN OPTION-FREE BOND WITH A BINOMIAL TREE

- ☐ describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node
- ☐ compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice

Our next task is twofold. First, we calculate the arbitrage-free value of an option-free, fixed-rate coupon bond. Second, we compare the pricing using the zero-coupon yield curve with the pricing using an arbitrage-free binomial lattice. Because these two valuation methods are arbitrage free, these two values must be the same.

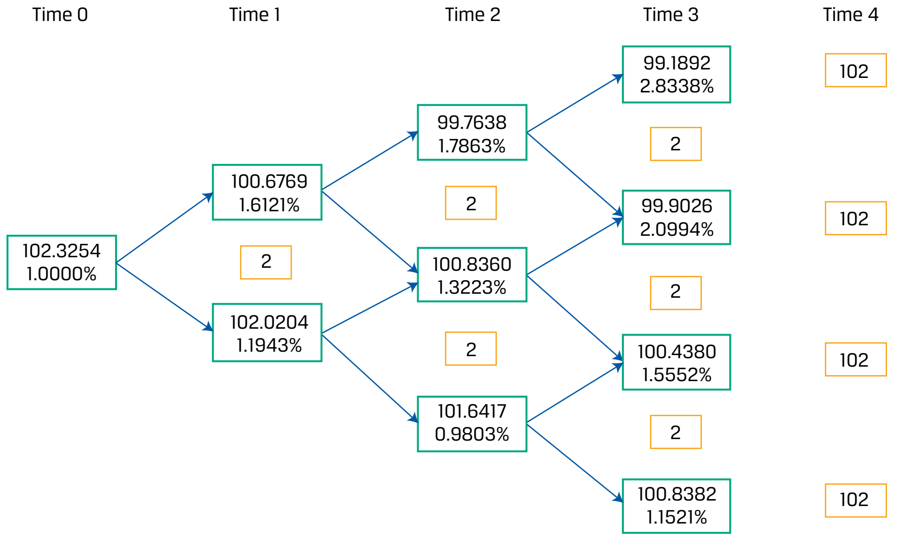
Now, consider an option-free bond with four years remaining to maturity and a coupon rate of 2%. Note that this is not a benchmark bond and it carries a higher coupon and price than the four-year benchmark bond, which is priced at par. The value of this bond can be calculated by discounting the cash flow at the spot rates in Exhibit 3 as shown in the following equation:

$$\frac{2}{(1.01)^1} + \frac{2}{(1.012012)^2} + \frac{2}{(1.012515)^3} + \frac{102}{(1.014044)^4} = 102.3254.$$

The binomial interest rate tree should produce the same value as when discounting the cash flows with the spot rates. An option-free bond that is valued by using the binomial interest rate tree should have the same value as when discounting by the spot rates, which is true because the binomial interest rate tree is arbitrage free.

Let us give the tree a test run and use the 2% option-free bond with four years remaining to maturity. Also assume that the issuer's benchmark yield curve is the one given in Exhibit 2; hence the appropriate binomial interest rate tree is the one in Exhibit 10. Exhibit 21 shows the various values in the discounting process and obtains a bond value of 102.3254. The tree produces the same value for the bond as the spot rates produce and is therefore consistent with our standard valuation model.

Exhibit 21: Sample Valuation for an Option-Free Bond using a Binomial Tree



EXAMPLE 5

Confirming the Arbitrage-Free Value of a Bond

1. Using the par curve from Example 2 and Example 4, the yield-to -maturity for a one-year annual coupon bond is 2%, for a two-year annual coupon bond is 3%, and for a three-year annual coupon bond is 4%. Because this is the same curve as that used in Example 4, we can use the calibrated tree from that example to price a bond. Let us use a three-year annual coupon bond with a 5% coupon, just as we did in Example 2. We know that if the

calibrated tree was built correctly and we perform calculations to value the bond with the tree shown in Exhibit 22, its price should be 102.8105.

Exhibit 22: Binomial Tree from Example 5

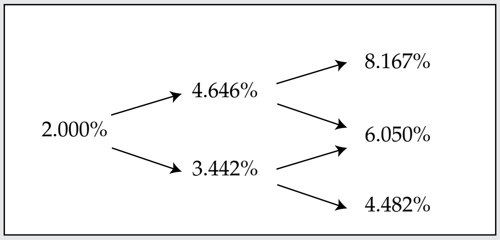
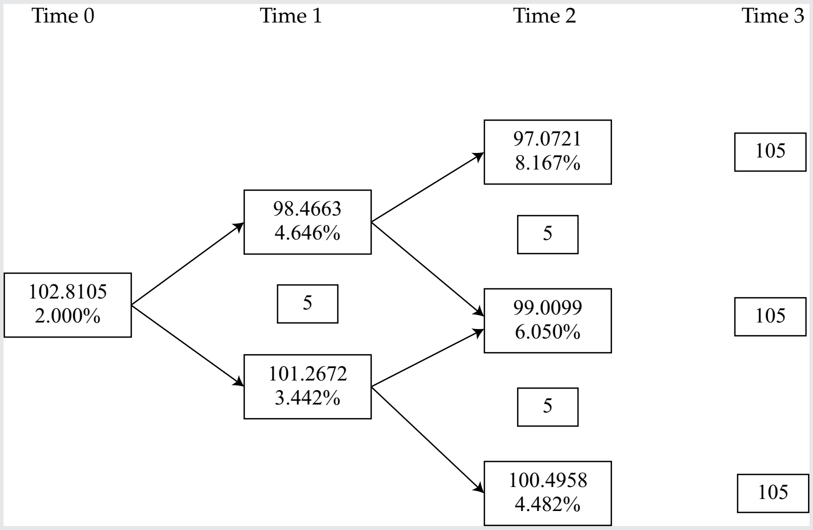


Exhibit 23: Valuing a 5%, Three-Year Bond



Because the tree was calibrated to the same par curve (and spot curve) that was used to price this option-free bond using spot rates only, the tree gives the same price as the spot rate pricing (the small difference is due to rounding).

6 VALUING AN OPTION-FREE BOND WITH PATHWISE VALUATION

☐ describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path

Pathwise valuation is an alternative approach to backward induction in a binomial tree. The binomial interest rate tree specifies all potential rate paths in the model, whereas an interest rate path is the route an interest rate takes from the current time to the security's maturity. Pathwise valuation calculates the present value of a bond for each possible interest rate path and takes the average of these values across paths. We will use the pathwise valuation approach to produce the same value as the backward induction method for an option-free bond. Pathwise valuation involves the following steps: (1) Specify a list of all potential paths through the tree, (2) determine the present value of a bond along each potential path, and (3) calculate the average across all possible paths.

Determining all potential paths is similar to the following experiment. Suppose you are tossing a fair coin and tracking how many ways heads and tails can be combined. We will use a device called Pascal's Triangle, displayed in Exhibit 24. Pascal's Triangle can be built as follows: Start with the number 1 at the top of the triangle. The numbers in the boxes below are the sum of the two numbers above it except that the edges on each side are all 1. The shaded numbers show that 3 is the sum of 2 and 1. Now toss the coin while keeping track of the possible outcomes. The possible groupings are listed in Exhibit 25, where H stands for heads and T stands for tails.

Exhibit 24: Pascal's Triangle

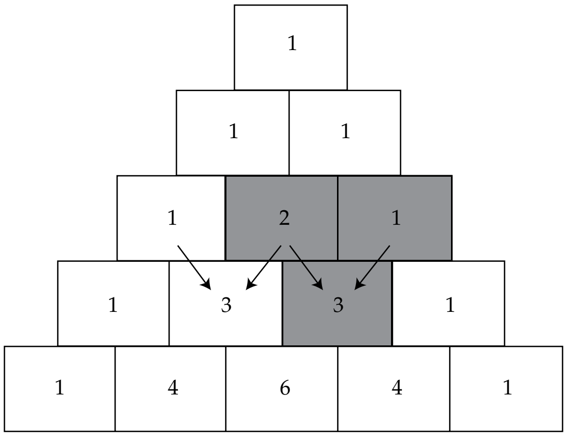


Exhibit 25: Possible Outcomes of Coin Tosses

Number of Tosses	Possible Outcomes	Pascal's Triangle
1	H T	1, 1
2	HH HT TH TT	1,2,1
3	HHH HHT HTH THH HTT THT TTH TTT	1, 3, 3, 1

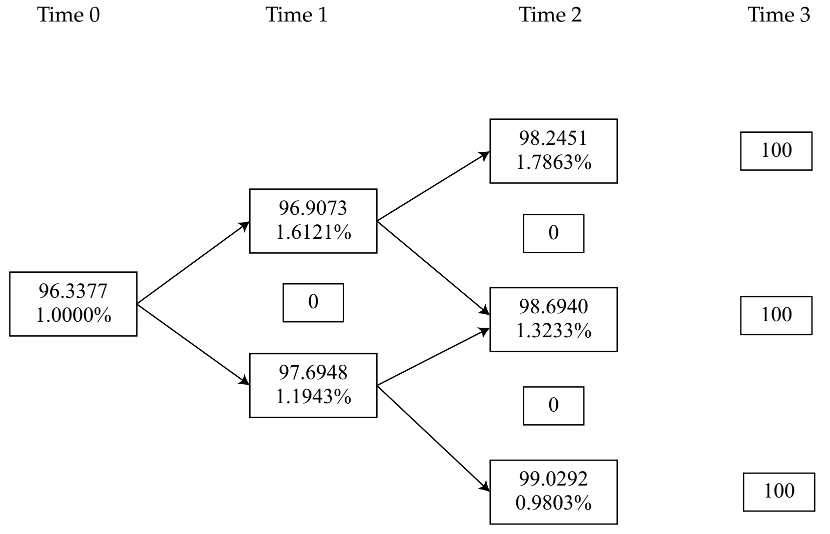
This experiment mirrors exactly the number of interest rate paths in our binomial interest rate tree. The total number of paths for each period/year can be easily determined by using Pascal's Triangle. Let us work through an example for a three-year zero-coupon bond. From Pascal's Triangle, there are four possible paths to arrive at Year 3: HH, HT, TH, TT. Using the same binomial tree from Exhibit 21, we specify the four paths as well as the possible forward rates along those paths. In Exhibit 26, the last column on the right shows the present value for each path. For example, 100/(1.01000 × 1.016121 × 1.017863) = 95.7291. In the bottom right corner is the average present value across all paths.

Exhibit 26: Four Interest Rate Paths for a Three-Year Zero-Coupon Bond

Path	Forward Rate Year 1	Forward Rate Year 2	Forward Rate Year 3	Present Value
1	1.0000%	1.6121%	1.7863%	95.7291
2	1.0000%	1.6121%	1.3233%	96.1665
3	1.0000%	1.1943%	1.3233%	96.5636
4	1.0000%	1.1943%	0.9803%	96.8916
				96.3377

Now, we can use the binomial tree to confirm our calculations for the three-year zero-coupon bond. The analysis is presented in Exhibit 27. The interest rate tree does indeed produce the same value.

Exhibit 27: Binomial Tree to Confirm Bond's Value



EXAMPLE 6

Pathwise Valuation Based on a Binomial Interest Rate Tree

1. Using the par curve from Example 2, Example 4, and Example 5, the yield-to-maturity for a one-year annual coupon bond is 2%, for a two-year annual coupon bond is 3%, and for a three-year annual coupon bond is 4%. We know that if we generate the paths in the tree correctly and discount the

cash flows directly, the three-year, 5% annual coupon bond should still be priced at 102.8105, as calculated in Example 5.

There are four paths through the three-year tree. We discount the cash flows along each of the four paths and take their average, as shown in Exhibit 28, Exhibit 29, and Exhibit 30.

Exhibit 28: Cash Flows

Path	Time 0	Time 1	Time 2	Time 3
1	0	5	5	105
2	0	5	5	105
3	0	5	5	105
4	0	5	5	105

Exhibit 29: Discount Rates

Path	Time 0	Time 1	Time 2	Time 3
1	2.000%	4.646%	8.167%	
2	2.000%	4.646%	6.050%	
3	2.000%	3.442%	6.050%	
4	2.000%	3.442%	4.482%	

Exhibit 30: Present Values

Path	Time 0
1	100.5298
2	102.3452
3	103.4794
4	104.8877
Average	**102.8105**

The present values are calculated by discounting the cash flows in Exhibit 28 by the forward rates in Exhibit 29. For example, the present value for the bond along Path 1 is 100.5298:

$$\frac{5}{1.02} + \frac{5}{(1.02)\ (1.04646)} + \frac{105}{(1.02)\ (1.04646)\ (1.08167)} = 100.5298.$$

The present value along Path 3 is 103.4794:

$$\frac{5}{1.02} + \frac{5}{(1.02)\ (1.03442)} + \frac{105}{(1.02)\ (1.03442)\ (1.06050)} = 103.4794.$$

The average for the bond prices using pathwise valuation is 102.8105, which matches the result obtained using backward induction in Exhibit 23.

7 THE MONTE CARLO METHOD

☐ describe a Monte Carlo forward-rate simulation and its application

The Monte Carlo method is an alternative method for simulating a sufficiently large number of potential interest rate paths to discover how the value of a security is affected. This method involves randomly selecting paths to approximate the results of a complete pathwise valuation. Monte Carlo methods are often used when a security's cash flows are path dependent. Cash flows are path dependent when the cash flow to be received depends on the path followed to reach its current level as well as the current level itself. For example, the valuation of mortgage-backed securities depends to a great extent on the level of prepayments. As mentioned in an earlier reading, prepayments tend to increase when interest rates fall, because borrowers are more likely to pay off mortgage loans and refinance at lower interest rates. Interest rate paths are generated on the basis of some probability distribution and a volatility assumption, and the model is fit to the current benchmark term structure of interest rates. The benchmark term structure is represented by the current spot rate curve such that the average present value across all scenario interest rate paths for each benchmark bond equals its actual market value. By using this approach, the model is rendered arbitrage free, which is equivalent to calibrating the interest rate tree as discussed in Section 3.

Suppose we intend to value with the Monte Carlo method a 30-year bond that has monthly coupon payments (e.g., mortgage-backed securities). The following steps are taken: (1) Simulate numerous (say, 500) paths of one-month interest rates under a volatility assumption and probability distribution, (2) generate spot rates from the simulated future one-month interest rates, (3) determine the cash flow along each interest rate path, (4) calculate the present value for each path, and (5) calculate the average present value across all interest rate paths.

Using the procedure just described, the model will produce benchmark bond values equal to the market prices only by chance. We want to ensure this is the case; otherwise the model will neither fit the current spot curve nor be arbitrage free. A constant is added to all interest rates on all paths such that the average present value for each benchmark bond equals its market value. The constant added to all short interest rates is called a drift term. When this technique is used, the model is said to be drift adjusted.

How many paths are appropriate for the Monte Carlo method? More paths increase the accuracy of the estimate in a statistical sense, but this does not mean the model is closer to the true fundamental value of the security. The Monte Carlo method is only as good as the valuation model used and the accuracy of the inputs.

Yield curve modelers also often include mean reversion in their Monte Carlo estimation. Mean reversion starts with the common-sense notion that history suggests that interest rates almost never get "too high" or "too low." What is meant by "too high" and "too low" is left to the discretion of the modeler. We implement mean reversion by implementing upper and lower bounds on the random process generating future interest rates. Mean reversion has the effect of moving the interest rate toward the implied forward rates from the yield curve.

EXAMPLE 7

The Application of Monte Carlo Simulation to Bond Pricing

1. Replace the interest rate paths from Example 6 with randomly generated paths calibrated to the same initial par and spot curves, as shown in Exhibit 31.

Exhibit 31: Discount Rates

Path	Time 0	Time 1	Time 2
1	2.000%	2.500%	4.548%
2	2.000%	3.600%	6.116%
3	2.000%	4.600%	7.766%
4	2.000%	5.500%	3.466%
5	2.000%	3.100%	8.233%
6	2.000%	4.500%	6.116%
7	2.000%	3.800%	5.866%
8	2.000%	4.000%	8.233%

Exhibit 32: Present Values

Path	Time 0
1	105.7459
2	103.2708
3	100.9104
4	103.8543
5	101.9075
6	102.4236
7	103.3020
8	101.0680
Average	**102.8103**

Because we continue to get 102.8103, as shown in Exhibit 32, as the price for our three-year, 5% annual coupon bond, we know that the Monte Carlo simulation has been calibrated correctly. The paths are now different enough such that path-dependent securities, such as mortgage-backed securities, can be analyzed in ways that provide insights not possible in binomial trees, because Monte Carlo techniques provide greater flexibility to change parameters over time.

8 TERM STRUCTURE MODELS

- [ ] describe term structure models and how they are used

Term structure models provide quantitatively precise descriptions of how interest rates evolve. A model provides a simplified description of a real-world phenomenon on the basis of a set of assumptions. These assumptions cannot be completely accurate in depicting the real world but are necessary for analytical tractability. Despite simplifying assumptions, models explain real-world phenomena sufficiently well to be useful for pricing and hedging.

The binomial tree and Monte Carlo simulation valuation approaches for complex fixed-income instruments described earlier rely on specific assumptions about the underlying asset properties. For example, how do we establish the node values in the binomial trees, and what determines the dispersion in rates from the top to the bottom nodes? This answer comes from term structure models, which make assumptions about the properties of rates over time and then use those properties to "fit," or determine the values of the rates at each node, binomial lattices used for pricing and risk management applications. The following section introduces common term structure models, with an emphasis on the underlying assumptions about the statistical properties of interest rates. Each of the models can be "fit" to lattice models for valuation and risk management applications.

Modeling the future path of interest rates is not only critical for scenario analysis and stress testing individual bonds and bond portfolio values but also important in the valuation of complex fixed-income instruments. A detailed description of these models depends on mathematical and statistical knowledge beyond the scope of this reading, but fixed-income practitioners will often find that these or other term structure models are embedded in many of the desktop tools and data analytics software they may use during their investment industry career. Thus, we provide a broad overview of these models in this reading.

Model Choice

Term structure models go beyond the lognormal random walk approach used earlier to describe the dynamics of the term structure for the purpose of pricing and hedging fixed-income securities and derivatives. All term structure models make simplifying assumptions about the evolution of rates over time. Many different interest rate models that differ in their assumptions exist. Arguably, there are many models, since no one model perfectly captures interest rate dynamics. Modelers face a trade-off between simplicity and accuracy when selecting a term structure model. Practitioners should be aware of the categories of models and their important features (which stem from their assumptions) as well as how those features affect pricing and hedging.

Interest rate factors

The valuation and hedging of fixed-income securities and their derivatives require information across the entire term structure. To develop a term structure model useful for pricing and hedging applications, we focus on modeling the factors that determine the term structure. The simplest class of models use one factor—the short rate, or the one-period rate—as the factor that drives the term structure. Although the use of one factor may seem limiting, because it implies all rates move in the same direction during any short time interval, it does not mean they have to move by the same amounts. Multi-factor models incorporate additional factors, such as the slope of the term structure, with the complexity of the models increasing in the number of factors.

Interest rate process

Term structure models use stochastic processes to describe interest rate dynamics. These stochastic processes have two components: a drift term and an uncertain, or stochastic, term. Although the stochastic processes are continuous time, the models can be "fit" to binomial or trinomial interest rate lattices using a discrete version of the models (integrating over time to obtain rates that span time intervals).

For a one-factor model, the general form of the process describing the short rate's (r) dynamics is

$$dr = \theta_t dt + \sigma_t dZ.$$

The drift term, $\theta_t dt$, describes the expected (zero-volatility) rate path. For example, in a one-factor model of the short rate, the drift describes the expected evolution of the short rate over time. The drift term may be constant or mean reverting.

The second term, $\sigma_t dZ$, adds randomness, or volatility, to the process. This dispersion term allows for the pricing of bonds with option features as well as interest rate derivatives and may take a variety of forms. The term Z is a Weiner process that is distributed normally. Given the symmetry of the normal distribution, it is possible and quite common for these models to produce interest rate paths with negative rates.

Within classes of models, such as one-factor no-arbitrage models, the key differences between the various models involve the stochastic difference equation.

Class of model

One class of models uses the arbitrage-free approach combined with assumptions about the statistical properties of interest rates. This class of models is referred to as no-arbitrage term structure models, where no-arbitrage is synonymous with arbitrage free. No-arbitrage term structure models begin with a set of assumptions about the term structure—a factor (or factors) and the stochastic process describing the factor evolution(s)—and take the term structure as given, assuming that both bond prices and the term structure bootstrapped from those prices are correct. The no-arbitrage models are "parameterized," which is the process of determining the values of the variables in the model such that those parameters produce bond prices that match current market prices. These models are used widely in practice and are often favored by practitioners since their pricing results are consistent with market prices.

Equilibrium term structure models seek to describe term structure dynamics using fundamental economic variables that are assumed to affect interest rates. The modeling process imposes restrictions that allow for the derivation of equilibrium prices for bonds and interest rate options.

Although equilibrium models use similar continuous stochastic difference equations to describe interest rate changes, equilibrium model parameters are not forced to values that produce bond prices consistent with current market prices. This property is seen by some market participants as a significant drawback in a static setting, such as pricing and hedging for the current time. However, other practitioners prefer equilibrium models since they capture not just the current market environment as reflected in the term structure but also the possibility of many different future paths. For more dynamic applications, equilibrium models may be preferred.

The best-known equilibrium models are the **Cox-Ingersoll-Ross model** (Cox, Ingersoll, and Ross 1985) and the **Vasicek model** (Vasicek 1977), discussed in the next two sections. Both the Vasicek and Cox–Ingersoll–Ross (CIR) models assume a single factor, the short-term interest rate, r_t. This approach is plausible because empirically, parallel shifts are often found to explain more than 90% of yield changes. In contrast, multifactor models may be able to model the curvature of a yield curve more accurately, but at the cost of greater complexity.

The reason that no-arbitrage models fit the current term structure is their greater number of parameters. These added parameters increase the computational requirements for estimation, which some practitioners find to be undesirable.

Other contrasts are more technical. They include that equilibrium models use real probabilities, whereas arbitrage-free models use so-called risk-neutral probabilities. An excellent example of an equilibrium term structure model is the Cox–Ingersoll–Ross model, discussed next.

Equilibrium Models

This section introduces the Cox–Ingersoll–Ross and Vasicek interest rate models.

The Cox–Ingersoll–Ross model

The Cox–Ingersoll–Ross (CIR) model assumes interest rates follow a mean-reverting process. However, the variance of rate changes differs depending on the level of rates. The CIR model uses the following formula to describe the interest rate process:

$$dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dZ.$$

Note that the drift term has three components. The level of rates at time t is r_t, and θ is the long-run mean rate, so their difference is the distance of the rate from its mean. The drift term equals zero if the rate is at the long-run mean, or $r_t = \theta$. The remaining drift term parameter, k, modulates the speed at which the rate reverts to its mean.

Another important feature of the CIR model is that the random component varies as rates change. In other words, the short-rate volatility is a function of the short rate. Importantly, at low rates, r_t, the term becomes small, which prevents rates from turning negative.

The Vasicek model

Although not developed in the context of a general equilibrium of individuals seeking to optimize consumption and investment decisions, as was the case for the CIR model, the Vasicek model is viewed as an equilibrium term structure model. Similar to the CIR model, the Vasicek model includes mean reversion. The Vasicek model uses the following equation to describe the interest rate process:

$$dr_t = k\left(\theta - r_t\right)dt + \sigma dZ.$$

The Vasicek model has the same drift term as the CIR model and thus tends toward mean reversion in the short rate. The stochastic or volatility term follows a random normal distribution for which the mean is zero and the standard deviation is 1. Unlike the CIR model, interest rates are calculated assuming constant volatility over the period of analysis. As with the CIR model, there is only one stochastic driver of the interest rate process. A key characteristic of the Vasicek model worth noting is that it is theoretically possible for the interest rate to become negative.

Arbitrage-Free Models

We will next illustrate two foundational no-arbitrage term structure models. There are many additional no-arbitrage models, but the basic features are similar, with differences stemming from different assumed interest rate processes.

The Ho–Lee model

In **arbitrage-free models**, the analysis begins with the current term structure, extrapolated from the market prices of a reference set of financial instruments. A maintained assumption is that the reference bonds are priced correctly. Unlike general equilibrium models, which have only a few parameters and can thus match only a few term structure points, arbitrage-free models allow the parameters to vary deterministically with time, creating a greater number of parameters and thus more points of match. As a result, the market yield curve can be modeled with the accuracy needed for such applications as valuing derivatives and bonds with embedded options.

The first arbitrage-free model was introduced by Ho and Lee (1986). The model is calibrated to market data and uses a binomial lattice approach to generate a distribution of possible future interest rates. In the **Ho–Lee model**, the short rate follows a normal process, as follows:

$$dr_t = \theta_t dt + \sigma dZ.$$

We see that the drift term, θ_t, is time dependent. This time dependency means there is a value for θ_t at each time step, which is critical for the model to produce prices that match market prices.

The Ho–Lee model, similar to the Vasicek model, has constant volatility, and interest rates may become negative because of the symmetry of the normal distribution and the model's use of constant volatility.

The Kalotay–Williams–Fabozzi model

The **Kalotay–Williams–Fabozzi (KWF) model** is analogous to the Ho–Lee model in that it assumes no mean reversion and constant volatility. However, the stochastic differential equation describes the dynamics of the log of the short rate, and as a result, the log of the short rate is distributed normally, meaning the short rate itself is distributed lognormally.

The differential process for the KWF model is

$$d \ln\left(r_t\right) = \theta_t dt + \sigma dZ.$$

At first glance, the main implication of modeling the log of the short rate is that it will prevent negative rates. After further analysis, it becomes evident that there are pricing implications where interest rate option values are influenced by the tails of the rate distributions. Exhibit 33 summarizes the key differences between these term structure models.

Exhibit 33: Term Structure Model Summary

Model	Type	Short Rate	Drift Term	Volatility
CIR	Equilibrium	dr_t	Mean reversion at speed k	Varies with $\sqrt{r}_t$
Vasicek	Equilibrium	dr_t	Mean reversion at speed k	Constant
Ho–Lee	Arbitrage free	dr_t	Time dependent	Constant
KWF	Arbitrage free	$d\ln(r_t)$	Time dependent	Constant

Modern Models

The one-factor models presented thus far are the building blocks on which modern interest rate models rely. Some current models extend those models to include multiple factors, while others use sophisticated approaches that combine observed forward curves with volatilities extracted from interest rate option prices.

The Gauss+ model is a multi-factor interest rate model used extensively in valuation and hedging. The Gauss+ model incorporates short-, medium- and long-term rates. The long-term factor is mean reverting and reflects trends in macroeconomic variables. The medium-term rate also reverts to the long-run rate. The short-term rate does not exhibit a random component, which is consistent with the central bank controlling the short end of the rate curve. This results in a hump-shaped volatility curve across tenors, with medium-term rates being the most volatile.

Although there are many different term structure models, knowledge of the basic assumptions and design of the classic models helps professionals understand and adapt more sophisticated modern models.

Example 8 addresses several basic points about modern term structure models.

EXAMPLE 8

Term Structure Models

1. Which of the following would be expected to provide the *most* accurate modeling with respect to the observed term structure?

   A. CIR model

   B. Ho–Lee model

   C. Vasicek model

Solution:

B is correct. The CIR model and the Vasicek model are examples of equilibrium term structure models, whereas the Ho–Lee model is an example of an arbitrage-free term structure model. A benefit of arbitrage-free term structure models is that they are calibrated to the current term structure. In other words, the starting prices ascribed to securities are those currently found in the market. In contrast, equilibrium term structure models frequently generate term structures that are inconsistent with current market data.

2. Which of the following statements about the Vasicek model is *most* accurate? It has:

   A. a single factor, the long rate.

   B. a single factor, the short rate.

   C. two factors, the short rate and the long rate.

Solution:

B is correct. Use of the Vasicek model requires assumptions for the short-term interest rate, which are usually derived from more general assumptions about the state variables that describe the overall economy. Using the assumed process for the short-term rate, one can determine the yield on longer-term bonds by looking at the expected path of interest rates over time.

3. The CIR model:

   A. assumes interest rates are not mean reverting.

B. has a drift term that differs from that of the Vasicek model.

C. assumes interest rate volatility increases with increases in the level of interest rates.

Solution:

C is correct. The drift term of the CIR model is identical to that of the Vasicek model, and both models assume that interest rates are mean reverting. The major difference between the two models is that the CIR model assumes a rise in interest rate volatility as rates increase, while the Vasicek model assumes interest rate volatility is constant.

SUMMARY

This reading presents the principles and tools for arbitrage valuation of fixed-income securities. Much of the discussion centers on the binomial interest rate tree, which can be used extensively to value both option-free bonds and bonds with embedded options. The following are the main points made in the reading:

- A fundamental principle of valuation is that the value of any financial asset is equal to the present value of its expected future cash flows.
- A fixed-income security is a portfolio of zero-coupon bonds, each with its own discount rate that depends on the shape of the yield curve and when the cash flow is delivered in time.
- In well-functioning markets, prices adjust until there are no opportunities for arbitrage, or a transaction that involves no cash outlay yet results in a riskless profit.
- Using the arbitrage-free approach, viewing a security as a package of zero-coupon bonds means that two bonds with the same maturity and different coupon rates are viewed as different packages of zero-coupon bonds and valued accordingly.
- For bonds that are option-free, an arbitrage-free value is simply the present value of expected future values using the benchmark spot rates.
- A binomial interest rate tree permits the short interest rate to take on one of two possible values consistent with the volatility assumption and an interest rate model based on a lognormal random walk.
- An interest rate tree is a visual representation of the possible values of interest rates (forward rates) based on an interest rate model and an assumption about interest rate volatility.
- The possible interest rates for any following period are consistent with the following three assumptions: (1) an interest rate model that governs the random process of interest rates, (2) the assumed level of interest rate volatility, and (3) the current benchmark yield curve.
- From the lognormal distribution, adjacent interest rates on the tree are multiples of e raised to the 2σ power, with the absolute change in interest rates becoming smaller and smaller as rates approach zero.
- We use the backward induction valuation methodology that involves starting at maturity, filling in those values, and working back from right to left to find the bond's value at the desired node.

- The interest rate tree is fit to the current yield curve by choosing interest rates that result in the benchmark bond value. By doing this, the bond value is arbitrage free.
- An option-free bond that is valued by using the binomial interest rate tree should have the same value as when discounting by the spot rates.
- Pathwise valuation calculates the present value of a bond for each possible interest rate path and takes the average of these values across paths.
- The Monte Carlo method is an alternative method for simulating a sufficiently large number of potential interest rate paths in an effort to discover how the value of a security is affected, and it involves randomly selecting paths in an effort to approximate the results of a complete pathwise valuation.
- Term structure models seek to explain the yield curve shape and are used to value bonds (including those with embedded options) and bond-related derivatives. General equilibrium and arbitrage-free models are the two major types of such models.
- Arbitrage-free models are frequently used to value bonds with embedded options. Unlike equilibrium models, arbitrage-free models begin with the observed market prices of a reference set of financial instruments, and the underlying assumption is that the reference set is correctly priced.

PRACTICE PROBLEMS

The following information relates to questions 1-6

Katrina Black, a portfolio manager at Coral Bond Management, Ltd., is conducting a training session with Alex Sun, a junior analyst in the fixed-income department. Black wants to explain to Sun the arbitrage-free valuation framework used by the firm. Black presents Sun with Exhibit 1, showing a fictitious bond being traded on three exchanges, and asks Sun to identify the arbitrage opportunity of the bond. Sun agrees to ignore transaction costs in his analysis.

Exhibit 1: Three-Year, €100 par, 3.00% Coupon, Annual Pay Option-Free Bond

	Eurex	NYSE Euronext	Frankfurt
Price	€103.7956	€103.7815	€103.7565

Black shows Sun some exhibits that were part of a recent presentation. Exhibit 3 presents most of the data of a binomial lognormal interest rate tree fit to the yield curve shown in Exhibit 2. Exhibit 4 presents most of the data of the implied values for a four-year, option-free, annual pay bond with a 2.5% coupon based on the information in Exhibit 3.

Exhibit 2: Yield-to-Maturity Par Rates for One-, Two-, and Three-Year Annual Pay Option-Free Bonds

One-year	Two-year	Three-year
1.25%	1.50%	1.70%

Exhibit 3: Binomial Interest Rate Tree Fit to the Yield Curve (Volatility = 10%)

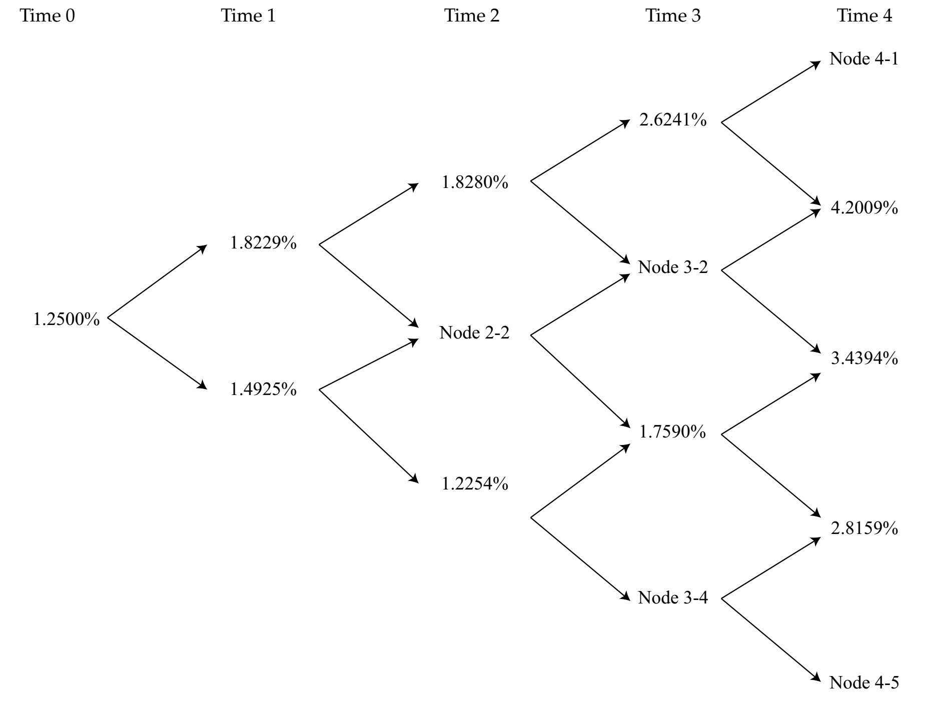
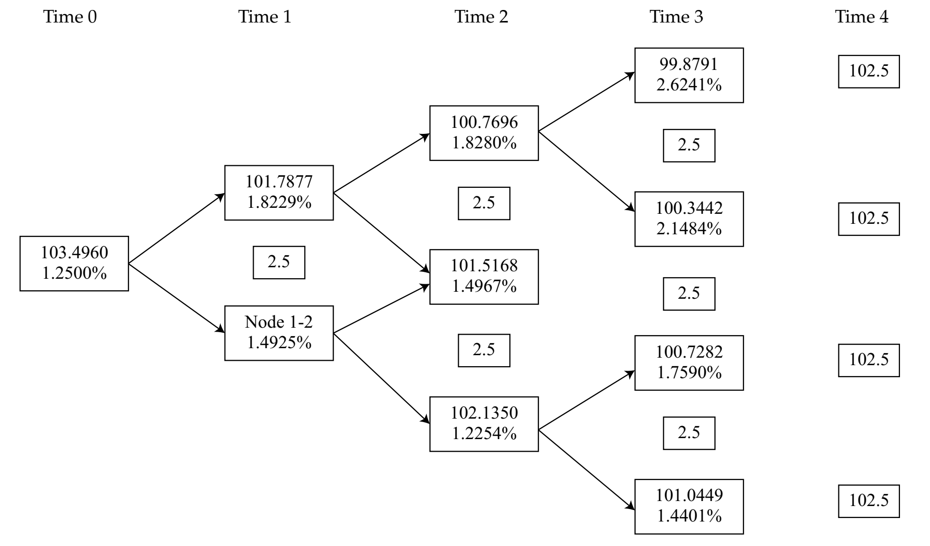


Exhibit 4: Implied Values (in Euros) for a 2.5%, Four-Year, Option-Free, Annual Pay Bond Based on Exhibit 3



Black asks about the missing data in Exhibits 3 and 4 and directs Sun to complete the following tasks related to those exhibits:

Task 1	Test that the binomial interest tree has been properly calibrated to be arbitrage free.
Task 2	Develop a spreadsheet model to calculate pathwise valuations. To test the accuracy of the spreadsheet, use the data in Exhibit 3 and calculate the value of the bond if it takes a path of lowest rates in Year 1 and Year 2 and the second lowest rate in Year 3.
Task 3	Identify a type of bond where the Monte Carlo calibration method should be used in place of the binomial interest rate method.
Task 4	Update Exhibit 3 to reflect the current volatility, which is now 15%.

1. Based on Exhibit 1, the *best* action that an investor should take to profit from the arbitrage opportunity is to:

   A. buy on Frankfurt, sell on Eurex.

   B. buy on NYSE Euronext, sell on Eurex.

   C. buy on Frankfurt, sell on NYSE Euronext.

2. Based on Exhibits 1 and 2, the exchange that reflects the arbitrage-free price of the bond is:

   A. Eurex.

   B. Frankfurt.

   C. NYSE Euronext.

3. Recall from the reading that each node is represented by both a time element and a rate change component. Which of the following statements about the missing data in Exhibit 3 is correct?

   A. Node 3–2 can be derived from Node 2–2.

   B. Node 4–1 should be equal to Node 4–5 multiplied by $e^{0.4}$.

   C. Node 2–2 approximates the implied one-year forward rate two years from now.

4. Based on the information in Exhibits 3 and 4, the bond price in euros at Node 1–2 in Exhibit 4 is *closest* to:

   A. 102.7917.

   B. 104.8640.

   C. 105.2917.

5. A benefit of performing Task 1 is that it:

   A. enables the model to price bonds with embedded options.

   B. identifies benchmark bonds that have been mispriced by the market.

   C. allows investors to realize arbitrage profits through stripping and reconstitution.

6. If the assumed volatility is changed as Black requested in Task 4, the forward

rates shown in Exhibit 3 will *most likely*:

A. spread out.

B. remain unchanged.

C. converge to the spot rates.

The following information relates to questions 7-10

Betty Tatton is a fixed-income analyst with the hedge fund Sailboat Asset Management (SAM). SAM invests in a variety of global fixed-income strategies, including fixed-income arbitrage. Tatton is responsible for pricing individual investments and analyzing market data to assess the opportunity for arbitrage. She uses two methods to value bonds:

Method 1 Discount each year's cash flow separately using the appropriate interest rate curve.

Method 2 Build and use a binomial interest rate tree.

Tatton compiles pricing data for a list of annual pay bonds (Exhibit 1). Each of the bonds will mature in two years, and Tatton considers the bonds risk-free; both the one-year and two-year benchmark spot rates are 2%. Tatton calculates the arbitrage-free prices and identifies an arbitrage opportunity to recommend to her team.

Exhibit 1: Market Data for Selected Bonds

Asset	Coupon	Market Price
Bond A	1%	98.0584
Bond B	3%	100.9641
Bond C	5%	105.8247

Next, Tatton uses the benchmark yield curve provided in Exhibit 2 to consider arbitrage opportunities of both option-free corporate bonds and corporate bonds with embedded options. The benchmark bonds in Exhibit 2 pay coupons annually, and the bonds are priced at par.

Exhibit 2: Benchmark Par Curve

Maturity (years)	Yield-to-Maturity (YTM)
1	3.0%
2	4.0%
3	5.0%

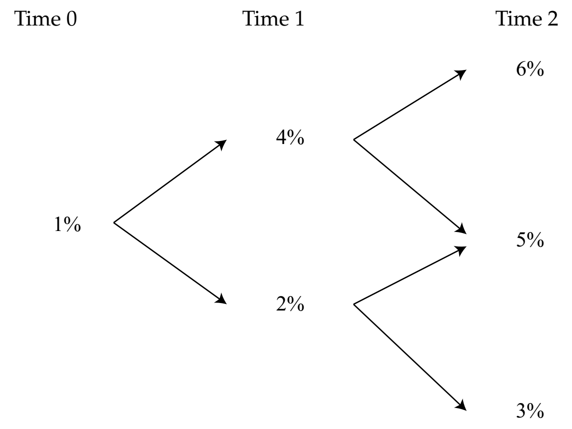
Tatton then identifies three mispriced three-year annual coupon bonds and compiles data on the bonds (see Exhibit 3).

Exhibit 3: Market Data of Annual Pay Corporate Bonds

Company	Coupon	Market Price	Yield	Embedded Option?
Hutto-Barkley Inc.	3%	94.9984	5.6%	No
Luna y Estrellas Intl.	0%	88.8996	4.0%	Yes
Peaton Scorpio Motors	0%	83.9619	6.0%	No

Lastly, Tatton identifies two mispriced Swiss bonds, Bond X, a three-year bond, and Bond Y, a five-year bond. Both are 6% annual coupon bonds. To calculate the bonds' values, Tatton devises the first three years of the interest rate lognormal tree presented in Exhibit 4 using historical interest rate volatility data. Tatton considers how these data would change if implied volatility, which is higher than historical volatility, were used instead.

Exhibit 4: Interest Rate Tree—Forward Rates Based on Swiss Market



7. Based on Exhibit 1, which of the following bonds *most likely* includes an arbitrage opportunity?
   A. Bond A
   B. Bond B
   C. Bond C

8. Based on Exhibits 2 and 3 and using Method 1, the amount (in absolute terms) by which the Hutto-Barkley Inc. corporate bond is mispriced is *closest* to:
   A. 0.3368 per 100 of par value.
   B. 0.4682 per 100 of par value.
   C. 0.5156 per 100 of par value.

9. Method 1 would *most likely* not be an appropriate valuation technique for the bond issued by:
   A. Hutto-Barkley Inc.

B. Luna y Estrellas Intl.

C. Peaton Scorpio Motors.

10. Based on Exhibit 4 and using Method 2, the correct price for Bond X is *closest* to:

A. 97.2998.

B. 109.0085.

C. 115.0085.

The following information relates to questions 11-19

Meredith Alvarez is a junior fixed-income analyst with Canzim Asset Management. Her supervisor, Stephanie Hartson, asks Alvarez to review the asset price and payoff data shown in Exhibit 1 to determine whether an arbitrage opportunity exists.

Exhibit 1: Price and Payoffs for Two Risk-Free Assets

Asset	Price Today	Payoff in One Year
Asset A	$500	$525
Asset B	$1,000	$1,100

Hartson also shows Alvarez data for a bond that trades in three different markets in the same currency. These data appear in Exhibit 2.

Exhibit 2: 2% Coupon, Five-Year Maturity, Annual Pay Bond

	New York	Hong Kong	Mumbai
Yield-to-Maturity	1.9%	2.3%	2.0%

Hartson asks Alvarez to value two bonds (Bond C and Bond D) using the binomial tree in Exhibit 3. Exhibit 4 presents selected data for both bonds.

Exhibit 3: Binomial Interest Rate Tree with Volatility = 25%

Time 0	Time 1	Time 2
		2.7183%
	2.8853%	
1.500%		1.6487%

Time 0	Time 1	Time 2
	1.7500%	
		1.0000%

Exhibit 4: Selected Data on Annual Pay Bonds

Bond	Maturity	Coupon Rate
Bond C	2 years	2.5%
Bond D	3 years	3.0%

Hartson tells Alvarez that she and her peers have been debating various viewpoints regarding the conditions underlying binomial interest rate trees. The following statements were made in the course of the debate.

Statement 1 The only requirements needed to create a binomial interest rate tree are current benchmark interest rates and an assumption about interest rate volatility.

Statement 2 Potential interest rate volatility in a binomial interest rate tree can be estimated using historical interest rate volatility or observed market prices from interest rate derivatives.

Statement 3 A bond value derived from a binomial interest rate tree with a relatively high volatility assumption will be different from the value calculated by discounting the bond's cash flows using current spot rates.

Based on data in Exhibit 5, Hartson asks Alvarez to calibrate a binomial interest rate tree starting with the calculation of implied forward rates shown in Exhibit 6.

Exhibit 5: Selected Data for a Binomial Interest Rate Tree

Maturity	Par Rate	Spot Rate
1	2.5000%	2.5000%
2	3.5000%	3.5177%

Exhibit 6: Calibration of Binomial Interest Rate Tree with Volatility = 25%

Time 0	Time 1
	5.8365%
2.500%	
	Lower one-period forward rate

Hartson mentions pathwise valuations as another method to value bonds using a binomial interest rate tree. Using the binomial interest rate tree in Exhibit 3, Alvarez calculates the possible interest rate paths for Bond D shown in Exhibit 7.

Exhibit 7: Interest Rate Paths for Bond D

Path	Time 0	Time 1	Time 2
1	1.500%	2.8853%	2.7183%
2	1.500	2.8853	1.6487
3	1.500	1.7500	1.6487
4	1.500	1.7500	1.0000

Before leaving for the day, Hartson asks Alvarez about the value of using the Monte Carlo method to simulate a large number of potential interest rate paths to value a bond. Alvarez makes the following statements.

Statement 4 Increasing the number of paths increases the estimate's statistical accuracy.

Statement 5 The bond value derived from a Monte Carlo simulation will be closer to the bond's true fundamental value.

11. Based on Exhibit 1, Alvarez finds that an arbitrage opportunity is:
    - **A.** not available.
    - **B.** available based on the dominance principle.
    - **C.** available based on the value additivity principle.

12. Based on the data in Exhibit 2, the *most* profitable arbitrage opportunity would be to buy the bond in:
    - **A.** Mumbai and sell it in Hong Kong.
    - **B.** Hong Kong and sell it in New York.
    - **C.** New York and sell it in Hong Kong.

13. Based on Exhibits 3 and 4, the value of Bond C at the upper node at Time 1 is *closest* to:
    - **A.** 97.1957.
    - **B.** 99.6255.
    - **C.** 102.1255.

14. Based on Exhibits 3 and 4, the price for Bond D is *closest* to:
    - **A.** 97.4785.
    - **B.** 103.3230.
    - **C.** 106.3230.

15. Which of the various statements regarding binomial interest rate trees is correct?
    - **A.** Statement 1
    - **B.** Statement 2

C. Statement 3

16. Based on Exhibits 5 and 6, the value of the lower one-period forward rate is *closest to*:

A. 3.5122%.

B. 3.5400%.

C. 4.8037%.

17. Based on Exhibits 4 and 7, the present value of Bond D's cash flows following Path 2 is *closest* to:

A. 97.0322.

B. 102.8607.

C. 105.8607.

18. Which of the statements regarding Monte Carlo simulation is correct?

A. Only Statement 4 is correct.

B. Only Statement 5 is correct.

C. Both Statement 4 and Statement 5 are correct.

19. Which term structure model can be calibrated to closely fit an observed yield curve?

A. The Ho–Lee model

B. The Vasicek model

C. The Cox–Ingersoll–Ross model

The following information relates to questions 20-21

Keisha Jones is a junior analyst at Sparling Capital. Julie Anderson, a senior partner and Jones's manager, meets with Jones to discuss interest rate models used for the firm's fixed-income portfolio.

Anderson begins the meeting by asking Jones to describe features of equilibrium and arbitrage-free term structure models. Jones responds by making the following statements:

Statement 1	Equilibrium term structure models are factor models that use the observed market prices of a reference set of financial instruments, assumed to be correctly priced, to model the market yield curve.
Statement 2	In contrast, arbitrage-free term structure models seek to describe the dynamics of the term structure by using fundamental economic variables that are assumed to affect interest rates.

Anderson then asks Jones about her preferences concerning term structure models. Jones states:

I prefer arbitrage-free models. Even though equilibrium models require fewer parameters to be estimated relative to arbitrage-free models, arbitrage-free models allow for time-varying parameters. In general, this allowance leads to arbitrage-free models being able to model the market yield curve more precisely than equilibrium models.

20. Which of Jones's statements regarding equilibrium and arbitrage-free term structure models is *incorrect*?

A. Statement 1 only

B. Statement 2 only

C. Both Statement 1 and Statement 2

21. Is Jones correct in describing key differences in equilibrium and arbitrage-free models as they relate to the number of parameters and model accuracy?

A. Yes

B. No, she is incorrect about which type of model requires fewer parameter estimates.

C. No, she is incorrect about which type of model is more precise at modeling market yield curves.

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22. Which of the following statements comparing the Ho–Lee and Kalotay–Williams–Fabozzi (KWF) equilibrium term structure models is *correct*?

A. The Ho–Lee model assumes constant volatility, while the KWF model does not.

B. The KWF model incorporates the possibility of negative rates, while the Ho–Lee model does not.

C. The KWF model describes the log of the dynamics of the short rate, while the Ho–Lee model does not.

SOLUTIONS

1. A is correct. This is the same bond being sold at three different prices, so an arbitrage opportunity exists by buying the bond from the exchange where it is priced lowest and immediately selling it on the exchange that has the highest price. Accordingly, an investor would maximize profit from the arbitrage opportunity by buying the bond on the Frankfurt exchange (which has the lowest price, €103.7565) and selling it on the Eurex exchange (which has the highest price, €103.7956) to generate a risk-free profit of €0.0391 (as mentioned, ignoring transaction costs) per €100 par.

   B is incorrect because buying on NYSE Euronext and selling on Eurex would result in a €0.0141 profit per €100 par (€103.7956 – €103.7815 = €0.0141), which is not the maximum arbitrage profit available. A greater profit would be realized if the bond were purchased in Frankfurt and sold on Eurex.

   C is incorrect because buying on Frankfurt and selling on NYSE Euronext would result in an €0.0250 profit per €100 par (€103.7815 – €103.7565 = €0.0250). A greater profit would be realized if the bond were purchased in Frankfurt and sold on Eurex.

2. C is correct. The bond from Exhibit 1 is selling for its calculated value on the NYSE Euronext exchange. The arbitrage-free value of a bond is the present value of its cash flows discounted by the spot rate for zero-coupon bonds maturing on the same date as each cash flow. The value of this bond, 103.7815, is calculated as follows:

	Year 1	Year 2	Year 3	Total PV
Yield-to-maturity	1.2500%	1.500%	1.700%	
Spot rate[1]	1.2500%	1.5019%	1.7049%	
Cash flow	3.00	3.00	103.00	
Present value of payment[2]	2.9630	2.9119	97.9066	103.7815

	Eurex	NYSE Euronext	Frankfurt
Price	€103.7956	€103.7815	€103.7565
Mispricing (per 100 par value)	0.141	0	–0.025

Notes:
(1) Spot rates are calculated using bootstrapping. For example, Year 2 spot rate (z_2): $100 = 1.5/1.0125 + 101.5/(1 + z_2)^2$; $z_2 = 0.015019$.
(2) Present value calculated using the formula $PV = FV/(1 + r)^n$, where n = number of years until cash flow, FV = cash flow amount, and r = spot rate.

   A is incorrect because the price on the Eurex exchange, €103.7956, was calculated using the yield-to-maturity rate to discount the cash flows when the spot rates should have been used. C is incorrect because the price on the Frankfurt exchange, €103.7565, uses the Year 3 spot rate to discount all the cash flows.

3. C is correct. Because Node 2–2 is the middle node rate in Year 2, it will be close to the implied one-year forward rate two years from now (as derived from the spot curve). Node 4–1 should be equal to the product of Node 4–5 and $e^{0.8}$. Lastly, Node 3–2 cannot be derived from Node 2–2; it can be derived from any other Year 3 node; for example, Node 3–2 can be derived from Node 3–4 (equal to the product of Node 3–4 and $e^{4\sigma}$).

4. A is correct. The value of a bond at a particular node, in this case Node 1–2, can be derived by determining the present value of the coupon payment and expected future bond values to the right of that node on the tree. In this case, those two nodes are the middle node in Year 2, equal to 101.5168, and the lower node in Year 2, equal to 102.1350. The coupon payment is 2.5. The bond value at Node 1–2 is calculated as follows:

$$\text{Value} = \frac{2.5 + (0.5 \times 101.5168 + 0.5 \times 102.1350)}{1.014925}$$

$$= 102.7917.$$

5. A is correct. Calibrating a binomial interest rate tree to match a specific term structure is important because we can use the known valuation of a benchmark bond from the spot rate pricing to verify the accuracy of the rates shown in the binomial interest rate tree. Once its accuracy is confirmed, the interest rate tree can then be used to value bonds with embedded options. While discounting with spot rates will produce arbitrage-free valuations for option-free bonds, this spot rate method will not work for bonds with embedded options where expected future cash flows are interest-rate dependent (because rate changes impact the likelihood of options being exercised). The interest rate tree allows for the alternative paths that a bond with embedded options might take.

   B is incorrect because calibration does not identify mispriced benchmark bonds. In fact, benchmark bonds are employed to prove the accuracy of the binomial interest rate tree, because they are assumed to be correctly priced by the market.

   C is incorrect because the calibration of the binomial interest rate tree is designed to produce an arbitrage-free valuation approach and such an approach does not allow a market participant to realize arbitrage profits through stripping and reconstitution.

6. A is correct. Volatility is one of the two key assumptions required to estimate rates for the binomial interest rate tree. Increasing the volatility from 10% to 15% would cause the possible forward rates to spread out on the tree because it increases the exponent in the relationship multiple between nodes ($e^{x\sigma}$, where x = 2 times the number of nodes above the lowest node in a given year in the interest rate tree). Conversely, using a lower estimate of volatility would cause the forward rates to narrow or converge to the implied forward rates from the prevailing yield curve.

   B is incorrect because volatility is a key assumption in the binomial interest rate tree model. Any change in volatility will cause a change in the implied forward rates.

   C is incorrect because increasing the volatility from 10% to 15% causes the possible forward rates to spread out on the tree, not converge to the implied forward rates from the current yield curve. Rates will converge to the implied forward rates when lower estimates of volatility are assumed.

7. B is correct. Bond B's arbitrage-free price is calculated as follows:

$$\frac{3}{1.02} + \frac{103}{1.02^2} = 101.9416,$$

   which is higher than the bond's market price of 100.9641. Therefore, an arbitrage opportunity exists. Since the bond's value (100.9641) is less than the sum of the values of its discounted cash flows individually (101.9416), a trader would perceive an arbitrage opportunity and could buy the bond while selling claims to the individual cash flows (zeros), capturing the excess value. The arbitrage-free prices of Bond A and Bond C are equal to the market prices of the respective bonds, so there is no arbitrage opportunity for these two bonds:

Bond A: $\frac{1}{1.02} + \frac{101}{1.02^2} = 98.0584.$

Bond C: $\frac{5}{1.02} + \frac{105}{1.02^2} = 105.8247.$

8. C is correct. The first step in the solution is to find the correct spot rate (zero-coupon rates) for each year's cash flow. The benchmark bonds in Exhibit 2 are conveniently priced at par so the yields-to-maturity and the coupon rates on the bonds are the same. Because the one-year issue has only one cash flow remaining, the YTM equals the spot rate of 3% (or z_1 = 3%). The spot rates for Year 2 (z_2) and Year 3 (z_3) are calculated as follows:

$$100 = \frac{4}{1.0300} + \frac{104}{(1 + z_2)^2};\ z_2 = 4.02\%.$$

$$100 = \frac{5}{1.0300} + \frac{5}{(1.0402)^2} + \frac{105}{(1 + z_3)^3};\ z_3 = 5.07\%.$$

The correct arbitrage-free price for the Hutto-Barkley Inc. bond is

$$P_0 = \frac{3}{(1.0300)} + \frac{3}{(1.0402)^2} + \frac{103}{(1.0507)^3} = 94.4828.$$

Therefore, the bond is mispriced by 94.9984 – 94.4828 = 0.5156 per 100 of par value.

A is incorrect because the correct spot rates are not calculated and instead the Hutto-Barkley Inc. bond is discounted using the respective YTM for each maturity. Therefore, this leads to an incorrect mispricing of 94.6616 – 94.9984 = –0.3368 per 100 of par value.

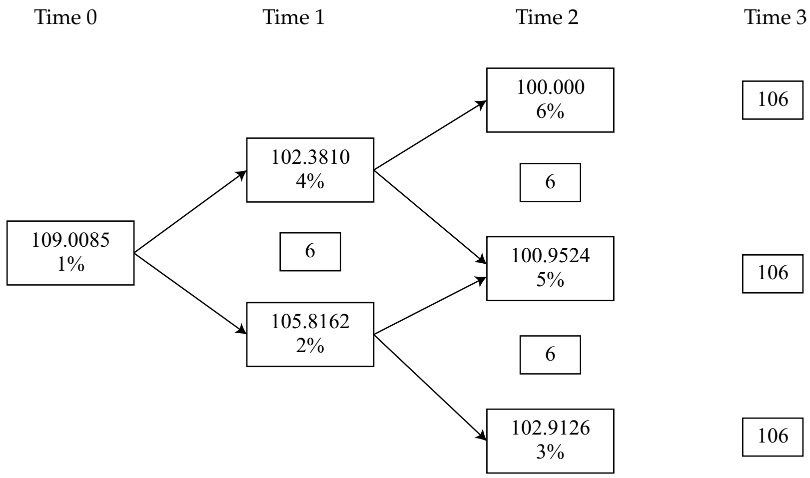
B is incorrect because the spot rates are derived using the coupon rate for Year 3 (maturity) instead of using each year's respective coupon rate to employ the bootstrap methodology. This leads to an incorrect mispricing of 94.5302 – 94.9984 = –0.4682 per 100 of par value.

9. B is correct. The Luna y Estrellas Intl. bond contains an embedded option. Method 1 will produce an arbitrage-free valuation for option-free bonds; however, for bonds with embedded options, changes in future interest rates impact the likelihood the option will be exercised and so impact future cash flows. Therefore, to develop a framework that values bonds with embedded options, interest rates must be allowed to take on different potential values in the future based on some assumed level of volatility (Method 2).

A and C are incorrect because the Hutto-Barkley Inc. bond and the Peaton Scorpio Motors bond are both option-free bonds and can be valued using either Method 1 or Method 2 to produce an arbitrage-free valuation.

10. B is correct. This is the binomial tree that obtains a bond value of 109.0085.

Exhibit 5: Valuing a 6%, Three-Year Bond



These are the calculations:

106/1.06 = 100.0000.

106/1.05 = 100.9524.

106/1.03 = 102.9126.

$$\frac{6 + (0.5\text{x}100.0000 + 0.5\text{x}100.9524)}{1.04} = 102.3810.$$

$$\frac{6 + (0.5\text{x}100.9524 + 0.5\text{x}102.9126)}{1.02} = 105.8162.$$

$$\frac{6 + (0.5\text{x}102.3810 + 0.5\text{x}105.8162)}{1.01} = 109.0085.$$

A is incorrect because the Time T coupon payment is subtracted from the value in each node calculation for Time T. C is incorrect because it assumes that a coupon is paid at Time 0.

11. B is correct. Based on the dominance principle, an arbitrage opportunity exists. The dominance principle asserts that a financial asset with a risk-free payoff in the future must have a positive price today. Because Asset A and Asset B are both risk-free assets, they should have the same discount rate. Relative to its payoff, Asset A is priced at $500/525, or 0.95238, and Asset B is priced at $1,000/1,100, or 0.90909. Given its higher implied discount rate (10%) and lower corresponding price, Asset B is cheap relative to Asset A, which has a lower implied discount rate (5%) and a higher corresponding price.

    The arbitrage opportunity based on dominance is to sell two units of Asset A for $1,000 and buy one unit of Asset B. There is no cash outlay today, and in one year, the portfolio delivers a net cash inflow of $50 [= $1,100 – (2 × $525)].

12. B is correct. Of the three markets, the New York bond has the lowest yield-to-maturity and, correspondingly, the highest bond price. Similarly, the Hong Kong bond has the highest yield-to-maturity and the lowest bond price of the three markets. Therefore, the most profitable arbitrage trade would be to buy the bond in Hong Kong and sell it in New York.

13. B is correct. The bond value at the upper node at Time 1 is closest to 99.6255. The cash flow at Time 2 is 102.5, the redemption of par value (100) plus the final coupon payment (2.5). Using backward induction, we calculate the present value of the bond at the upper node of Time 1 as 102.5/1.028853 = 99.6255.

14. B is correct. The price of Bond D is closest to 103.3230 and can be calculated using backward induction.

 Bond value at a node $= \dfrac{C + (0.5 \times VH + 0.5 \times VL)}{1 + i}$.

 Calculations:

 The cash flow at Time 3 is 103, the redemption of par value (100) plus the final coupon payment (3).

 Time 2 node values:

 Upper node: 103/1.027183 = 100.2742.

 Middle node: 103/1.016487 = 101.3294.

 Lower node: 103/1.010000 = 101.9802.

 Working back to Time 1 requires the use of the general expression above.

 Time 1 node values:

 Upper node: $\dfrac{3 + (0.5 \times 100.2742 + 0.5 \times 101.3294)}{1.028853} = 100.8908.$

 Lower node: $\dfrac{3 + (0.5 \times 101.3294 + 0.5 \times 101.9802)}{1.0175} = 102.8548.$

 Time 0 node value:

 $$\frac{3 + (0.5 \times 100.8908 + 0.5 \times 102.8548)}{1.015} = 103.3230.$$

 Therefore, the price of the bond is 103.3230.

15. B is correct. Two methods are commonly used to estimate potential interest rate volatility in a binomial interest rate tree. The first method bases estimates on historical interest rate volatility. The second method uses observed market prices of interest rate derivatives.

 Statement 1 is incorrect because there are three requirements to create a binomial interest rate tree, not two. The third requirement is an assumption regarding the interest rate model. Statement 3 is incorrect because the valuation of a bond using spot rates and the valuation of a bond from an interest rate tree will be the same regardless of the volatility assumption used in the model.

16. B is correct. The value of the lower one-period forward rate is closest to 3.5400%. Since the higher one-period forward rate is 5.8365% and interest rate volatility is 25%, the lower rate equals the higher rate multiplied by $e^{-2\sigma}$. This is calculated as $0.058365 \times e^{-0.50} = 0.035400$.

17. B is correct. The present value of Bond D's cash flows following Path 2 is 102.8607 and can be calculated as follows:

 $$\frac{3}{1.015} + \frac{3}{(1.015)(1.028853)} + \frac{103}{(1.015)(1.028853)(1.016487)} = 102.8607.$$

18. A is correct. Increasing the number of paths using the Monte Carlo method does

increase the estimate's statistical accuracy. It does not, however, provide a value that is closer to the bond's true fundamental value.

19. A is correct. The Ho–Lee model is arbitrage free and can be calibrated to closely match the observed term structure.

20. C is correct. Both statements are incorrect because Jones incorrectly describes both types of model. Equilibrium term structure models are factor models that seek to describe the dynamics of the term structure by using fundamental economic variables that are assumed to affect interest rates. Arbitrage-free term structure models use observed market prices of a reference set of financial instruments, assumed to be correctly priced, to model the market yield curve.

21. A is correct. Consistent with Jones's statement, equilibrium term structure models require fewer parameters to be estimated relative to arbitrage-free models, and arbitrage-free models allow for time-varying parameters. Consequently, arbitrage-free models can model the market yield curve more precisely than equilibrium models.

22. C is correct. The Kalotay–Williams–Fabozzi equilibrium term structure model is similar to the Ho–Lee model in that it assumes constant drift, no mean reversion, and constant volatility, but the KWF model describes the log of the dynamics of the short rate, while the Ho–Lee model does not.

LEARNING MODULE

3

Valuation and Analysis of Bonds with Embedded Options

by Leslie AbreoMFE, Ioannis Georgiou, CFA, and Andrew Kalotay, PhD.

Leslie Abreo, MFE, is at ICE Data Analytics LLC (USA). Ioannis Georgiou, CFA, is at Finovex.com (Cyprus). Andrew Kalotay, PhD, is at Andrew Kalotay Advisors, Inc. (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	describe fixed-income securities with embedded options
☐	explain the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option
☐	describe how the arbitrage-free framework can be used to value a bond with embedded options
☐	explain how interest rate volatility affects the value of a callable or putable bond
☐	explain how changes in the level and shape of the yield curve affect the value of a callable or putable bond
☐	calculate the value of a callable or putable bond from an interest rate tree
☐	explain the calculation and use of option-adjusted spreads
☐	explain how interest rate volatility affects option-adjusted spreads
☐	calculate and interpret effective duration of a callable or putable bond
☐	compare effective durations of callable, putable, and straight bonds
☐	describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options
☐	compare effective convexities of callable, putable, and straight bonds
☐	calculate the value of a capped or floored floating-rate bond
☐	describe defining features of a convertible bond

The presentation of the binomial trees in this reading was revised to conform with other readings in 2018 & 2019 by Donald J. Smith, PhD, Boston University (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	calculate and interpret the components of a convertible bond's value
☐	describe how a convertible bond is valued in an arbitrage-free framework
☐	compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock

1 INTRODUCTION

☐	describe fixed-income securities with embedded options

The valuation of a fixed-rate, option-free bond generally requires determining its future cash flows and discounting them at the appropriate rates. Valuation becomes more complicated when a bond has one or more embedded options because the values of embedded options are typically contingent on interest rates.

Understanding how to value and analyze bonds with embedded options is important for practitioners. Issuers of bonds often manage interest rate exposure with embedded options, such as call provisions. Investors in callable bonds must appreciate the risk of being called. The perception of this risk is collectively represented by the premium, in terms of increased coupon or yield, that the market demands for callable bonds relative to otherwise identical option-free bonds. Issuers and investors must also understand how other types of embedded options—such as put provisions, conversion options, caps, and floors—affect bond values and the sensitivity of these bonds to interest rate movements.

We first provide a brief overview of various types of embedded options. We then discuss bonds that include a call or put provision. Taking a building-block approach, we show how the arbitrage-free valuation framework discussed earlier can be applied to the valuation of callable and putable bonds—first in the absence of interest rate volatility, and then when interest rates fluctuate. We also discuss how option-adjusted spreads are used to value risky callable and putable bonds. We then turn to interest rate sensitivity. It highlights the need to use effective duration, including one-sided durations and key rate durations, as well as effective convexity to assess the effect of interest rate movements on the value of callable and putable bonds. We also explain the valuation of capped and floored floating-rate bonds (floaters) and convertible bonds.

Overview of Embedded Options

The term "embedded bond options" or **embedded options** refers to contingency provisions found in the bond's indenture or offering circular. These options represent rights that enable their holders to take advantage of interest rate movements. They can be exercised by the issuer or the bondholder, or they may be exercised automatically depending on the course of interest rates. For example, a call option allows the issuer to benefit from lower interest rates by retiring the bond issue early and refinancing at a lower cost. In contrast, a put option allows the bondholder to benefit from higher interest rates by putting back the bonds to the issuer and reinvesting the proceeds of the retired bond at a higher yield. These options are not independent of the bond and thus cannot be traded separately—hence the adjective "embedded." In this section, we provide a review of familiar embedded options.

Corresponding to every embedded option, or combination of embedded options, is an underlying bond with a specified issuer, issue date, maturity date, principal amount and repayment structure, coupon rate and payment structure, and currency denomination. We also refer to this underlying option-free bond as the **straight bond**. The coupon of an underlying bond can be fixed or floating. Fixed-coupon bonds may have a single rate for the life of the bond, or the rate may step up or step down according to a coupon schedule. The coupons of floaters are reset periodically according to a formula based on a reference rate plus a credit spread—for example, Market reference rate + 100 basis points (bps). Except when we discuss capped and floored floaters, our focus is on fixed-coupon, single-rate bonds, also referred to as fixed-rate bonds.

Simple Embedded Options

Call and put options are standard examples of embedded options. In fact, the vast majority of bonds with embedded options are callable, putable, or both. The call provision is by far the most prevalent type of embedded option.

Call Options

A **callable bond** is a bond that includes an embedded call option. The call option is an issuer option; that is, the right to exercise the option is at the discretion of the bond's issuer. The call provision allows the issuer to redeem the bond issue prior to maturity. Early redemption usually happens when the issuer has the opportunity to replace a high-coupon bond with another bond that has more favorable terms, typically when interest rates have fallen or when the issuer's credit quality has improved.

Until the 1990s, most long-term corporate bonds in the United States were callable after either 5 or 10 years. The initial call price (exercise price) was typically at a premium above par, the premium depended on the coupon, and the call price gradually declined to par a few years prior to maturity. Today, most investment-grade corporate bonds are essentially non-refundable. They may have a "make-whole call," so named because the call price is such that the bondholders are more than "made whole" (compensated) in exchange for surrendering their bonds. The call price is calculated at a narrow spread to a benchmark security—usually an on-the-run sovereign bond, such as Treasuries in the United States or gilts in the United Kingdom. Thus, economical refunding is virtually out of the question. Investors need have no fear of receiving less than their bonds are worth.

Most callable bonds include a call **protection period** during which the issuer cannot call the bond. For example, a 10-year callable bond may have a call protection period of three years, meaning that the first potential call date is three years after the bond's issue date. Call protection periods may be as short as one month or extend to several years. For example, high-yield corporate bonds are often callable a few years after issuance. Holders of such bonds are usually less concerned about early redemption than about possible default. Of course, this perspective can change over the life of the bond—for example, if the issuer's credit quality improves.

Callable bonds include different types of call features. The issuer of a European-style callable bond can exercise the call option only once on the call date. An American-style callable bond is continuously callable at any time starting on the first call date. A Bermudan-style call option can be exercised only on a predetermined schedule on specified dates following the call protection period. These dates are specified in the bond's indenture or offering circular.

With a few exceptions, bonds issued by government-sponsored enterprises in the United States (e.g., Fannie Mae, Freddie Mac, Federal Home Loan Banks, and Federal Farm Credit Banks) are callable. These bonds tend to have relatively short maturities (5–10 years) and very short call protection periods (three months to one year). The call price is almost always at 100% of par, and the call option is often Bermudan style.

Tax-exempt municipal bonds (often called "munis"), a type of non-sovereign (local) government bond issued in the United States, are almost always callable at 100% of par any time after the end of the 10th year. They may also be eligible for advance refunding—a highly specialized topic that is not discussed here.

Although the bonds of US government-sponsored enterprises and municipal issuers account for most of the callable bonds issued and traded globally, bonds that include call provisions are also found in other countries in Asia Pacific, Europe, Canada, and Central and South America. The vast majority of callable bonds are denominated in US dollars or euros because of investors' demand for securities issued in these currencies. Australia, the United Kingdom, Japan, and Norway are examples of countries that have a market for callable bonds denominated in local currency.

Put Options and Extension Options

A **putable bond** is a bond that includes an embedded put option. The put option is an investor option; that is, the right to exercise the option is at the discretion of the bondholder. The put provision allows the bondholders to put back the bonds to the issuer prior to maturity, usually at par. This usually happens when interest rates have risen and higher-yielding bonds are available.

Similar to callable bonds, most putable bonds include protection periods. They can be European or, rarely, Bermudan style, but there are no American-style putable bonds.

Another type of embedded option that resembles a put option is an extension option. At maturity, the holder of an **extendible bond** (sometimes spelled "extendable") has the right to keep the bond for a number of years after maturity, possibly with a different coupon. In this case, the terms of the bond's indenture or offering circular are modified, but the bond remains outstanding. An example of a corporate extendible is an offering from Heathrow Funding Ltd. It pays a 0.50% coupon and matures on 17 May 2024. However, it is extendible to 7 May 2026 as a floating-rate note paying 12-month MRR plus 4.00%. We will discuss the resemblance between a putable and an extendible bond later.

Complex Embedded Options

Although callable and putable bonds are the most common types of bonds with embedded options, there are bonds with other types of options or combinations of options. For instance, some bonds can be both callable and putable. These bonds can be either called by the issuer or put by the bondholders.

Convertible bonds are another type of bond with an embedded option. The conversion option allows bondholders to convert their bonds into the issuer's common stock. Convertible bonds are usually also callable by the issuer; the call provision enables the issuer to take advantage of lower interest rates or to force conversion.

Another layer of complexity is added when the option is contingent on some particular event. An example is the estate put or survivor's option that may be available to retail investors. In the event of the holder's death, this bond can be put at par by the heir(s). Because the estate put comes into play only in the event of the bondholder's death, the value of a bond with an estate put is contingent on the life expectancy of its holder, which is uncertain.

Bonds may contain several interrelated issuer options without any investor option. A prime example is a **sinking fund bond** (sinker). A sinker requires the issuer to set aside funds over time to retire the bond issue, thus reducing credit risk. Such a bond may be callable and may also include options unique to sinking fund bonds, such as an acceleration provision and a delivery option.

SINKING FUND BONDS

The underlying bond has an amortizing structure—for example, a 30-year maturity with level annual principal repayments beginning at the end of the 11th year. In this case, each payment is 5% of the original principal amount. A typical sinking fund bond may include the following options:

- A standard *call option* above par, with declining premiums, starting at the end of Year 10. Thus, the entire bond issue could be called from Year 10 onward.
- An *acceleration provision*, such as a "triple up." Such a provision allows the issuer to repurchase at par three times the mandatory amount, or in this case 15% of the original principal amount, on any scheduled sinking fund date. Assume that the issuer wants to retire the bonds at the end of Year 11. Instead of calling the entire outstanding amount at a premium, it would be more cost effective to "sink" 15% at par and call the rest at a premium. Thus, the acceleration provision provides an additional benefit to the issuer if interest rates decline.
- A *delivery option*, which allows the issuer to satisfy a sinking fund payment by delivering bonds to the bond's trustee in lieu of cash. The bond's trustee is appointed by the issuer but acts in a fiduciary capacity with the bondholders. If the bonds are currently trading below par, say at 90% of par, it is more cost effective for the issuer to buy back bonds from investors to meet the sinking fund requirements than to pay par. The delivery option benefits the issuer if interest rates rise. Of course, the benefit can be materialized only if there is a liquid market for the bonds. Investors can take defensive action by accumulating the bonds and refusing to sell them at a discount.

From the issuer's perspective, the combination of the call option and the delivery option is effectively a "long straddle"—an option strategy involving the purchase of a put option and a call option on the same underlying with the same exercise price and expiration date. At expiration, if the underlying price is above the exercise price, the put option is worthless but the call option is in the money. In contrast, if the underlying price is below the exercise price, the call option is worthless but the put option is in the money. Thus, a long straddle benefits the investor when the underlying price moves up or down. The greater the move up or down (i.e., the greater the volatility), the greater the benefit for the investor. As a consequence, a sinking fund bond benefits the issuer not only if interest rates decline but also if they rise. Determining the combined value of the underlying bond and the three options is quite challenging.

EXAMPLE 1

Types of Embedded Options

1. Investors in putable bonds *most likely* seek to take advantage of:

   A. higher interest rates.

   B. improvements in the issuer's credit rating.

   C. movements in the price of the issuer's common stock.

Solution:

A is correct. A putable bond offers the bondholder the ability to take advantage of a rise in interest rates by putting back the bond to the issuer and reinvesting the proceeds of the retired bond in a higher-yielding bond.

2. The conversion option in a convertible bond is a right held by:

   A. the issuer.

   B. the bondholders.

   C. the issuer and the bondholders jointly.

Solution:

B is correct. A conversion option is a call option that gives the bondholders the right to convert their bonds into the issuer's common stock.

The presence of embedded options affects a bond's value. To quantify this effect, financial theory and financial technology come into play. The following section presents basic valuation and analysis concepts for bonds with embedded options.

2 CALLABLE AND PUTABLE BONDS

- ☐ explain the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the embedded option
- ☐ describe how the arbitrage-free framework can be used to value a bond with embedded options

Under the arbitrage-free framework, the value of a bond with embedded options is equal to the sum of the arbitrage-free values of its parts. We first identify the relationships between the values of a callable or putable bond, the underlying option-free (straight) bond, and the call or put option. We then discuss how to value callable and putable bonds under different risk and interest rate volatility scenarios.

Relationships between the Values of a Callable or Putable Bond, Straight Bond, and Embedded Option

The value of a bond with embedded options is equal to the sum of the arbitrage-free value of the straight bond and the arbitrage-free values of the embedded options.

For a callable bond, the decision to exercise the call option is made by the issuer. Thus, the investor is long the bond but short the call option. From the investor's perspective, therefore, the value of the call option *decreases* the value of the callable bond relative to the value of the straight bond:

Value of callable bond = Value of straight bond – Value of issuer call option.

The value of the straight bond can be obtained by discounting the bond's future cash flows at the appropriate rates. The hard part is valuing the call option because its value is contingent on future interest rates. Specifically, the issuer's decision to call the bond depends on its ability to refinance at a lower cost. In practice, the value of the call option is often calculated as the difference between the value of the straight bond and the value of the callable bond:

Value of issuer call option

= Value of straight bond – Value of callable bond. (1)

For a putable bond, the decision to exercise the put option is made by the investor. Thus, the investor has a long position in both the bond and the put option. As a consequence, the value of the put option *increases* the value of the putable bond relative to the value of the straight bond.

Value of putable bond = Value of straight bond + Value of investor put option.

It follows that

Value of investor put option

= Value of putable bond – Value of straight bond. (2)

Although most investment professionals do not need to be experts in bond valuation, they should have a solid understanding of the basic analytical approach, which is presented in the following sections.

Valuation of Default-Free and Option-Free Bonds: A Refresher

An asset's value is the present value of the cash flows the asset is expected to generate in the future. In the case of a default-free and option-free bond, the future cash flows are, by definition, certain. Thus, the question is, at which rates should these cash flows be discounted? The answer is that each cash flow should be discounted at the spot rate corresponding to the cash flow's payment date. Although spot rates might not be directly observable, they can be inferred from readily available information, usually from the market prices of actively traded on-the-run sovereign bonds of various maturities. These prices can be transformed into spot rates, par rates (i.e., coupon rates of hypothetical bonds of various maturities selling at par), or forward rates. Recall from Level I that spot rates, par rates, and forward rates are equivalent ways of conveying the same information; knowing any one of them is sufficient to determine the others.

Suppose we want to value a three-year 4.25% annual coupon bond. Exhibit 1 provides the equivalent forms of a yield curve with maturities of one, two, and three years.

Exhibit 1: Equivalent Forms of a Yield Curve

Maturity (year)	Par Rate (%)	Spot Rate (%)	One-Year Forward Rate (%)	
1	2.500	2.500	0 years from now	2.500
2	3.000	3.008	1 year from now	3.518
3	3.500	3.524	2 years from now	4.564

We start with the par rates provided in the second column of Exhibit 1. Because we are assuming annual coupons and annual compounding, the one-year spot rate is simply the one-year par rate. The hypothetical one-year par bond implied by the given par rate has a single cash flow of 102.500 (principal plus coupon) in Year 1. In order to have a present value of par, this future cash flow must be divided by 1.025. Thus, the one-year spot rate or discount rate is 2.500% (*Note:* All cash flows and values are expressed as a percentage of par).

A two-year 3.000% par bond has two cash flows: 3 in Year 1 and 103 in Year 2. By definition, the sum of the two discounted cash flows must equal 100. We know that the discount rate appropriate for the first cash flow is the one-year spot rate (2.500%). We now solve the following equation to determine the two-year spot rate (z_2):

$$\frac{3}{(1.025)} + \frac{103}{(1+z_2)^2} = 100.$$

We can follow a similar approach to determine the three-year spot rate (z_3):

$$\frac{3.500}{(1.02500)} + \frac{3.500}{(1.03008)^2} + \frac{103.500}{(1+z_3)^3} = 100.$$

The one-year forward rates are determined by using indifference equations. Assume an investor has a two-year horizon. She could invest for two years either at the two-year spot rate or at the one-year spot rate for one year and then reinvest the proceeds at the one-year forward rate one year from now ($F_{1,1}$). The result of investing using either of the two approaches should be the same. Otherwise, there would be an arbitrage opportunity. Thus,

$$(1 + 0.03008)^2 = (1 + 0.02500) \times (1 + F_{1,1}).$$

Similarly, the one-year forward rate two years from now ($F_{2,1}$) can be calculated using the following equation:

$$(1 + 0.03524)^3 = (1 + 0.03008)^2 \times (1 + F_{2,1}).$$

The three-year 4.25% annual coupon bond can now be valued using the spot rates:

$$\frac{4.25}{(1.02500)} + \frac{4.25}{(1.03008)^2} + \frac{104.25}{(1.03524)^3} = 102.114.$$

An equivalent way to value this bond is to discount its cash flows one year at a time using the one-year forward rates:

$$\frac{4.25}{(1.02500)} + \frac{4.25}{(1.02500)\,(1.03518)} + \frac{104.25}{(1.02500)\,(1.03518)\,(1.04564)} = 102.114.$$

Valuation of Default-Free Callable and Putable Bonds in the Absence of Interest Rate Volatility

When valuing bonds with embedded options, the approach relying on one-period forward rates provides a better framework than that relying on the spot rates because we need to know the value of the bond at different points in time in the future to determine whether the embedded option will be exercised at those points in time.

Valuation of a Callable Bond at Zero Volatility

Let us apply this framework to the valuation of a Bermudan-style three-year 4.25% annual coupon bond that is callable at par one year and two years from now. The decision to exercise the call option is made by the issuer. Because the issuer borrowed money, it will exercise the call option when the value of the bond's future cash flows is higher than the call price (exercise price). Exhibit 2 shows how to calculate the value of this callable bond using the one-year forward rates calculated in Exhibit 1.

Exhibit 2: Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at Zero Volatility

	Today	Year 1	Year 2	Year 3
Cash flow		4.250	4.250	104.250
Discount rate		2.500%	3.518%	4.564%
Value of the callable bond	$\frac{100 + 4.250}{1.02500} = 101.707$	$\frac{99.700 + 4.250}{1.03518} =$ ~~100.417~~ Called at 100	$\frac{104.250}{1.04564} = 99.700$ Not called	

We start by discounting the bond's cash flow at maturity (104.250) to Year 2 using the one-year forward rate two years from now (4.564%). The present value at Year 2 of the bond's future cash flows is 99.700. This value is lower than the call price of 100, so a rational borrower will not call the bond at that point in time. Next, we add the cash flow in Year 2 (4.250) to the present value of the bond's future cash flows at Year 2 (99.700) and discount the sum to Year 1 using the one-year forward rate one year from now (3.518%). The present value at Year 1 of the bond's future cash flows is 100.417. Here, a rational borrower will call the bond at 100 because leaving it outstanding would be more expensive than redeeming it. Last, we add the cash flow in Year 1 (4.250) to the present value of the bond's future cash flows at Year 1 (100.000) then discount the sum to today at 2.500%. The result (101.707) is the value of the callable bond (*Note:* For the purpose of coverage of this topic, all cash flows and values are expressed as a percentage of par).

We can apply Equation 1 to calculate the value of the call option embedded in this callable bond. The value of the straight bond is the value of the default-free and option-free three-year 4.25% annual coupon bond calculated earlier (102.114). Thus,

Value of issuer call option = 102.114 – 101.707 = 0.407.

Recall from the earlier discussion about the relationships between the value of a callable bond, straight bond, and call option that the investor is long the bond and short the call option. Thus, the value of the call option decreases the value of the callable bond relative to that of an otherwise identical option-free bond.

Valuation of a Putable Bond at Zero Volatility

We now apply this framework to the valuation of a Bermudan-style three-year 4.25% annual coupon bond that is putable at par one year and two years from now. The decision to exercise the put option is made by the investor. Because the investor lent money, he will exercise the put option when the value of the bond's future cash flows is lower than the put price (exercise price). Exhibit 3 shows how to calculate the value of the three-year 4.25% annual coupon bond putable at par one year and two years from today.

Exhibit 3: Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Putable at Par One Year and Two Years from Now at Zero Volatility

	Today	Year 1	Year 2	Year 3
Cash flow		4.250	4.250	104.250
Discount rate		2.500%	3.518%	4.564%
Value of the putable bond	$\frac{100.707 + 4.250}{1.02500} = 102.397$	$\frac{100 + 4.250}{1.03518} = 100.707$ Not put	$\frac{104.250}{1.04564} =$ ~~99.700~~ Put at 100	

We can apply Equation 2 to calculate the value of the put option:

Value of investor put option = 102.397 – 102.114 = 0.283.

Because the investor is long the bond and the put option, the value of the put option increases the value of the putable bond relative to that of an otherwise identical option-free bond.

OPTIMAL EXERCISE OF OPTIONS

The holder of an embedded bond option can extinguish (or possibly modify the terms of) the bond. Assuming that the option is currently exercisable, the obvious question is, does it pay to exercise? Assuming that the answer is affirmative, the follow-up question is whether it is better to exercise the option at present or to wait.

Let us consider the first question: Would it be profitable to exercise the option? The answer is usually straightforward: Compare the value of exercising with the value of not exercising. For example, suppose that a bond is currently putable at 100. If the bond's market price is above 100, putting the bond makes no sense because the cash value from selling the bond would exceed 100. In contrast, if the bond's market price is 100, putting the bond should definitely be considered. Note that the market price of the bond cannot be less than 100 because such a situation creates an arbitrage opportunity: Buy the bond below 100 and immediately put it at 100.

The logic of a call decision by the issuer is similar. If a bond's market price is significantly less than the call price, calling is foolish because the bond could be simply repurchased in the market at a lower price. Alternatively, if the price is very close to the call price, calling may make sense.

Assume that we have determined that exercising the option would be profitable. If the option under consideration is European style, it is obvious that it should in fact be exercised: There is no justification for not doing so. But if it is an American-style or Bermudan-style option, the challenge is to determine whether it is better to act now or to wait for a better opportunity. The problem is that although circumstances may become more favorable, they may also get worse. So, option holders must consider the odds and decide to act or wait, depending on their risk preference.

The approach presented here for valuing bonds with embedded options assumes that the option holders, be they issuers or investors, are risk neutral. They exercise if, and only if, the benefit from exercise exceeds the expected benefit from waiting. In reality, option holders may be risk averse and may exercise early even if the option is worth more alive than dead.

EXAMPLE 2

Valuation of Default-Free Callable and Putable Bonds

George Cahill, a portfolio manager, has identified three five-year annual coupon bonds issued by a sovereign government. The three bonds have identical characteristics. The exceptions are that Bond A is an option-free bond; Bond B is callable at par two years and three years from today; and Bond C is also callable at par two years and three years from today as well as putable at par one year from today.

1. Relative to the value of Bond A, the value of Bond B is:

   A. lower.

   B. the same.

   C. higher.

Solution:

A is correct. Bond B is a callable bond, and Bond A is the underlying option-free (straight) bond. The call option embedded in Bond B is an issuer option that decreases the bond's value for the investor. If interest rates decline, bond prices usually increase; however, the price appreciation of Bond B will be capped relative to the price appreciation of Bond A because the issuer will call the bond to refinance at a lower cost.

2. Relative to the value of Bond B, the value of Bond C is:

   A. lower.

   B. the same.

   C. higher.

Solution:

C is correct. Relative to Bond B, Bond C includes a put option. A put option is an investor option that increases the bond's value for the investor. Thus, the value of Bond C is higher than that of Bond B.

3. Given an anticipation of rising interest rates, Bond C will be expected to:

   A. be called by the issuer.

   B. be put by the bondholders.

   C. mature without exercise of any of the embedded options.

Solution:

B is correct. As interest rates rise, bond prices decrease. Thus, the bondholders will have an incentive to exercise the put option so that they can reinvest the proceeds of the retired bond at a higher yield.

Exhibit 2 and Exhibit 3 show how callable and putable bonds are valued in the absence of interest rate volatility. In real life, however, interest rates do fluctuate. Thus, the option holder must consider possible evolutions of the yield curve over time.

3 EFFECT OF INTEREST RATE VOLATILITY

- ☐ explain how interest rate volatility affects the value of a callable or putable bond
- ☐ explain how changes in the level and shape of the yield curve affect the value of a callable or putable bond

In this section, we discuss the effects of interest rate volatility as well as the level and shape of the yield curve on the value of embedded options.

Interest Rate Volatility

The value of any embedded option, regardless of the type of option, increases with interest rate volatility. The greater the volatility, the more opportunities for the embedded option to be exercised. Thus, it is critical for issuers and investors to understand the effect of interest rate volatility on the value of bonds with embedded options.

The effect of interest rate volatility is represented in an interest rate tree or lattice, as illustrated in Exhibit 4. From each node on the tree starting from today, interest rates could go up or down. From these two states, interest rates could again go up or down. The dispersion between these up and down states anywhere on the tree is determined by the process generating interest rates based on a given yield curve and interest rate volatility assumptions.

Exhibit 4: Building an Interest Rate Tree

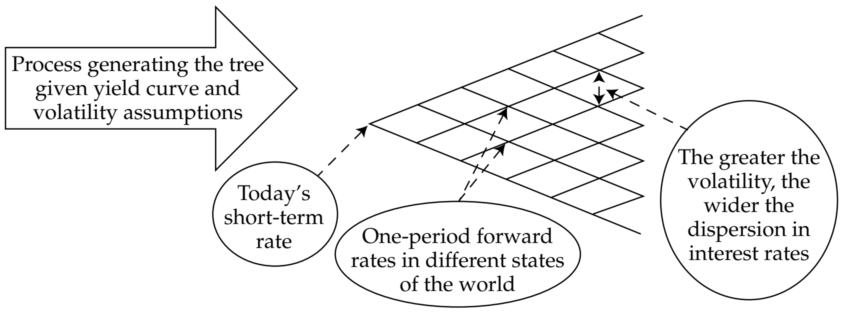
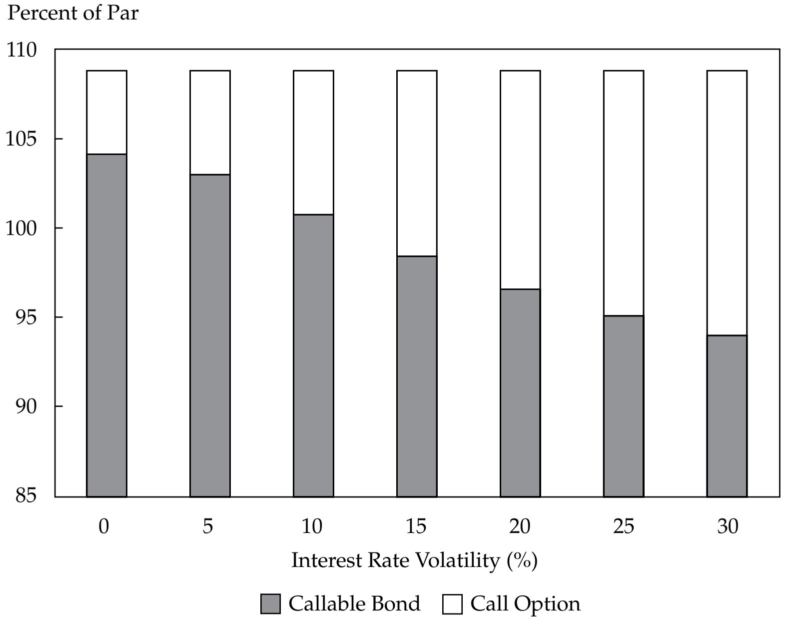


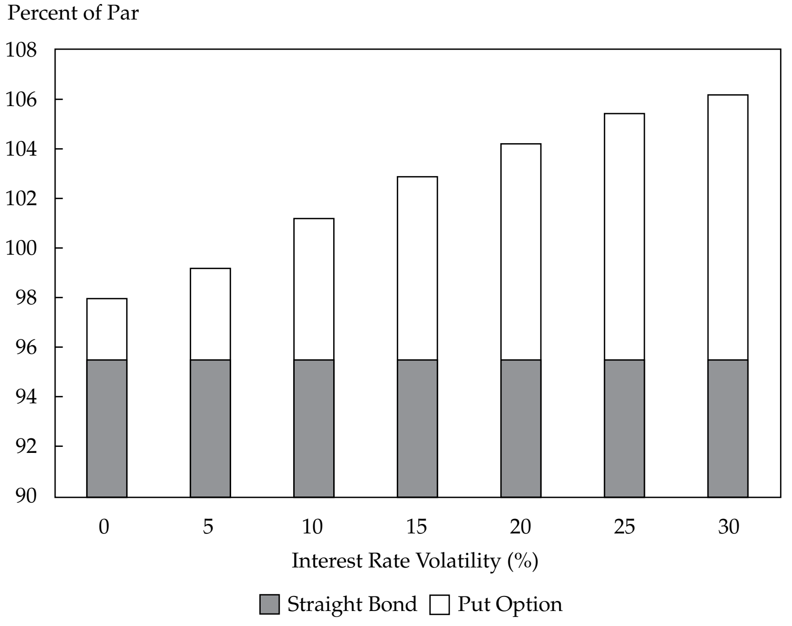
Exhibit 5 and Exhibit 6 show the effect of interest rate volatility on the value of a callable bond and putable bond, respectively.

Exhibit 5: Value of a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Volatility Scenarios Assuming a 4% Flat Yield Curve



The stacked bars in Exhibit 5 represent the value of the straight bond, which is unaffected by interest rate volatility. The white component is the value of the call option; taking it away from the value of the straight bond gives the value of the callable bond—the shaded component. All else being equal, the call option increases in value with interest rate volatility. At zero volatility, the value of the call option is 4.60% of par; at 30% volatility, it is 14.78% of par. Thus, as interest rate volatility increases, the value of the callable bond decreases.

Exhibit 6: Value of a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Volatility Scenarios Assuming a 4% Flat Yield Curve



In Exhibit 6, the shaded component is the value of the straight bond, and the white component is the value of the put option; thus, the stacked bars represent the value of the putable bond. All else being equal, the put option increases in value with interest rate volatility. At zero volatility, the value of the put option is 2.30% of par; at 30% volatility, it is 10.54% of par. Thus, as interest rate volatility increases, the value of the putable bond increases.

Level and Shape of the Yield Curve

The value of a callable or putable bond is also affected by changes in the level and shape of the yield curve.

Effect on the Value of a Callable Bond

Exhibit 7 shows the value of the same callable bond as in Exhibit 5 under different flat yield curve levels assuming an interest rate volatility of 15%.

Exhibit 7: Value of a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Flat Yield Curve Levels at 15% Interest Rate Volatility

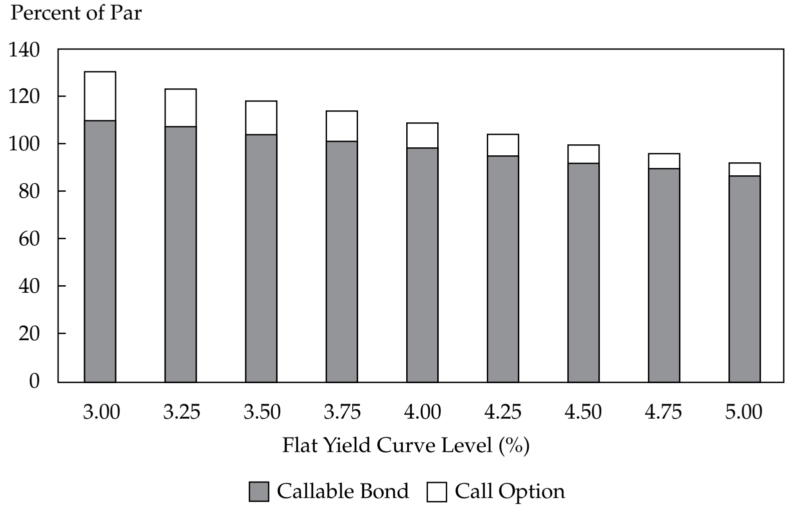
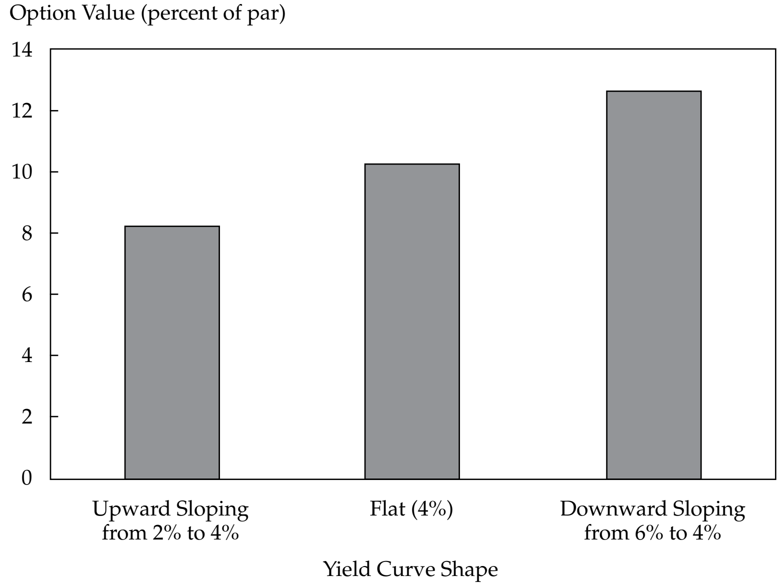


Exhibit 7 shows that as interest rates decline, the value of the straight bond rises; however, the rise is partially offset by the increase in the value of the call option. For example, if the yield curve is 5% flat, the value of the straight bond is 92.27% of par and the value of the call option is 5.37% of par; thus, the value of the callable bond is 86.90% of par. If the yield curve declines to 3% flat, the value of the straight bond rises by 40% to 129.54% of par, but the value of the callable bond increases by only 27% to 110.43% of par. Thus, the value of the callable bond rises less rapidly than the value of the straight bond, limiting the upside potential for the investor.

The value of a call option, and thus the value of a callable bond, is also affected by changes in the shape of the yield curve, as illustrated in Exhibit 8.

Exhibit 8: Value of a Call Option Embedded in a 30-Year 4.50% Bond Callable at Par in 10 Years under Different Yield Curve Shapes at 15% Interest Rate Volatility



All else being equal, the value of the call option increases as the yield curve flattens. If the yield curve is upward sloping with short-term rates at 2% and long-term rates at 4% (the first bar), the value of the call option represents approximately 8% of par. It rises to approximately 10% of par if the yield curve flattens to 4% (the second bar). The value of the call option increases further if the yield curve actually inverts. Exhibit 8 shows that it exceeds 12% of par if the yield curve is downward sloping with short-term rates at 6% and long-term rates at 4% (the third bar). An inverted yield curve is rare but does happen from time to time.

The intuition to explain the effect of the shape of the yield curve on the value of the call option is as follows. When the yield curve is upward sloping, the one-period forward rates on the interest rate tree are high and opportunities for the issuer to call the bond are fewer. When the yield curve flattens or inverts, many nodes on the tree have lower forward rates that increase the opportunities to call.

Assuming a normal, upward-sloping yield curve at the time of issue, the call option embedded in a callable bond issued at par is out of the money. It would not be called if the arbitrage-free forward rates at zero volatility prevailed. Callable bonds issued at a large premium, as happens frequently in the municipal sector in the United States, are in the money. They will be called if the arbitrage-free forward rates prevail.

Effect on the Value of a Putable Bond

Exhibit 9 and Exhibit 10 show how changes in the level and shape of the yield curve affect the value of the putable bond used in Exhibit 6.

Exhibit 9: Value of a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Flat Yield Curve Levels at 15% Interest Rate Volatility

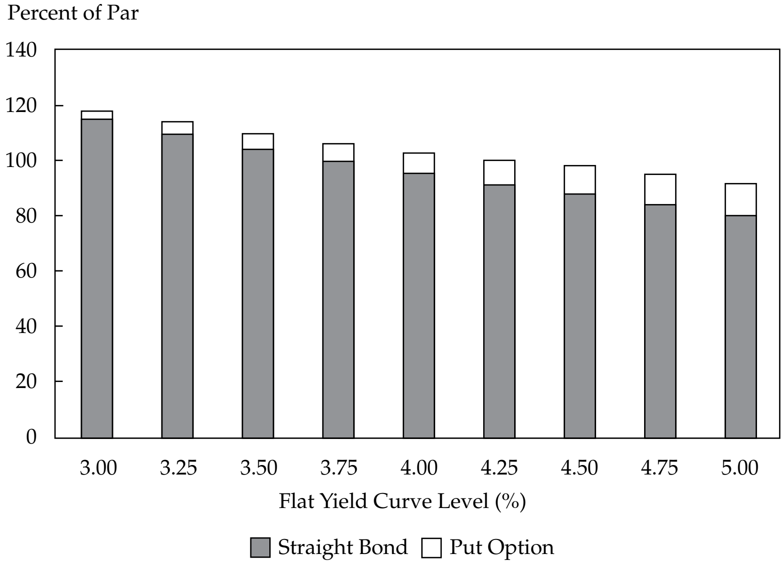
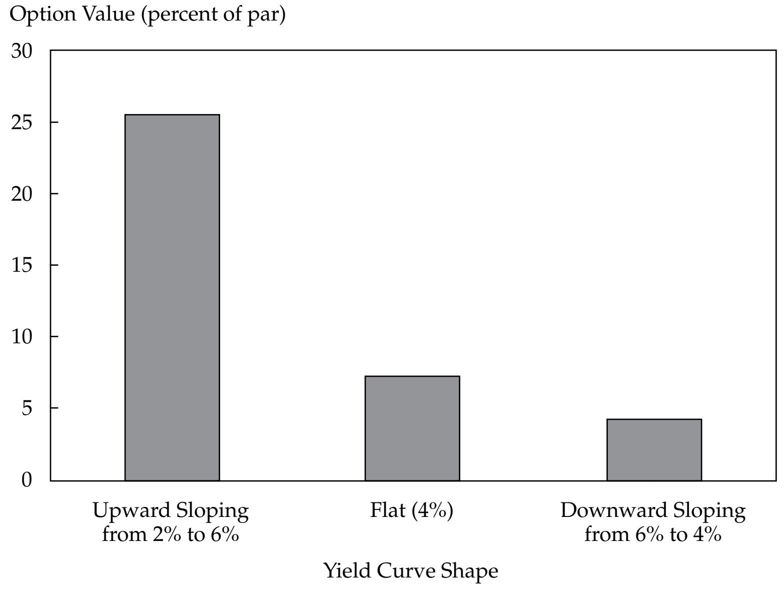


Exhibit 9 illustrates why the put option is considered a hedge against rising interest rates for investors. As interest rates rise, the value of the straight bond declines; however, the decline is partially offset by the increase in the value of the put option. For example, if the yield curve moves from 3% flat to 5% flat, the value of the straight bond falls by 30% while the fall in the value of the putable bond is limited to 22%.

Exhibit 10: Value of the Put Option Embedded in a 30-Year 3.75% Bond Putable at Par in 10 Years under Different Yield Curve Shapes at 15% Interest Rate Volatility



All else being equal, the value of the put option decreases as the yield curve moves from being upward sloping, to flat, to downward sloping. When the yield curve is upward sloping, the one-period forward rates in the interest rate tree are high, which creates more opportunities for the investor to put the bond. As the yield curve flattens or inverts, the number of opportunities declines.

4 VALUATION OF DEFAULT-FREE CALLABLE AND PUTABLE BONDS WITH INTEREST RATE VOLATILITY

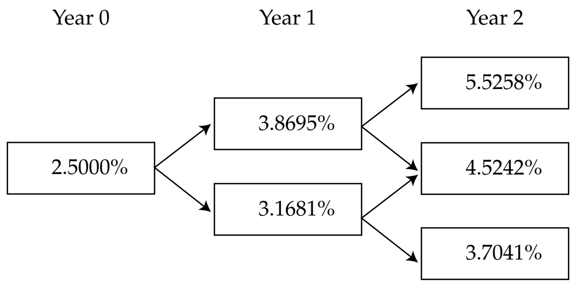
☐ calculate the value of a callable or putable bond from an interest rate tree

The procedure to value a bond with an embedded option in the presence of interest rate volatility is as follows:

- Generate a tree of interest rates based on the given yield curve and interest rate volatility assumptions.
- At each node of the tree, determine whether the embedded option will be exercised.
- Apply the backward induction valuation methodology to calculate the bond's present value. This methodology involves starting at maturity and working back from right to left to find the bond's present value.

Let us return to the default-free three-year 4.25% annual coupon bonds discussed earlier to illustrate how to apply this valuation procedure. The bonds' characteristics are identical. The yield curve given in Exhibit 1 remains the same—with one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively. But we now assume an interest rate volatility of 10% instead of 0%. The resulting binomial interest rate tree showing the one-year forward rates zero, one, and two years from now is shown in Exhibit 11. The branching from each node to an up state and a down state is assumed to occur with equal probability.

Exhibit 11: Binomial Interest Rate Tree at 10% Interest Rate Volatility



The calibration of a binomial interest rate tree was discussed in earlier coverage of fixed-income concepts. As mentioned before, the one-year par rate, the one-year spot rate, and the one-year forward rate zero years from now are identical (2.500%). Because there is no closed-form solution, the one-year forward rates one year from now in the two states are determined iteratively by meeting the following two constraints:

1. The rate in the up state (R_u) is given by

$$R_u = R_d \times e^{2\sigma\sqrt{t}},$$

   where R_d is the rate in the down state, σ is the interest rate volatility (10% here), and t is the time in years between "time slices" (a year, so here t = 1).

2. The discounted value of a two-year par bond (bearing a 3.000% coupon rate in this example) equals 100.

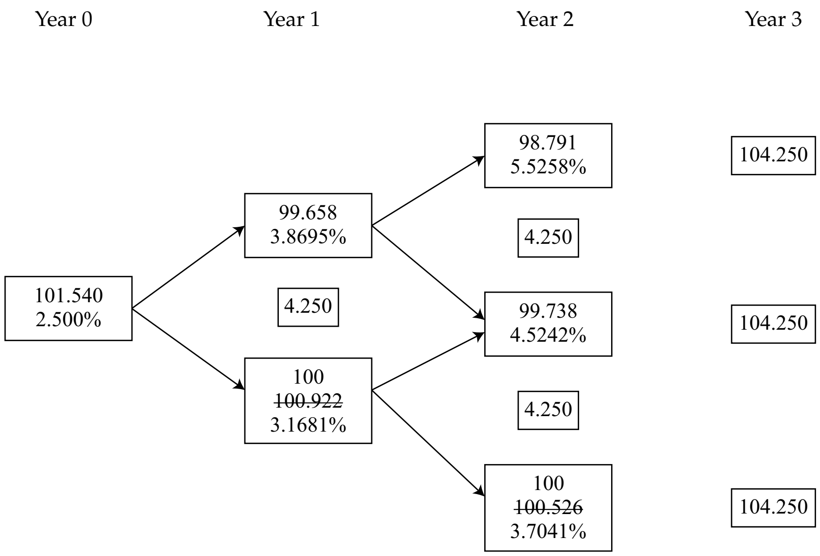
In Exhibit 11 at the one-year time slice, R_d is 3.1681% and R_u is 3.8695%. Having established the rates that correctly value the one-year and two-year par bonds implied by the given par yield curve, we freeze these rates and proceed to iterate the rates in the next time slice to determine the one-year forward rates in the three states two years from now. The same constraints as before apply: (1) Each rate must be related to its neighbor by the factor $e^{2\sigma\sqrt{t}}$, and (2) the rates must discount a three-year par bond (bearing a 3.500% coupon rate in this example) to a value of 100.

Now that we have determined all the one-year forward rates, we can value the three-year 4.25% annual coupon bonds that are either callable or putable at par one year and two years from now.

Valuation of a Callable Bond with Interest Rate Volatility

Exhibit 12 depicts the valuation of a callable bond at 10% volatility.

Exhibit 12: Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



The coupon and principal cash flows are placed directly to the right of the interest rate nodes. The calculated bond values at each node are placed above the interest rate. We start by calculating the bond values at Year 2 by discounting the cash flow for Year 3 with the three possible rates.

$$98.791 = \frac{104.250}{1.055258}$$

$$99.738 = \frac{104.250}{1.045242}$$

$$100.526 = \frac{104.250}{1.037041}$$

Because the bond is callable at par in Year 2, we check each scenario to determine whether the present value of the future cash flows is higher than the call price, in which case the issuer calls the bond. Exercise happens only at the bottom of the tree, where the rate is 3.7041%, and so we reset the value from 100.526 to 100 in that state.

The value in each state of Year 1 is calculated by discounting the values in the two future states emanating from the present state plus the coupon at the appropriate rate in the present state:

$$99.658 = \frac{4.250 + (0.5 \times 98.791 + 0.5 \times 99.738)}{1.038695}.$$

The first term in the numerator is the coupon payment, and the second term is the expected bond value at Year 2. In this model, the probabilities for moving to the higher and lower node are the same (0.5):

$$100.922 = \frac{4.250 + (0.5 \times 99.738 + 0.5 \times 100)}{1.031681}.$$

Notice that the reset value of 100 is used to get the expected bond value. Once again the bond will be callable at the lower node where the interest rate is 3.1681%.

At Year 0, the value of the callable bond is 101.540:

$$101.540 = \frac{4.250 + (0.5 \times 99.658 + 0.5 \times 100)}{1.025000}.$$

The value of the call option, obtained by taking the difference between the value of the straight bond and the value of the callable bond, is now 0.574 (102.114 – 101.540). The fact that the value of the call option is larger at 10% volatility than at 0% volatility (0.407) is consistent with our earlier discussion that option value increases with interest rate volatility.

EXAMPLE 3

Valuation of a Callable Bond Assuming Interest Rate Volatility

Return to the valuation of the Bermudan-style three-year 4.25% annual coupon bond callable at par one year and two years from now as depicted in Exhibit 12. The one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively, and the interest rate volatility is 10%.

1. Assume that nothing changes relative to the initial setting except that the interest rate volatility is now 15% instead of 10%. The new value of the callable bond is:

   A. less than 101.540.

   B. equal to 101.540.

   C. more than 101.540.

Solution:

A is correct. A higher interest rate volatility increases the value of the call option. Because the value of the call option is subtracted from the value of the straight bond to obtain the value of the callable bond, a higher value for the call option leads to a lower value for the callable bond. Thus, the value of the callable bond at 15% volatility is less than that at 10% volatility—that is, less than 101.540.

2. Assume that nothing changes relative to the initial setting except that the bond is now callable at 102 instead of 100. The new value of the callable bond is *closest to*:

   A. 100.000.

   B. 102.000.

   C. 102.114.

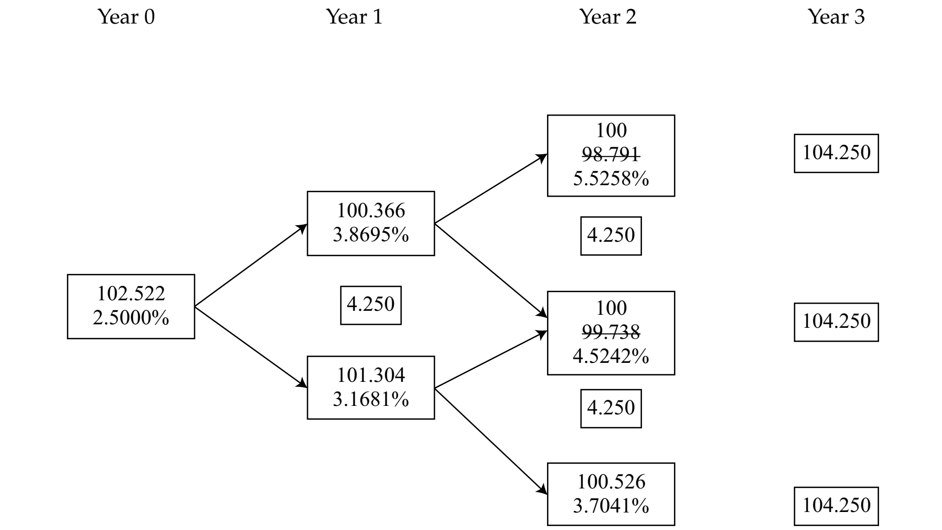
Solution:

C is correct. Looking at Exhibit 12, the call price is too high for the call option to be exercised in any scenario. Thus, the value of the call option is zero, and the value of the callable bond is equal to the value of the straight bond—that is, 102.114.

Valuation of a Putable Bond with Interest Rate Volatility

The valuation of the three-year 4.25% annual coupon bond putable at par one year and two years from now at 10% volatility is depicted in Exhibit 13. The procedure for valuing a putable bond is very similar to that described earlier for valuing a callable bond, except that in each state, the bond's value is compared with the put price. The investor puts the bond only when the present value of the bond's future cash flows is lower than the put price. In this case, the value is reset to the put price (100). It happens twice in Year 2, in the states where the interest rates are 5.5258% and 4.5242%. The investor would not exercise the put option in Year 1 because the values for the bond exceed the put price.

Exhibit 13: Valuation of a Default-Free Three-Year 4.25% Annual Coupon Bond Putable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



The value of the putable bond is 102.522. The value of the put option, obtained by taking the difference between the value of the putable bond and the value of the straight bond, is now 0.408 (102.522 – 102.114). As expected, the value of the put option is larger at 10% volatility than at 0% volatility (0.283).

EXAMPLE 4

Valuation of a Putable Bond Assuming Interest Rate Volatility

Return to the valuation of the Bermudan-style three-year 4.25% annual coupon bond putable at par one year and two years from now, as depicted in Exhibit 13. The one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively, and the interest rate volatility is 10%.

1. Assume that nothing changes relative to the initial setting except that the interest rate volatility is now 20% instead of 10%. The new value of the putable bond is:

   A. less than 102.522.

   B. equal to 102.522.

   C. more than 102.522.

Solution:

C is correct. A higher interest rate volatility increases the value of the put option. Because the value of the put option is added to the value of the straight bond to obtain the value of the putable bond, a higher value for the put option leads to a higher value for the putable bond. Thus, the value of the putable bond at 20% volatility is more than that at 10% volatility—that is, more than 102.522.

2. Assume that nothing changes relative to the initial setting except that the bond is now putable at 95 instead of 100. The new value of the putable bond is *closest to*:

   A. 97.522.

   B. 102.114.

   C. 107.522.

Solution:

B is correct. Looking at Exhibit 13, the put price is too low for the put option to be exercised in any scenario. Thus, the value of the put option is zero, and the value of the putable bond is equal to the value of the straight bond—that is, 102.114.

PUTABLE VS. EXTENDIBLE BONDS

Putable and extendible bonds are equivalent, except that their underlying option-free bonds are different. Consider a three-year 3.30% bond putable in Year 2. Its value should be exactly the same as that of a two-year 3.30% bond extendible by one year. Otherwise, there would be an arbitrage opportunity. Clearly, the cash flows of the two bonds are identical up to Year 2. The cash flows in Year 3 are dependent on the one-year forward rate two years from now. These cash flows will also be the same for both bonds regardless of the level of interest rates at the end of Year 2.

If the one-year forward rate at the end of Year 2 is higher than 3.30%, the putable bond will be put because the bondholder can reinvest the proceeds of the retired bond at a higher yield and the extendible bond will not be extended for the same reason. So, both bonds pay 3.30% for two years and are then redeemed. Alternatively, if the one-year forward rate at the end of Year 2 is lower than 3.30%, the putable bond will not be put because the bondholder would not want to reinvest at a lower yield and the extendible bond will be extended to hold onto the higher interest rate. Thus, both bonds pay 3.30% for three years and are then redeemed.

EXAMPLE 5

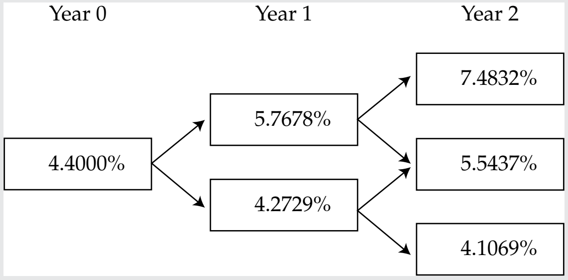
Valuation of Bonds with Embedded Options Assuming Interest Rate Volatility

Sidley Brown, a fixed-income associate at KMR Capital, is analyzing the effect of interest rate volatility on the values of callable and putable bonds issued by Weather Analytics (WA). WA is owned by the sovereign government, so its bonds are considered default free. Brown is currently looking at three of WA's bonds and has gathered the following information about them:

Characteristic	Bond X	Bond Y	Bond Z
Time to maturity	Three years from today	Three years from today	Three years from today
Coupon	5.2% annual	Not available	4.8% annual

Characteristic	Bond X	Bond Y	Bond Z
Type of bond	Callable at par one year and two years from today	Callable at par one year and two years from today	Putable at par two years from today
Price (as a % of par)	Not available	101.325	Not available

The one-year, two-year, and three-year par rates are 4.400%, 4.700%, and 5.000%, respectively. Based on an estimated interest rate volatility of 15%, Brown has constructed the following binomial interest rate tree:



Brown is now analyzing the effect of interest rate volatility on the price of WA's bonds.

1. The price of Bond X is *closest to*:

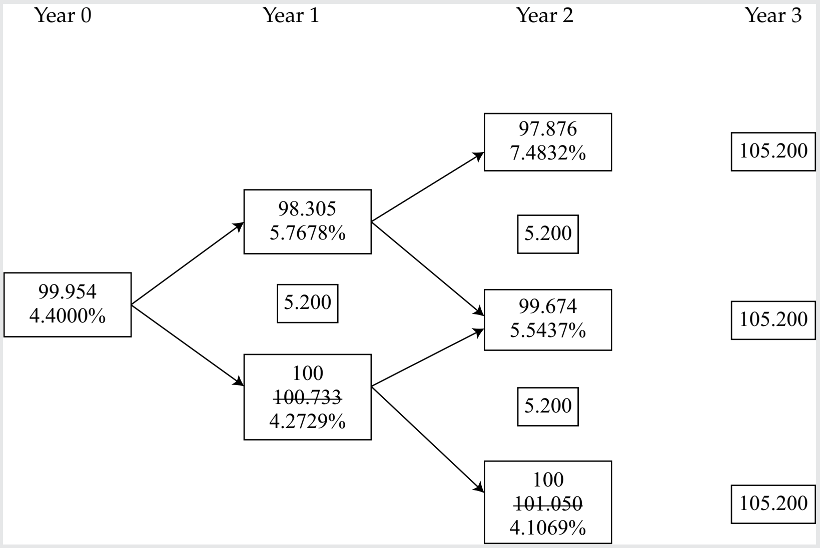
   A. 96.057% of par.

   B. 99.954% of par.

   C. 100.547% of par.

Solution:

B is correct.



2. The coupon rate of Bond Y is *closest to*:

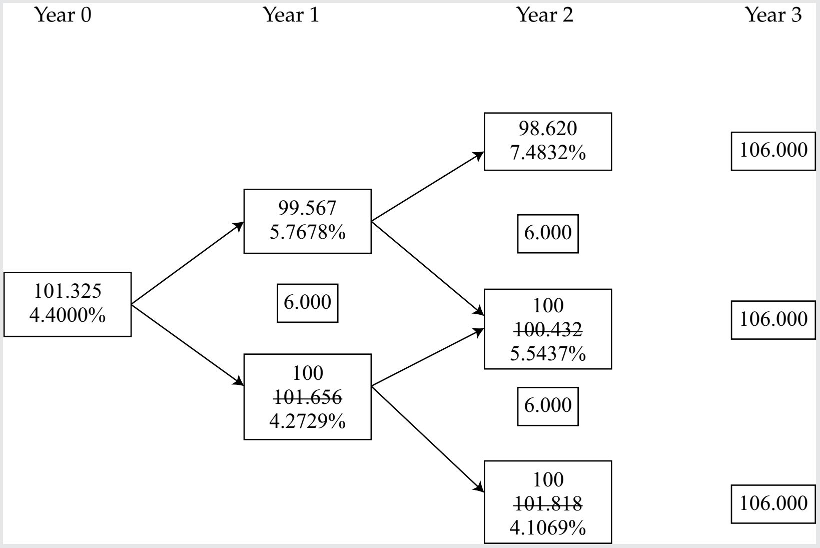
   A. 4.200%.

   B. 5.000%.

   C. 6.000%.

Solution:

C is correct.



Although the correct answer can be found by using the interest rate tree depicted, it is possible to identify it by realizing that the other two answers are clearly incorrect. The three-year 5% straight bond is worth par given that the three-year par rate is 5%. Because the presence of a call option reduces the price of a callable bond, a three-year 5% bond callable at par can only be worth less than par—and certainly less than 101.325 given the yield curve and interest rate volatility assumptions—so B is incorrect. The value of a bond with a coupon rate of 4% is even less, so A is incorrect. Thus, C must be the correct answer.

3. The price of Bond Z is *closest to*:

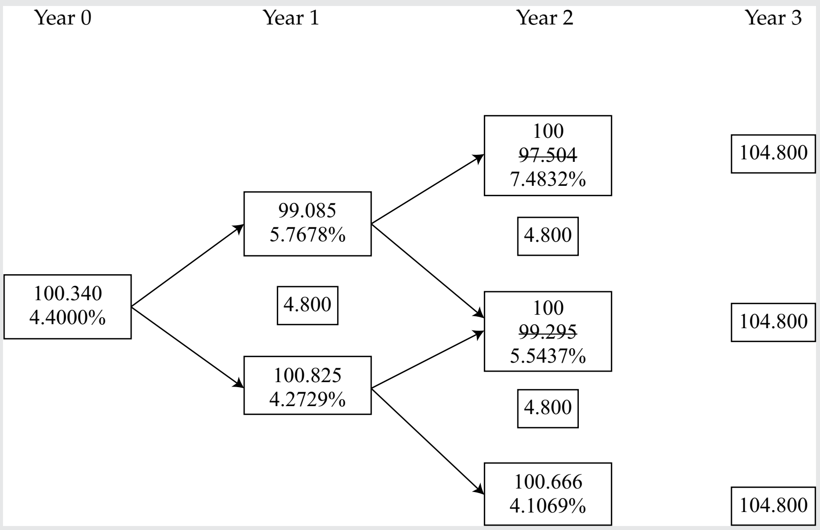
   A. 99.638% of par.

   B. 100.340% of par.

   C. 100.778% of par.

Solution:

B is correct.



4. Relative to its price at 15% interest rate volatility, the price of Bond X at a lower interest rate volatility will be:

   A. lower.

   B. the same.

   C. higher.

Solution:

C is correct. Bond X is a callable bond. As shown in Equation 1, the value of the call option decreases the value of Bond X relative to the value of the underlying option-free bond. As interest rate volatility decreases, the value of the call option decreases; thus, the value of Bond X increases.

5. Relative to its price at 15% interest rate volatility, the price of Bond Z at a higher interest rate volatility will be:

   A. lower.

   B. the same.

   C. higher.

Solution:

C is correct. Bond Z is a putable bond. As shown in Equation 2, the value of the put option increases the value of Bond Z relative to the value of the underlying option-free bond. As interest rate volatility increases, the value of the put option increases; thus, the value of Bond Z increases.

5 VALUATION OF RISKY CALLABLE AND PUTABLE BONDS

- ☐ explain the calculation and use of option-adjusted spreads
- ☐ explain how interest rate volatility affects option-adjusted spreads

Although the approach described earlier for default-free bonds may apply to securities issued by sovereign governments in their local currency, the fact is that most bonds are subject to default. Accordingly, we have to extend the framework to the valuation of risky bonds.

Two distinct approaches to valuing bonds are subject to default risk. The industry-standard approach is to increase the discount rates above the default-free rates to reflect default risk. Higher discount rates imply lower present values, and thus the value of a risky bond will be lower than that of an otherwise identical default-free bond.

The second approach to valuing risky bonds is to make the default probabilities explicit—that is, assigning a probability to each time period going forward. For example, the probability of default in Year 1 may be 1%; the probability of default in Year 2, conditional on surviving Year 1, may be 1.25%; and so on. This approach requires specifying the recovery value given default (e.g., 40% of par). Information about default probabilities and recovery values may be accessible from credit default swaps. This important topic is covered elsewhere.

Option-Adjusted Spread

Depending on available information, two standard approaches are used to construct a suitable yield curve for a risky bond. The more satisfactory but less convenient one is to use an issuer-specific curve, which represents the issuer's borrowing rates over the relevant range of maturities. Unfortunately, most bond professionals do not have access to such a level of detail. A more convenient and relatively satisfactory alternative is to uniformly raise the one-year forward rates derived from the default-free benchmark yield curve by a fixed spread, which is estimated from the market prices of suitable bonds of similar credit quality. This fixed spread is known as the zero-volatility spread, or *Z*-spread.

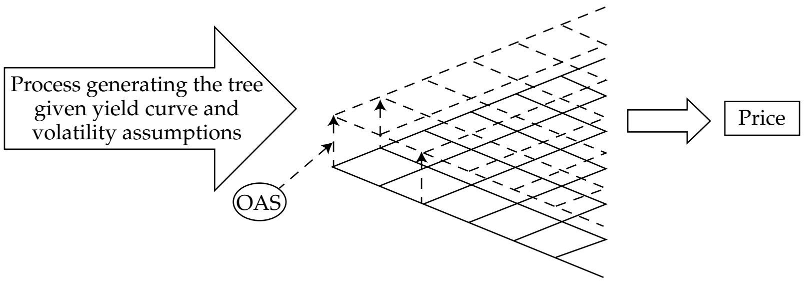
To illustrate, we return to the three-year 4.25% option-free bond introduced earlier, but now we assume that it is a risky bond and that the appropriate *Z*-spread is 100 bps. To calculate the arbitrage-free value of this bond, we have to increase each of the one-year forward rates given in Exhibit 1 by the *Z*-spread of 100 bps:

$$\frac{4.25}{(1.03500)} + \frac{4.25}{(1.03500)(1.04518)} + \frac{104.25}{(1.03500)(1.04518)(1.05564)} = 99.326.$$

As expected, the value of this risky bond (99.326) is considerably lower than the value of an otherwise identical but default-free bond (102.114).

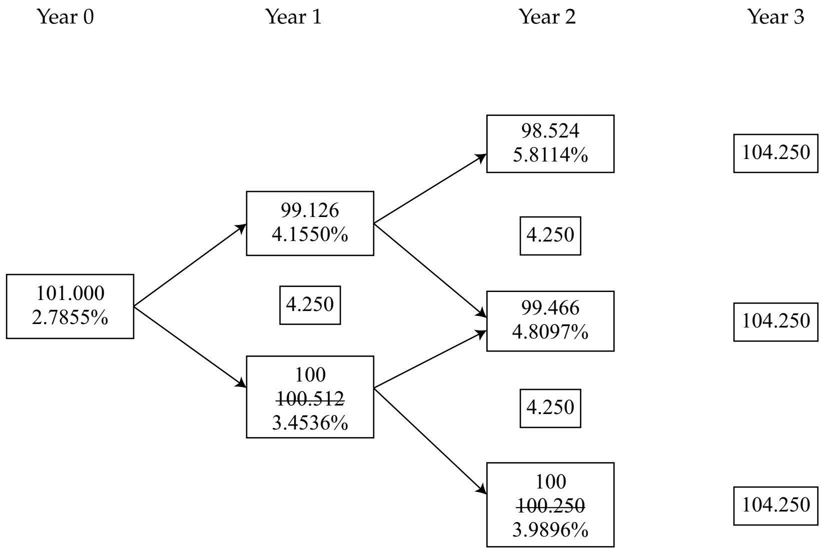
The same approach can be applied to the interest rate tree when valuing risky bonds with embedded options. In this case, an **option-adjusted spread** (OAS) is used. As depicted in Exhibit 14, the OAS is the constant spread that when added to all the one-period forward rates on the interest rate tree, makes the arbitrage-free value of the bond equal to its market price. Note that the *Z*-spread for an option-free bond is simply its OAS at zero volatility.

Exhibit 14: Interest Rate Tree and OAS



If the bond's price is given, the OAS is determined by trial and error. For example, suppose that the market price of a three-year 4.25% annual coupon bond callable in one year and two years from now (identical to the one valued in Exhibit 12 except that it is risky instead of default-free) is 101.000. To determine the OAS, we try shifting all the one-year forward rates in each state by adding a constant spread. For example, when we add 30 bps to all the one-year forward rates, we obtain a value for the callable bond of 100.973, which is lower than the bond's price. Because of the inverse relationship between a bond's price and its yield, this result means that the discount rates are too high, so we try a slightly lower spread. Adding 28 bps results in a value for the callable bond of 101.010, which is slightly too high. As illustrated in Exhibit 15, the constant spread added uniformly to all the one-period forward rates that justifies the given market price of 101.000 is 28.55 bps; this number is the OAS.

Exhibit 15: OAS of a Risky Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility



As illustrated in Exhibit 15, the value at each node is adjusted based on whether the call option is exercised. Thus, the OAS removes the amount that results from the option risk, which is why this spread is called "option adjusted."

OAS is often used as a measure of value relative to the benchmark. An OAS lower than that for a bond with similar characteristics and credit quality indicates that the bond is likely overpriced (rich) and should be avoided. A larger OAS than that of a bond with similar characteristics and credit quality means that the bond is likely underpriced (cheap). If the OAS is close to that of a bond with similar characteristics and credit quality, the bond looks fairly priced. In our example, the OAS at 10% volatility is 28.55 bps. This number should be compared with the OAS of bonds with similar characteristics and credit quality to make a judgment about the bond's attractiveness.

Effect of Interest Rate Volatility on Option-Adjusted Spread

The dispersion of interest rates on the tree is volatility dependent, and so is the OAS. Exhibit 16 shows the effect of volatility on the OAS for a callable bond. The bond is a 5% annual coupon bond with 23 years left to maturity, callable in three years, priced at 95% of par, and valued assuming a flat yield curve of 4%.

Exhibit 16: Effect of Interest Rate Volatility on the OAS for a Callable Bond

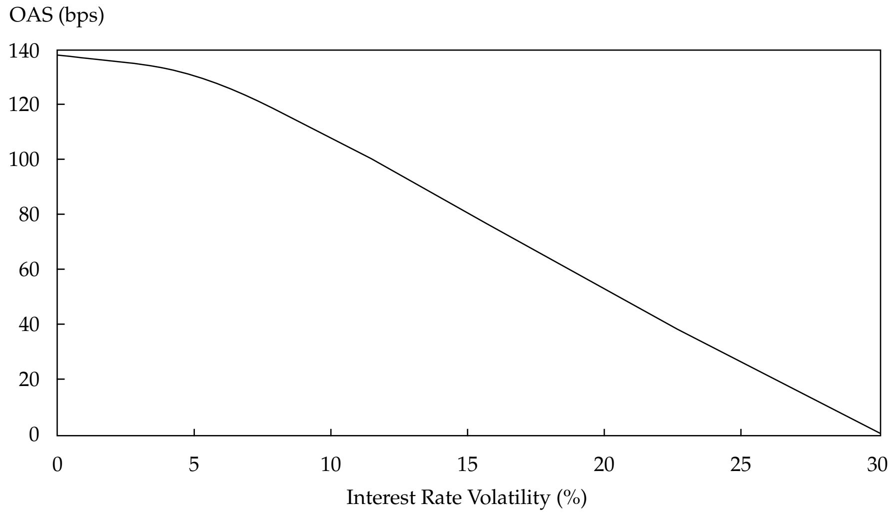
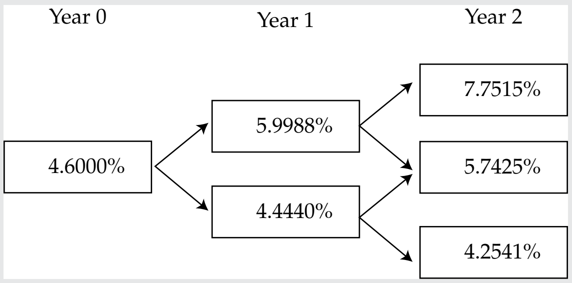


Exhibit 16 shows that as interest rate volatility increases, the OAS for the callable bond decreases. The OAS drops from 138.2 bps at 0% volatility to 1.2 bps at 30% volatility. This exhibit clearly demonstrates the importance of the interest rate volatility assumption. Returning to the example in Exhibit 15, the callable bond may look underpriced at 10% volatility. If an investor assumes a higher volatility, however, the OAS and thus relative cheapness will decrease.

EXAMPLE 6

Option-Adjusted Spread

Robert Jourdan, a portfolio manager, has just valued a 7% annual coupon bond that was issued by a French company and has three years remaining until maturity. The bond is callable at par one year and two years from now. In his valuation, Jourdan used the yield curve based on the on-the-run French government bonds. The one-year, two-year, and three-year par rates are 4.600%, 4.900%, and 5.200%, respectively. Based on an estimated interest rate volatility of 15%, Jourdan constructed the following binomial interest rate tree:



Jourdan valued the callable bond at 102.294% of par. However, Jourdan's colleague points out that because the corporate bond is riskier than French government bonds, the valuation should be performed using an OAS of 200 bps.

1. To update his valuation of the French corporate bond, Jourdan should:

   A. subtract 200 bps from the bond's annual coupon rate.

   B. add 200 bps to the rates in the binomial interest rate tree.

C. subtract 200 bps from the rates in the binomial interest rate tree.

Solution:

B is correct. The OAS is the constant spread that must be *added* to all the one-period forward rates given in the binomial interest rate tree to justify a bond's given market price.

2. All else being equal, the value of the callable bond at 15% volatility is *closest to*:

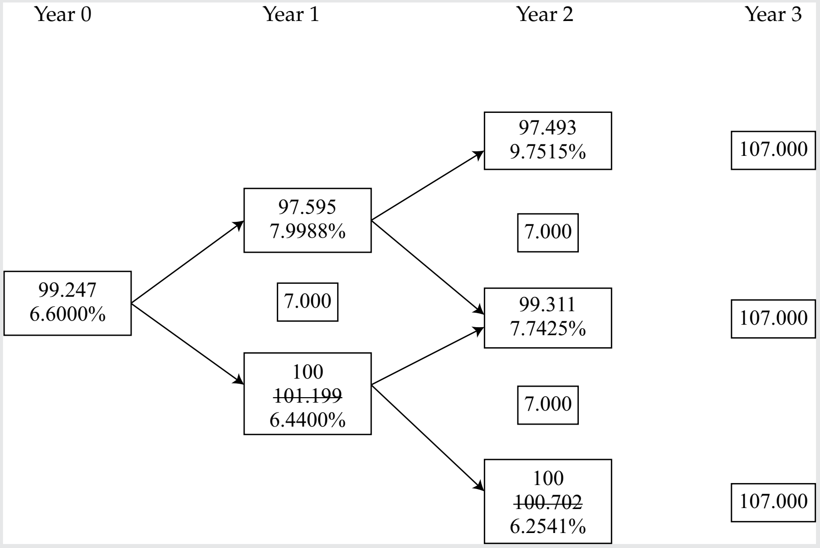
A. 99.198% of par.

B. 99.247% of par.

C. 104.288% of par.

Solution:

B is correct.



3. Holding the price calculated in the previous question, the OAS for the callable bond at 20% volatility will be:

A. lower.

B. the same.

C. higher.

Solution:

A is correct. If interest rate volatility increases from 15% to 20%, the OAS for the callable bond will decrease.

SCENARIO ANALYSIS OF BONDS WITH OPTIONS

Another application of valuing bonds with embedded options is scenario analysis over a specified investment horizon. In addition to reinvestment of interest and principal, option valuation comes into play in that callable and putable bonds can be redeemed and their proceeds reinvested during the holding period.

Making scenario-dependent, optimal option-exercise decisions involves computationally intensive use of OAS technology because the call or put decision must be evaluated considering the evolution of interest rate scenarios during the holding period.

Performance over a specified investment horizon entails a trade-off between reinvestment of cash flows and change in the bond's value. Let us take the example of a 4.5% bond with five years left to maturity and assume that the investment horizon is one year. If the bond is option free, higher interest rates increase the reinvestment income but result in lower principal value at the end of the investment horizon. Because the investment horizon is short, reinvestment income is relatively insignificant and performance will be dominated by the change in the value of the principal. Accordingly, lower interest rates will result in superior performance.

If the bond under consideration is callable, however, it is not at all obvious how the interest rate scenario affects performance. Suppose, for example, that the bond is first callable six months from now and that its current market price is 99.74. Steeply rising interest rates would depress the bond's price, and performance would definitely suffer. But steeply declining interest rates would also be detrimental because the bond would be called and *both interest and principal* would have to be reinvested at lower interest rates. Exhibit 17 shows the return over the one-year investment horizon for the 4.5% bond first callable in six months with five years left to maturity and valued on a 4% flat yield curve.

Exhibit 17: Effect of Interest Rate Changes on a Callable Bond's Total Return

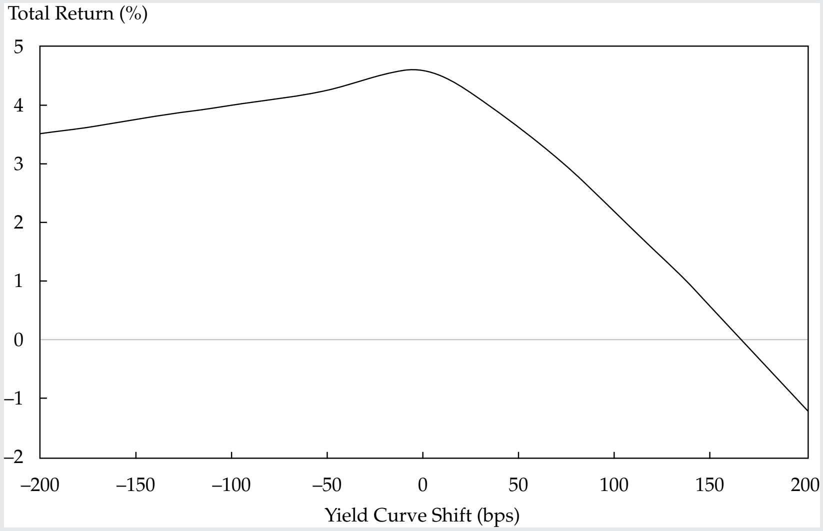


Exhibit 17 clearly shows that lower interest rates do not guarantee higher returns for callable bonds. The point to keep in mind is that the bond may be called long before the end of the investment horizon. Assuming that it is called on the horizon date would overestimate performance. Thus, a realistic prediction of option exercise is essential when performing scenario analysis of bonds with embedded options.

6 BONDS WITH EMBEDDED OPTIONS: EFFECTIVE DURATION

- ☐ calculate and interpret effective duration of a callable or putable bond
- ☐ compare effective durations of callable, putable, and straight bonds

Measuring and managing exposure to interest rate risk are two essential tasks of fixed-income portfolio management. Applications range from hedging a portfolio to asset–liability management of financial institutions. Portfolio managers, whose performance is often measured against a benchmark, also need to monitor the interest rate risk of both their portfolio and the benchmark. In this section, we cover two key measures of interest rate risk: duration and convexity.

Duration

The duration of a bond measures the sensitivity of the bond's full price (including accrued interest) to changes in the bond's yield to maturity (in the case of *yield* duration measures) or to changes in benchmark interest rates (in the case of yield-curve or *curve* duration measures). Yield duration measures, such as modified duration, can be used only for option-free bonds because these measures assume that a bond's expected cash flows do not change when the yield changes. This assumption is in general false for bonds with embedded options because the values of embedded options are typically contingent on interest rates. Thus, for bonds with embedded options, the only appropriate duration measure is the curve duration measure known as effective (or option-adjusted) duration. Because effective duration works for straight bonds as well as for bonds with embedded options, practitioners tend to use it regardless of the type of bond being analyzed.

Effective Duration

Effective duration indicates the sensitivity of the bond's price to a 100 bps parallel shift of the benchmark yield curve—in particular, the government par curve—assuming no change in the bond's credit spread (*Note:* Although it is possible to explore how arbitrary changes in interest rates affect the bond's price, in practice the change is usually specified as a parallel shift of the benchmark yield curve). The formula for calculating a bond's effective duration is

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)}, \quad (3)$$

where

ΔCurve = the magnitude of the parallel shift in the benchmark yield curve (in decimal)

PV_- = the full price of the bond when the benchmark yield curve is shifted down by ΔCurve

PV_+ = the full price of the bond when the benchmark yield curve is shifted up by ΔCurve

PV_0 = the current full price of the bond (i.e., with no shift)

How is this formula applied in practice? Without a market price, we would need an issuer-specific yield curve to compute PV_0, PV_-, and PV_+. But practitioners usually have access to the bond's current price and thus use the following procedure:

1. Given a price (PV_0), calculate the implied OAS to the benchmark yield curve at an appropriate interest rate volatility.
2. Shift the benchmark yield curve down, generate a new interest rate tree, and then revalue the bond using the OAS calculated in Step 1. This value is PV_-.
3. Shift the benchmark yield curve up by the same magnitude as in Step 2, generate a new interest rate tree, and then revalue the bond using the OAS calculated in Step 1. This value is PV_+.
4. Calculate the bond's effective duration using Equation 3.

Let us illustrate using the same three-year 4.25% bond callable at par one year and two years from now, the same par yield curve (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively), and the same interest rate volatility (10%) as before. Also as before, we assume that the bond's current full price is 101.000. We apply the procedure just described:

1. As shown in Exhibit 15, given a price (PV_0) of 101.000, the OAS at 10% volatility is 28.55 bps.
2. We shift the par yield curve down by, say, 30 bps, generate a new interest rate tree, and then revalue the bond at an OAS of 28.55 bps. As shown in Exhibit 18, PV_- is 101.599.
3. We shift the par yield curve up by the same 30 bps, generate a new interest rate tree, and then revalue the bond at an OAS of 28.55 bps. As shown in Exhibit 19, PV_+ is 100.407.
4. Thus,

$$\text{EffDur} = \frac{101.599 - 100.407}{2 \times 0.0030 \times 101.000} = 1.97.$$

An effective duration of 1.97 indicates that a 100 bps increase in interest rate would reduce the value of the three-year 4.25% callable bond by 1.97%.

Exhibit 18: Valuation of a Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility with an OAS of 28.55 bps When Interest Rates Are Shifted Down by 30 bps

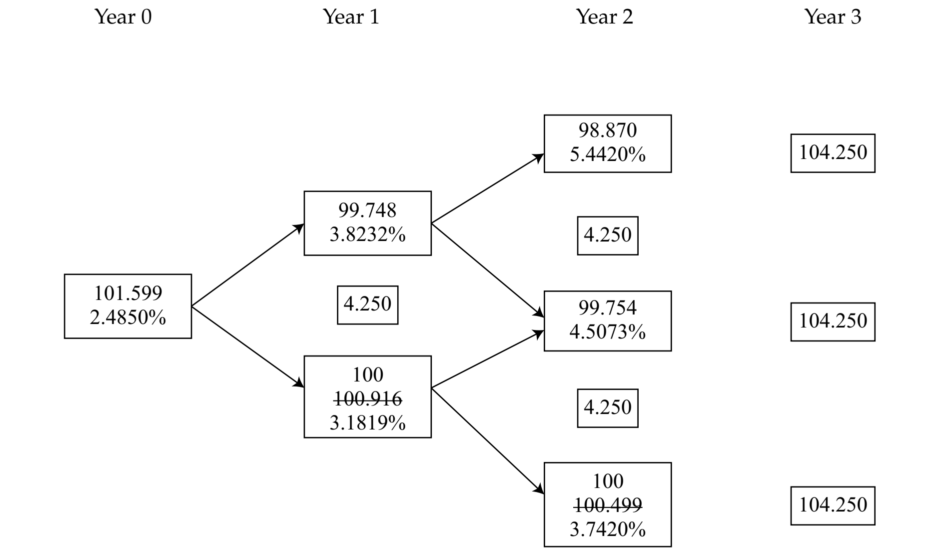
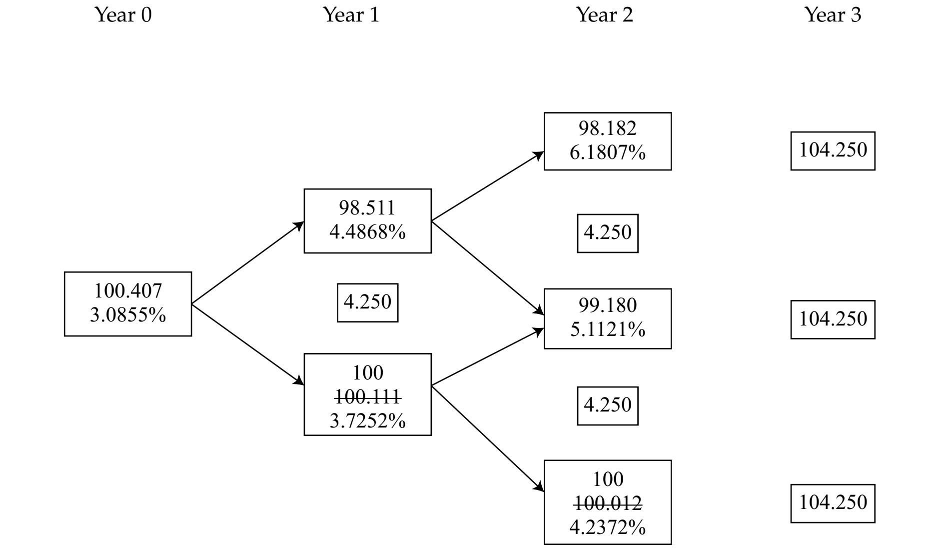


Exhibit 19: Valuation of a Three-Year 4.25% Annual Coupon Bond Callable at Par One Year and Two Years from Now at 10% Interest Rate Volatility with an OAS of 28.55 bps When Interest Rates Are Shifted Up by 30 bps



The effective duration of a callable bond cannot exceed that of the straight bond. When interest rates are high relative to the bond's coupon, the call option is out of the money so the bond is unlikely to be called. Thus, the effect of an interest rate change on the price of a callable bond is very similar to that on the price of an otherwise identical option-free bond; the callable and straight bonds have very similar effective durations. In contrast, when interest rates fall, the call option moves into the money.

Remember that the call option gives the issuer the right to retire the bond at the call price and thus limits the price appreciation when interest rates decline. As a consequence, the call option reduces the effective duration of the callable bond relative to that of the straight bond.

The effective duration of a putable bond also cannot exceed that of the straight bond. When interest rates are low relative to the bond's coupon, the put option is out of the money so the bond is unlikely to be put. Thus, the effective duration of the putable bond is in this case very similar to that of an otherwise identical option-free bond. In contrast, when interest rates rise, the put option moves into the money and limits the price depreciation because the investor can put the bond and reinvest the proceeds of the retired bond at a higher yield. Thus, the put option reduces the effective duration of the putable bond relative to that of the straight bond.

When the embedded option (call or put) is deep in the money, the effective duration of the bond with an embedded option resembles that of the straight bond maturing on the first exercise date, reflecting the fact that the bond is highly likely to be called or put on that date.

Exhibit 20 compares the effective durations of option-free, callable, and putable bonds. All bonds are 4% annual coupon bonds with a maturity of 10 years. Both the call option and the put option are European-like and exercisable two months from now. The bonds are valued assuming a 4% flat yield curve and an interest rate volatility of 10%.

Exhibit 20: Comparison of the Effective Durations of Option-Free, Callable, and Putable Bonds

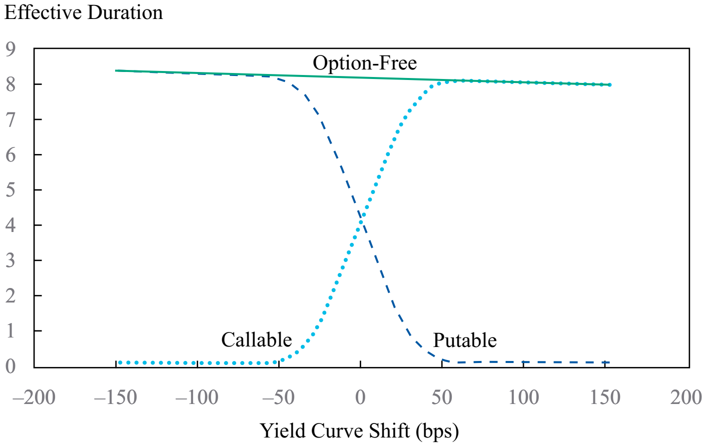


Exhibit 20 shows that the effective duration of an option-free bond changes very little in response to interest rate movements. As expected, when interest rates rise the put option moves into the money, which limits the price depreciation of the putable bond and shortens its effective duration. In contrast, the effective duration of the callable bond shortens when interest rates fall, which is when the call option moves into the money and thus limits the price appreciation of the callable bond.

EFFECTIVE DURATION IN PRACTICE

Effective duration is a concept most practically used in the context of a portfolio. Thus, an understanding of the effective durations of various types of instruments helps manage portfolio duration. In the following table, we show some properties of the effective duration of cash and the common types of bonds:

Type of Bond	Effective Duration
Cash	0
Zero-coupon bond	≈ Maturity
Fixed-rate bond	< Maturity
Callable bond	≤ Duration of straight bond
Putable bond	≤ Duration of straight bond
Floater (MRR flat)	≈ Time (in years) to next reset

In general, a bond's effective duration does not exceed its maturity. There are a few exceptions, however, such as tax-exempt bonds when analyzed on an after-tax basis.

Knowing the effective duration of each type of bond is useful when one needs to change portfolio duration. For example, a portfolio manager who wants to shorten the effective duration of a portfolio of fixed-rate bonds can add floaters. For the debt manager of a company or other issuing entity, another way of shortening effective duration is to issue callable bonds. The topic of changing portfolio duration is covered thoroughly in Level III.

7 ONE-SIDED AND KEY RATE DURATION

☐ describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options

Effective durations are normally calculated by averaging the changes resulting from shifting the benchmark yield curve up and down by the same amount. This calculation works well for option-free bonds, but the results can be misleading in the presence of embedded options. The problem is that when the embedded option is in the money, the price of the bond has limited upside potential if the bond is callable or limited downside potential if the bond is putable. Thus, the price sensitivity of bonds with embedded options is not symmetrical to positive and negative changes in interest rates of the same magnitude.

Consider, for example, a 4.5% bond maturing in five years, which is currently callable at 100. On a 4% flat yield curve at 15% volatility, the value of this callable bond is 99.75. If interest rates declined by 30 bps, the price would rise to 100. In fact, no matter how far interest rates decline, the price of the callable bond cannot exceed 100 because no investor will pay more than the price at which the bond can be immediately called. In contrast, the price decline has no limit if interest rates rise. Thus, the average price response to up- and down-shifts of interest rates (effective duration) is not as informative as the price responses to the up-shift (one-sided up-duration) and the down-shift (one-sided down-duration) of interest rates.

Exhibit 21 and Exhibit 22 illustrate why **one-sided durations**—that is, the effective durations when interest rates go up or down—are better at capturing the interest rate sensitivity of a callable or putable bond than the (two-sided) effective durations, particularly when the embedded option is near the money.

Exhibit 21: Durations for a 4.5% Annual Coupon Bond Maturing in Five Years and Immediately Callable at Par on a 4% Flat Yield Curve at 15% Interest Rate Volatility

	At a 4% Flat Yield Curve	Interest Rate up by 30 bps	Interest Rate down by 30 bps
Value of the bond	99.75	99.17	100.00
Duration measure	Effective duration 1.39	One-sided up-duration 1.94	One-sided down-duration 0.84

Exhibit 21 shows that a 30 bps increase in the interest rate has a greater effect on the value of the callable bond than a 30 bps decrease in the interest rate. The fact that the one-sided up-duration is higher than the one-sided down-duration confirms that the callable bond is more sensitive to interest rate rises than to interest rate declines.

Exhibit 22: Durations for a 4.1% Annual Coupon Bond Maturing in Five Years and Immediately Putable at Par on a 4% Flat Yield Curve at 15% Interest Rate Volatility

	At a 4% Flat Yield Curve	Interest Rate up by 30 bps	Interest Rate down by 30 bps
Value of the bond	100.45	100.00	101.81
Duration measure	Effective duration 3.00	One-sided up-duration 1.49	One-sided down-duration 4.51

The one-sided durations in Exhibit 22 indicate that the putable bond is more sensitive to interest rate declines than to interest rate rises.

Key Rate Durations

Effective duration is calculated by assuming parallel shifts in the benchmark yield curve. In reality, however, interest rate movements are not as neat. Many portfolio managers and risk managers like to isolate the price responses to changes in the rates of key maturities on the benchmark yield curve. For example, how would the price of a bond be expected to change if only the two-year benchmark rate moved up by 5 bps? The answer is found by using **key rate durations** (also known as partial durations), which reflect the sensitivity of the bond's price to changes in specific maturities on the benchmark yield curve. Thus, key rate durations help portfolio managers and risk managers identify the "shaping risk" for bonds—that is, the bond's sensitivity to changes in the shape of the yield curve (e.g., steepening and flattening).

The valuation procedure and formula applied in the calculation of key rate durations are identical to those used in the calculation of effective duration, but instead of shifting the entire benchmark yield curve, only key points are shifted one at a time. Thus, the effective duration for each maturity point shift is calculated in isolation.

Exhibit 23, Exhibit 24, and Exhibit 25 show the key rate durations for bonds valued at a 4% flat yield curve. Exhibit 23 examines option-free bonds (assuming semi-annual coupons), and Exhibit 24 and Exhibit 25 extend the analysis to callable and putable bonds, respectively.

Exhibit 23: Key Rate Durations of 10-Year Option-Free Bonds Valued at a 4% Flat Yield Curve

Coupon (%)	Price (% of par)	Key Rate Durations				
		Total	2-Year	3-Year	5-Year	10-Year
0	67.30	9.81	–0.07	–0.34	–0.93	11.15
2	83.65	8.83	–0.03	–0.13	–0.37	9.37
4	100.00	8.18	0.00	0.00	0.00	8.18
6	116.35	7.71	0.02	0.10	0.27	7.32
8	132.70	7.35	0.04	0.17	0.47	6.68
10	149.05	7.07	0.05	0.22	0.62	6.18

As shown in Exhibit 23, for option-free bonds not trading at par (the white rows), shifting any par rate has an effect on the value of the bond, but shifting the maturity-matched (10-year in this example) par rate has the greatest effect. This is simply because the largest cash flow of a fixed-rate bond occurs at maturity with the payment of both the final coupon and the principal.

For an option-free bond trading at par (the shaded row), the maturity-matched par rate is the only rate that affects the bond's value. It is a definitional consequence of "par" rates. If the 10-year par rate on a curve is 4%, then a 10-year 4% bond valued on that curve at zero OAS will be worth par regardless of the par rates of the other maturity points on the curve. In other words, shifting any rate other than the 10-year rate on the par yield curve will not change the value of a 10-year bond trading at par. Shifting a par rate up or down at a particular maturity point, however, respectively increases or decreases the *discount rate* at that maturity point. These facts will be useful to remember in the following paragraph.

As illustrated in Exhibit 23, key rate durations can sometimes be negative for maturity points that are shorter than the maturity of the bond being analyzed if the bond is a zero-coupon bond or has a very low coupon. We can explain why this is the case by using the zero-coupon bond (the first row of Exhibit 23). As discussed in the previous paragraph, if we increase the five-year par rate, the value of a 10-year bond trading at par must remain unchanged because the 10-year par rate has not changed. But the five-year zero-coupon rate has increased because of the increase in the five-year par rate. Thus, the value of the five-year coupon of the 10-year bond trading at par will be lower than before the increase. But because the value of the 10-year bond trading at par must remain par, the remaining cash flows, including the cash flow occurring in Year 10, must be discounted at slightly *lower* rates to compensate. This results in a lower 10-year zero-coupon rate, which makes the value of a 10-year zero-coupon bond (whose only cash flow is in Year 10) *rise* in response to an *upward* change in the five-year par rate. Consequently, the five-year key rate duration for a 10-year zero-coupon bond is negative (–0.93).

Unlike for option-free bonds, the key rate durations of bonds with embedded options depend not only on the *time to maturity* but also on the *time to exercise*. Exhibit 24 and Exhibit 25 illustrate this phenomenon for 30-year callable and putable bonds. Both the call option and the put option are European-like exercisable 10 years from now, and the bonds are valued assuming a 4% flat yield curve and a volatility of 15%.

Exhibit 24: Key Rate Durations of 30-Year Bonds Callable in 10 Years Valued at a 4% Flat Yield Curve with 15% Interest Rate Volatility

Coupon (%)	Price (% of par)	Key Rate Durations					
		Total	2-Year	3-Year	5-Year	10-Year	30-Year
2	64.99	19.73	−0.02	−0.08	−0.21	−1.97	22.01
4	94.03	13.18	0.00	0.02	0.05	3.57	9.54
6	114.67	9.11	0.02	0.10	0.29	6.00	2.70
8	132.27	7.74	0.04	0.17	0.48	6.40	0.66
10	148.95	7.14	0.05	0.22	0.62	6.06	0.19

The bond with a coupon of 2% (the first row of Exhibit 24) is unlikely to be called, and thus it behaves more like a 30-year option-free bond, whose effective duration depends primarily on movements in the 30-year par rate. Therefore, the rate that has the highest effect on the value of the callable bond is the maturity-matched (30-year) rate. As the bond's coupon increases, however, so does the likelihood of the bond being called. Thus, the bond's total effective duration shortens, and the rate that has the highest effect on the callable bond's value gradually shifts from the 30-year rate to the 10-year rate. At the very high coupon of 10%, because of the virtual certainty of being called, the callable bond behaves like a 10-year option-free bond; the 30-year key rate duration is negligible (0.19) relative to the 10-year key rate duration (6.06).

Exhibit 25: Key Rate Durations of 30-Year Bonds Putable in 10 Years Valued at a 4% Flat Yield Curve with 15% Interest Rate Volatility

Coupon (%)	Price (% of par)	Key Rate Durations					
		Total	2-Year	3-Year	5-Year	10-Year	30-Year
2	83.89	9.24	−0.03	−0.14	−0.38	8.98	0.81
4	105.97	12.44	0.00	−0.01	−0.05	4.53	7.97
6	136.44	14.75	0.01	0.03	0.08	2.27	12.37
8	169.96	14.90	0.01	0.06	0.16	2.12	12.56
10	204.38	14.65	0.02	0.07	0.21	2.39	11.96

If the 30-year bond putable in 10 years has a high coupon, its price is more sensitive to the 30-year rate because it is unlikely to be put and thus behaves like an otherwise identical option-free bond. The 10% putable bond (the last row of Exhibit 25), for example, is most sensitive to changes in the 30-year rate, as illustrated by a 30-year key rate duration of 11.96. At the other extreme, a low-coupon bond is most sensitive to movements in the 10-year rate. It is almost certain to be put and so behaves like an option-free bond maturing on the put date.

EFFECTIVE CONVEXITY

8

☐ compare effective convexities of callable, putable, and straight bonds

Duration is an approximation of the expected bond price responses to changes in interest rates because actual changes in bond prices are not linear, particularly for bonds with embedded options. Thus, it is useful to measure **effective convexity**—that is, the sensitivity of duration to changes in interest rates—as well. The formula to calculate a bond's effective convexity is

$$\text{EffCon} = \frac{(PV_{-})+(PV_{+})-[2\times(PV_0)]}{(\Delta\text{Curve})^2\times(PV_0)}, \quad (4)$$

where

ΔCurve = the magnitude of the parallel shift in the benchmark yield curve (in decimal)

PV_{-} = the full price of the bond when the benchmark yield curve is shifted down by ΔCurve

PV_{+} = the full price of the bond when the benchmark yield curve is shifted up by ΔCurve

PV_0 = the current full price of the bond (i.e., with no shift)

Let us return to the three-year 4.25% bond callable at par one year and two years from now. We still use the same par yield curve (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively) and the same interest rate volatility (10%) as before, but we now assume that the bond's current full price is 100.785 instead of 101.000. Thus, the implied OAS is 40 bps. Given 30 bps shifts in the benchmark yield curve, the resulting PV_{-} and PV_{+} are 101.381 and 100.146, respectively. Using Equation 4, the effective convexity is:

$$\text{EffCon} = \frac{101.381 + 100.146 - 2\times 100.785}{(0.003)^2\times 100.785} = -47.41.$$

[Note that there are two different conventions for reporting convexity in practice; "raw" convexity figures, such as in this example, are sometimes scaled (divided) by 100.]

Exhibit 20, shown earlier, displays effective durations but also illustrates the effective convexities of callable and putable bonds. When interest rates are high and the value of the call option is low, the callable and straight bond experience very similar effects from changes in interest rates. They both have positive convexity. However, the effective convexity of the callable bond turns negative when the call option is near the money, as in the example just presented, which indicates that the upside for a callable bond is much smaller than the downside. The reason is because when interest rates decline, the price of the callable bond is capped by the price of the call option if it is near the exercise date.

Conversely, putable bonds always have positive convexity. When the option is near the money, the upside for a putable bond is much larger than the downside because the price of a putable bond is floored by the price of the put option if it is near the exercise date.

Compared side by side, putable bonds have more upside potential than otherwise identical callable bonds when interest rates decline. Putable bonds also have less downside risk than otherwise identical callable bonds when interest rates rise.

EXAMPLE 7

Interest Rate Sensitivity

Erna Smith, a portfolio manager, has two fixed-rate bonds in her portfolio: a callable bond (Bond X) and a putable bond (Bond Y). She wants to examine the interest rate sensitivity of these two bonds to a parallel shift in the benchmark yield curve. Assuming an interest rate volatility of 10%, her valuation software shows how the prices of these bonds change for 30 bps shifts up or down:

	Bond X	Bond Y
Time to maturity	Three years from today	Three years from today
Coupon	3.75% annual	3.75% annual
Type of bond	Callable at par one year from today	Putable at par one year from today
Current price (% of par)	100.594	101.330
Price (% of par) when shifting the benchmark yield curve down by 30 bps	101.194	101.882
Price (% of par) when shifting the benchmark yield curve up by 30 bps	99.860	100.924

1. The effective duration for Bond X is *closest* to:

   A. 0.67.
   B. 2.21.
   C. 4.42.

Solution:

B is correct. The effective duration for Bond X is

$$\text{EffDur} = \frac{101.194 - 99.860}{2 \times 0.003 \times 100.594} = 2.21.$$

A is incorrect because the duration of a bond with a single cash flow one year from now is approximately one year, so 0.67 is too low—even assuming that the bond will be called in one year with certainty. C is incorrect because 4.42 exceeds the maturity of Bond X (three years).

2. The effective duration for Bond Y is *closest* to:

   A. 0.48.
   B. 0.96.
   C. 1.58.

Solution:

C is correct. The effective duration for Bond Y is

$$\text{EffDur} = \frac{101.882 - 100.924}{2 \times 0.003 \times 101.330} = 1.58.$$

3. When interest rates rise, the effective duration of:

   A. Bond X shortens.
   B. Bond Y shortens.

C. the underlying option-free (straight) bond corresponding to Bond X lengthens.

Solution:

B is correct. When interest rates rise, a put option moves into the money and the putable bond is more likely to be put. Thus, it behaves like a shorter-maturity bond, and its effective duration shortens. A is incorrect because when interest rates rise, a call option moves out of the money; so, the callable bond is less likely to be called. C is incorrect because the effective duration of an option-free bond goes down as interest rates rise.

4. When the option embedded in Bond Y is in the money, the one-sided durations *most likely* show that the bond is:

   A. more sensitive to a decrease in interest rates.
   B. more sensitive to an increase in interest rates.
   C. equally sensitive to a decrease or to an increase in interest rates.

Solution:

A is correct. If interest rates rise, the investor's ability to put the bond at par limits the price depreciation. In contrast, the increase in the bond's price has no limit when interest rates decline. Thus, the price of a putable bond whose embedded option is in the money is more sensitive to a decrease in interest rates.

5. The price of Bond X is affected:

   A. only by a shift in the one-year par rate.
   B. only by a shift in the three-year par rate.
   C. by all par rate shifts but is most sensitive to shifts in the one-year and three-year par rates.

Solution:

C is correct. The main driver of the call decision is the two-year forward rate one year from now. This rate is most significantly affected by changes in the one-year and three-year par rates.

6. The effective convexity of Bond X:

   A. cannot be negative.
   B. turns negative when the embedded option is near the money.
   C. turns negative when the embedded option moves out of the money.

Solution:

B is correct. The effective convexity of a callable bond turns negative when the call option is near the money because the price response of a callable bond to lower interest rates is capped by the call option. That is, in case of a decline in interest rates, the issuer will call the bonds and refund at lower rates, thus limiting the upside potential for the investor.

7. Which of the following statements is *most* accurate?

   A. Bond Y exhibits negative convexity.
   B. For a given decline in interest rate, Bond X has less upside potential than Bond Y.

C. The underlying option-free (straight) bond corresponding to Bond Y exhibits negative convexity.

Solution:

B is correct. As interest rates decline, the value of a call option increases whereas the value of a put option decreases. The call option embedded in Bond X limits its price appreciation, but Bond Y has no such cap. Thus, Bond X has less upside potential than Bond Y. A is incorrect because a putable bond always has positive convexity; that is, Bond Y has more upside than downside potential. C is incorrect because an option-free bond exhibits low positive convexity.

CAPPED AND FLOORED FLOATING-RATE BONDS

9

☐ calculate the value of a capped or floored floating-rate bond

Options in floating-rate bonds (floaters) are exercised automatically depending on the course of interest rates; if the coupon rate rises or falls below the threshold, the cap or floor automatically applies. Similar to callable and putable bonds, capped and floored floaters can be valued by using the arbitrage-free framework.

Valuation of a Capped Floater

The cap provision in a floater prevents the coupon rate from increasing above a specified maximum rate. As a consequence, a **capped floater** protects the issuer against rising interest rates and is thus an issuer option. Because the investor is long the bond but short the embedded option, the value of the cap decreases the value of the capped floater relative to the value of the straight bond:

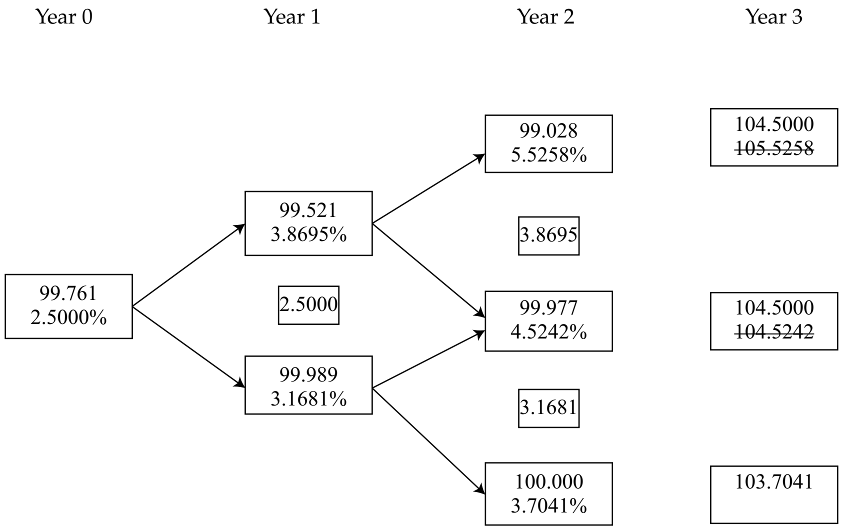
Value of capped floater

= Value of straight bond – Value of embedded cap. (5)

To illustrate how to value a capped floater, consider a floating-rate bond that has a three-year maturity. The floater's coupon pays the one-year reference rate annually, set in arrears, and is capped at 4.500%. The term "set in arrears" means that the coupon rate is set at the *end* of the coupon period; the payment date and the setting date are one and the same. For simplicity, we assume that the issuer's credit quality closely matches the reference rate swap curve (i.e., there is no credit spread) and that the reference rate swap curve is the same as the par yield curve given in Exhibit 1 (i.e., one-year, two-year, and three-year par yields of 2.500%, 3.000%, and 3.500%, respectively). We also assume that the interest rate volatility is 10%.

The valuation of the capped floater is depicted in Exhibit 26.

Exhibit 26: Valuation of a Three-Year Reference Rate Floater Capped at 4.500% at 10% Interest Rate Volatility



Without a cap, the value of this floater would be 100 because in every scenario, the coupon paid would be equal to the discount rate. But because the coupon rate is capped at 4.500%, which is lower than the highest interest rates in the tree, the value of the capped floater will be lower than the value of the straight bond.

For each scenario, we check whether the cap applies; if it does, the cash flow is adjusted accordingly. For example, at the top of the tree at Year 2, the reference rate (5.5258%) is higher than the 4.500% cap. Thus, the coupon payment at Year 3 is capped at the 4.500 maximum amount, and the cash flow is adjusted downward from the uncapped amount (105.5258) to the capped amount (104.5000). The coupon is also capped when the reference rate is 4.5242% at Year 2.

As expected, the value of the capped floater is lower than 100 (99.761). The value of the cap can be calculated by using Equation 5:

Value of embedded cap = 100 – 99.761 = 0.239.

Valuation of a Floored Floater

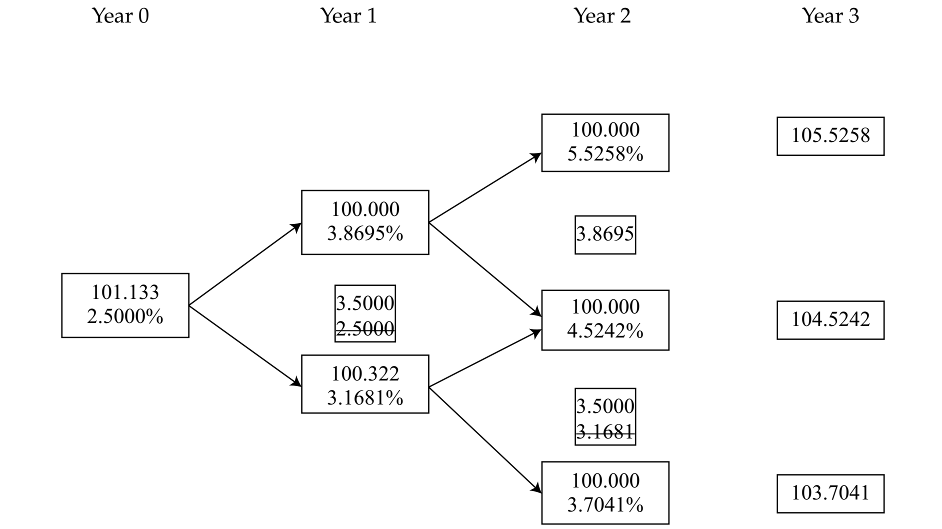
The floor provision in a floater prevents the coupon rate from decreasing below a specified minimum rate. As a consequence, a **floored floater** protects the investor against declining interest rates and is thus an investor option. Because the investor is long both the bond and the embedded option, the value of the floor increases the value of the floored floater relative to the value of the straight bond:

Value of floored floater

= Value of straight bond + Value of embedded floor. (6)

To illustrate how to value a floored floater, we return to the example we used for the capped floater but assume that the embedded option is now a 3.500% floor instead of a 4.500% cap. The other assumptions remain the same. The valuation of the floored floater is depicted in Exhibit 27.

Exhibit 27: Valuation of a Three-Year Reference Rate Floater Floored at 3.500% at 10% Interest Rate Volatility



Recall from the discussion about the capped floater that if there were no cap, the value of the floater would be 100 because the coupon paid would equal the discount rate. The same principle applies here: If there were no floor, the value of this floater would be 100. Because the presence of the floor potentially increases the cash flows, however, the value of the floored floater must be equal to or higher than the value of the straight bond.

Exhibit 27 shows that the floor is binding at Year 0 because the reference rate (2.5000%) is less than the cap rate (3.5000%) and at Year 1 at the lower node where the reference rate is 3.1681%. Thus, the corresponding interest payments at Year 1 and 2 are increased to the minimum amount of 3.5000. As a consequence, the value of the floored floater exceeds 100 (101.133). The value of the floor can be calculated by using Equation 6:

Value of embedded floor = 101.133 – 100 = 1.133.

EXAMPLE 8

Valuation of Capped and Floored Floaters

1. A three-year floating rate bond pays annual coupons of one-year reference rate (set in arrears) and is capped at 5.600%. The reference rate swap curve is as given in Exhibit 1 (i.e., the one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively), and interest rate volatility is 10%. The value of the capped floater is *closest to*:

   A. 100.000.

   B. 105.600.

   C. 105.921.

Solution:

A is correct. As illustrated in Exhibit 26, the cap is higher than any of the rates at which the floater is reset on the interest rate tree. Thus, the value of the bond is the same as if it had no cap—that is, 100.

2. A three-year floating-rate bond pays annual coupons of one-year reference rate (set in arrears) and is floored at 3.000%. The reference swap curve is as given in Exhibit 1 (i.e., the one-year, two-year, and three-year par yields are 2.500%, 3.000%, and 3.500%, respectively), and interest rate volatility is 10%. The value of the floored floater is *closest to*:

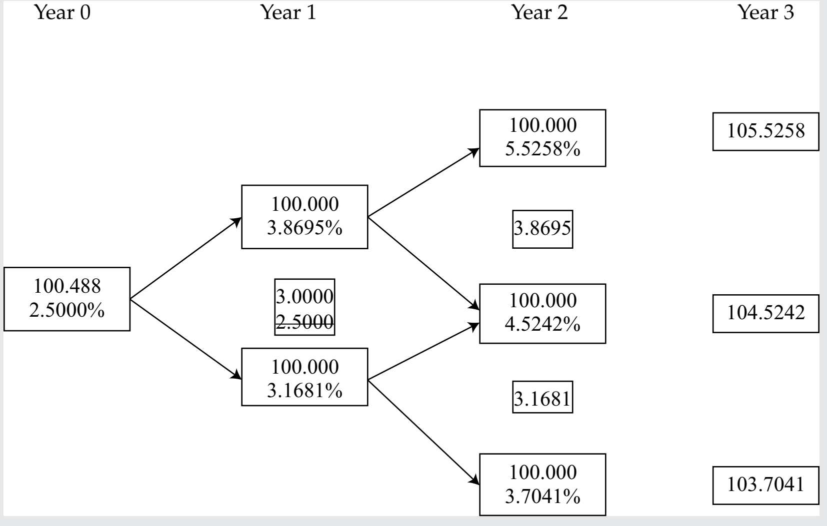
   A. 100.000.

   B. 100.488.

   C. 103.000.

Solution:

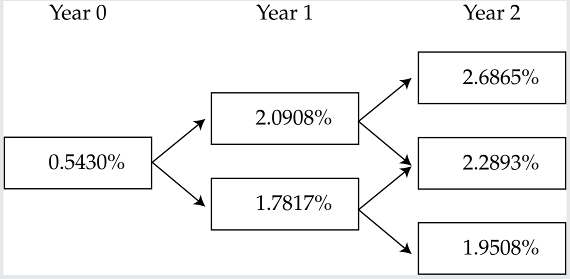
B is correct. One can eliminate C because as illustrated in Exhibit 27, all else being equal, the bond with a higher floor (3.500%) has a value of 101.133. The value of a bond with a floor of 3.000% cannot be higher. Intuitively, B is the likely correct answer because the straight bond is worth 100. However, it is still necessary to calculate the value of the floored floater because if the floor is low enough, it could be worthless.



Here, it turns out that the floor adds 0.488 in value to the straight bond. Had the floor been 2.500%, the floored floater and the straight bond would both be worth par.

3. An issuer in the eurozone wants to sell a three-year floating-rate note at par with an annual coupon based on the 12-month Euribor + 300 bps. Because the 12-month Euribor is currently at a historic low and the issuer wants to protect itself against a sudden increase in interest cost, the issuer's advisers recommend increasing the credit spread to 320 bps and capping the coupon

at 5.50%. Assuming an interest rate volatility of 8%, the advisers have constructed the following binomial interest rate tree:



The value of the capped floater is *closest to*:

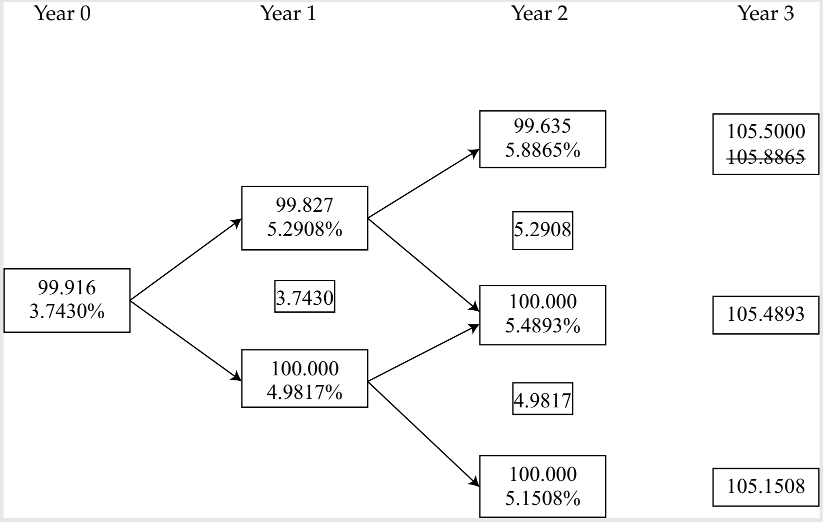
A. 92.929.

B. 99.916.

C. 109.265.

Solution:

B is correct.



CONVERTIBLE BONDS 10

- ☐ describe defining features of a convertible bond
- ☐ calculate and interpret the components of a convertible bond's value

So far, we have discussed bonds for which the exercise of the option is at the discretion of the issuer (callable bond), at the discretion of the bondholder (putable bond), or set through a pre-defined contractual arrangement (capped and floored floaters). What distinguishes a convertible bond from the bonds discussed earlier is that exercising the option results in the change of the security from a bond to a common stock. This section describes defining features of convertible bonds and discusses how to analyze and value these bonds.

Defining Features of a Convertible Bond

A **convertible bond** presents the characteristics of an option-free bond and an embedded conversion option, which gives bondholders the right to convert their debt into equity during the **conversion period** at a pre-determined **conversion price**.

Investors usually accept a lower coupon for convertible bonds than for otherwise identical non-convertible bonds because they can participate in the potential upside through the conversion mechanism that allows the bondholders to convert their bonds into shares at a cost lower than market value. The issuer benefits from paying a lower coupon. In case of conversion, an added benefit for the issuer is that it no longer has to repay the debt that was converted into equity.

However, what might appear as a win–win situation for both the issuer and the investors is not a "free lunch" because the issuer's existing shareholders face dilution in case of conversion. In addition, if the underlying share price remains below the conversion price and the bond is not converted, the issuer must repay the debt or refinance it, potentially at a higher cost. If conversion is not achieved, the bondholders will have lost interest income relative to an otherwise identical non-convertible bond that would have been issued with a higher coupon and would have thus offered investors an additional spread.

We will use the information provided in Exhibit 28 to describe the features of a convertible bond and then illustrate how to analyze it. Exhibit 28 is based on a $1 billion convertible bond issued in June 2018 by Twitter, Inc. (TWTR), a company listed on the New York Stock Exchange. Some features of the actual convertible bond, such as the presence of a make-whole call option, have been dropped for simplicity.

Exhibit 28: Twitter, Inc., $1 billion, 0.25% Convertible Bonds Due 15 June 2024

- **Issue Date:** 11 June 2018
- **Ranking:** Senior unsecured
- **Interest:** 0.25% per year. Interest will accrue from 11 June 2018 and will be payable semiannually in arrears on 15 June and 15 December of each year, beginning on 15 December 2018.
- **Issue Price:** 100% of par value
- **Maturity:** 15 June 2024
- **Conversion Rate:** Each bond of par value of $1,000 is convertible to 17.5 shares of common stock.
- **Conversion Price:** $57.14 per share
- **Share Price at Issuance:** $40.10
- (Assumed) **Share Price on 15 June 2019:** $35.14
- (Assumed) **Convertible Bond Price on 15 June 2019:** 95.225% of par value
- **Conversion Premium:** 42.5%

The applicable share price at which the investor can convert the bonds into ordinary (common) shares is called the conversion price. In the Twitter example provided in Exhibit 28, the conversion price is $57.14 per share.

The number of shares of common stock that the bondholder receives from converting the bonds into shares is called the **conversion rate (or ratio)**. In the Twitter example, bondholders who hold $10,000 in par value can convert their bonds into shares and receive 175 shares ($10,000/$57.14). The conversion rate is 17.5 per $1,000 in par value. The conversion may be exercised during a particular period or at set intervals during the life of the bond.

The conversion price in Exhibit 28 is referred to as the *initial* conversion price because it reflects the conversion price *at issuance*. Corporate actions—such as stock splits, bonus share issuances, and rights or warrants issuances—affect a company's share price and may reduce the benefit of conversion for the convertible bondholders. Thus, the terms of issuance of the convertible bond contain detailed information defining how the conversion price and conversion ratio are adjusted should such a corporate action occur during the life of the bond. For example, suppose that Twitter performs a 2:1 stock split to its common shareholders. In this case, the conversion price would be adjusted to $28.57 (i.e., $57.14/2) per share and the conversion rate adjusted to 35 (i.e., 17.5 × 2) shares per $1,000 of nominal value.

As long as the convertible bond is still outstanding and has not been converted, the bondholders receive interest payments (semiannually in the Twitter example). Meanwhile, if the issuer declares and pays dividends, common shareholders receive dividend payments. The terms of issuance may offer no compensation to convertible bondholders for dividends paid out during the life of the bond at one extreme, or they may offer full protection by adjusting the conversion price downward for any dividend payments at the other extreme. Typically, a threshold dividend is defined in the terms of issuance. Annual dividend payments below the threshold dividend have no effect on the conversion price. In contrast, the conversion price is adjusted downward for annual dividend payments above the threshold dividend to offer compensation to convertible bondholders.

Should the issuer be acquired by or merged with another company during the life of the bond, bondholders might no longer be willing to continue lending to the new entity. Change-of-control events are defined in the prospectus or offering circular, and if such an event occurs, convertible bondholders usually have the choice between

- a put option that can be exercised during a specified period following the change-of-control event and that provides full redemption of the nominal value of the bond; or
- an adjusted conversion price that is lower than the initial conversion price. This downward adjustment gives the convertible bondholders the opportunity to convert their bonds into shares earlier and at more advantageous terms—thus allowing them to participate in the announced merger or acquisition as common shareholders.

In addition to a put option in case of a change-of-control event, it is not unusual for a convertible bond to include a put option that convertible bondholders can exercise during specified periods. Put options can be classified as "hard" puts or "soft" puts. In the case of a hard put, the issuer must redeem the convertible bond for cash. In the case of a soft put, the investor has the right to exercise the put but the issuer chooses how the payment will be made. The issuer may redeem the convertible bond for cash, common stock, subordinated notes, or a combination of the three.

It is more frequent for convertible bonds to include a call option that gives the issuer the right to call the bond during a specified period and at specified times. As discussed earlier, the issuer may exercise the call option and redeem the bond early if interest rates are falling or if its credit rating is revised upward—thus enabling the issuance of debt at a lower cost. The issuer may also believe that its share price will increase significantly in the future because of its performance or because of events that will take place in the economy or in its sector. In this case, the issuer may try to maximize the benefit to its existing shareholders relative to convertible bondholders and call the bond. To offer convertible bondholders protection against early repayment, convertible bonds usually have a protection period. Subsequently, they can be called but at a premium, which decreases as the maturity of the bond approaches.

If a convertible bond is callable, the issuer has an incentive to call the bond when the underlying share price increases above the conversion price in order to avoid paying further coupons. Such an event is called **forced conversion** because it forces bondholders to convert their bonds into shares. Otherwise, the redemption value that bondholders would receive from the issuer calling the bond would result in a disadvantageous position and a loss compared with conversion. Even if interest rates have not fallen or the issuer's credit rating has not improved, thus not allowing refinancing at a lower cost, the issuer might still proceed with calling the bond when the underlying share price exceeds the conversion price. Doing so allows the issuer to take advantage of the favorable equity market conditions and force the bondholders to convert their bonds into shares. The forced conversion strengthens the issuer's capital structure and eliminates the risk that a subsequent correction in equity prices prevents conversion and requires redeeming the convertible bonds at maturity.

Analysis of a Convertible Bond

A number of investment metrics and ratios help analyze and value a convertible bond.

Conversion Value

The **conversion value**, or parity value, of a convertible bond indicates the value of the bond if it is converted at the market price of the shares.

Conversion value = Underlying share price × Conversion ratio.

Based on the information provided in Exhibit 28, we can calculate the conversion value for Twitter's convertible bonds at the issuance date and on 15 June 2019 (*Note:* The assumed prices actually pertain to 11 April 2019 to simplify the calculation of the straight bond values as there are then five full years to maturity):

Conversion value at the issuance date = \$40.10 × 17.5 = \$701.75.

Conversion value on 15 June 2019 = \$35.14 × 17.5 = \$614.95.

Minimum Value of a Convertible Bond

The minimum value of a convertible bond is equal to the greater of

- the conversion value and
- the value of the underlying option-free bond. Theoretically, the value of the straight bond (straight value) can be estimated by using the market value of a non-convertible bond of the issuer with the same characteristics as the convertible bond but without the conversion option. In practice, such a bond rarely exists. Thus, the straight value is found by using the arbitrage-free framework and by discounting the bond's future cash flows at the appropriate rates.

The minimum value of a convertible bond can also be described as a floor value. It is a *moving* floor, however, because the straight value is not fixed; it changes with fluctuations in interest rates and credit spreads. If interest rates rise, the value of the straight bond falls, making the floor fall. Similarly, if the issuer's credit spread increases—as a result, for example, of a downgrade of its credit rating from investment grade to non-investment grade—the floor value will fall too.

Using the conversion values calculated earlier, the minimum value of Twitter's convertible bonds at the issuance date is

Minimum value at the issuance date = Maximum ($701.75; $1,000)

= $1,000.

The straight value at the issuance date is $1,000 because the issue price is set at 100% of par. But after this date, this value will fluctuate. Thus, to calculate the minimum value of Twitter's convertible bond on 15 June 2019, it is first necessary to calculate the value of the straight bond that day using the arbitrage-free framework. From Exhibit 28, the coupon is 0.25%, paid semiannually. Assuming a 2.5% flat yield curve, the straight value on 15 June 2019 when five years remain until maturity is $894.86 per $1,000 in par value:

$$\frac{\$1.25}{\left(1+\frac{0.025}{2}\right)^{1}}+\frac{\$1.25}{\left(1+\frac{0.025}{2}\right)^{2}}+\ldots+\frac{\$1,001.25}{\left(1+\frac{0.025}{2}\right)^{10}} = \$894.86.$$

It follows that the minimum value of Twitter's convertible bonds on 15 June 2019 is:

Minimum value = Maximum ($614.95; $894.86) = $894.86.

If the value of the convertible bond were lower than the greater of the conversion value and the straight value, an arbitrage opportunity would ensue. Two scenarios help illustrate this concept. Returning to the Twitter example, suppose that the convertible bond is selling for $850.00 on 15 June 2019—that is, at a price that is lower than the straight value of $894.86. In this scenario, the convertible bond is cheap relative to the straight bond; put another way, the convertible bond offers a higher yield than an otherwise identical non-convertible bond. Thus, investors will find the convertible bond attractive, buy it, and push its price up until the convertible bond price returns to the straight value and the arbitrage opportunity disappears.

Alternatively, assume that on 15 June 2019 the yield on otherwise identical non-convertible bonds is 12.00% instead of 2.50%. Using the arbitrage-free framework, the straight value is $567.59 per $1,000 in par value. Suppose that the convertible bond is selling at this straight value—that is, at a price that is lower than its conversion value of $614.95. In this case, an arbitrageur can buy the convertible bond for $567.59, convert it into 17.5 shares, and sell the shares at $35.14 each or $614.95 in total. The arbitrageur makes a profit equal to the difference between the conversion value and the straight value—that is, $47.36 ($614.95 – $567.59). As more arbitrageurs follow the same strategy, the convertible bond price will increase until it reaches the conversion value and the arbitrage opportunity disappears.

Market Conversion Price, Market Conversion Premium per Share, and Market Conversion Premium Ratio

Many investors do not buy a convertible bond at issuance on the primary market but instead buy such a bond later in its life on the secondary market. The **market conversion premium per share** allows investors to identify the premium or discount payable when buying the convertible bond rather than the underlying common stock:

Market conversion premium per share

= Market conversion price – Underlying share price,

where

$$\text{Market conversion price} = \frac{\text{Convertible bond price}}{\text{Conversion ratio}}.$$

The market conversion price represents the price that investors effectively pay for the underlying common stock if they buy the convertible bond and then convert it into shares. It can be viewed as a break-even price. Once the underlying share price exceeds the market conversion price, any further rise in the underlying share price is certain to increase the value of the convertible bond by at least the same percentage (we will discuss why at a later stage).

Based on the information provided in Exhibit 28,

$$\text{Marketconversionpriceon15June2019} = \frac{\$952.25}{17.5} = \$54.40$$

and

Market conversion premium per share on 15 June 2019

= \$54.40 – \$35.14

= \$19.26.

The **market conversion premium ratio** expresses the premium, or discount, investors have to pay as a percentage of the current market price of the shares:

$$\text{Market conversion premium ratio} = \frac{\text{Market conversion premium per share}}{\text{Underlying share price}}.$$

In the Twitter example,

$$\text{Market conversion premium ratio on 15 June 2019} = \frac{\$19.26}{\$35.14}$$

= 54.8%.

Why would investors be willing to pay a premium to buy the convertible bond? Recall that the straight value acts as a floor for the convertible bond price. Thus, as the underlying share price falls, the convertible bond price will not fall below the straight value. Viewed in this context, the market conversion premium per share resembles the price of a call option. Investors who buy a call option limit their downside risk to the price of the call option (premium). Similarly, the premium paid when buying a convertible bond allows investors to limit their downside risk to the straight value. There is a fundamental difference, however, between the buyers of a call option and the buyers of a convertible bond. The former know exactly the amount of the downside risk, whereas the latter know only that the most they can lose is the difference between the convertible bond price and the straight value because the straight value is not fixed.

Market conversion discounts per share are rare, but they can theoretically happen given that the convertible bond and the underlying common stock trade in different markets with different types of market participants. For example, highly volatile share prices may result in the market conversion price being lower than the underlying share price.

Downside Risk with a Convertible Bond

Many investors use the straight value as a measure of the downside risk of a convertible bond and calculate the following metric:

$$\text{Premium over straight value} = \frac{\text{Convertible bond price}}{\text{Straight value}} - 1.$$

All else being equal, the higher the premium over straight value, the less attractive the convertible bond. In the Twitter example,

$$\text{Premium over straight value} = \frac{\$952.25}{\$894.86} - 1$$

=6.41%

Despite its use in practice, the premium over straight value is a flawed measure of downside risk because, as mentioned earlier, the straight value is not fixed but rather fluctuates with changes in interest rates and credit spreads.

Upside Potential of a Convertible Bond

The upside potential of a convertible bond depends primarily on the prospects of the underlying common stock. Thus, convertible bond investors should be familiar with the techniques used to value and analyze common stocks. These techniques are covered elsewhere.

COMPARISON OF RISK–RETURN CHARACTERISTICS 11

- ☐ describe how a convertible bond is valued in an arbitrage-free framework
- ☐ compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock

Historically, the valuation of convertible bonds has been challenging because these securities combine characteristics of bonds, stocks, and options—thus requiring an understanding of what affects the value of fixed income, equity, and derivatives. The complexity of convertible bonds has also increased over time as a result of market innovations and additions to the terms and conditions of these securities. For example, there are now contingent convertible bonds and convertible contingent convertible bonds, which are even more complex to value and analyze.

CONTINGENT CONVERTIBLES

Contingent convertible bonds, or "CoCos," pay a higher coupon than otherwise identical non-convertible bonds; however, they usually are deeply subordinated and may be converted into equity or face principal write-downs if regulatory capital ratios are breached. Convertible contingent convertible bonds, or "CoCoCos," combine a traditional convertible bond and a CoCo. They are convertible at the discretion of the investor, thus offering upside potential if the share price increases. They are also converted into equity or face principal write-downs in the event of a regulatory capital breach. CoCos and CoCoCos are usually issued by financial institutions, particularly in Europe.

The fact that many bond's prospectuses or offering circulars frequently provide for an independent financial valuer to determine the conversion price (and, in essence, the value of the convertible bond) under different scenarios is evidence of the complexity associated with valuing convertible bonds. Because of this complexity, convertible bonds in many markets come with selling restrictions. They are typically offered in very high denominations and only to professional or institutional investors. Regulators perceive them as securities that are too risky for retail investors to invest in directly.

As with any fixed-income instrument, convertible bond investors should perform a diligent risk–reward analysis of the issuer, including its ability to service the debt and repay the principal, as well as a review of the bond's terms of issuance (e.g., collateral, credit enhancements, covenants, and contingent provisions). In addition, convertible bond investors must analyze the factors that typically affect bond prices, such as interest rate movements. Because most convertible bonds have lighter covenants than otherwise similar non-convertible bonds and are frequently issued as subordinated securities, the valuation and analysis of some convertible bonds can be complex.

The investment characteristics of a convertible bond depend on the underlying share price, so convertible bond investors must also analyze factors that may affect the issuer's common stock, including dividend payments and the issuer's actions (e.g., acquisitions or disposals, rights issues). Even if the issuer is performing well, adverse market conditions might depress share prices and prevent conversion. Thus, convertible bond investors must also identify and analyze the exogenous reasons that might ultimately have a negative effect on convertible bonds.

Academics and practitioners have developed advanced models to value convertible bonds, but the most commonly used model remains the arbitrage-free framework. A traditional convertible bond can be viewed as a straight bond and a call option on the issuer's common stock, so

Value of convertible bond

= Value of straight bond + Value of call option on the issuer's stock.

Many convertible bonds include a call option that gives the issuer the right to call the bond during a specified period and at specified times. The value of such bonds is

Value of callable convertible bond

= Value of straight bond + Value of call option on the issuer's stock – Value of issuer call option.

Suppose that the callable convertible bond also includes a put option that gives the bondholder the right to require that the issuer repurchase the bond. The value of such a bond is

Value of callable putable convertible bond

= Value of straight bond + Value of call option on the issuer's stock – Value of issuer call option + Value of investor put option.

No matter how many options are embedded into a bond, the valuation procedure remains the same. It relies on generating a tree of interest rates based on the given yield curve and interest rate volatility assumptions, determining at each node of the tree whether the embedded options will be exercised, and then applying the backward induction valuation methodology to calculate the present value of the bond.

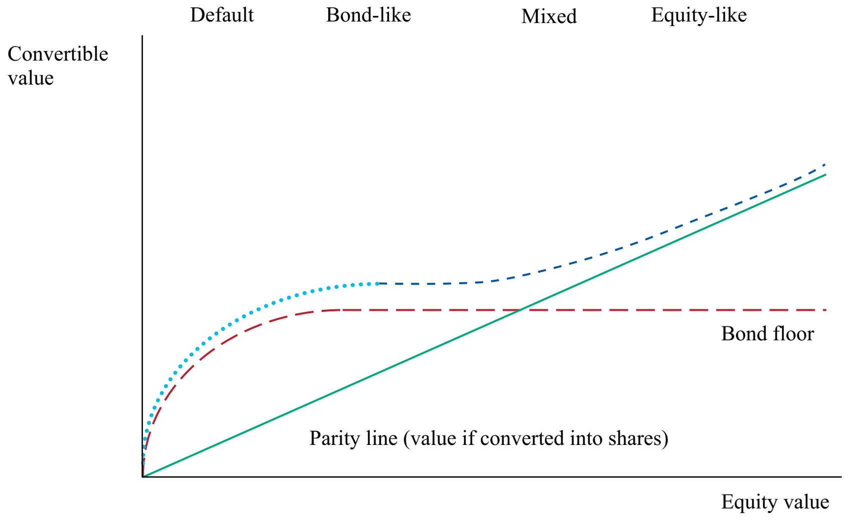
Comparison of the Risk–Return Characteristics of a Convertible Bond, the Straight Bond, and the Underlying Common Stock

In its simplest form, a convertible bond can be viewed as a straight bond and a call option on the issuer's common stock. When the underlying share price is well below the conversion price, the convertible bond is described as "busted convertible" and exhibits mostly bond risk–return characteristics. That is, the risk–return characteristics of the convertible bond resemble those of the underlying option-free (straight) bond. In this case, the call option is out of the money, so share price movements do not significantly affect the price of the call option and, thus, the price of the convertible bond. Consequently, the price movement of the convertible bond closely follows that of the straight bond, and such factors as interest rate movements and credit spreads significantly affect the convertible bond price. As the share price approaches zero, the value of the bond will fall to approach the present value of the recovery rate in bankruptcy. The convertible bond exhibits even stronger bond risk–return characteristics when the call option is out of the money and the conversion period is approaching its end because the time value component of the option decreases toward zero, making it highly likely that the conversion option will expire worthless. This scenario is shown in Exhibit 29 on the left.

In contrast, when the underlying share price is above the conversion price, a convertible bond exhibits mostly stock risk–return characteristics (see the right-hand side of Exhibit 29). That is, the risk–return characteristics of the convertible bond resemble those of the underlying common stock. In this case, the call option is in the money, so the price of the call option—and thus the price of the convertible bond—is significantly affected by share price movements but mostly unaffected by factors driving the value of an otherwise identical option-free bond, such as interest rate movements. When the call option is in the money, it is more likely to be exercised by the bondholder and the value of the shares resulting from the conversion is higher than the redemption value of the bond. Such convertible bonds trade at prices that closely follow the conversion value of the convertible bond, and their price exhibits similar movements to that of the underlying stock.

In between the bond and the stock extremes, the call option component increases in value as the underlying share price approaches the conversion price. The return on the convertible bond during such periods increases significantly but at a lower rate than the increase in the underlying share price because the conversion price has not yet been reached. When the share price exceeds the conversion price and goes higher, the change in the convertible bond price converges toward the change in the underlying share price. This is why we noted earlier that when the underlying share price exceeds the market conversion price, any further rise in the underlying share price is certain to increase the value of the convertible bond by at least the same percentage.

Exhibit 29: Price Behavior of a Convertible Bond and the Underlying Common Stock



Why would an investor not exercise the conversion option when the underlying share price is above the conversion price? The call option on the issuer's common stock may be a European-style option that cannot be exercised now but only at the end of a pre-determined period. Even if the call option is an American-style option, making it possible to convert the bond into equity, it may not be optimal for the convertible bondholder to exercise prior to the expiry of the conversion period. As discussed earlier, it is sometimes better to wait than to exercise an option that is in the money. The investor may also prefer to sell the convertible bond instead of exercising the conversion option.

Except for busted convertibles, the most important factor in the valuation of convertible bonds is the underlying share price. However, it is worth mentioning that large movements in interest rates or in credit spreads may significantly affect the value of convertible bonds. For a convertible bond with a fixed coupon, all else being equal, a significant fall in interest rates would result in an increase in its value and price, whereas a significant rise in interest rates would lead in a decrease in its value and price. Similarly, all else being equal, a significant improvement in the issuer's credit quality would result in an increase in the value and price of its convertible bonds, whereas a deterioration of the issuer's credit quality would lead to a decrease in the value and price of its convertible bonds.

EXAMPLE 9

Valuation of Convertible Bonds

Nick Andrews, a fixed-income investment analyst, has been asked by his supervisor to prepare an analysis of the convertible bond issued by Heavy Element Inc., a chemical industry company, for presentation to the investment committee. Andrews has gathered the following data from the convertible bond's prospectus and market information:

Issuer: Heavy Element Inc.

Issue Date: 15 September 2020

Maturity Date: 15 September 2025

Interest: 3.75% payable annually

Issue Size: $100,000,000

Issue Price: $1,000 at par

Conversion Ratio: 23.26

Convertible Bond Price on 16 September 2022: $1,230

Share Price on 16 September 2022: $52

1. The conversion price is *closest to*:

   A. $19.
   B. $43.
   C. $53.

Solution:

B is correct. The conversion price is equal to the par value of the convertible bond divided by the conversion ratio—that is, $1,000/23.26 = $43 per share.

2. The conversion value on 16 September 2022 is *closest to*:

   A. $24.
   B. $230.
   C. $1,209.

Solution:

C is correct. The conversion value is equal to the underlying share price multiplied by the conversion ratio—that is, \$52 × 23.26 = \$1,209.

3. The market conversion premium per share on 16 September 2022 is *closest to*:

   A. \$0.88.

   B. \$2.24.

   C. \$9.00.

Solution:

A is correct. The market conversion premium per share is equal to the convertible bond price divided by the conversion ratio, minus the underlying share price—that is, (\$1,230/23.26) – \$52 = \$52.88 – \$52 = \$0.88.

4. The risk–return characteristics of the convertible bond on 16 September 2022 *most likely* resemble that of:

   A. a busted convertible.

   B. Heavy Element's common stock.

   C. a bond of Heavy Element that is identical to the convertible bond but without the conversion option.

Solution:

B is correct. The underlying share price (\$52) is well above the conversion price (\$43). Thus, the convertible bond exhibits risk–return characteristics that are similar to those of the underlying common stock. A is incorrect because a busted convertible is a convertible bond for which the underlying common stock trades at a significant discount relative to the conversion price. C is incorrect because it describes a busted convertible.

5. As a result of favorable economic conditions, credit spreads for the chemical industry narrow, resulting in lower interest rates for the debt of such companies as Heavy Element. All else being equal, the price of Heavy Element's convertible bond will *most likely*:

   A. decrease significantly.

   B. not change significantly.

   C. increase significantly.

Solution:

B is correct. The underlying share price (\$52) is well above the conversion price (\$43). Thus, the convertible bond exhibits mostly stock risk–return characteristics, and its price is mainly driven by the underlying share price. Consequently, the decrease in credit spreads will have little effect on the convertible bond price.

6. Suppose that on 16 September 2022 the convertible bond is available in the secondary market at a price of \$1,050. An arbitrageur can make a risk-free profit by:

   A. buying the underlying common stock and shorting the convertible bond.

B. buying the convertible bond, exercising the conversion option, and selling the shares resulting from the conversion.

C. shorting the convertible bond and buying a call option on the underlying common stock exercisable at the conversion price on the conversion date.

Solution:

B is correct. The convertible bond price ($1,050) is lower than its minimum value ($1,209). Thus, the arbitrageur can buy the convertible bond for $1,050; convert it into 23.26 shares; and sell the shares at $52 each, or $1,209 in total, making a profit of $159. A and C are incorrect because in both scenarios, the arbitrageur is short the underpriced asset (convertible bond) and long an overpriced asset, resulting in a loss.

7. A few months have passed. Because of chemical spills in lake water at the site of a competing facility, the government has introduced very costly environmental legislation. As a result, share prices of almost all publicly traded chemical companies, including Heavy Element, have decreased sharply. Heavy Element's share price is now $28. Now, the risk–return characteristics of the convertible bond *most likely* resemble that of:

A. a bond.

B. a hybrid instrument.

C. Heavy Element's common stock.

Solution:

A is correct. The underlying share price ($28) is now well below the conversion price ($43), so the convertible bond is a busted convertible and exhibits mostly bond risk–return characteristics. B is incorrect because the underlying share price would have to be close to the conversion price for the risk–return characteristics of the convertible bond to resemble that of a hybrid instrument. C is incorrect because the underlying share price would have to be in excess of the conversion price for the risk–return characteristics of the convertible bond to resemble that of the company's common stock.

SUMMARY

- An embedded option represents a right that can be exercised by the issuer, by the bondholder, or automatically depending on the course of interest rates. It is attached to, or embedded in, an underlying option-free bond called a straight bond.
- Simple embedded option structures include call options, put options, and extension options. Callable and putable bonds can be redeemed prior to maturity, at the discretion of the issuer in the former case and of the bondholder in the latter case. An extendible bond gives the bondholder the right to keep the bond for a number of years after maturity. Putable and extendible bonds are equivalent, except that their underlying option-free bonds are different.

- Complex embedded option structures include bonds with other types of options or combinations of options. For example, a convertible bond includes a conversion option that allows the bondholders to convert their bonds into the issuer's common stock. A bond with an estate put can be put by the heirs of a deceased bondholder. Sinking fund bonds make the issuer set aside funds over time to retire the bond issue and are often callable, may have an acceleration provision, and may also contain a delivery option. Valuing and analyzing bonds with complex embedded option structures is challenging.
- According to the arbitrage-free framework, the value of a bond with an embedded option is equal to the arbitrage-free values of its parts—that is, the arbitrage-free value of the straight bond and the arbitrage-free values of each of the embedded options.
- Because the call option is an issuer option, the value of the call option decreases the value of the callable bond relative to an otherwise identical but non-callable bond. In contrast, because the put option is an investor option, the value of the put option increases the value of the putable bond relative to an otherwise identical but non-putable bond.
- In the absence of default and interest rate volatility, the bond's future cash flows are certain. Thus, the value of a callable or putable bond can be calculated by discounting the bond's future cash flows at the appropriate one-period forward rates, taking into consideration the decision to exercise the option. If a bond is callable, the decision to exercise the option is made by the issuer, which will exercise the call option when the value of the bond's future cash flows is higher than the call price. In contrast, if the bond is putable, the decision to exercise the option is made by the bondholder, who will exercise the put option when the value of the bond's future cash flows is lower than the put price.
- In practice, interest rates fluctuate and interest rate volatility affects the value of embedded options. Thus, when valuing bonds with embedded options, it is important to consider the possible evolution of the yield curve over time.
- Interest rate volatility is modeled using a binomial interest rate tree. The higher the volatility, the lower the value of the callable bond and the higher the value of the putable bond.
- Valuing a bond with embedded options assuming an interest rate volatility requires three steps: (1) Generate a tree of interest rates based on the given yield curve and volatility assumptions; (2) at each node of the tree, determine whether the embedded options will be exercised; and (3) apply the backward induction valuation methodology to calculate the present value of the bond.
- The option-adjusted spread is the single spread added uniformly to the one-period forward rates on the tree to produce a value or price for a bond. OAS is sensitive to interest rate volatility: The higher the volatility, the lower the OAS for a callable bond.
- For bonds with embedded options, the best measure to assess the sensitivity of the bond's price to a parallel shift of the benchmark yield curve is effective duration. The effective duration of a callable or putable bond cannot exceed that of the straight bond.

- When the option is near the money, the convexity of a callable bond is negative, indicating that the upside for a callable bond is much smaller than the downside, whereas the convexity of a putable bond is positive, indicating that the upside for a putable bond is much larger than the downside.
- Because the prices of callable and putable bonds respond asymmetrically to upward and downward interest rate changes of the same magnitude, one-sided durations provide a better indication regarding the interest rate sensitivity of bonds with embedded options than (two-sided) effective duration.
- Key rate durations show the effect of shifting only key points, one at a time, rather than the entire yield curve.
- The arbitrage-free framework can be used to value capped and floored floaters. The cap provision in a floater is an issuer option that prevents the coupon rate from increasing above a specified maximum rate. Thus, the value of a capped floater is equal to or less than the value of the straight bond. In contrast, the floor provision in a floater is an investor option that prevents the coupon from decreasing below a specified minimum rate. Thus, the value of a floored floater is equal to or higher than the value of the straight bond.
- The characteristics of a convertible bond include the conversion price, which is the applicable share price at which the bondholders can convert their bonds into common shares, and the conversion ratio, which reflects the number of shares of common stock that the bondholders receive from converting their bonds into shares. The conversion price is adjusted in case of corporate actions, such as stock splits, bonus share issuances, and rights and warrants issuances. Convertible bondholders may receive compensation when the issuer pays dividends to its common shareholders, and they may be given the opportunity to either put their bonds or convert their bonds into shares earlier and at more advantageous terms in the case of a change of control.
- A number of investment metrics and ratios help analyze and value convertible bonds. The conversion value indicates the value of the bond if it is converted at the market price of the shares. The minimum value of a convertible bond sets a floor value for the convertible bond at the greater of the conversion value or the straight value. This floor is moving, however, because the straight value is not fixed. The market conversion premium represents the price investors effectively pay for the underlying shares if they buy the convertible bond and then convert it into shares. Scaled by the market price of the shares, it represents the premium payable when buying the convertible bond rather than the underlying common stock.
- Because convertible bonds combine characteristics of bonds, stocks, and options, as well as potentially other features, their valuation and analysis are challenging. Convertible bond investors should consider the factors that affect not only bond prices but also the underlying share price.
- The arbitrage-free framework can be used to value convertible bonds, including callable and putable ones. Each component (straight bond, call option of the stock, and call and/or put option on the bond) can be valued separately.
- The risk–return characteristics of a convertible bond depend on the underlying share price relative to the conversion price. When the underlying share price is well below the conversion price, the convertible bond is "busted" and exhibits mostly bond risk–return characteristics. Thus, it is mainly

sensitive to interest rate movements. In contrast, when the underlying share price is well above the conversion price, the convertible bond exhibits mostly stock risk–return characteristics. Thus, its price follows similar movements to the price of the underlying stock. In between these two extremes, the convertible bond trades like a hybrid instrument.

PRACTICE PROBLEMS

The following information relates to questions 1-10

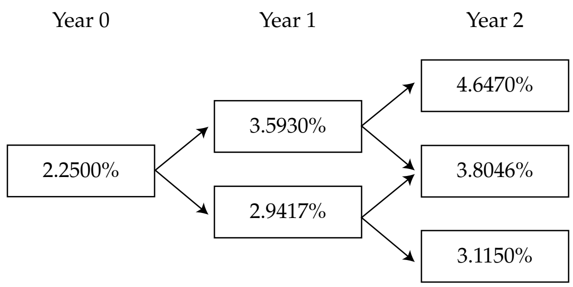
Samuel & Sons is a fixed-income specialty firm that offers advisory services to investment management companies. On 1 October 20X0, Steele Ferguson, a senior analyst at Samuel, is reviewing three fixed-rate bonds issued by a local firm, Pro Star, Inc. The three bonds, whose characteristics are given in Exhibit 1, carry the highest credit rating.

Exhibit 1: Fixed-Rate Bonds Issued by Pro Star, Inc.

Bond	Maturity	Coupon	Type of Bond
Bond #1	1 October 20X3	4.40% annual	Option-free
Bond #2	1 October 20X3	4.40% annual	Callable at par on 1 October 20X1 and on 1 October 20X2
Bond #3	1 October 20X3	4.40% annual	Putable at par on 1 October 20X1 and on 1 October 20X2

The one-year, two-year, and three-year par rates are 2.250%, 2.750%, and 3.100%, respectively. Based on an estimated interest rate volatility of 10%, Ferguson constructs the binomial interest rate tree shown in Exhibit 2.

Exhibit 2: Binomial Interest Rate Tree



On 19 October 20X0, Ferguson analyzes the convertible bond issued by Pro Star given in Exhibit 3. That day, the option-free value of Pro Star's convertible bond is $1,060 and its stock price $37.50.

Exhibit 3: Convertible Bond Issued by Pro Star, Inc.

Issue Date:	6 December 20X0
Maturity Date:	6 December 20X4
Coupon Rate:	2%
Issue Price:	$1,000
Conversion Ratio:	31

1. The call feature of Bond #2 is *best* described as:

   A. European style.

   B. American style.

   C. Bermudan style.

2. The bond that would *most likely* protect investors against a significant increase in interest rates is:

   A. Bond #1.

   B. Bond #2.

   C. Bond #3.

3. A fall in interest rates would *most likely* result in:

   A. a decrease in the effective duration of Bond #3.

   B. Bond #3 having more upside potential than Bond #2.

   C. a change in the effective convexity of Bond #3 from positive to negative.

4. The value of Bond #2 is *closest* to:

   A. 102.103% of par.

   B. 103.121% of par.

   C. 103.744% of par.

5. The value of Bond #3 is *closest* to:

   A. 102.103% of par.

   B. 103.688% of par.

   C. 103.744% of par.

6. All else being equal, a rise in interest rates will *most likely* result in the value of the option embedded in Bond #3:

   A. decreasing.

   B. remaining unchanged.

   C. increasing.

7. All else being equal, if Ferguson assumes an interest rate volatility of 15% instead of 10%, the bond that would *most likely* increase in value is:

   A. Bond #1.

   B. Bond #2.

   C. Bond #3.

8. All else being equal, if the shape of the yield curve changes from upward sloping to flattening, the value of the option embedded in Bond #2 will *most likely*:

   A. decrease.

   B. remain unchanged.

   C. increase.

9. The conversion price of the bond in Exhibit 3 is closest to:

   A. $26.67.

   B. $32.26.

   C. $34.19.

10. If the market price of Pro Star's common stock falls from its level on 19 October 20X0, the price of the convertible bond will *most likely*:

    A. fall at the same rate as Pro Star's stock price.

    B. fall but at a slightly lower rate than Pro Star's stock price.

    C. be unaffected until Pro Star's stock price reaches the conversion price.

The following information relates to questions 11-18

John Smith, an investment adviser, meets with Lydia Carter to discuss her pending retirement and potential changes to her investment portfolio. Domestic economic activity has been weakening recently, and Smith's outlook is that equity market values will be lower during the next year. He would like Carter to consider reducing her equity exposure in favor of adding more fixed-income securities to the portfolio.

Government yields have remained low for an extended period, and Smith suggests considering investment-grade corporate bonds to provide additional yield above government debt issues. In light of recent poor employment figures and two consecutive quarters of negative GDP growth, the consensus forecast among economists is that the central bank, at its next meeting this month, will take actions that will lead to lower interest rates.

Smith and Carter review par, spot, and one-year forward rates (Exhibit 1) and four fixed-rate investment-grade bonds issued by Alpha Corporation that are being considered for investment (Exhibit 2).

Exhibit 1: Par, Spot, and One-Year Forward Rates (annual coupon payments)

Maturity (Years)	Par Rate (%)	Spot Rate (%)	One-Year Forward (%)
1	1.0000	1.0000	1.0000
2	1.2000	1.2012	1.4028
3	1.2500	1.2515	1.3522

Exhibit 2: Selected Fixed-Rate Bonds of Alpha Corporation

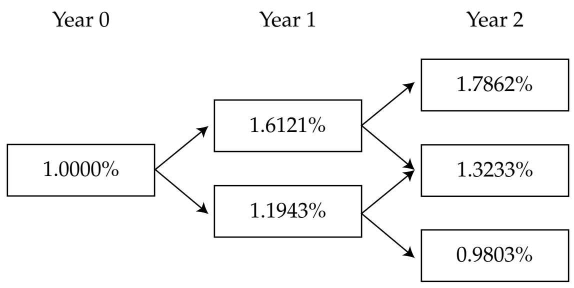
Bond	Annual Coupon	Type of Bond
Bond 1	1.5500%	Straight bond
Bond 2	1.5500%	Convertible bond: currently trading out of the money
Bond 3	1.5500%	Putable bond: putable at par one year and two years from now
Bond 4	1.5500%	Callable bond: callable at par without any protection periods

Note: All bonds in Exhibit 2 have remaining maturities of exactly three years.

Carter tells Smith that the local news media have been reporting that housing starts, exports, and demand for consumer credit are all relatively strong, even in light of other poor macroeconomic indicators. Smith explains that the divergence in economic data leads him to believe that volatility in interest rates will increase. Smith also states that he recently read a report issued by Brown and Company forecasting that the yield curve could invert within the next six months.

Smith develops a binomial interest rate tree with a 15% interest rate volatility assumption to assess the value of Alpha Corporation's bonds. Exhibit 3 presents the interest rate tree.

Exhibit 3: Binomial Interest Rate Tree for Alpha Corporation with 15% Interest Rate Volatility



Carter asks Smith about the possibility of analyzing bonds that have lower credit ratings than the investment-grade Alpha bonds. Smith discusses four other corporate bonds with Carter. Exhibit 4 presents selected data on the four bonds.

Exhibit 4: Selected Information on Fixed-Rate Bonds for Beta, Gamma, Delta, and Rho Corporations

Bond	Issuer	Bond Features	Credit Rating
Bond 5	Beta Corporation	Coupon 1.70% Callable in Year 2 OAS of 45 bps	B
Bond 6	Gamma Corporation	Coupon 1.70% Callable in Year 2 OAS of 65 bps	B
Bond 7	Delta Corporation	Coupon 1.70% Callable in Year 2 OAS of 85 bps	B
Bond 8	Rho Corporation	Coupon 1.70% Callable in Year 2 OAS of 105 bps	CCC

Notes: All bonds have remaining maturities of three years. OAS stands for option-adjusted spread.

11. Based on Exhibit 2, and assuming that the forecast for interest rates and Smith's outlook for equity returns are validated, which bond's option is *most likely* to be exercised?

A. Bond 2

B. Bond 3

C. Bond 4

12. Based on Exhibit 2, the current price of Bond 1 is *most likely* greater than the current price of:

A. Bond 2.

B. Bond 3.

C. Bond 4.

13. Assuming the forecast for interest rates is proven accurate, which bond in Exhibit 2 will likely experience the smallest price increase?

A. Bond 1

B. Bond 3

C. Bond 4

14. Based on the information in Exhibit 1 and Exhibit 2, the value of the embedded option in Bond 4 is *closest* to:

A. nil.

B. 0.1906.

C. 0.8789.

15. If Smith's interest rate volatility forecast turns out to be true, which bond in Exhibit 2 is likely to experience the greatest price increase?

A. Bond 2

B. Bond 3

C. Bond 4

16. If the Brown and Company forecast comes true, which of the following is *most* likely to occur? The value of the embedded option in:

A. Bond 3 decreases.

B. Bond 4 decreases.

C. both Bond 3 and Bond 4 increases.

17. Based on Exhibit 2 and Exhibit 3, the market price of Bond 4 is *closest* to:

A. 100.0000.

B. 100.5123.

C. 100.8790.

18. Which of the following conclusions regarding the bonds in Exhibit 4 is correct?

A. Bond 5 is relatively cheaper than Bond 6.

B. Bond 7 is relatively cheaper than Bond 6.

C. Bond 8 is relatively cheaper than Bond 7.

The following information relates to questions 19-27

Rayes Investment Advisers specializes in fixed-income portfolio management. Meg Rayes, the owner of the firm, would like to add bonds with embedded options to the firm's bond portfolio. Rayes has asked Mingfang Hsu, one of the firm's analysts, to assist her in selecting and analyzing bonds for possible inclusion in the firm's bond portfolio.

Hsu first selects two corporate bonds that are callable at par and have the same characteristics in terms of maturity, credit quality, and call dates. Hsu uses the option adjusted spread (OAS) approach to analyze the bonds, assuming an interest rate volatility of 10%. The results of his analysis are presented in Exhibit 1.

Exhibit 1: Summary Results of Hsu's Analysis Using the OAS Approach

Bond	OAS (in bps)
Bond #1	25.5
Bond #2	30.3

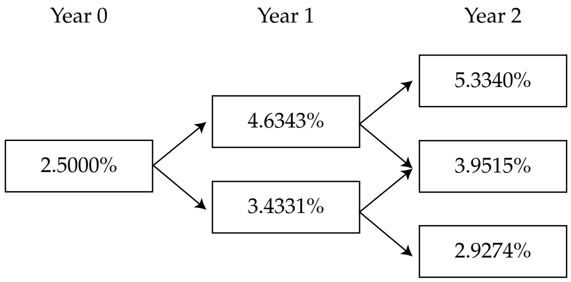
Hsu then selects the four bonds issued by RW, Inc., given in Exhibit 2. These bonds all have a maturity of three years and the same credit rating. Bonds #4 and #5 are identical to Bond #3, an option-free bond, except that they each include an embedded option.

Exhibit 2: Bonds Issued by RW, Inc.

Bond	Coupon	Special Provision
Bond #3	4.00% annual	
Bond #4	4.00% annual	Callable at par at the end of years 1 and 2
Bond #5	4.00% annual	Putable at par at the end of years 1 and 2
Bond #6	One-year reference rate annually, set in arrears	

To value and analyze RW's bonds, Hsu uses an estimated interest rate volatility of 15% and constructs the binomial interest rate tree provided in Exhibit 3.

Exhibit 3: Binomial Interest Rate Tree Used to Value RW's Bonds



Rayes asks Hsu to determine the sensitivity of Bond #4's price to a 20 bps parallel shift of the benchmark yield curve. The results of Hsu's calculations are shown in Exhibit 4.

Exhibit 4: Summary Results of Hsu's Analysis about the Sensitivity of Bond #4's Price to a Parallel Shift of the Benchmark Yield Curve

Magnitude of the Parallel Shift in the Benchmark Yield Curve	+20 bps	–20 bps
Full Price of Bond #4 (% of par)	100.478	101.238

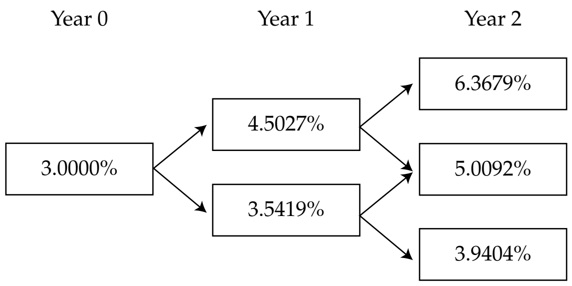
Hsu also selects the two floating-rate bonds issued by Varlep, plc, given in Exhibit 5. These bonds have a maturity of three years and the same credit rating.

Exhibit 5: Floating-Rate Bonds Issued by Varlep, plc

Bond	Coupon
Bond #7	One-year reference rate annually, set in arrears, capped at 5.00%
Bond #8	One-year reference rate annually, set in arrears, floored at 3.50%

To value Varlep's bonds, Hsu constructs the binomial interest rate tree provided in Exhibit 6.

Exhibit 6: Binomial Interest Rate Tree Used to Value Varlep's Bonds



Last, Hsu selects the two bonds issued by Whorton, Inc., given in Exhibit 7. These bonds are close to their maturity date and are identical, except that Bond #9 includes a conversion option. Whorton's common stock is currently trading at $30 per share.

Exhibit 7: Bonds Issued by Whorton, Inc.

Bond	Type of Bond
Bond #9	Convertible bond with a conversion price of $50
Bond #10	Identical to Bond #9 except that it does not include a conversion option

19. Based on Exhibit 1, Rayes would *most likely* conclude that relative to Bond #1, Bond #2 is:

   A. overpriced.

   B. fairly priced.

   C. underpriced.

20. The effective duration of Bond #6 is:

   A. close to 1.

   B. higher than 1 but lower than 3.

   C. higher than 3.

21. In Exhibit 2, the bond whose effective duration might lengthen if interest rates

rise is:

A. Bond #3.

B. Bond #4.

C. Bond #5.

22. The effective duration of Bond #4 is *closest* to:

A. 0.76.

B. 1.88.

C. 3.77.

23. The value of Bond #7 is *closest* to:

A. 99.697% of par.

B. 99.936% of par.

C. 101.153% of par.

24. The value of Bond #8 is *closest* to:

A. 98.116% of par.

B. 100.000% of par.

C. 100.485% of par.

25. The value of Bond #9 is equal to the value of Bond #10:

A. plus the value of a put option on Whorton's common stock.

B. plus the value of a call option on Whorton's common stock.

C. minus the value of a call option on Whorton's common stock.

26. The minimum value of Bond #9 is equal to the *greater* of:

A. the conversion value of Bond #9 and the current value of Bond #10.

B. the current value of Bond #10 and a call option on Whorton's common stock.

C. the conversion value of Bond #9 and a call option on Whorton's common stock.

27. The factor that is currently *least likely* to affect the risk–return characteristics of Bond #9 is:

A. interest rate movements.

B. Whorton's credit spreads.

C. Whorton's common stock price movements.

The following information relates to questions 28-36

Jules Bianchi is a bond analyst for Maneval Investments, Inc. Bianchi gathers data on three corporate bonds, as shown in Exhibit 1.

Exhibit 1: Selected Bond Data

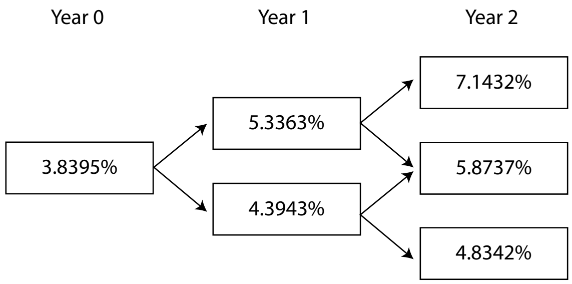
Issuer	Coupon Rate	Price	Bond Description
Ayrault, Inc. (AI)	5.25%	100.200	Callable at par in one year and two years from today
Blum, Inc. (BI)	5.25%	101.300	Option-free
Cresson Enterprises (CE)	5.25%	102.100	Putable at par in one year from today

Note: Each bond has a remaining maturity of three years, annual coupon payments, and a credit rating of BBB.

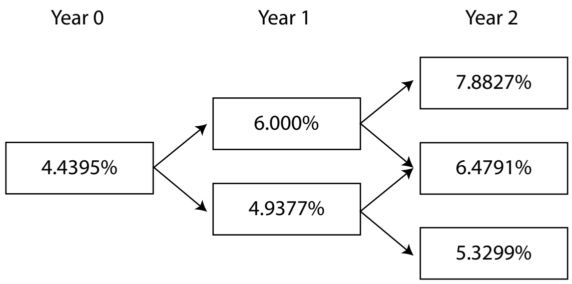
To assess the interest rate risk of the three bonds, Bianchi constructs two binomial interest rate trees based on a 10% interest rate volatility assumption and a current one-year rate of 4%. Panel A of Exhibit 2 provides an interest rate tree assuming the benchmark yield curve shifts down by 30 bps, and Panel B provides an interest rate tree assuming the benchmark yield curve shifts up by 30 bps. Bianchi determines that the AI bond is currently trading at an option-adjusted spread (OAS) of 13.95 bps relative to the benchmark yield curve.

Exhibit 2: Binomial Interest Rate Trees

Interest Rates Shift Down by 30 bps



Interest Rates Shift Up by 30 bps



Armand Gillette, a convertible bond analyst, stops by Bianchi's office to discuss two convertible bonds. One is issued by DeLille Enterprises (DE), and the other is issued by Raffarin Incorporated (RI). Selected data for the two bonds are presented in Exhibit 3 and Exhibit 4.

Exhibit 3: Selected Data for DE Convertible Bond

Issue price	€1,000 at par
Conversion period	13 September 20X5 to 12 September 20X8
Initial conversion price	€10.00 per share
Threshold dividend	€0.50 per share
Change of control conversion price	€8.00 per share
Common stock share price on issue date	€8.70
Share price on 17 September 20X5	€9.10
Convertible bond price on 17 September 20X5	€1,123

Exhibit 4: Selected Data for RI Convertible Bond

Straight bond value	€978
Value of embedded issuer call option	€43
Value of embedded investor put option	€26
Value of embedded call option on issuer's stock	€147
Conversion price	€12.50
Current common stock share price	€11.75

Gillette makes the following comments to Bianchi:

- "The DE bond does not contain any call or put options, but the RI bond contains both an embedded call option and put option. I expect that DeLille Enterprises will soon announce a common stock dividend of €0.70 per share."
- "My belief is that, over the next year, Raffarin's share price will appreciate toward the conversion price but not exceed it."

28. Based on Exhibit 1 and Exhibit 2, the effective duration for the AI bond is *closest to*:

A. 1.98.

B. 2.15.

C. 2.73.

29. If benchmark yields were to fall, which bond in Exhibit 1 would *most likely* experience a decline in effective duration?

A. AI bond

B. BI bond

C. CE bond

30. Based on Exhibit 1, for the BI bond, one-sided:

   A. up-duration will be greater than one-sided down-duration.

   B. down-duration will be greater than one-sided up-duration.

   C. up-duration and one-sided down-duration will be about equal.

31. Based on Exhibit 1, which key rate duration is the largest for the BI bond?

   A. One-year key rate duration

   B. Two-year key rate duration

   C. Three-year key rate duration

32. Which bond in Exhibit 1*most likely* has the lowest effective convexity?

   A. AI bond

   B. BI bond

   C. CE bond

33. Based on Exhibit 3, if DeLille Enterprises pays the dividend expected by Gillette, the conversion price of the DE bond will:

   A. be adjusted downward.

   B. not be adjusted.

   C. be adjusted upward.

34. Based on Exhibit 3, the market conversion premium per share for the DE bond on 17 September 20X5 is *closest* to:

   A. €0.90.

   B. €2.13.

   C. €2.53.

35. Based on Exhibit 4, the arbitrage-free value of the RI bond is *closest* to:

   A. €814.

   B. €1,056.

   C. €1,108.

36. Based on Exhibit 4 and Gillette's forecast regarding Raffarin's share price, the return on the RI bond over the next year is *most likely* to be:

   A. lower than the return on Raffarin's common shares.

   B. the same as the return on Raffarin's common shares.

   C. higher than the return on Raffarin's common shares.

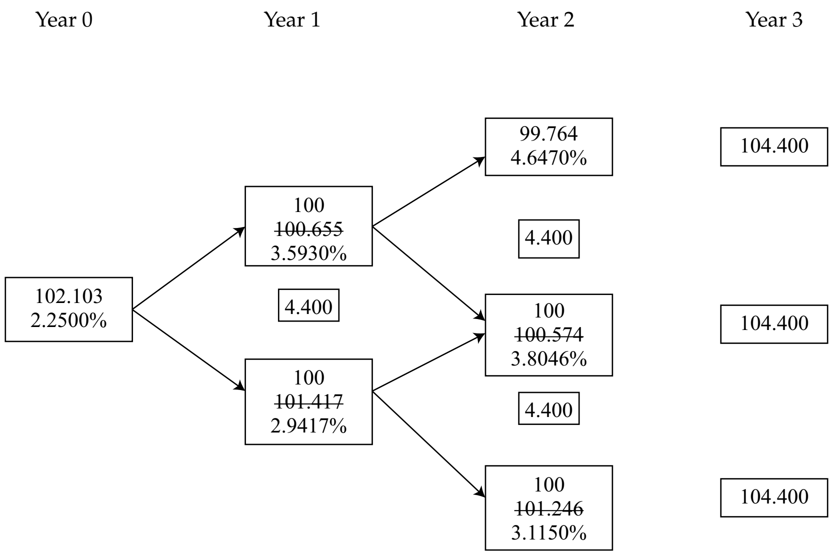
SOLUTIONS

1. C is correct. The call option embedded in Bond #2 can be exercised only at two predetermined dates: 1 October 20X1 and 1 October 20X2. Thus, the call feature is Bermudan style.

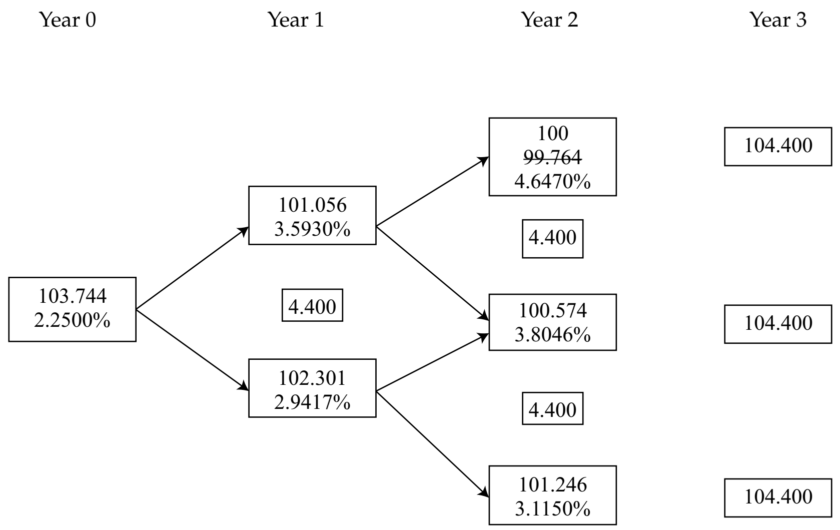
2. C is correct. The bond that would most likely protect investors against a significant increase in interest rates is the putable bond (i.e., Bond #3). When interest rates have risen and higher-yield bonds are available, a put option allows the bondholders to put back the bonds to the issuer prior to maturity and to reinvest the proceeds of the retired bonds in higher-yielding bonds.

3. B is correct. A fall in interest rates results in a rise in bond values. For a callable bond, such as Bond #2, the upside potential is capped because the issuer is more likely to call the bond. In contrast, the upside potential for a putable bond, such as Bond #3, is uncapped. Thus, a fall in interest rates would result in a putable bond having more upside potential than an otherwise identical callable bond. Note that A is incorrect because the effective duration of a putable bond increases, not decreases, with a fall in interest rates; the bond is less likely to be put and thus behaves more like an option-free bond. C is also incorrect because the effective convexity of a putable bond is always positive. It is the effective convexity of a callable bond that will change from positive to negative if interest rates fall and the call option is near the money.

4. A is correct:



5. C is correct:



6. C is correct. Bond #3 is a putable bond, and the value of a put option increases as interest rates rise. At higher interest rates, the value of the underlying option-free bond (straight bond) declines, but the decline is offset partially by the increase in the value of the embedded put option, which is more likely to be exercised.

7. C is correct. Regardless of the type of option, an increase in interest rate volatility results in an increase in option value. Because the value of a putable bond is equal to the value of the straight bond *plus* the value of the embedded put option, Bond #3 will increase in value if interest rate volatility increases. Put another way, an increase in interest rate volatility will most likely result in more scenarios where the put option is exercised, which increases the values calculated in the interest rate tree and, thus, the value of the putable bond.

8. C is correct. Bond #2 is a callable bond, and the value of the embedded call option increases as the yield curve flattens. When the yield curve is upward sloping, the one-period forward rates on the interest rate tree are high and opportunities for the issuer to call the bond are fewer. When the yield curve flattens or inverts, many nodes on the tree have lower forward rates, which increase the opportunities to call and, thus, the value of the embedded call option.

9. B is correct. The conversion price of a convertible bond is equal to the par value divided by the conversion ratio—that is, $1,000/31 = $32.26 per share.

10. B is correct. The conversion value of the bond is 31 × $37.50 or $1,162.50, which represents its minimum value. Thus, the convertible bond exhibits mostly stock risk–return characteristics; a fall in the stock price will result in a fall in the convertible bond price. However, the change in the convertible bond price is less than the change in the stock price because the convertible bond has a floor. That floor is the value of the straight (option-free) bond.

11. C is correct. If the central bank takes actions that lead to lower interest rates, the yields on Alpha's bonds are likely to decrease. If the yield to maturity on Bond 4 (callable) falls below the 1.55% coupon rate, the call option will become valuable and Alpha may call the bond because it is in the money.

    A is incorrect because if the equity market declines, the market value of Alpha stock will also likely decrease. Therefore, Bond 2 (convertible) would have a lower conversion value; hence, the conversion option likely would not be exercised. Because Bond 2 is currently trading out of the money, it will likely trade further out of the money once the price of Alpha stock decreases.

B is incorrect because Bond 3 (putable) is more likely to be exercised in an increasing rather than a decreasing interest rate environment.

12. C is correct. All four bonds in Exhibit 2 issued by Alpha Corporation offer the same coupon rate and have the same remaining term to maturity. Bond 4 (callable) most likely has a current price that is less than Bond 1 (straight or option free) because investors are short the call option and must be compensated for bearing call risk. Bond 2 (convertible) most likely has a current price that is greater than Bond 1 because investors are paying for the conversion option embedded in Bond 2 and the option has time value associated with it, even though the option is trading out of the money. Similarly, Bond 3 (putable) most likely has a current price that is greater than Bond 1 because investors are paying for the put option.

13. C is correct. The consensus economic forecast is for interest rates to decrease. In an environment of decreasing interest rates, all bond prices should rise, ignoring any price impact resulting from any embedded options. When interest rates fall, the value of the embedded call option in Bond 4 (callable) increases, causing an opposing effect on price. The put option of putable bonds, by contrast, increases in value when interest rates rise rather than decline.

14. C is correct. Bond 4 is a callable bond. Value of an issuer call option = Value of straight bond – Value of callable bond. The value of the straight bond may be calculated using the spot rates or the one-year forward rates.

    Value of an option-free (straight) bond with a 1.55% coupon using spot rates:

    $1.55/(1.0100)^1 + 1.55/(1.012012)^2 + 101.55/(1.012515)^3 = 100.8789.$

    The value of a callable bond (at par) with no call protection period cannot exceed 100, as at that price or higher the bond would be called. The value of the call option = 100.8789 – 100 = 0.8789.

15. B is correct. An increase in interest rate volatility will cause the value of the put and call options embedded in Bond 3 and Bond 4 to increase. Bond 3 (putable) would experience an increase in price because the increased value of the put option increases the bond's value. In contrast, Bond 4 (callable) will experience a price decrease because the increased value of the call option reduces the callable bond's value. Bond 2, an out-of-the-money convertible, will resemble the risk–return characteristics of a straight bond and will thus be unaffected by interest rate volatility.

16. A is correct. All else being equal, the value of a put option decreases as the yield curve moves from being upward sloping to flat to downward sloping (inverted). Alternatively, a call option's value increases as the yield curve flattens and increases further if the yield curve inverts. Therefore, if the yield curve became inverted, the value of the embedded option in Bond 3 (putable) would decrease and the value of the embedded option in Bond 4 (callable) would increase.

17. A is correct. The market price of callable Bond 4 with no protection period cannot exceed 100.

18. B is correct. A bond with a larger option-adjusted spread (OAS) than that of a bond with similar characteristics and credit quality means that the bond is likely underpriced (cheap). Bond 7 (OAS 85 bps) is relatively cheaper than Bond 6 (OAS 65 bps).

    C is incorrect because Bond 8 (CCC) has a lower credit rating than Bond 7 (B) and the OAS alone cannot be used for the relative value comparison. The larger OAS (105 bps) incorporates compensation for the difference between the B and

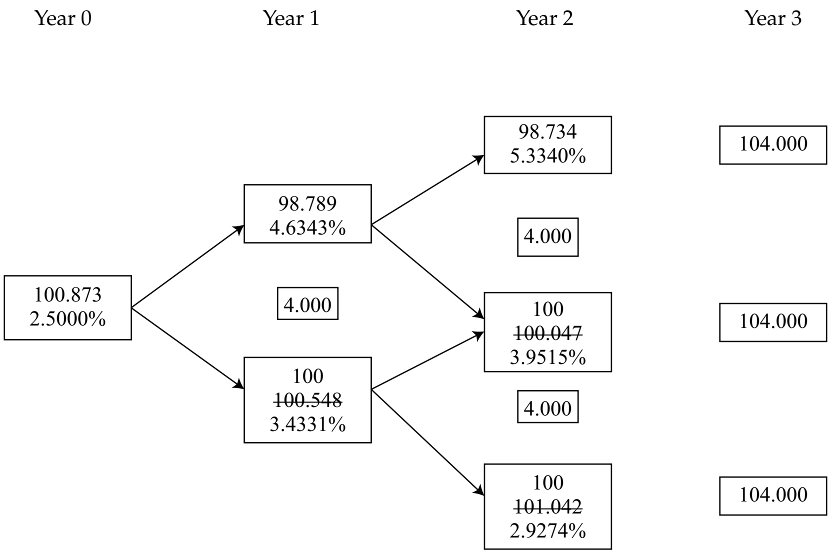
CCC bond credit ratings. Therefore, there is not enough information to draw a conclusion about relative value.

19. C is correct. The option-adjusted spread (OAS) is the constant spread added to all the one-period forward rates that makes the arbitrage-free value of a risky bond equal to its market price. The OAS approach is often used to assess bond relative values. If two bonds have the same characteristics and credit quality, they should have the same OAS. If this is not the case, the bond with the largest OAS (i.e., Bond #2) is likely to be underpriced (cheap) relative to the bond with the smallest OAS (i.e., Bond #1).

20. A is correct. The effective duration of a floating-rate bond is close to the time to next reset. As the reset for Bond #6 is annual, the effective duration of this bond is close to 1.

21. B is correct. Effective duration indicates the sensitivity of a bond's price to a 100 bps parallel shift of the benchmark yield curve assuming no change in the bond's credit spread. The effective duration of an option-free bond, such as Bond #3, goes down as interest rates rise. As interest rates rise, a call option moves out of the money, which increases the value of the callable bond and lengthens its effective duration. In contrast, as interest rates rise, a put option moves into the money, which limits the price depreciation of the putable bond and shortens its effective duration. Thus, the bond whose effective duration might lengthen if interest rates rise is the callable bond (i.e., Bond #4).

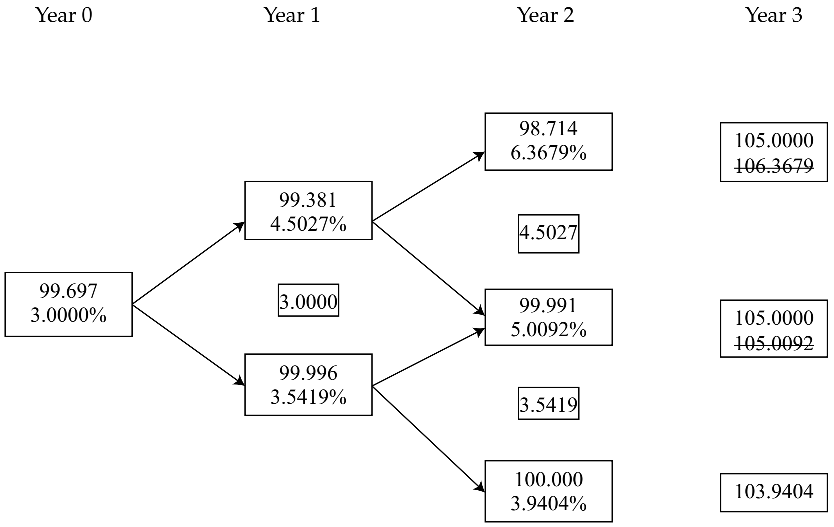
22. B is correct. The effective duration of Bond #4 can be calculated using Equation 3, where ΔCurve is 20 bps, PV_- is 101.238, and PV_+ is 100.478. PV_0, the current full price of the bond (i.e., with no shift), is not given but can be calculated using Exhibit 3 as follows:



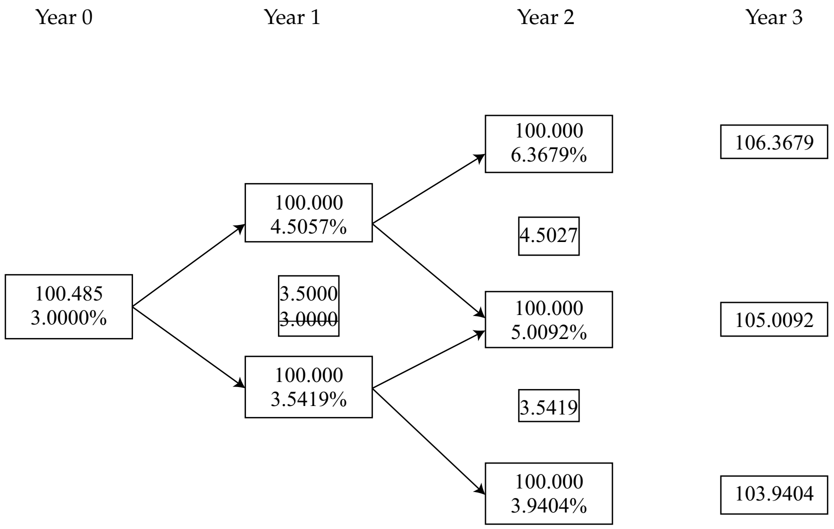
Thus, the effective duration of Bond #4 is:

$$\text{EffDur} = \frac{101.238 - 100.478}{2 \times (0.0020) \times (100.873)} = 1.88.$$

23. A is correct:



24. C is correct:



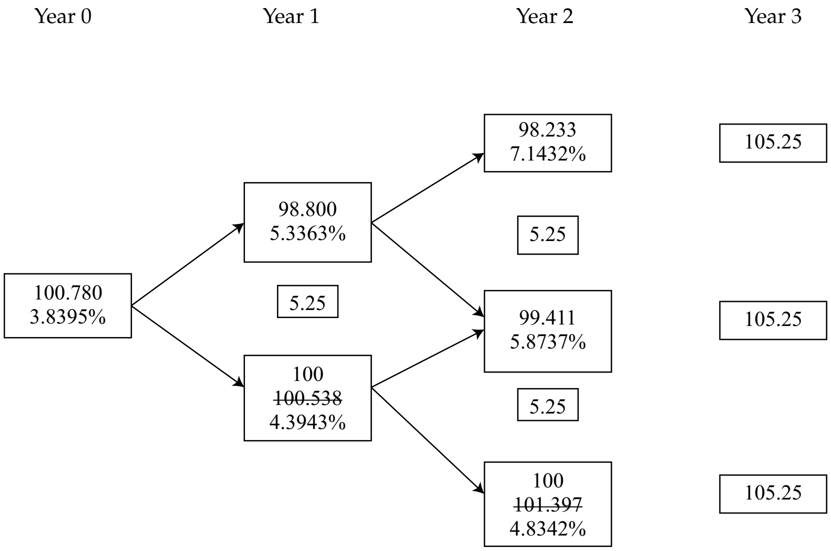
25. B is correct. A convertible bond includes a conversion option, which is a call option on the issuer's common stock. This conversion option gives the bondholders the right to convert their debt into equity. Thus, the value of Bond #9, the convertible bond, is equal to the value of Bond #10, the underlying option-free bond (straight bond), plus the value of a call option on Whorton's common stock.

26. A is correct. The minimum value of a convertible bond is equal to the greater of the conversion value of the convertible bond (i.e., Bond #9) and the current value of the straight bond (i.e., Bond #10).

27. C is correct. The risk–return characteristics of a convertible bond depend on the market price of the issuer's common stock (underlying share price) relative to the bond's conversion price. When the underlying share price is well below the conversion price, the convertible bond exhibits mostly bond risk–return characteristics. In this case, the price of the convertible bond is mainly affected by interest rate movements and the issuer's credit spreads. In contrast, when the underlying share price is above the conversion price, the convertible bond exhibits mostly stock risk–return characteristics. In this case, the price of the convertible bond is mainly affected by the issuer's common stock price movements. The underlying share price ($30) is lower than the conversion price of Bond #9 ($50). Thus, Bond

#9 exhibits mostly bond risk–return characteristics and is least affected by Whorton's common stock price movements.

28. B is correct. The AI bond's value if interest rates shift down by 30 bps (PV_-) is 100.78:



The AI bond's value if interest rates shift up by 30 bps (PV_+) is 99.487:

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Curve}) \times (PV_0)} = \frac{100.780 - 99.487}{2 \times 0.003 \times 100.200} = 2.15.$$

29. A is correct. The AI bond is a callable bond, and the effective duration of a callable bond decreases when interest rates fall. The reason is because a decline in interest rates may result in the call option moving into the money, which limits the price appreciation of the callable bond. Exhibit 1 also shows that the price of the AI bond is 100.200 and that it is callable at par in one year and two years. Thus, the call option is already in the money and would likely be exercised in response to increases in the AI bond's price.

30. C is correct. The BI bond is an option-free bond, and one-sided up-duration and one-sided down-duration will be about equal for option-free bonds.

31. C is correct. The BI bond is an option-free bond. Its longest key rate duration will be in the year of its maturity because the largest cash flow (payment of both coupon and principal) occurs in that year.

32. A is correct. All else being equal, a callable bond will have lower effective convexity than an option-free bond when the call option is in the money. Similarly, when the call option is in the money, a callable bond will also have lower effective convexity than a putable bond if the put option is out of the money. Exhibit 1 shows that the callable AI bond is currently priced slightly higher than its call price of par value, which means the embedded call option is in the money. The put option embedded in the CE bond is not in the money; the bond is currently priced 2.1% above par value. Thus, at the current price, the putable CE bond is more likely to behave like the option-free BI bond. Consequently, the effective convexity of the AI bond will likely be lower than the option-free BI bond and the putable CE bond.

33. A is correct. The conversion price would be adjusted downward because Gillette's expected dividend payment of €0.70 is greater than the threshold dividend of

€0.50.

34. B is correct. The market conversion premium per share is equal to the market conversion price minus the underlying share price. The market conversion price is calculated as follows:

$$\text{Market conversion price} = \frac{\text{Convertible bond price}}{\text{Conversion ratio}}$$
$$= \frac{€1,123}{€1,000/€10 \text{ per share}} = €11.23 \text{ per share.}$$

The market conversion premium per share is then calculated as follows:

Market conversion premium per share = Market conversion price – Underlying share price.

= €11.23 – €9.10 = €2.13.

35. C is correct. The value of a convertible bond with both an embedded call option and a put option can be determined using the following formula:

Value of callable putable convertible bond = Value of straight bond + Value of call option on the issuer's stock – Value of issuer call option + Value of investor put option.

Value of callable putable bond = €978 + €147 – €43 + €26 = €1,108.

36. A is correct. Over the next year, Gillette believes that Raffarin's share price will continue to increase toward the conversion price but not exceed it. If Gillette's forecast becomes true, the return on the RI bond will increase but at a lower rate than the increase in Raffarin's share price because the conversion price is not expected to be reached.

LEARNING MODULE

4

Credit Analysis Models

by James F. Adams, PhD, CFA, and Donald J. Smith, PhD.

James Adams, PhD, CFA, is at New York University (USA). Donald J. Smith, PhD, is at Boston University Questrom School of Business (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment
☐	explain credit scores and credit ratings
☐	calculate the expected return on a bond given transition in its credit rating
☐	explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses
☐	calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters
☐	interpret changes in a credit spread
☐	explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads
☐	compare the credit analysis required for securitized debt to the credit analysis of corporate debt

INTRODUCTION

1

Credit analysis plays an important role in the broader fixed-income space. Our coverage will go over important concepts, tools, and applications of credit analysis. We first look at modeling credit risk. The inputs to credit risk modeling are the expected exposure to default loss, the loss given default, and the probability of default. We explain these terms and use a numerical example to illustrate the calculation of the credit valuation adjustment for a corporate bond and its credit spread over a government bond yield taken as a proxy for a default-risk-free rate (or default-free rate).

We then discuss credit scoring and credit ratings. Credit scoring is a measure of credit risk used in retail loan markets, and ratings are used in the wholesale bond market. We explain two types of credit analysis models used in practice—structural models and reduced-form models. Both models are highly mathematical and beyond the scope of our coverage. Therefore, we provide only an overview to highlight the key ideas and the similarities and differences between them. We then use the arbitrage-free framework and a binomial interest rate tree to value risky fixed-rate and floating-rate bonds for different assumptions about interest rate volatility. We also build on the credit risk model to interpret changes in credit spreads that arise from changes in the assumed probability of default, the recovery rate, or the exposure to default loss. We also explain the term structure of credit spreads and finally compare the credit analysis required for securitized debt with the credit analysis of corporate bonds.

2 MODELING CREDIT RISK AND THE CREDIT VALUATION ADJUSTMENT

☐ explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment

The difference between the yields to maturity on a corporate bond and a government bond with the same maturity is the most commonly used measure of credit risk. It is called the *credit spread* and is also known in practice as the G-spread. It reveals the compensation to the investor for bearing the default risk of the issuer—the possibility that the issuer fails to make a scheduled payment in full on the due date—and for losses incurred in the event of default.

The terms "default risk" and "credit risk" are sometimes used interchangeably in practice, but we will distinguish between the two in our coverage. Default risk is the narrower term because it addresses the likelihood of an event of default. Credit risk is the broader term because it considers both the default probability and how much is expected to be lost if default occurs. For example, it is possible that the default risk on a collateralized loan is high while the credit risk is low, especially if the value of the collateral is high relative to the amount that is owed.

We assume that the corporate bond and the default-risk-free government bond have the same taxation and liquidity. This is a simplifying assumption, of course. In reality, government bonds typically are more liquid than corporate bonds. Also, differences in liquidity within the universe of corporate bonds are great. Government bonds are available in greater supply than even the most liquid corporates and have demand from a wider set of institutional investors. In addition, government bonds can be used more readily as collateral in repo transactions and for centrally cleared derivatives. Also, there are differences in taxation in some markets. For example, interest income on US corporate bonds is taxable by both the federal and state governments. Government debt, however, is exempt from taxes at the state level. Disregarding tax and liquidity differences allows us to focus on default risk and expected loss as the determining factors for the credit spread.

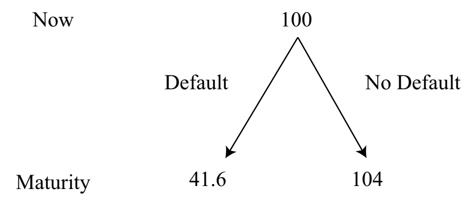
The first factor to consider in modeling credit risk is the **expected exposure** to default loss. This quantity is the projected amount of money the investor could lose if an event of default occurs, before factoring in possible recovery. Although the most common event of default is nonpayment leading to bankruptcy proceedings, the bond prospectus might identify other events of default, such as the failure to meet a different obligation or the violation of a financial covenant.

Consider a one-year, 4% annual payment corporate bond priced at par value. The expected exposure to default loss at the end of the year is simply 104 (per 100 of par value). Later, we will include multiple time periods and volatility in interest rates. That complicates the calculation of expected exposure because we will need to consider the likelihood that the bond price varies as interest rates vary. In this initial example, the exposure is simply the final coupon payment plus the redemption of principal.

The second factor is the assumed **recovery rate**, which is the percentage of the loss recovered from a bond in default. The recovery rate varies by industry, the degree of seniority in the capital structure, the amount of leverage in the capital structure in total, and whether a particular security is secured or otherwise collateralized. We assume a 40% recovery rate for this corporate bond, which is a common baseline assumption in practice. Given the recovery rate assumption, we can determine the assumed **loss given default** (the amount of loss if a default occurs). This is 62.4 per 100 of par value: 104 × (1 – 0.40) = 62.4. A related term is *loss severity*; if the recovery rate is 40%, the assumed loss severity is 60%.

Exhibit 1 illustrates the projected cash flows on the corporate bond. If there is no default, the investor receives 104. If default occurs, the investor receives 41.6: 104 – 62.4 = 41.6. We assume instantaneous recovery, which surely is another simplifying assumption. In practice, lengthy time delays can occur between the event of default and eventual recovery of cash. Notice that we assume that the recovery rate applies to interest as well as principal. One last note is that in the exhibits that we use, calculations may slightly differ on occasion due to rounding at intermediate steps.

Exhibit 1: A Simple Credit Risk Example



The third factor is the assumed **probability of default**, which is the probability that a bond issuer will not meet its contractual obligations on schedule. It is important in credit risk modeling to distinguish *risk-neutral* probabilities of default and *actual* (or historical) default probabilities. "Risk-neutral" follows the usage of the term in option pricing. In the risk-neutral option pricing methodology, the expected value for the payoffs is discounted using the risk-free interest rate. The key point is that in getting the expected value of the option, the risk-neutral probabilities associated with the payoffs need to be used. The same idea applies to valuing corporate bonds.

Suppose that a credit rating agency has collected an extensive dataset on the historical default experience for one-year corporate bonds issued by companies having the same business profile as the issuer in this example. It is observed that 99% of the bonds survive and make the full coupon and principal payment at maturity. Just 1% of the bonds default, resulting in an average recovery rate of 40%. Based on these data, the actual default probability for the corporate bond can reasonably be assumed to be 1%.

If the actual probability of default is used to get the expected future value for the corporate bond, the result is 103.376: (104 × 0.99) + (41.6 × 0.01) = 103.376. Discounting that amount at an assumed risk-free rate of 3% gives a present value of 100.365: 103.376/1.03 = 100.365. Note that in risk-neutral valuation, the expected value is discounted using the risk-free rate and not the bond's yield to maturity. The key point is that 100.365 overstates the observed value of the bond, which is 100. The issue is to determine the default probability that does produce a value of 100.

Denote the risk-neutral default probability to be P^*. The probability of survival is $1 - P^*$. Given that the corporate bond is priced at 100, $P^* = 1.60\%$. This is found as the solution to P^* in

$$100 = \frac{[104 \times (1 - P^*)] + (41.6 \times P^*)}{1.03}.$$

One reason for the difference between actual (or historical) and risk-neutral default probabilities is that actual default probabilities do not include the default risk premium associated with uncertainty over the timing of possible default loss. Another reason is that the observed spread over the yield on a risk-free bond in practice also includes liquidity and tax considerations in addition to credit risk.

To further see the interaction between the credit risk parameters—the expected exposure, the loss given default, and the probability of default—we consider a five-year, zero-coupon corporate bond. Our goal is to determine the fair value for the bond given its credit risk, its yield to maturity, and its spread over a maturity-matching government bond.

Exhibit 2 displays the calculation of the **credit valuation adjustment** (CVA). The CVA is the value of the credit risk in present value terms. In Exhibit 2, LGD stands for the loss given default, POD stands for the probability of default on the given date, POS stands for the probability of survival as of the given date, DF stands for the discount factor, and PV stands for the present value.

Exhibit 2: A Five-Year, Zero-Coupon Corporate Bond

Date (1)	Exposure (2)	Recovery (3)	LGD (4)	POD (5)	POS (6)	Expected Loss (7)	DF (8)	PV of Expected Loss (9)
0								
1	88.8487	35.5395	53.3092	1.2500%	98.7500%	0.6664	0.970874	0.6470
2	91.5142	36.6057	54.9085	1.2344%	97.5156%	0.6778	0.942596	0.6389
3	94.2596	37.7038	56.5558	1.2189%	96.2967%	0.6894	0.915142	0.6309
4	97.0874	38.8350	58.2524	1.2037%	95.0930%	0.7012	0.888487	0.6230
5	100.0000	40.0000	60.0000	1.1887%	93.9043%	0.7132	0.862609	0.6152
				6.0957%			CVA =	3.1549

The first step is to get the exposures to default loss. These are shown in Column 2 of Exhibit 2. We assume a flat government bond yield curve at 3.00%. Also, we assume that default occurs only at year-end—on Dates 1, 2, 3, 4, and 5—and that default will not occur on Date 0, the current date. The exposure on Date 5 is 100. For the other dates, we discount using the risk-free rate and the remaining number of years until maturity. For example, exposure at Date 1 is $100/(1.0300)^4 = 88.8487$.

Note that there is no interest rate volatility in this example. In a later section, we will use the arbitrage-free framework to build a binomial interest rate tree for a specified level of volatility. Then, knowing the probability of attaining each node in the tree, we will calculate the *expected exposure* for each date.

Column 3 of Exhibit 2 projects the assumed recovery if default occurs. Here, the recovery rate is a percentage of the exposure. In general, it will be a percentage of the expected exposure, including coupon interest payments, when the model allows for interest rate volatility. We assume for this example that the recovery rate is 40%. The amounts shown in Column 3 are the exposures in Column 2 times 0.40.

Column 4 shows the loss given default. It is the exposure for each date minus the assumed recovery. If the issuer defaults on Date 4, the investor's loss is projected to be 58.2524 (= 97.0874 – 38.8350) per 100 of par value.

The next parameter is the risk-neutral probability of default for each date. In Column 5 of Exhibit 2, we assume that the POD on Date 1 is 1.25%. We use *conditional probabilities of default*, meaning that each year-by-year POD assumes no prior default. These are called hazard rates in statistics. Column 6 reports the probability of survival for each year. The probability of surviving past Date 1 and arriving at Date 2 is 98.75% (= 100% – 1.25%). Therefore, the POD for Date 2 is 1.2344% (= 1.25% × 98.75%), and the POS is 97.5156% (= 98.75% – 1.2344%). The POD for Date 3 is 1.2189% (= 1.25% × 97.5156%), and the POS is 96.2967% (= 97.5156% – 1.2189%). The cumulative probability of default over the five-year lifetime of the corporate bond is 6.0957%, the sum of the PODs in Column 5.

Another method to calculate the POS for each year—a method that is used later in our discussion—is 100% minus the annual default probability raised to the power of the number of years. For example, the probability of the bond surviving until maturity is $(100\% - 1.25\%)^5 = 93.9043\%$. Note that 6.0957% plus 93.9043% equals 100%.

The assumed annual default probability does not need to be the same each year. Later we will show some examples of it changing over the lifetime of the bond.

Column 7 gives the *expected loss* for each date. This is the LGD times the POD. For example, on Date 3, the expected loss is 0.6894 per 100 of par value. The exposure is 94.2596. At 40% recovery, the LGD is 56.5558. Assuming no prior default, the POD for that date is 1.2189%. The expected loss of 0.6894 is calculated as 56.5558 times 1.2189%.

Column 8 presents the default-risk-free *discount factors* based on the flat government bond yield curve at 3.00%. The Date 5 discount factor is 0.862609 [$= 1/(1.0300)^5$]. Finally, Column 9 shows the present value of the expected loss for each year. This is the expected loss times the discount factor. The present value of the expected Date 5 loss is 0.6152 per 100 of par value, the expected loss of 0.7132 times 0.862609.

The sum of Column 9 is 3.1549. This amount is known as the credit valuation adjustment. It allows us to calculate the *fair value* of the five-year, zero-coupon corporate bond. If the bond were default free, its price would be 86.2609—that is, the par value of 100 times the Date 5 discount factor. Subtracting the CVA from this amount gives a fair value of 83.1060 (= 86.2609 – 3.1549).

We can now calculate the credit spread on the corporate bond. Given a price of 83.1060, its yield to maturity is 3.77%. The solution for *yield* in this expression is

$$\frac{100}{(1 + \text{Yield})^5} = 83.1060.$$

The yield on the five-year, zero-coupon government bond is 3.00%. Therefore, the credit spread is 77 bps: 3.77% – 3.00% = 0.77%. (Note that an approximation for the credit spread commonly used in practice is the annual default probability times 1 minus the recovery rate. In this case, the approximate credit spread is 0.75% [= 1.25% × (1 – 0.40)].) A key point is that the compensation for credit risk received by the investor can be expressed in two ways: (1) as the CVA of 3.1549 in terms of a present value per 100 of par value on Date 0 and (2) as a credit spread of 77 bps in terms of an annual percentage rate for five years.

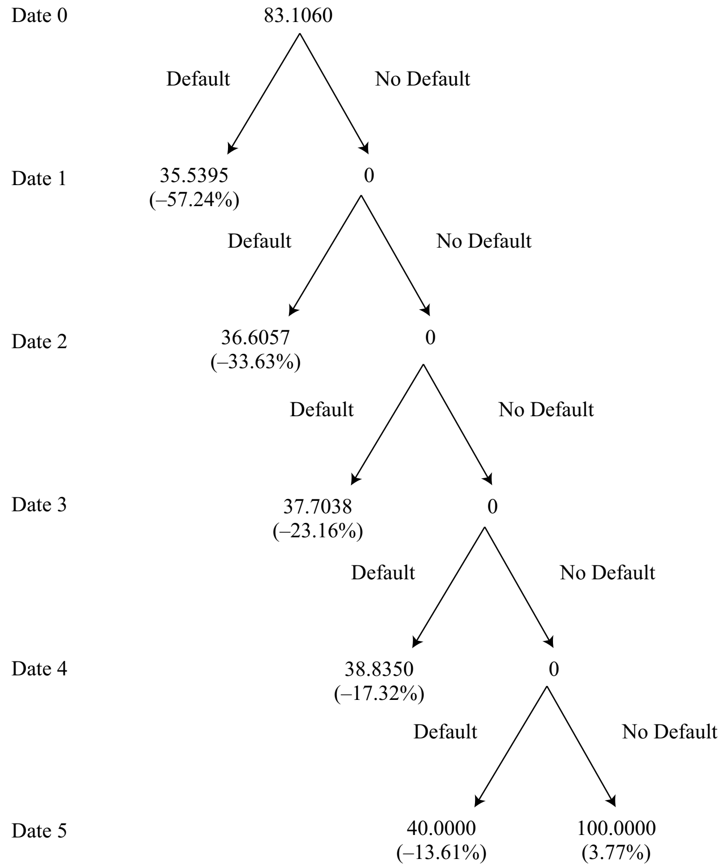
Exhibit 3 provides a display of the projected cash flows and annual rates of return depending on when and if default occurs. On Date 0, the five-year, zero-coupon corporate bond is worth its fair value, 83.1060 per 100 of par value. If on Date 1 the issuer defaults, the investor gets the recoverable amount of 35.5395. The annual rate of return is –57.24%, the solution for the internal rate of return (IRR):

$$83.1060 = \frac{35.5395}{1 + \text{IRR}}.$$

$$\text{IRR} = -0.5724.$$

If there is no default, the investor receives the coupon payment on that date, which in this case is zero.

Exhibit 3: Projected Annual Rates of Return



If the issuer defaults on Date 2, the annual rate of return is –33.63%.

$$83.1060 = \frac{0}{(1+\text{IRR})^1} + \frac{36.6057}{(1+\text{IRR})^2}.$$
$$\text{IRR} = -0.3363.$$

If the default occurs on the maturity date, the annual rate of return "improves" to –13.61%:

$$83.1060 = \frac{0}{(1+\text{IRR})^1} + \frac{0}{(1+\text{IRR})^2} + \frac{0}{(1+\text{IRR})^3} + \frac{0}{(1+\text{IRR})^4} + \frac{40.0000}{(1+\text{IRR})^5}.$$
$$\text{IRR} = -0.1361.$$

If there is no default, which is most likely because the probability of survival to Date 5 is 93.9043%, the realized rate of return is 3.77%. This reminds us that a yield to maturity on a risky bond is a measure of return to the investor, assuming no default.

The key observation from this example is that the investor faces a wide range of outcomes on the bond depending critically on the *timing* of default. This is a source of the default risk premium that typically is built into the pricing of the bond. Stated differently, the probability of default in credit risk models incorporates the likely time of incidence of default events as well as uncertainty over the timing of the events.

Although this is clearly a simple example of a credit risk model, it does serve to illustrate the interaction between the exposure to default loss for each date, the recovery rate, the loss given default, the probability of default, the expected loss, and the present value of expected loss. It can be made more complex and realistic. Here, the initial probability of default (the hazard rate) used to calculate the conditional PODs and the recovery rate is the same for each year, but these parameters could vary year by year. The government bond yield curve is flat, but it could be upward or downward sloping. Then, the discount factors would need to be calculated sequentially by a process known as "bootstrapping." An example of this process is included later.

In this example, we assume an annual default probability and a recovery rate to get the fair value for the risky corporate bond. This could be reversed. Suppose that we observe that the market price for the five-year, zero-coupon bond is 83.1060 and its credit spread is 77 bps. Then, the same table could be used to get—by trial-and-error search—the annual probability of default that is consistent with the bond price and a recovery rate of 40%. That default probability, which is used to calculate the year-by-year PODs, would be 1.25%. Another possibility is to change the assumed recovery rate. Suppose it is 30% of the exposure. Given the observed bond price and credit spread, the default probability would turn out to be 1.0675%. In that case, the lower recovery rate is offset by the lower probability of default. A higher recovery rate would need to be offset by a higher default probability. In general, for a given price and credit spread, the assumed probability of default and the recovery rate are positively correlated.

EXAMPLE 1

Analysis of Credit Risk (1)

1. A fixed-income analyst is considering the credit risk over the next year for three corporate bonds currently held in her bond portfolio. Her assessment for the exposure, probability of default, and recovery is summarized in this table:

Corporate Bond	Exposure (per 100 of par value)	Probability of Default	Recovery (per 100 of par value)
A	104	0.75%	40
B	98	0.90%	35
C	92	0.80%	30

Although all three bonds have very similar yields to maturity, the differences in the exposures arise because of differences in their coupon rates.

Based on these assumptions, how would she rank the three bonds, from highest to lowest, in terms of credit risk over the next year?

Solution:

She needs to get the loss given default for each bond and multiply that by the probability of default to get the expected loss. The LGD is the exposure minus the assumed recovery.

Corporate Bond	LGD (per 100 of par value)	POD	Expected Loss
A	64	0.75%	0.480
B	63	0.90%	0.567
C	62	0.80%	0.496

Based on the expected losses, Bond B has the highest credit risk and Bond A, the lowest. The ranking is B, C, and A. Note that there is not enough information to recommend a trading strategy because the current prices of the bonds are not given.

EXAMPLE 2

Analysis of Credit Risk (2)

1. A fixed-income trader at a hedge fund observes a three-year, 5% annual payment corporate bond trading at 104 per 100 of par value. The research team at the hedge fund determines that the risk-neutral annual probability of default used to calculate the conditional POD for each date for the bond, given a recovery rate of 40%, is 1.50%. The government bond yield curve is flat at 2.50%.

   Based on these assumptions, does the trader deem the corporate bond to be overvalued or undervalued? By how much? If the trader buys the bond at 104, what are the projected annual rates of return?

Solution:

The trader needs to build a table similar to that shown in Exhibit 2; this table is presented in Exhibit 4.

Exhibit 4: CVA Calculation for Example 2

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	109.8186	43.9274	65.8911	1.5000%	98.5000%	0.9884	0.975610	0.9643
2	107.4390	42.9756	64.4634	1.4775%	97.0225%	0.9524	0.951814	0.9066
3	105.0000	42.0000	63.0000	1.4553%	95.5672%	0.9169	0.928599	0.8514
				4.4328%			CVA =	2.7222

The exposures are the values for the bond plus the coupon payment for each date assuming a yield to maturity of 2.50%. The exposure is 109.8186 for Date 1 when two years to maturity remain:

$$5 + \frac{5}{(1.0250)^1} + \frac{105}{(1.0250)^2} = 109.8186.$$

The assumed recovery for Date 1 is 43.9274 (= 109.8186 × 0.40) for a loss given default of 65.8911 (= 109.8186 – 43.9274). (Note that all calculations are carried out on spreadsheets to preserve precision. The rounded results are reported in the text.) The expected loss is 0.9884 (= 65.8911 × 0.0150). The discount factor for Date 1 is 0.975610 = $1/(1.0250)^1$. The present value of the expected loss is 0.9643 (= 0.9884 × 0.975610).

The credit valuation adjustment for the bond is 2.7222, the sum of the present values of expected loss. If this five-year, 5% bond were default free, its price would be 107.1401.

$$\frac{5}{(1.0250)^1} + \frac{5}{(1.0250)^2} + \frac{105}{(1.0250)^3} = 107.1401.$$

Therefore, the fair value of the bond given the assumed credit risk parameters is 104.4178 (= 107.1401 – 2.7222). If this three-year, 5% bond were default free, its price would be 107.1401.

The projected annual rates of return for default on Dates 1, 2, and 3 are –57.76%, –33.27%, and –22.23%, respectively. If there is no default, the rate of return is 3.57%, which is the yield to maturity. Note that these rates of return neglect coupon reinvestment risk because internal rate of return calculations implicitly assume reinvestment at the same rate. The calculations are as follows:

$$104 = \frac{43.9274}{(1 + \text{IRR})^1}.$$
$$\text{IRR} = -0.5776.$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{42.9756}{(1 + \text{IRR})^2}.$$
$$\text{IRR} = -0.3327.$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{5}{(1 + \text{IRR})^2} + \frac{42.0000}{(1 + \text{IRR})^3}.$$
$$\text{IRR} = -0.2223.$$

$$104 = \frac{5}{(1 + \text{IRR})^1} + \frac{5}{(1 + \text{IRR})^2} + \frac{105}{(1 + \text{IRR})^3}.$$
$$\text{IRR} = 0.0357.$$

Environmental, social, and governance (ESG) considerations may also play a role in credit risk assessment. For example, companies responsible for pollution run the risk of fines or other business sanctions, those with poor labor practices risk their reputation and may face customer boycotts or lawsuits, and firms with weak governance are more likely to engage in aggressive or even fraudulent accounting. Estimated probabilities of default and loss given default should incorporate these potential impacts.

Recent years have also seen several types of bond with explicit links to ESG matters. Climate, or green, bonds are typically issued with proceeds earmarked for environmentally beneficial purposes and may come with tax incentives to enhance their attractiveness to investors.

Another category of fixed-income instruments whose special features affect credit risk assessment are catastrophe and pandemic bonds. They resemble an insurance product, rather than a traditional debt instrument. For example, the World Bank issued pandemic bonds in 2017, offering investors high interest payments in return for taking on the risk of losing capital should a pandemic occur, in which case they would pay out aid to poor nations suffering from a serious outbreak of infectious disease. At the time of this writing (July 2020), nearly all the principal from those bonds has been wiped out because caseloads and deaths from COVID-19 have exceeded the bonds' thresholds.

3 CREDIT SCORES AND CREDIT RATINGS

- ☐ explain credit scores and credit ratings
- ☐ calculate the expected return on a bond given transition in its credit rating

Credit scores and ratings are used by lenders in deciding to extend credit to a borrower and in determining the terms of the contract. Credit scores are used primarily in the retail lending market for small businesses and individuals. Credit ratings are used in the wholesale market for bonds issued by corporations and government entities, as well as for asset-backed securities (ABS).

Credit scoring methodologies can vary. In some countries, only negative information, such as delinquent payments or outright default, is included. Essentially, everyone has a good credit score until proven otherwise. In other countries, a broader set of information is used to determine the score. A score reflects actual observed factors. In general, credit reporting agencies are national in scope because of differences in legal systems and privacy concerns across countries.

The FICO score, which is the federally registered trademark of the Fair Isaac Corporation, is used in the United States by about 90% of lenders to retail customers. FICO scores are computed using data from consumer credit files collected by three national credit bureaus: Experian, Equifax, and TransUnion. Five primary factors are included in the proprietary algorithm used to get the score:

- 35% for the payment history: This includes the presence or lack of such information as delinquency, bankruptcy, court judgments, repossessions, and foreclosures.
- 30% for the debt burden: This includes credit card debt-to-limit ratios, the number of accounts with positive balances, and the total amount owed.
- 15% for the length of credit history: This includes the average age of accounts on the credit file and the age of the oldest account.
- 10% for the types of credit used: This includes the use of installment payments, consumer finance, and mortgages.
- 10% for recent searches for credit: This includes "hard" credit inquiries when consumers apply for new loans but not "soft" inquiries, such as for employee verification or self-checking one's score.

Fair Isaac Corporation, on its website, notes items that are not included in the FICO credit score: race, color, national origin, sex, marital status, age, salary, occupation, employment history, home address, and child/family support obligations. The company also reports from time to time the distribution across scores, which range from a low of 300 to a perfect score of 850. Exhibit 5 shows the distribution for three particular months: October 2005, before the global financial crisis; April 2009, in the depths of the crisis; and April 2017, well after the crisis. It is evident that the percentage of weak scores increased as economic conditions worsened but has gone down since then. The average FICO score varied from 688 to 687 to 700 during these months.

Exhibit 5: Distribution of FICO Scores

FICO Score	October 2005	April 2009	April 2017
300–499	6.6%	7.3%	4.7%
500–549	8.0%	8.7%	6.8%
550–599	9.0%	9.1%	8.5%
600–649	10.2%	9.5%	10.0%
650–699	12.8%	12.0%	13.2%
700–749	16.4%	15.9%	17.1%
750–799	20.1%	19.3%	19.0%
800–850	16.9%	18.2%	20.7%

Source: Fair Isaac Corporation.

EXAMPLE 3

Credit Scoring

1. Tess Waresmith is a young finance professional who plans to eventually buy a two-family house, live in one unit, and rent the other to help cover the mortgage payments. She is a careful money manager and every year checks her FICO credit score. She is pleased to see that it has improved from 760 last year to 775 this year. Which of these factors can explain the improvement?

   A. She is now one year older and has not had any late payments on credit cards during the year.

   B. Her bank on its own raised her limit on a credit card from $1,000 to $2,500, but she has maintained the same average monthly balance.

   C. She applied for and received a new car loan from her credit union.

   D. She refrained from checking her FICO score monthly, which some of her friends do.

Solution:

Factors A, B, and C help explain the improvement. Going down the list:

A. Age itself is not a factor used by Fair Isaac to determine the credit score. However, the average age of the accounts is a factor, as is the age of the oldest account. Therefore, other things being equal, the passage of time tends to improve the score. In general, age and credit score are highly correlated.

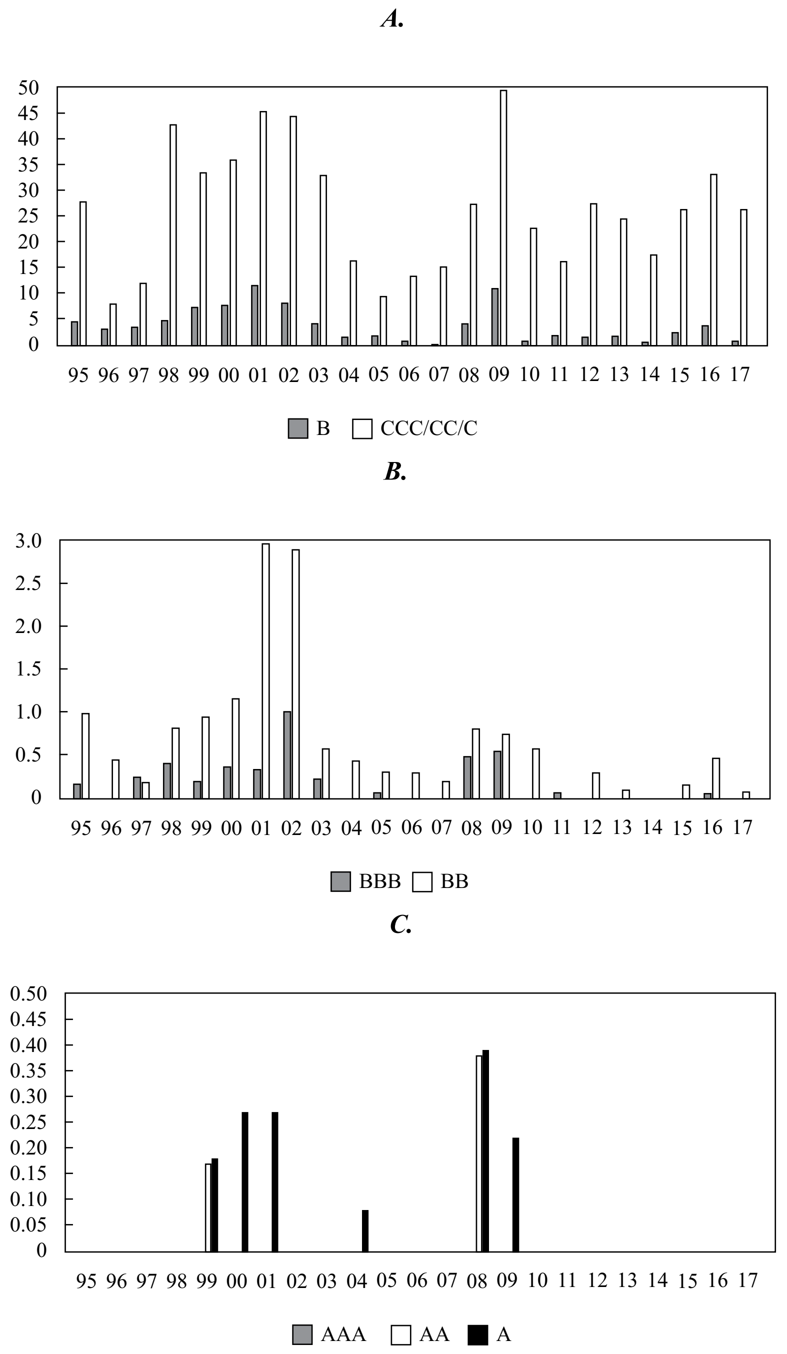
B. The credit card debt-to-limit ratio is a component of the debt burden. Having a higher limit for the same average balance reduces the ratio and improves the credit score.

C. Because the car loan is a new type of credit usage and thus does not have any late payments, it has a positive impact on the score.

D. Refraining from self-checking one's credit score has no impact. Self-checking is deemed to be a "soft inquiry" and does not factor into the calibration of the FICO score.

Whereas credit scores are the primary measure of credit risk in retail lending, credit ratings are widely used in corporate and sovereign bond markets. The three major global credit rating agencies are Moody's Investors Service, Standard & Poor's, and Fitch Ratings. Each provides quality ratings for issuers as well as specific issues. Similar to credit scores, these are ordinal ratings focusing on the probability of default. The historical corporate default experience by various ratings for 1995 to 2017 is shown in Exhibit 6.

Exhibit 6: Historical Corporate Default Experience by Rating (entries are in %)



The credit rating agencies consider the expected loss given default by means of *notching*, which is a rating adjustment methodology (covered earlier in the CFA Program curriculum) to reflect the priority of claim for specific debt issues of that issuer and to reflect any subordination. The issuer rating is typically for senior unsecured debt. The rating on subordinated debt is then adjusted, or "notched," by lowering it one or two levels. This inclusion of loss given default in addition to the probability of default explains why they are called "credit ratings" and not just "default ratings."

In addition to the "letter grade," the rating agencies provide an outlook (positive, stable, or negative) for the issuer as well as when the issuer is under "watch." For example, what follows is the history of Standard & Poor's issuer rating for RadioShack Corporation as it moved from BBB– in 1969 to BB+ in 1978, to AAA in 1983, to BB in 2006, and finally to default in 2015:

• 2 May 1969	BBB–
• 13 October 1978	BB+
• 12 December 1980	BB
• 1 April 1981	BBB+
• 7 January 1982	A
• 10 January 1983	AAA
• 28 November 1984	A+/Watch Negative
• 8 August 1991	A/Stable
• 4 January 1993	A/Watch Negative
• 25 February 1993	A–/Stable
• 27 May 1993	A–/Watch Positive
• 17 January 1994	A–/Stable
• 17 October 1996	A–/Negative
• 24 February 1999	A–/Stable
• 13 May 2005	A–/Watch Negative
• 8 August 2005	BBB+/Stable
• 21 April 2006	BBB–/Stable
• 24 July 2006	BBB–/Negative
• 25 October 2006	BB/Negative
• 12 August 2008	BB/Stable
• 21 November 2011	BB–/Stable
• 2 March 2012	B+/Negative
• 30 July 2012	B–/Negative
• 21 November 2012	CCC+/Negative
• 1 August 2013	CCC/Negative
• 20 December 2013	CCC+/Negative
• 16 June 2014	CCC/Negative
• 11 September 2014	CCC–/Negative
• 6 February 2015	D

Source: Standard & Poor's, "2014 Annual Global Corporate Default Study and Rating Transitions," Table 54 (30 April 2015).

The history of RadioShack illustrates that the rating can remain the same for prolonged periods of time. The company was A+ from 1984 to 1991 and A– from 1993 to 2005. The rating agencies report *transition matrixes* based on their historical experience. Exhibit 7 is a representative example. It shows the probabilities of a particular rating transitioning to a different rating over the course of the following year. An A rated issuer has an 87.50% probability of remaining at that level; a 0.05% probability of moving up to AAA (such as RadioShack did in 1983); a 2.50% probability of moving up to AA; an 8.40% probability of moving down to BBB; 0.75% down to BB; 0.60% to B; 0.12% to CCC, CC, or C; and 0.08% to D, where it is in default.

Exhibit 7: Representative One-Year Corporate Transition Matrix (entries are in %)

From/To	AAA	AA	A	BBB	BB	B	CCC, CC, C	D
AAA	**90.00**	9.00	0.60	0.15	0.10	0.10	0.05	0.00
AA	1.50	**88.00**	9.50	0.75	0.15	0.05	0.03	0.02
A	0.05	2.50	**87.50**	8.40	0.75	0.60	0.12	0.08
BBB	0.02	0.30	4.80	**85.50**	6.95	1.75	0.45	0.23
BB	0.01	0.06	0.30	7.75	**79.50**	8.75	2.38	1.25
B	0.00	0.05	0.15	1.40	9.15	**76.60**	8.45	4.20
CCC, CC, C	0.00	0.01	0.12	0.87	1.65	18.50	**49.25**	29.60
Credit Spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%	

Exhibit 7 also shows representative credit spreads for a 10-year corporate bond. The credit transition matrix and the credit spreads allow a fixed-income analyst to estimate a one-year rate of return given the possibility of credit rating migration but still no default. Assume that an A rated 10-year corporate bond will have a modified duration of 7.2 at the end of the year given stable yields and spreads. For each possible transition, the analyst can calculate the expected percentage price change as the product of the modified duration and the change in the spread:

From A to AAA:	–7.2 × (0.60% – 1.10%) = +3.60%.
From A to AA:	–7.2 × (0.90% – 1.10%) = +1.44%.
From A to BBB:	–7.2 × (1.50% – 1.10%) = –2.88%.
From A to BB:	–7.2 × (3.40% – 1.10%) = –16.56%.
From A to B:	–7.2 × (6.50% – 1.10%) = –38.88%.
From A to CCC, CC, or C:	–7.2 × (9.50% – 1.10%) = –60.48%.

The probabilities of migration now can be used to calculate the expected percentage change in the bond value over the year. The expected percentage change in bond value for an A rated corporate bond is found by multiplying each expected percentage price change for a possible credit transition by its respective transition probability found in the row associated with the A rating and summing the products:

$$(0.0005 \times 3.60\%) + (0.0250 \times 1.44\%) + (0.8750 \times 0\%) + (0.0840 \times -2.88\%) + (0.0075 \times -16.56\%) + (0.0060 \times -38.88\%) + (0.0012 \times -60.48\%)$$

$$= -0.6342\%.$$

Therefore, the expected return on the bond over the next year is its yield to maturity minus 0.6342%, assuming no default. If the bond was not investment grade, the small probability of a transition to default would need to be taken into consideration.

Credit spread migration typically reduces the expected return for two reasons. First, the probabilities for change are not symmetrically distributed around the current rating. They are skewed toward a downgrade rather than an upgrade. Second, the increase in the credit spread is much larger for downgrades than the decrease in the spread for upgrades.

EXAMPLE 4

The Impact of Credit Migration on Expected Return

1. Manuel Perello is a wealth manager for several Latin American families who seek to keep a portion of their assets in very high-quality corporate bonds. Mr. Perello explains that the yields to maturity on the bonds should be adjusted for possible *credit spread widening* to measure the expected rate of return over a given time horizon. In his presentation to one of the families, he uses a 10-year, AAA rated corporate bond that would have a modified duration of 7.3 at the end of the year. Using the corporate transition matrix in Exhibit 7, Mr. Perello concludes that the expected return on the bond over the next year can be approximated by the yield to maturity less 32.5 bps to account for a possible credit downgrade even if there is no default. Demonstrate how he arrives at that conclusion.

Solution:

First, calculate the expected percentage price change using the modified duration for the bond and the change in the credit spread:

From AAA to AA:	–7.3 × (0.90% – 0.60%) = –2.19%.
From AAA to A:	–7.3 × (1.10% – 0.60%) = –3.65%.
From AAA to BBB:	–7.3 × (1.50% – 0.60%) = –6.57%.
From AAA to BB:	–7.3 × (3.40% – 0.60%) = –20.44%.
From AAA to B:	–7.3 × (6.50% – 0.60%) = –43.07%.
From AAA to CCC, CC, or C:	–7.3 × (9.50% – 0.60%) = –64.97%.

Second, calculate the expected percentage change in bond value over the year using the probabilities associated with the AAA rating row in the corporate transition matrix:

$$(0.9000 \times 0\%) + (0.0900 \times -2.19\%) + (0.0060 \times -3.65\%) + (0.0015 \times -6.57\%) + (0.0010 \times -20.44\%) + (0.0010 \times -43.07\%) + (0.0005 \times -64.97\%)$$

$$= -0.3249\%.$$

STRUCTURAL AND REDUCED-FORM CREDIT MODELS 4

- [ ] explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses

Credit analysis models fall into two broad categories—structural models and reduced-form models (Fabozzi 2013). Structural models of credit risk date back to the 1970s and the seminal contributions to finance theory by Fischer Black, Myron Scholes, and Robert Merton (Black and Scholes 1973; Merton 1974). Their key insights were that a company defaults on its debt if the value of its assets falls below the amount of its liabilities and that the probability of that event has the features of an option.

Reduced-form varieties emerged in the 1990s (Jarrow and Turnbull 1995; Duffie and Singleton 1999) and avoid a fundamental problem with the structural models. The Black–Scholes–Merton option pricing model explicitly assumes that the assets on which the options are written (i.e., the shares of a company) are actively traded. That assumption is fine for stock options; however, the assets of the company typically do not trade. Reduced-form models get around this problem by not treating default as an endogenous (internal) variable. Instead, the default is an exogenous (external) variable that occurs randomly. Unlike structural models that aim to explain *why* default occurs (i.e., when the asset value falls below the amount of liabilities), reduced-form models aim to explain statistically *when*. This is known as the *default time* and can be modeled using a Poisson stochastic process. The key parameter in this process is the *default intensity*, which is the probability of default over the next time increment. Reduced-form credit risk models are thus also called *intensity-based* and *stochastic default rate* models.

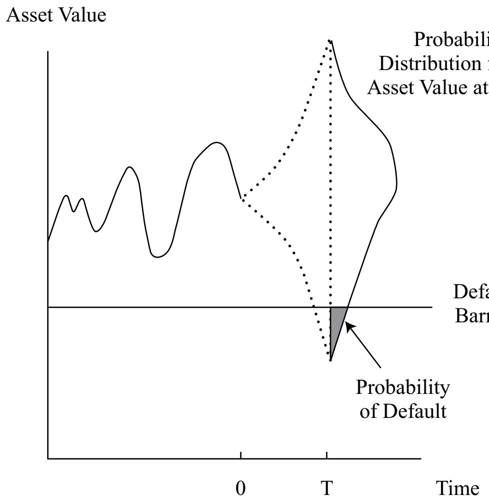
Both types of credit risk model have advantages and disadvantages. Structural models provide insight into the nature of credit risk but can be burdensome to implement. The modeler needs to determine the value of the company, its volatility, and the default barrier that is based on the liabilities of the company. In the model, the company defaults when the value of its assets dips below this default barrier. Although straightforward in theory, it can be difficult in practice because of limitations in available data. Examples of companies hiding debt (Enron Corporation, Tyco International, WorldCom, Parmalat, and Lehman Brothers, to name a few) highlight the challenge to measure the default barrier, especially in times when knowing changes in default probabilities would be most beneficial to investors (Smith 2011).

Reduced-form models have the advantage that the inputs are observable variables, including historical data. The default intensity is estimated using regression analysis on *company-specific* variables (e.g., leverage ratio, net-income-to-assets ratio, and cash-to-assets ratio), and *macroeconomic* variables (e.g., unemployment rate, GDP growth rate, measures of stock market volatility). This flexibility allows the model to directly reflect the business cycle in the credit risk measure.

A disadvantage of reduced-form models is that, unlike structural models, they do not explain the economic reasons for default. Also, reduced-form models assume that default comes as a "surprise" and can occur at any time. In reality, default is rarely a surprise because the issuer usually has been downgraded several times before the final event, as we saw with the RadioShack experience in the previous section.

Exhibit 8 depicts a structural model of default. The vertical axis measures the asset value of the company. It is called a structural model because it depends on the structure of the company's balance sheet—its assets, liabilities, and equity. It also can be called a *company-value* model because the key variable is the asset value of the company. In Exhibit 8, the asset value has been volatile prior to now, time 0, but has remained above the horizontal line that represents the default barrier. If the asset value falls below the barrier, the company defaults on the debt.

Exhibit 8: A Structural Model of Default



Source: This exhibit is adapted from Duffie and Singleton (2003, p. 54).

There is a probability distribution for the asset value as of some future date, time *T*. The probability of default is endogenous to this structural model. It is the portion of the probability distribution that lies below the default barrier. This default probability increases with the variance of the future asset value, with greater time to *T*, and with greater financial leverage. Less debt in the capital structure lowers the horizontal line and reduces the probability of default. These factors indicate that credit risk is linked to option pricing theory.

An important feature of the structural credit models is that they allow interpretation of debt and equity values in terms of options. Let $A(T)$ be the random asset value as of time *T*. To simplify, we can assume that the debt liabilities are zero-coupon bonds that mature at time *T*. These bonds have a face value of *K*, which represents the default barrier in Exhibit 8. The values for debt and equity at time *T* are denoted $D(T)$ and $E(T)$ and depend on the relationship between $A(T)$ and *K*:

$$D(T) + E(T) = A(T). \quad (1)$$

$$E(T) = \max[A(T) - K, 0]. \quad (2)$$

$$D(T) = A(T) - \max[A(T) - K, 0]. \quad (3)$$

Equation 1 is the balance sheet identity: The market values of debt and equity at time *T* equal the asset value. Equation 2 indicates that equity is essentially a purchased call option on the assets of the company whereby the strike price is the face value of the debt. It is a long position in a call option because the value of equity goes up when the asset value goes up. Moreover, like options, equity does not take on negative values. Equation 3 shows that in this formulation, the debtholders own the assets of the company and have written the call option held by the shareholders. We can interpret the premium that the debtholders receive for writing the option as the value of having priority of claim in the event that the asset value falls below *K*. In that case, the value of equity falls to zero and the debtholders own the remaining assets.

Suppose that at time *T*, $A(T) > K$ so that the call option is in the money to the shareholders. Then, $E(T) = A(T) - K$ and $D(T) = A(T) - [A(T) - K] = K$. Instead, suppose that $A(T) < K$ so that the call option is out of the money and the debt is in default.

In this case, $E(T) = 0$ and $D(T) = A(T) - 0 = A(T)$. In both situations, as well as when $A(T) = K$, the balance sheet identity holds. Notice that *limited liability* is an inherent assumption in this model. Equity, like options, does not take on negative values.

EXAMPLE 5

An Equivalent Option Interpretation of Debt and Equity

1. Carol Feely is a junior credit analyst at one of the major international credit rating agencies. She understands that in the standard structural models, equity is interpreted as a call option on the asset value of the company. However, she is not comfortable with the assumption that it is the debtholders who implicitly own the assets and write a call option on them. She claims that the model should start with the understanding that the shareholders own the net value of the company, which is $A(T) - K$, and that their limited liability is essentially the value of a long position in a put option at a strike price of K. Furthermore, the debtholders own a "risk-free" bond having a value of K at time T and a short position in the put that is held by the shareholders.

   Demonstrate that Ms. Feely's "embedded put option" interpretation provides the same values for debt and equity at time T as does the more customary call option structural model.

Solution:

A long position in a put option on the asset value at a strike price of K takes the form $\max[K - A(T), 0]$. This put option has intrinsic value to its holder when $K > A(T)$ and is worthless when $K \leq A(T)$. The values for $E(T)$ and $D(T)$ according to Ms. Feely at time T are as follows:

$$E(T) = A(T) - K + \max[K - A(T), 0].$$

$$D(T) = K - \max[K - A(T), 0].$$

If $A(T) > K$ at time T, the put option is out of the money, $E(T) = A(T) - K + 0 = A(T) - K$, and $D(T) = K - 0 = K$. If $A(T) < K$, the put is in the money, $E(T) = A(T) - K + [K - A(T)] = 0$, and $D(T) = K - [K - A(T)] = A(T)$. This interpretation indicates that the value of limited liability to shareholders is the value of the put option that they purchase from the debtholders. Ms. Feely is correct in that the same payoffs as the embedded call option interpretation are obtained.

Although credit risk is inherently linked to option pricing, it is the implementation of structural models that has provided practical value to fixed-income analysis. Many credit rating agencies and consultancies, most notably Moody's KMV Corporation, use option pricing methodologies to estimate such credit risk parameters as the probability of default and the loss given default. Building on the classic Black–Scholes–Merton model and later variants, the model builders use historical data on the company's equity price to estimate volatility, which is a key element in option pricing models.

These advantages and disadvantages indicate that the choice of credit risk model depends on how it is to be used and by whom. Structural models require information best known to the managers of the company (and perhaps their commercial bankers and the credit rating agencies). Therefore, they can be used for internal risk management, for banks' internal credit risk measures, and for publicly available credit ratings. Reduced-form models require only information generally available in financial markets, which suggests that they should be used to value risky debt securities and credit derivatives.

VALUING RISKY BONDS IN AN ARBITRAGE-FREE FRAMEWORK

5

☐ calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters

In this section, we use the arbitrage-free framework to analyze the credit risk of a corporate bond in the context of volatile interest rates (based on Smith 2017). Earlier, we solved for the credit valuation adjustment and the credit spread under the assumptions of no interest rate volatility and a flat government bond yield curve. A binomial interest rate tree for benchmark bond yields allows us to calculate the *expected exposure* to default loss. In addition, we have an upward-sloping yield curve for benchmark bonds. We take the risk-neutral probability of default as given, as if it has been determined using a structural or reduced-form credit model. We also assume a recovery rate if default were to occur that conforms to the seniority of the debt issue and the nature of the issuer's assets.

The first step is to build the binomial interest rate tree under the assumption of no arbitrage. Exhibit 9 displays the data on annual payment benchmark government bonds that are used to build the binomial interest rate tree. This is the *par curve* because each bond is priced at par value. The coupon rates are equal to the yields to maturity because the years to maturity are whole numbers (integers) so that there is no accrued interest. The one-year government bond has a negative yield to reflect the conditions seen in some financial markets. Note that the actual one-year security is likely to be a zero-coupon bond priced at a premium, at 100.2506 per 100 of par value: (100/100.2506) – 1 = –0.0025. However, on a par curve for which all the bonds are priced at 100, it is shown as having a negative coupon rate.

Exhibit 9: Par Curve for Annual Payment Benchmark Government Bonds, Spot Rates, Discount Factors, and Forward Rates

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	–0.25%	100	1.002506	–0.2500%	
2	0.75%	100	0.985093	0.7538%	1.7677%
3	1.50%	100	0.955848	1.5166%	3.0596%
4	2.25%	100	0.913225	2.2953%	4.6674%
5	2.75%	100	0.870016	2.8240%	4.9664%

Note: All calculations in this and subsequent exhibits were completed on a spreadsheet; rounded results are reported in the text.

The discount factors and spot rates are bootstrapped using the cash flows on the underlying benchmark bonds in this sequence of equations:

$$100 = (100 - 0.25) \times DF_1.$$

$DF_1 = 1.002506.$

$100 = (0.75 \times 1.002506) + (100.75 \times DF_2).$

$DF_2 = 0.985093.$

$100 = (1.50 \times 1.002506) + (1.50 \times 0.985093) + (101.50 \times DF_3).$

$DF_3 = 0.955848.$

$100 = (2.25 \times 1.002506) + (2.25 \times 0.985093) + (2.25 \times 0.955848) + (102.25 \times DF_4).$

$DF_4 = 0.913225.$

$100 = (2.75 \times 1.002506) + (2.75 \times 0.985093) + (2.75 \times 0.955848) + (2.75 \times 0.913225) + (102.75 \times DF_5).$

$DF_5 = 0.870016.$

The spot (i.e., implied zero-coupon) rates can be calculated from the discount factors; for instance, the two-year spot rate is 0.7538% and the four-year spot rate is 2.2953%:

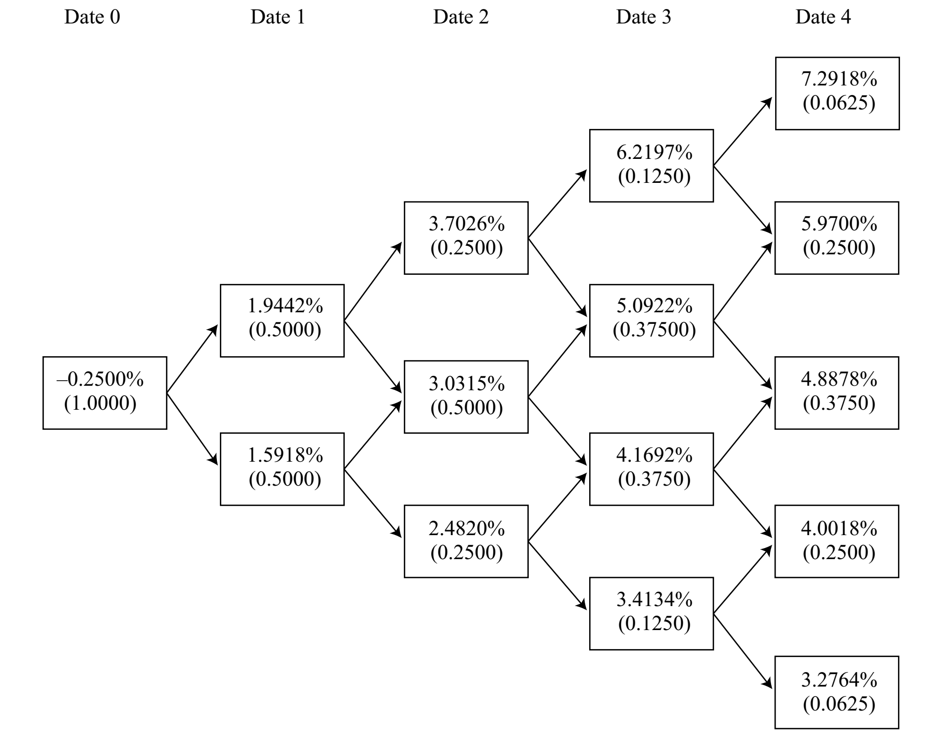
$$\left(\frac{1}{0.985093}\right)^{1/2} - 1 = 0.007538.$$

$$\left(\frac{1}{0.913225}\right)^{1/4} - 1 = 0.022953.$$

The forward rates are calculated as the ratios of the discount factors. The one-year forward rate two years into the future is 3.0596%: 0.985093/0.955848 – 1 = 0.030596. The one-year forward rate four years into the future is 4.9665%: 0.913225/0.870016 – 1 = 0.049665.

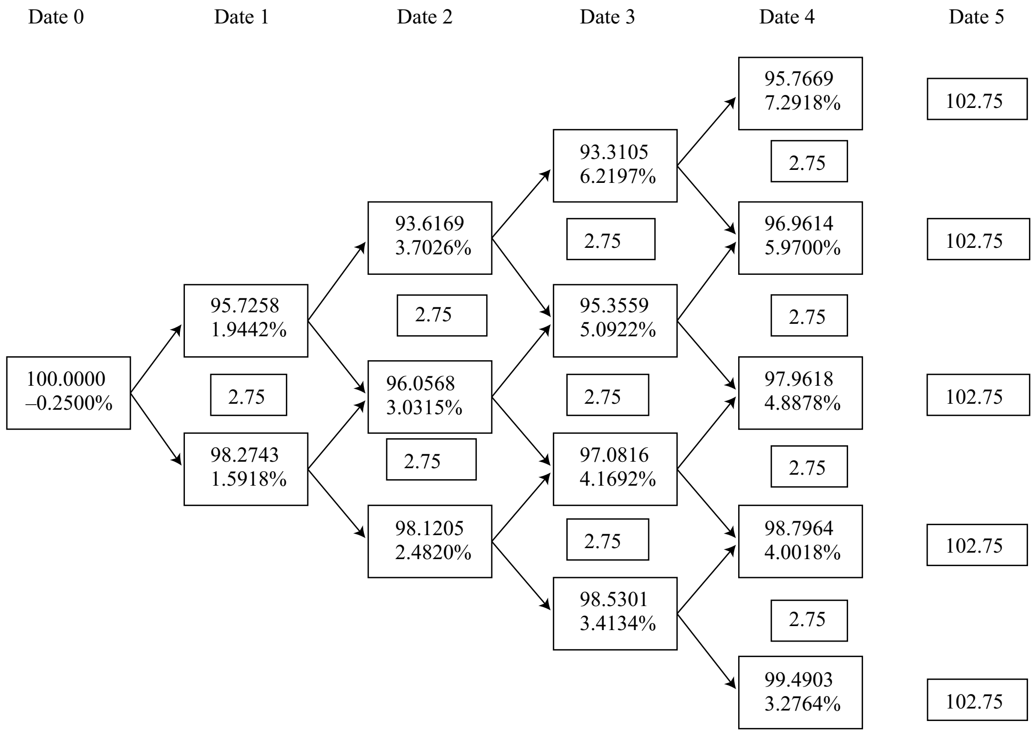
Following the methodology detailed in the "Arbitrage-Free Valuation Framework" topic, we build a binomial interest rate tree for one-year forward rates consistent with the pricing of the benchmark government bonds and an assumption of future interest rate volatility. Here we assume 10% volatility. The resulting binomial interest rate tree is presented in Exhibit 10. Below each rate is the probability of attaining that node in the tree. The current (Date 0) one-year rate of –0.25% will rise to 1.9442% or "fall" to 1.5918% by the end of the year (Date 1) with equal probability. On Date 2, at the end of the second year, the one-year rate will be 3.7026%, 3.0315%, or 2.4820% with probabilities of 0.25, 0.50, and 0.25, respectively. On Date 4, the forward rate will fall within the range of a high of 7.2918% to a low of 3.2764%. For each date, the possible rates are spread out around the forward rates shown in Exhibit 9.

Exhibit 10: One-Year Binomial Interest Rate Tree for 10% Volatility



To demonstrate that this is an arbitrage-free binomial interest rate tree, we calculate the Date 0 value of a 2.75% annual payment government bond. We know from Exhibit 9 that this bond is priced at par value. Exhibit 11 shows that the Date 0 value is indeed 100.0000. Notice that the scheduled year-end coupon and principal payments are placed to the right of each forward rate in the tree.

Exhibit 11: Valuation of a 2.75% Annual Payment Government Bond



These are the five Date 4 values for the government bond, shown above the interest rate at each node:

$$102.75/1.072918 = 95.7669.$$

$$102.75/1.059700 = 96.9614.$$

$$102.75/1.048878 = 97.9618.$$

$$102.75/1.040018 = 98.7964.$$

$$102.75/1.032764 = 99.4903.$$

These are the four Date 3 values:

$$\frac{[(0.5 \times 95.7669) + (0.5 \times 96.9614)] + 2.75}{1.062197} = 93.3105.$$

$$\frac{[(0.5 \times 96.9614) + (0.5 \times 97.9618)] + 2.75}{1.050922} = 95.3559.$$

$$\frac{[(0.5 \times 97.9618) + (0.5 \times 98.7964)] + 2.75}{1.041692} = 97.0816.$$

$$\frac{[(0.5 \times 98.7964) + (0.5 \times 99.4903)] + 2.75}{1.034134} = 98.5301.$$

Continuing with backward induction, the Date 0 value turns out to be 100.0000, confirming that the binomial interest rate tree has been correctly calibrated.

Now consider a five-year, 3.50% annual payment corporate bond. A fixed-income analyst assigns an annual default probability of 1.25% and a recovery rate of 40% to this bond and assumes 10% volatility in benchmark interest rates. The problem at hand for the analyst is to assess the fair value for the bond under these assumptions. This is done in two steps:

- First, determine the value for the corporate bond assuming no default (VND).
- Second, calculate the credit valuation adjustment.

The fair value of the bond is the VND minus the CVA.

The binomial interest rate tree for the benchmark rates in Exhibit 10 can be used to calculate the VND for the bond. Exhibit 12 shows that the VND is 103.5450 per 100 of par value. This number could also have been obtained more directly by using the benchmark discount factors:

$$(3.50 \times 1.002506) + (3.50 \times 0.985093) + (3.50 \times 0.955848) + (3.50 \times 0.913225) + (103.50 \times 0.870016)$$

$$= 103.5450.$$

The advantage of using the binomial interest rate tree to get the VND is that the same tree is used to calculate the expected exposure to default loss, which is a key element in the credit risk model.

Exhibit 12: Value of a 3.50% Annual Payment Corporate Bond Assuming No Default

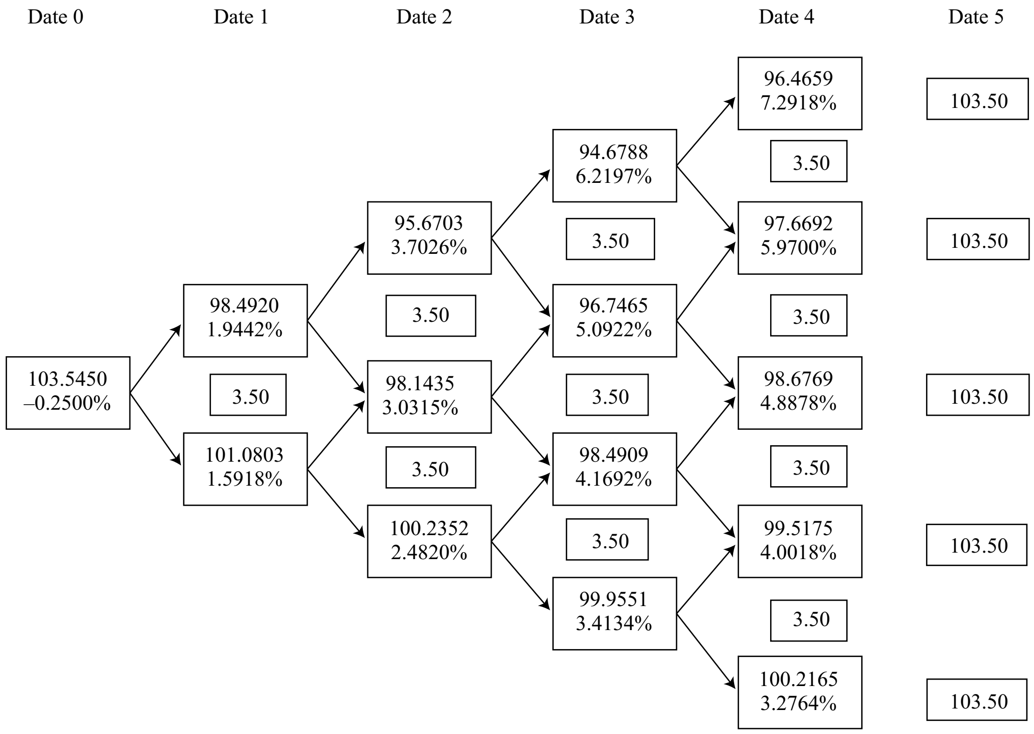


Exhibit 13 shows that the credit valuation adjustment to the value assuming no default is 3.5394 per 100 of par value. The expected exposure for Date 4 is 102.0931, calculated using the bond values at each node, the probability of attaining the node, and the coupon payment:

$(0.0625 \times 96.4659) + (0.25 \times 97.6692) + (0.375 \times 98.6769) + (0.25 \times 99.5175) + (0.0625 \times 100.2165) + 3.50$

$= 102.0931.$

(Note again that all calculations are done on a spreadsheet to maintain precision; only the rounded results are reported in the text.) The loss given default for Date 4 is 61.2559 [= 102.0931 × (1 – 0.40)] because the assumed recovery rate is 40% of the exposure. The probability of default at Date 4 is 1.2037%, assuming no prior default. This is based on the probability of survival into the fourth year. It is calculated as

$1.25\% \times (100\% - 1.25\%)^3 = 1.2037\%.$

The probability of survival after Date 3 is $(100\% - 1.25\%)^3$, and the probability of default on Date 4 is 1.25%. The product of the LGD and the POD is the expected loss. The present value of the expected loss, 0.6734, is the contribution to total CVA for Date 4. The sum of the CVAs for each year is the overall CVA.

Exhibit 13: Credit Valuation Adjustment for the 3.50% Annual Payment Corporate Bond

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.2500%	1.002506	0.7766
2	101.5481	60.9289	1.2344%	0.985093	0.7409
3	101.0433	60.6260	1.2189%	0.955848	0.7064
4	102.0931	61.2559	1.2037%	0.913225	0.6734
5	103.5000	62.1000	1.1887%	0.870016	0.6422
			6.0957%	CVA =	3.5394

The fixed-income analyst concludes that the fair value of the corporate bond is 100.0056 per 100 of par value: 103.5450 – 3.5394 = 100.0056. Depending on the current market price for the bond, the analyst might recommend a buy or sell decision.

The yield to maturity (YTM) for the corporate bond given a fair value of 100.0056 is 3.4988%:

$$100.0056 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}.$$

$$\text{YTM} = 0.034988.$$

The five-year par yield for the government bond in Exhibit 9 is 2.75%. Therefore, the credit spread over the benchmark bond is 0.7488% (= 3.4988% – 2.75%). In practice, the credit spread is typically measured against the actual yield on the comparable-maturity government bond, which might be trading at a premium or a discount.

We can say that the credit risk on this corporate bond is captured by a CVA of 3.5394 per 100 in par value as of Date 0 or as an annual spread of 74.88 bps per year for five years. This conclusion, however, assumes that the observed credit spread is based entirely on credit risk. In fact, there usually are liquidity and tax differences between government and corporate bonds. Those differences are neglected in this analysis to focus on credit risk. Stated differently, the liquidity and tax differences are represented in the credit spread.

EXAMPLE 6

Using Credit Analysis in Decision Making

1. Lori Boller is a fixed-income money manager specializing in taking long positions on high-yield corporate bonds that she deems to be undervalued. In particular, she looks for bonds for which the credit spread over government securities appears to indicate too high a probability of default or too low a recovery rate if default were to occur. Currently, she is looking at a three-year, 4.00% annual payment bond that is priced at 104 (per 100 of par value). In her opinion, this bond should be priced to reflect an annual default probability of 2.25% given a recovery rate of 40%. Ms. Boller is comfortable with an assumption of 10% volatility in government bond yields over the next few years. Should she consider buying this bond for her portfolio? Use the government par curve in Exhibit 9 and the binomial interest rate tree in Exhibit 10 in the solution.

Solution:

Ms. Boller needs to calculate the fair value of the three-year, 4% annual payment corporate bond given her assumptions about the credit risk parameters. The results are shown in Exhibit 14.

Exhibit 14: Fair Value of the Three-Year, 4% Annual Payment Corporate Bond

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	64.2541	2.2500%	1.002506	1.4493
2	104.9120	62.9472	2.1994%	0.985093	1.3638
3	104.0000	62.4000	2.1499%	0.955848	1.2823
			6.5993%	CVA =	4.0954

The VND for the bond is 107.3586. The calculations for the bond values in the binomial interest rate tree are as follows:

$$104/1.037026 = 100.2868.$$

$$104/1.030315 = 100.9400.$$

$$104/1.024820 = 101.4812.$$

$$\frac{(0.5 \times 100.2868) + (0.5 \times 100.9400) + 4}{1.019442} = 102.6183.$$

$$\frac{(0.5 \times 100.9400) + (0.5 \times 101.4812) + 4}{1.015918} = 103.5621.$$

$$\frac{(0.5 \times 102.6183) + (0.5 \times 103.5621) + 4}{0.997500} = 107.3586.$$

The CVA for the bond is 4.0954 given the assumption of an annual default probability of 2.25% and a recovery rate of 40% of the expected exposure. The following are calculations for the Date 1 and Date 2 expected exposures:

$$(0.50 \times 102.6183) + (0.50 \times 103.5621) + 4 = 107.0902.$$

(0.25 × 100.2868) + (0.50 × 100.9400) + (0.25 × 101.4812) + 4 = 104.9120.

The calculations for the LGD are as follows:

107.0902 × (1 – 0.40) = 64.2541.

104.9120 × (1 – 0.40) = 62.9472.

104 × (1 – 0.40) = 62.4000.

The following are calculations for the POD for Date 2 and Date 3:

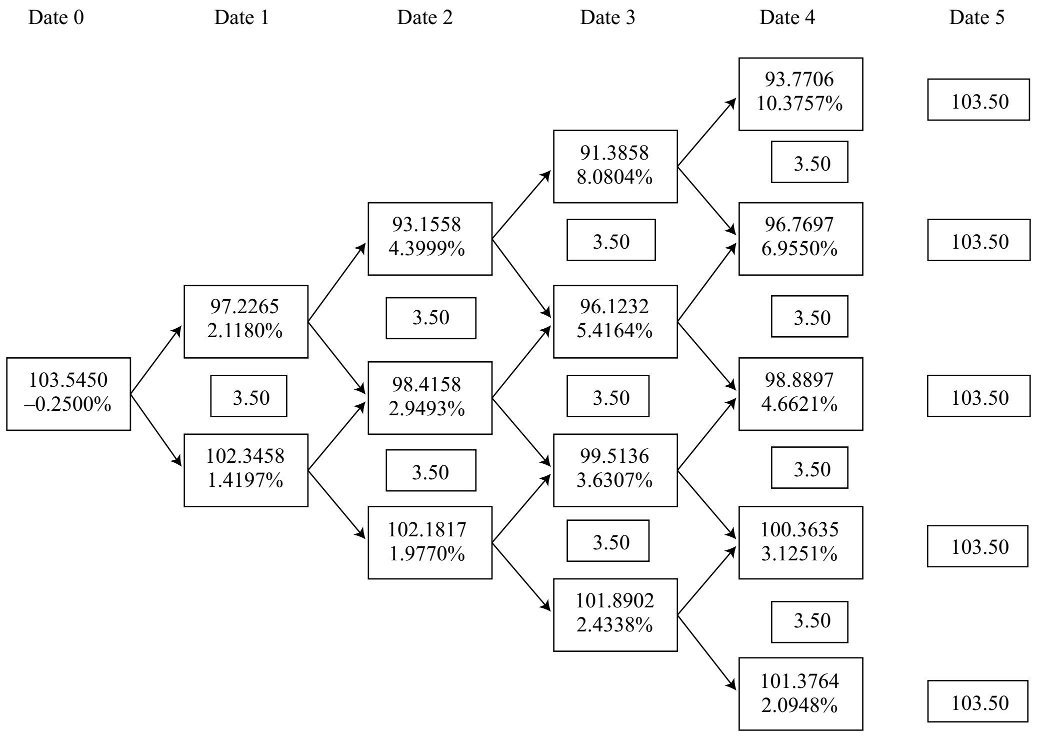
2.25% × (100% – 2.25%) = 2.1994%.

2.25% × (100% – 2.25%)^2 = 2.1499%.

Ms. Boller determines, on the basis of her assumed credit risk parameters, that the fair value for the high-yield corporate bond is 103.2632 (= 107.3586 – 4.0954). Given that the bond is trading at 104, she would likely decline to purchase because in her opinion the bond is overvalued.

A change in the assumed level of interest rate volatility can be shown to have a small impact on the fair value of the corporate bond. Usually the effect of a change in volatility is demonstrated with a bond having an embedded option, such as a callable or putable bond. Here we see an impact of the calculation of CVA on a bond having no embedded options. This is illustrated with Exhibit 15 and Exhibit 16, which use a no-arbitrage binomial interest rate tree for 20% volatility to value the five-year, 3.50% annual payment corporate bond using the same credit risk parameters as in the previous calculations.

Exhibit 15: VND Calculation for the 3.50% Corporate Bond Assuming No Default and 20% Volatility



Notice in Exhibit 15 that with 20% volatility, the range in forward rates for each date is now wider. With 10% volatility, the Date 4 rates go from a low of 3.2764% to a high of 7.2918%. Now, with 20% volatility, the range is from 2.0948% to 10.3757%. The key point is that changing all the bond values still results in a VND of 103.5450. This confirms that the tree has been correctly calibrated and that the assumed level of future interest rate volatility has no impact on the value of a default-risk-free government bond. Changes in the fair value of a corporate bond arising from a change in the assumed rate volatility occur only when there are embedded options and, as demonstrated in Exhibit 16, when there is credit risk.

Exhibit 16: CVA Calculation for the 3.50% Corporate Bond Assuming 20% Volatility

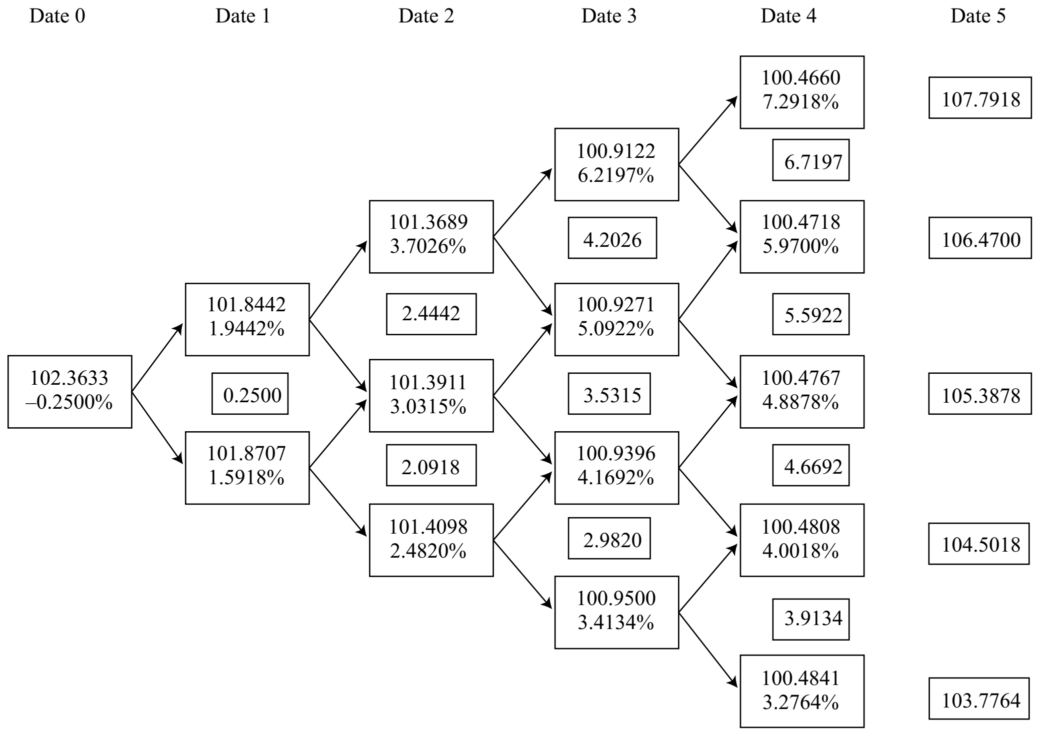
Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.2500%	1.002506	0.7766
2	101.5423	60.9254	1.2344%	0.985093	0.7408
3	101.0233	60.6140	1.2189%	0.955848	0.7062
4	102.0636	61.2382	1.2037%	0.913225	0.6732
5	103.5000	62.1000	1.1887%	0.870016	0.6422
			6.0957%	CVA =	3.5390

Exhibit 16 presents the table to calculate the CVA for 20% volatility. The expected exposures to default loss are slightly lower for Dates 2, 3, and 4 compared with Exhibit 13 for 10% volatility. These small changes feed through the table, reducing the loss given default and the contribution to total CVA for those dates. Overall, the CVA is 3.5390 per 100 of par value. The fair value of the bond is now slightly higher at 100.0060 (= 103.5450 – 3.5390), compared with the value for 10% volatility of 100.0056 (= 103.5450 – 3.5394).

The reason for the small volatility impact on the fair value is the asymmetry in the forward rates produced by the lognormality assumption in the interest rate model. In building the tree, rates are spread out around the implied forward rate for each date—more so the greater the given level of volatility. However, the range is not symmetric about the implied forward rate. For example, the one-year forward rate four years into the future is 4.9665% in Exhibit 9. With 20% volatility, the Date 4 rate at the top of the tree is higher by 5.4092% (= 10.3757% – 4.9665%), while the rate at the bottom of the tree is lower by 2.8717% (= 4.9665% – 2.0948%). The net effect is to reduce the expected exposure to default loss. The top of the tree shows less potential loss because the current value of the bond is lower, which more than offsets the greater exposure to loss at the bottom of the tree.

The arbitrage-free framework can be adapted to value a risky floating-rate note. Consider a five-year "floater" that pays annually the one-year benchmark rate plus 0.50%. This 50 bp addition to the index rate is called the *quoted margin* and typically is fixed over the lifetime of the security. Exhibit 17 demonstrates that the VND for the floater is 102.3633 per 100 of par value, using the binomial interest rate tree for 10% interest rate volatility. Notice that the interest payment is "in arrears," meaning that the rate is set at the beginning of the period and paid at the end of the period. That is why the interest payments set to the right of each rate vary depending on the realized rate in the tree. The interest payment for Date 1 is 0.25 because the Date 0 reference rate is –0.25%: (–0.25% + 0.50%) × 100 = 0.25. The final payment on Date 5 when the floater matures is 105.3878 if the one-year rate is 4.8878% on Date 4: (4.8878% + 0.50%) × 100 + 100 = 105.3878.

Exhibit 17: Value of a Floating-Rate Note Paying the Benchmark Rate Plus 0.50% Assuming No Default and 10% Volatility



Notice that the bond values for each date are very similar for the various forward rates. That, of course, is the intent of a floating-rate note. The bond values would all be exactly 100.0000 if the note paid the benchmark rate "flat," meaning a quoted margin of zero. The VND of 102.3633 is obtained via backward induction (i.e., beginning at maturity and working backward in time). The following are the calculations for the bond values for Date 4:

107.7918/1.072918 = 100.4660.

106.4700/1.059700 = 100.4718.

105.3878/1.048878 = 100.4767.

104.5018/1.040018 = 100.4808.

103.7764/1.032764 = 100.4841.

These are the calculations for Date 3:

$$\frac{(0.5 \times 100.4660) + (0.5 \times 100.4718) + 6.7197}{1.062197} = 100.9122.$$

$$\frac{(0.50 \times 100.4718) + (0.5 \times 100.4767) + 5.5922}{1.050922} = 100.9271.$$

$$\frac{(0.5 \times 100.4767) + (0.5 \times 100.4808) + 4.6692}{1.041692} = 100.9396.$$

$$\frac{(0.5 \times 100.4808) + (0.5 \times 100.4841) + 3.9134}{1.034134} = 100.9500.$$

These are the calculations for the bond values for Date 2:

$$\frac{(0.5 \times 100.9122) + (0.5 \times 100.9271) + 4.2026}{1.037026} = 101.3689.$$

$$\frac{(0.5 \times 100.9271) + (0.5 \times 100.9396) + 3.5315}{1.030315} = 101.3911.$$

$$\frac{(0.5 \times 100.9396) + (0.5 \times 100.9500) + 2.9820}{1.024820} = 101.4098.$$

These are the calculations for the bond values for Date 1 and Date 0:

$$\frac{(0.5 \times 101.3689) + (0.5 \times 101.3911) + 2.4442}{1.019442} = 101.8442.$$

$$\frac{(0.5 \times 101.3911) + (0.5 \times 101.4098) + 2.0918}{1.015918} = 101.8707.$$

$$\frac{(0.5 \times 101.8442) + (0.5 \times 101.8707) + 0.2500}{0.997500} = 102.3633.$$

Exhibit 18 shows the credit risk table for the floating-rate note. For this example, we assume that for the first three years, the annual default probability is 0.50% and the recovery rate is 20%. The credit risk of the issuer then worsens: For the final two years, the annual probability of default goes up to 0.75% and the recovery rate goes down to 10%. This is an example in which the assumed annual default probability changes over the lifetime of the bond.

Exhibit 18: CVA Calculation for the Value of a Floating-Rate Note Paying the Benchmark Rate Plus 0.50%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	102.1074	81.6859	0.5000%	1.002506	0.4095
2	103.6583	82.9266	0.4975%	0.985093	0.4064
3	104.4947	83.5957	0.4950%	0.955848	0.3955
4	105.6535	95.0881	0.7388%	0.913225	0.6416
5	105.4864	94.9377	0.7333%	0.870016	0.6057
			2.9646%	CVA =	2.4586

Note: Credit risk parameter assumptions: for Dates 1–3, annual default probability = 0.50% and recovery rate = 20%; for Dates 4–5, annual default probability = 0.75% and recovery rate = 10%.

The calculation for the expected exposure recognizes that the bond values for each date follow the probabilities of attaining those rates, whereas possible interest payments use the probabilities for the prior date. For example, the expected exposure to default loss for Date 4 is 105.6535:

$$\begin{bmatrix}(0.0625 \times 100.4660) + (0.25 \times 100.4718) + (0.375 \times 100.4767)\\ + (0.25 \times 100.4808) + (0.0625 \times 100.4841)\end{bmatrix}$$
$$+ [(0.125 \times 6.7197) + (0.375 \times 5.5922) + (0.375 \times 4.6692) + (0.125 \times 3.9134)]$$
$$= 105.6535.$$

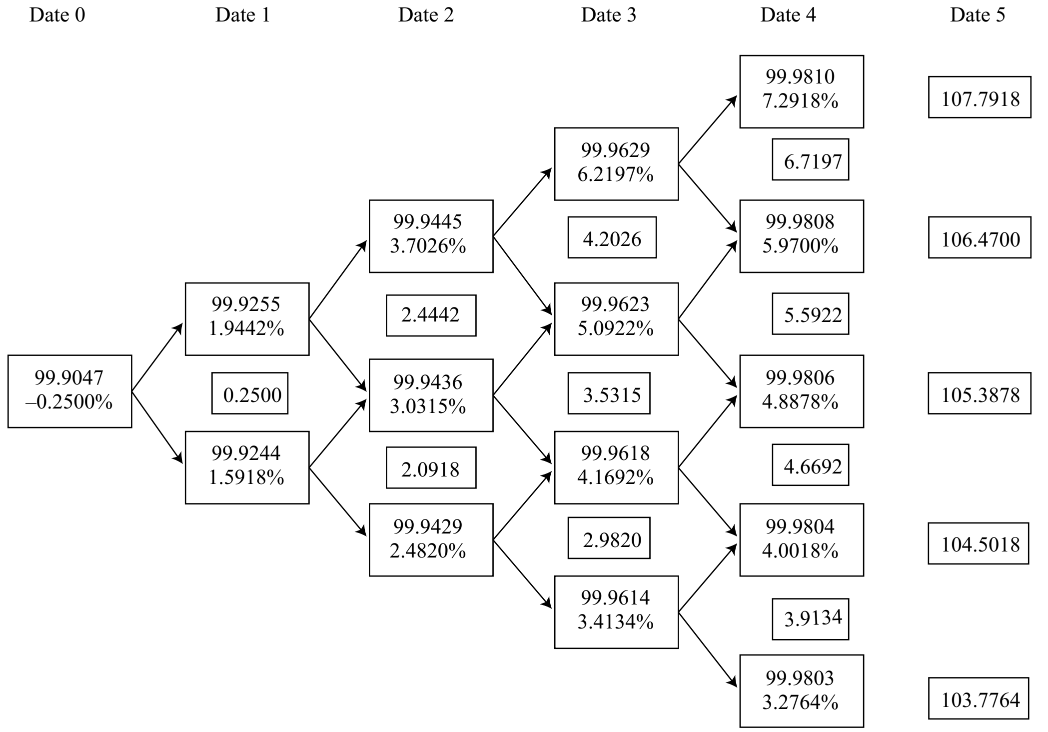
The first term in brackets is the expected bond value using the Date 4 probabilities for each of the five possible rates. The second term is the expected interest payment using the Date 3 probabilities for each of the four possible rates.

The expected LGD for Date 2 is 82.9266 [= 103.6583 × (1 – 0.20)]; for Date 4, it is 95.0881 [= 105.6535 × (1 – 0.10)]. The PODs in Exhibit 18 reflect the probability of default for each year. For Date 2, the POD is 0.4975%, conditional on no default on Date 1: 0.50% × (100% – 0.50%) = 0.4975%. For Date 3, the POD is 0.4950%: 0.50% × (100% – 0.50%)^2 = 0.4950%. The probability of survival into the fourth year is 98.5075%: (100% – 0.50%)^3 = 98.5075%. Therefore, the POD for Date 4 increases to 0.7388% because of the assumed worsening credit risk: 0.75% × 98.5075% = 0.7388%. The probability of survival into the fifth year is 97.7687% (= 98.5075% – 0.7388%). The POD for Date 5 is 0.7333% (= 0.75% × 97.7687%). The cumulative probability of default over the lifetime of the floater is 2.9646%.

Given these assumptions about credit risk, the CVA for the floater is 2.4586. The fair value is 99.9047, the VND of 102.3633 minus the CVA. Because the security is priced below par value, its *discount margin* (DM) must be higher than the quoted margin of 0.50%. The discount margin for a floating-rate note is a yield measure commonly used on floating-rate notes in the same manner that the credit spread is used with fixed-rate bonds.

The arbitrage-free framework can be used to determine the DM for this floater by trial-and-error search (or GoalSeek or Solver in Excel). We add a trial DM to benchmark rates that are used to get the bond values at each node in the tree. Then the trial DM is changed until the Date 0 value matches the fair value of 99.9047. Exhibit 19 shows that the DM for this floater is 0.52046%, slightly above the quoted margin because the security is priced at a small discount below par value.

Exhibit 19: The Discount Margin for the Floating-Rate Note Paying the Benchmark Rate Plus 0.50%, Assuming 10% Volatility



These are the calculations for the bond values for Date 2:

$$\frac{(0.5 \times 99.9629) + (0.5 \times 99.9623) + 4.2026}{1 + 0.037026 + 0.0052046} = 99.9445.$$

$$\frac{(0.5 \times 99.9623) + (0.5 \times 99.9618) + 3.5315}{1 + 0.030315 + 0.0052046} = 99.9436.$$

$$\frac{(0.5 \times 99.9618) + (0.5 \times 99.9614) + 2.9820}{1 + 0.024820 + 0.0052046} = 99.9429.$$

Throughout the binomial interest rate tree, the assumed DM is added to the benchmark rate to factor in credit risk. After a trial-and-error search, a DM of 0.52046% gives the same Date 0 value for the floating-rate note of 99.9047 as is obtained with the VND and CVA models.

EXAMPLE 7

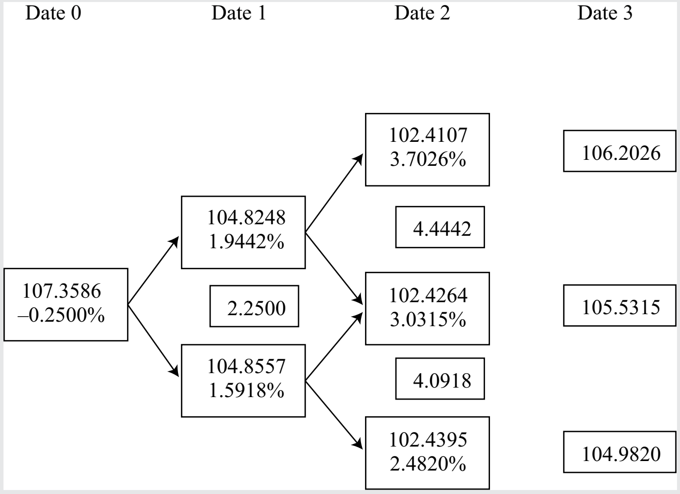
Evaluating a Floating-Rate Note

1. Omar Yassin is an experienced credit analyst at a fixed-income investment firm. His current assignment is to assess potential purchases of distressed high-yield corporate bonds. One intriguing prospect is a three-year, annual payment floating-rate note paying the one-year benchmark rate plus 2.50%. The floater is rated CCC and is priced at 84 per 100 of par value. Based on various research reports on and prices of the issuer's credit default swaps, Mr. Yassin believes the probability of default in the next year is about 30%. If the issuer goes into bankruptcy at any time, he expects the recovery rate to be at least 50%; it could be as high as 60% because of some valuable real estate holdings. He further believes that if the issuer is able to survive this next year, the default probability for the remaining two years will be only about 10% for each year. Based on these assumptions about the credit risk parameters and an expectation of 10% volatility for interest rates, should Mr. Yassin recommend purchasing the floating-rate note?

Solution:

Mr. Yassin calculates the fair value of the three-year, annual payment floating-rate note given his assumptions about the default probabilities and the recovery rate ranging between 50% and 60%. The results are shown in Exhibit 20.

Exhibit 20: Fair Value of the Three-Year, Annual Payment Floating-Rate Note Paying the One-Year Rate Plus 2.50%



Assumed 50% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	53.5451	30.0000%	1.002506	16.1038
2	106.6938	53.3469	7.0000%	0.985093	3.6786
3	105.5619	52.7810	6.3000%	0.955848	3.1784
			43.3000%	CVA =	22.9608

Fair value = 107.3586 – 22.9608 = 84.3978.

Assumed 60% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	107.0902	42.8361	30.0000%	1.002506	12.8830
2	106.6938	42.6775	7.0000%	0.985093	2.9429
3	105.5619	42.2248	6.3000%	0.955848	2.5427
			43.3000%	CVA =	18.3686

Fair value = 107.3586 – 18.3686 = 88.9900.

Each projected interest payment in the tree is the benchmark rate at the beginning of the year plus 2.50% times 100. The rate is –0.25% on Date 0; the "in-arrears" interest payment on Date 1 is 2.2500 [= (–0.25% + 2.50%) × 100]. If the rate is 2.4820% on Date 2, the payment at maturity on Date 3 is 104.9820 [= (2.4820% + 2.50%) × 100 + 100].

The VND for the floater is 107.3586. The calculations for the bond values in the binomial interest rate tree are as follows:

106.2026/1.037026 = 102.4107.

105.5315/1.030315 = 102.4264.

104.9820/1.024820 = 102.4395.

$$\frac{(0.5 \times 102.4107) + (0.5 \times 102.4264) + 4.4442}{1.019442} = 104.8248.$$

$$\frac{(0.5 \times 102.4264) + (0.5 \times 102.4395) + 4.0918}{1.015918} = 104.8557.$$

$$\frac{(0.5 \times 104.8248) + (0.5 \times 104.8557) + 2.2500}{0.997500} = 107.3586.$$

These are the calculations for the expected exposures to default loss:

(0.5 × 104.8248) + (0.5 × 104.8557) + 2.2500 = 107.0902.

(0.25 × 102.4107) + (0.5 × 102.4264) + (0.25 × 102.4395) + (0.5 × 4.4442) + (0.5 × 4.0918)

=106.6938.

(0.25 × 106.2026) + (0.5 × 105.5315) + (0.25 × 104.9820) = 105.5619.

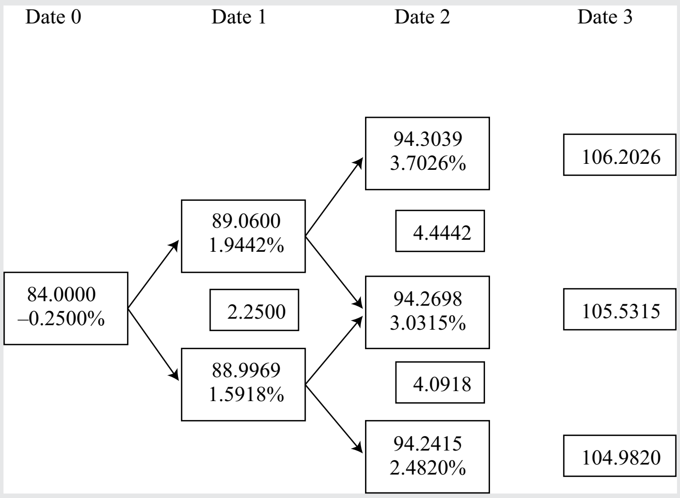
The assumed default probability for the first year is 30%. The POD for Date 2 is 7.00%, which is the probability of survival into the second year, 70%, times the 10% probability of default. The probability of survival into the third year is 63% (= 70% – 7%); the POD for Date 3 is 6.30% (= 10% × 63%).

The decision to consider purchase of the floating-rate note comes down to the assumption about recovery. Exhibit 20 first shows the results for 50% recovery of the expected exposure. The LGD on Date 2 is 53.3469 [= 106.6938 × (1 – 0.50)]. The overall CVA is 22.9608, giving a fair value of 84.3978 (= 107.3586 – 22.9608). Exhibit 20 next shows the results for 60% recovery. With this assumption, the LGD for Date 2 is just 42.6775 [= 106.6938 × (1 – 0.60)]. Stronger recovery reduces the overall CVA to 18.3686. The fair value for the floater is now 88.9900.

Mr. Yassin should recommend purchasing the distressed floating-rate note. Although there is a significant 43.3% probability of default at some point over the three years, the security appears to be fairly priced at 84 given a recovery rate of 50%. At 60% recovery, it is significantly undervalued.

In addition, there is still a 57.7% (= 100% – 43.3%) chance of no default. Exhibit 21 shows the calculation for the discount margin, which is a measure of the return to the investor assuming no default (like a yield to maturity on a fixed-rate bond). Found by a trial-and-error search, the DM is 8.9148%, considerably higher than the quoted margin because the floater is priced at a deep discount.

Exhibit 21: Discount Margin on the Three-Year, Annual Payment Floating-Rate Note Paying the One-Year Rate Plus 2.50%



These are the calculations for the bond values for Date 1 and Date 0:

$$\frac{(0.5 \times 94.3039) + (0.5 \times 94.2698) + 4.4442}{1 + 0.019442 + 0.089148} = 89.0600.$$

$$\frac{(0.5 \times 94.2698) + (0.5 \times 94.2415) + 4.0918}{1 + 0.015918 + 0.089148} = 88.9969.$$

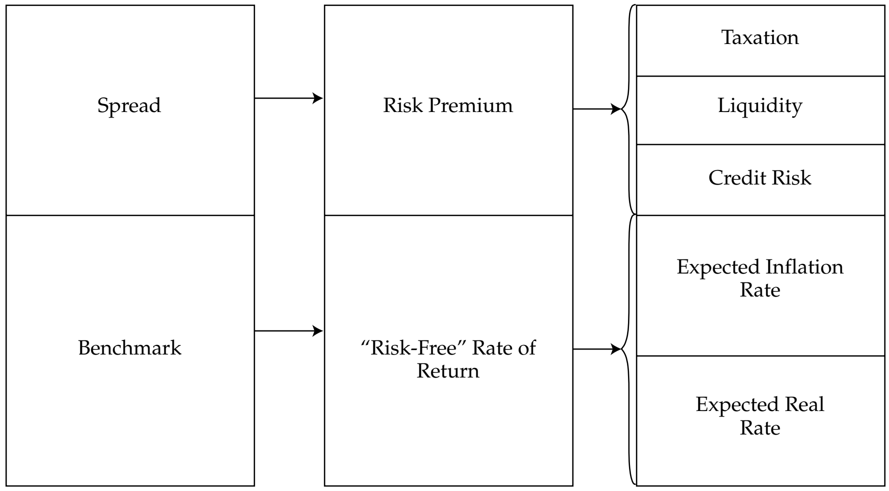
$$\frac{(0.5 \times 89.0600) + (0.5 \times 88.9969) + 2.2500}{1 - 0.0025 + 0.089148} = 84.0000.$$

6 INTERPRETING CHANGES IN CREDIT SPREADS

- [ ] interpret changes in a credit spread

Corporate and benchmark bond yields and the credit spread between them change from day to day. The challenge for a fixed-income analyst is to understand that and be able to explain *why* the yields and spreads change. Exhibit 22 offers a breakdown of the main components of bond yields. Benchmark bond yields, in general, capture the *macroeconomic* factors affecting all debt securities. These are the expected inflation rate and the expected real rate of return. Risk-averse investors in benchmark bonds also might require compensation for uncertainty regarding those variables.

Exhibit 22: Components of a Corporate Bond Yield



The spread over the benchmark bond yield captures the *microeconomic* factors that pertain to the corporate issuer and the specific issue itself. The chief microeconomic factor is the expected loss due to default. There also are liquidity and tax differences between the corporate and benchmark bonds. Moreover, it can be difficult to separate these factors. Securities for which it becomes more difficult for analysts to assess a probability of default and a recovery rate typically become less liquid. Similarly, an uncertain tax status on a bond's gains and losses will increase the time and cost to estimate value. That makes the bond less liquid. Another factor in the observed spread between the corporate and benchmark bond yields can be compensation to risk-averse investors for uncertainty regarding credit risk, as well as liquidity and tax factors.

Research groups at major banks and consultancies have been working on models to better include counterparty credit risk, funding costs, and liquidity and taxation effects in the valuations of derivatives. First, a value is obtained using benchmark discount factors, in practice, derived from rates on overnight indexed swaps (OIS). These are interest rate swaps that reference an average daily interest rate. For instance, in the United States this daily rate is the effective federal funds rate. Then this OIS value, which is comparable to the VND in the previous section, is adjusted for the other factors. These valuation adjustments collectively are known as the XVA. The credit valuation adjustment is the most developed and most used in practice. Others include a funding valuation adjustment (FVA), a liquidity valuation adjustment (LVA), and a taxation valuation adjustment (TVA). In principle, the same ideas apply to debt securities in that these XVA comprise the observed spread between corporate and benchmark bond yields. For the purposes of our coverage, we focus only on the credit risk component, the CVA.

We can use the arbitrage-free framework and the credit risk model to examine the connections between the default probability, the recovery rate, and the credit spread. To be sure, this is a simple model to illustrate the much more complex models used in practice. These (which are called *XVA engines*) typically use Monte Carlo simulations for thousands of possible paths for interest rates. Our binomial interest rate tree has only 16 paths for the five years; it's a model of the actual model.

Consider again the five-year, 3.50% annual payment corporate bond examined earlier. In Exhibit 12, the value assuming no default was determined to be 103.5450 per 100 of par value. Now let us use the credit risk model to find the probabilities of default that would be consistent with various credit spreads and a recovery rate of 40%. Suppose, as in Exhibit 7, the credit spread for an AAA rated bond is 0.60%. Using trial-and-error search, we find that an annual probability of default of 1.01% produces a 60 bp credit spread. The credit risk table is presented in Exhibit 23. Notice that the expected exposure to default loss and the loss given default are the same as in Exhibit 13. Only the default probabilities and the contributions to total CVA for each year change.

Exhibit 23: CVA Calculation for the 3.50% Corporate Bond Given a Default Probability of 1.01% and a Recovery Rate of 40%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	61.9717	1.0100%	1.002506	0.6275
2	101.5481	60.9289	0.9998%	0.985093	0.6001
3	101.0433	60.6260	0.9897%	0.955848	0.5735
4	102.0931	61.2559	0.9797%	0.913225	0.5481
5	103.5000	62.1000	0.9698%	0.870016	0.5240
			4.9490%	CVA =	2.8731

The CVA for the bond is 2.8731 per 100 of par. The fair value is 100.6719 (= 103.5450 – 2.8731). This gives a yield to maturity of 3.35%.

$$100.6719 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}.$$

$$\text{YTM} = 0.0335.$$

Given that the yield on the five-year benchmark bond is 2.75%, the credit spread is 0.60% (= 3.35% – 2.75%).

We can repeat this exercise for the other credit spreads and ratings shown in Exhibit 7. In each case, trial-and-error search is used to get the initial POD that corresponds to the CVA, the fair value, and the yield to maturity for each assumed spread. The results for the annual and cumulative default probabilities over the five years are shown in Exhibit 24.

Exhibit 24: Default Probabilities Consistent with Given Credit Ratings and Spreads and 40% Recovery

Credit Rating	Credit Spread	Annual Default Probability	Cumulative Default Probability
AAA	0.60%	1.01%	4.95%
AA	0.90%	1.49%	7.23%
A	1.10%	1.83%	8.82%
BBB	1.50%	2.48%	11.80%
BB	3.40%	5.64%	25.19%
B	6.50%	10.97%	44.07%
CCC, CC, C	9.50%	16.50%	59.41%

The default probabilities illustrated in Exhibit 24 might seem high, especially given the historical experience presented in Exhibit 6. Since 1995, no AAA rated company has defaulted; still, we model the likelihood to be over 1% for the first year and almost 5% for the next five years. However, as discussed earlier, these are *risk-neutral* probabilities of default and are higher than the actual probabilities because market prices reflect uncertainty over the timing of possible default. Investors are concerned about credit spread widening, especially if they do not intend to hold the bond to maturity. Credit rating migration from year to year, as illustrated in Exhibit 7, is a concern even for a high-quality investment-grade corporate bond. This is captured in the risk-neutral probability of default. Also, we must remember that observed credit spreads reflect more than just credit risk—there also are liquidity and tax differences. That further explains the difference between risk-neutral and actual default probabilities.

The relationship between the assumed recovery rate and the credit spread can be examined in the context of the credit risk model. Suppose that the five-year, 3.50% annual payment corporate bond has an initial probability of default of 1.83%. In Exhibit 24, we see that for a 40% recovery rate, the credit spread is 1.10%. What if the recovery rate is expected to be only 30%? Exhibit 25 shows the credit risk table for that assumption.

Exhibit 25: CVA Calculation for the 3.50% Corporate Bond Given a Default Probability of 1.83% and a Recovery Rate of 30%

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	103.2862	72.3003	1.8300%	1.002506	1.3264
2	101.5481	71.0837	1.7965%	0.985093	1.2580
3	101.0433	70.7303	1.7636%	0.955848	1.1923
4	102.0931	71.4652	1.7314%	0.913225	1.1300
5	103.5000	72.4500	1.6997%	0.870016	1.0714
			8.8212%	CVA =	5.9781

The reduction in the recovery rate from 40% to 30% has an impact on LGD and CVA for each year. The overall CVA is 5.9781 per 100 of par value. The fair value for the bond is 97.5670 (= 103.5450 – 5.9781), and the yield to maturity is 4.05%, giving a credit spread of 1.30% (= 4.05% – 2.75%).

$$97.5670 = \frac{3.50}{(1+\text{YTM})^1} + \frac{3.50}{(1+\text{YTM})^2} + \frac{3.50}{(1+\text{YTM})^3} + \frac{3.50}{(1+\text{YTM})^4} + \frac{103.50}{(1+\text{YTM})^5}.$$

$$\text{YTM} = 0.0405.$$

This example illustrates how a credit rating agency might use "notching" to combine the expected loss given default and the probability of default in setting the rating for a corporate bond. If the issuer were rated single A, associated with a default probability of 1.83% and a recovery rate of 40% on the company's senior unsecured debt, that debt might have a credit spread of 1.10%, comparable to other A rated companies. This particular bond is subordinated, leading analysts at the rating agency to believe that a lower recovery rate assumption of 30% is applicable. That could justify assigning a lower rating of A– or BBB+ on the subordinated debt, along with its 20 bp higher spread.

EXAMPLE 8

Evaluating Changes in Credit Risk Parameters

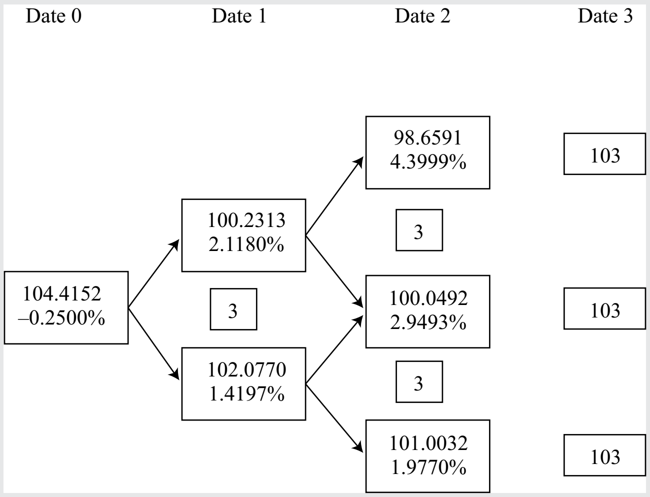
1. Edward Kapili is a summer intern working on a fixed-income trading desk at a major money-center bank. His supervisor asks him to value a three-year, 3% annual payment corporate bond using a binomial interest rate tree model for 20% volatility and the current par curve for benchmark government bonds. (This is the binomial tree in Exhibit 15.) The assumed annual probability of default is 1.50%, and the recovery rate is 40%.

   The supervisor asks Mr. Kapili if the credit spread over the yield on the three-year benchmark bond, which is 1.50% in Exhibit 9, is likely to go up more if the default probability doubles to 3.00% or if the recovery rate halves to 20%. Mr. Kapili's intuition is that doubling the probability of default has a larger impact on the credit spread. Is his intuition correct?

Solution:

Mr. Kapili first determines the fair value of the three-year, 3% annual payment bond given the assumptions for the original credit risk parameters. The binomial interest rate tree and credit risk table are presented in Exhibit 26.

Exhibit 26: Fair Value of the Three-Year, 3% Annual Payment Corporate Bond Assuming 20% Volatility



Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	62.4925	1.5000%	1.002506	0.9397
2	102.9402	61.7641	1.4775%	0.985093	0.8990
3	103.0000	61.8000	1.4553%	0.955848	0.8597
			4.4328%	CVA =	2.6984

Fair value = 104.4152 – 2.6984 = 101.7168.

The VND for the bond is 104.4152, the CVA is 2.6984, and the fair value is 101.7168 per 100 of par value. The yield to maturity is 2.40%, and the credit spread is 0.90% (= 2.40% – 1.50%).

$$101.7168 = \frac{3}{(1+\text{YTM})^1} + \frac{3}{(1+\text{YTM})^2} + \frac{103}{(1+\text{YTM})^3}.$$

$$\text{YTM} = 0.0240.$$

Next, Mr. Kapili calculates the fair values under the new credit risk parameters, first for doubling the default probability and second for halving the recovery rate. These tables are shown in Exhibit 27.

Exhibit 27: Fair Value Calculations for Doubling the Default Probability and Halving the Recovery Rate

3.00% Default Probability, 40% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	62.4925	3.0000%	1.002506	1.8795
2	102.9402	61.7641	2.9100%	0.985093	1.7705
3	103.0000	61.8000	2.8227%	0.955848	1.6674
			8.7327%	CVA =	5.3174

Fair value = 104.4152 – 5.3174 = 99.0978.

1.50% Default Probability, 20% Recovery Rate

Date	Expected Exposure	LGD	POD	Discount Factor	CVA per Year
0					
1	104.1541	83.3233	1.5000%	1.002506	1.2530
2	102.9402	82.3522	1.4775%	0.985093	1.1986
3	103.0000	82.4000	1.4553%	0.955848	1.1463
			4.4328%	CVA =	3.5978

Fair value = 104.4152 – 3.5978 = 100.8173.

The fair value of the corporate bond falls to 99.0978 when the default probability is raised to 3.00% and the recovery rate stays at 40%. The VND is the same, at 104.4152, and the CVA goes up to 5.3174. The yield to maturity increases to 3.32%, and the credit spread rises to 1.82% (= 3.32% – 1.50%).

$$99.0978 = \frac{3}{(1 + \text{YTM})^1} + \frac{3}{(1 + \text{YTM})^2} + \frac{103}{(1 + \text{YTM})^3}.$$

$$\text{YTM} = 0.0332.$$

The fair value of the corporate bond falls to 100.8173 when the recovery rate is reduced by half, to 20%, and the default probability is maintained at 1.50%. The VND is again the same, at 104.4152, and the CVA goes up to 3.5978. The yield to maturity increases to 2.71%, and the credit spread rises to 1.21% (= 2.71% – 1.50%).

$$100.8173 = \frac{3}{(1 + \text{YTM})^1} + \frac{3}{(1 + \text{YTM})^2} + \frac{103}{(1 + \text{YTM})^3}.$$

$$\text{YTM} = 0.0271.$$

Mr. Kapili's intuition is correct: Doubling the default probability has a greater impact on the credit spread than halving the recovery rate.

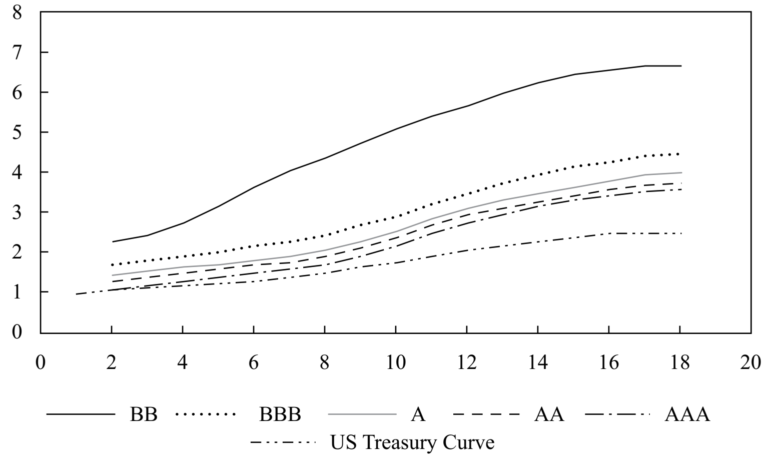
7 THE TERM STRUCTURE OF CREDIT SPREADS

☐ explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads

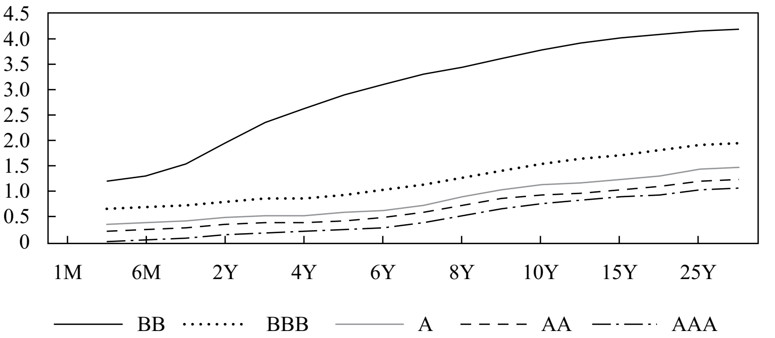
In the same way that the yield curve is composed of the interest rates on a single government issuer's debt across bond maturities, a credit curve shows the spread over a benchmark security for an issuer for outstanding fixed-income securities with shorter to longer maturities. For example, Exhibit 28 shows the relationship between US Treasury yields of a specific maturity and bonds rated AAA, AA, A, BBB, and BB. The total yields of the bonds are shown in Panel A, and spreads over the benchmark Treasury are shown in Panel B.

Exhibit 28: Composite Yield Graphs

A. Total Yields



B. Spreads



Source: Bloomberg.

The term structure of credit spreads is a useful gauge for issuers, underwriters, and investors in measuring the risk–return trade-off for a single issuer or a set of issuers across ratings and/or sectors across maturities. Issuers often work with their underwriter to consider the terms of a new issuance or a tender for existing debt based on relative credit spreads across maturities. For example, an investment-grade bond portfolio manager might use the existing credit curve for a particular issuer to determine a bid for a new primary debt issuance as well as to inform trading decisions for secondary debt positions. In some cases, investors, issuers, or underwriters might use the credit spread term structure for a particular rating or corporate sector either to derive prospective pricing for a new issuance or to determine fair value spreads for outstanding securities, which is an extension of matrix pricing. A high-yield debt investor might employ the term structure of credit spreads to gauge the risk/reward trade-offs between debt maturities. Given the impact of monetary and fiscal policies on risky debt markets, policymakers have extended their focus from default-risk-free yield curve dynamics to the term structure of credit spreads.

There are several key drivers of the term structure of credit spreads. First, credit quality is a key factor. For investment-grade securities with the highest credit ratings and extremely low spreads, credit spread migration is only possible in one direction given the implied lower bound of zero on credit spreads. As a result, the credit term structure for the most highly rated securities tends to be either flat or slightly upward sloping. Securities with lower credit quality, however, face greater sensitivity to the credit cycle. The greater likelihood of default associated with high-yield securities generally results in a steeper credit spread curve, both in cases where a weaker economy suggests credit spread widening and when an inverted credit spread curve suggests tighter spreads for longer maturities. As a high-yield bond moves further down the credit spectrum into a more distressed scenario, the contractual cash flows through maturity become less certain—with the value of distressed debt converging to a dollar price equal to the recovery rate as default becomes more certain, regardless of the remaining time to maturity. Such a scenario will result in a steeply inverted credit spread term structure. We now review the determinants of that term structure inversion and other implications of this scenario in more detail.

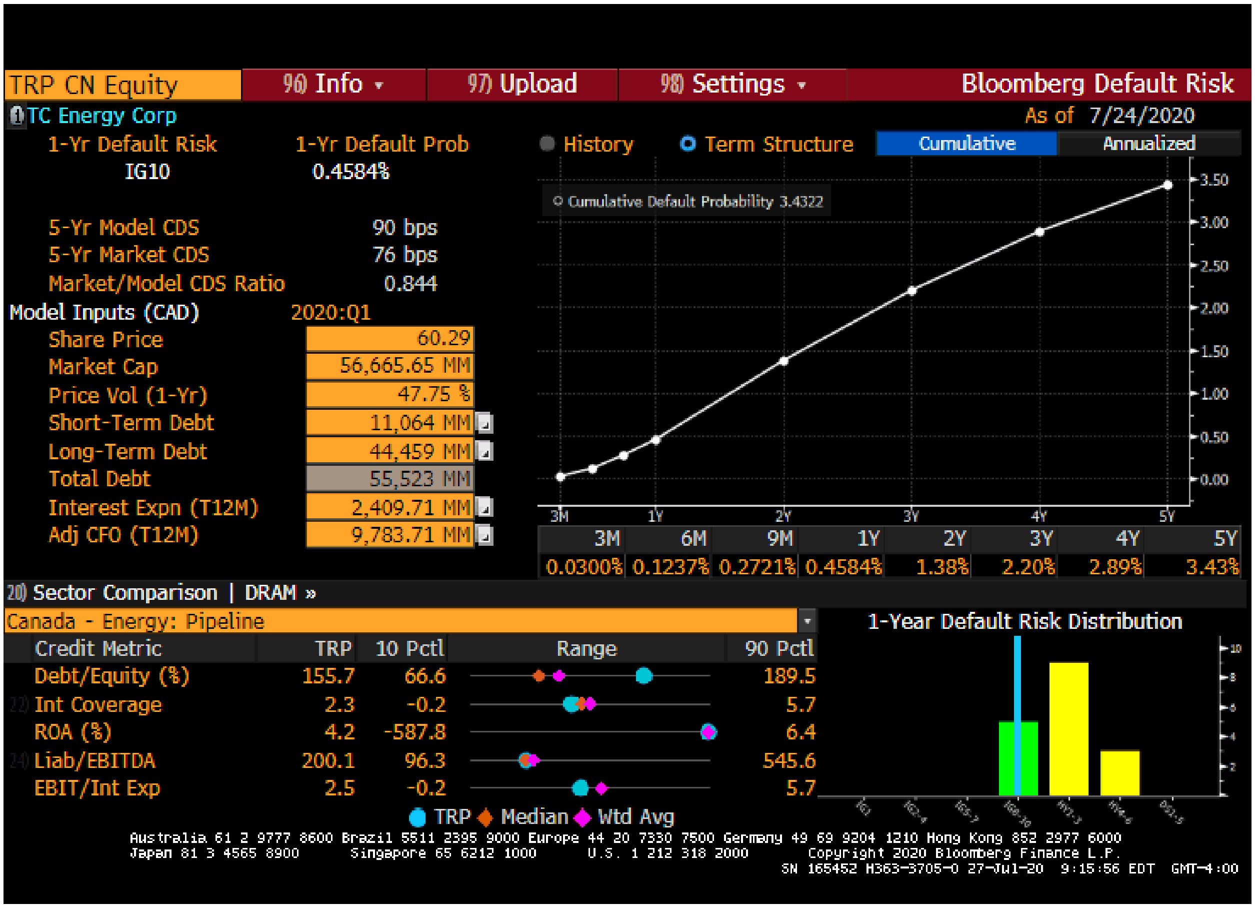
Financial conditions are another critical factor affecting the credit spread term structure. From a macroeconomic perspective, the credit risk of a bond is influenced by expectations for economic growth and inflation. A stronger economic climate is generally associated with higher benchmark yields but lower credit spreads for issuers whose default probability declines during periods of economic growth (cash flows tend to improve and profitability increases under such a scenario). The countercyclical relationship between spreads and benchmark rates is therefore commonly observed across the business cycle.

Market supply and demand dynamics are another critical factor influencing the credit curve term structure. Unlike default-risk-free government securities in developed markets, the relative liquidity of corporate bonds varies widely, with the vast majority of securities not trading on a daily basis. Given that new and most recently issued securities tend to represent the largest proportion of trading volume and are responsible for much of the volatility in credit spreads, the credit curve will be most heavily influenced by the most frequently traded securities. For example, although one might expect the credit curve to steepen for a borrower refinancing near-term maturities with long-term debt, this effect may be partially offset by a tighter bid–offer spread for longer credit maturities. This flattening may also occur within a specific rating or if market participants anticipate significant supply in a particular tenor. Infrequently traded bonds trading with wider bid–offer spreads can also impact the shape of the term structure, so it is important to gauge the size and frequency of trades in bonds across the maturity spectrum to ensure consistency.

Finally, from a microeconomic perspective, company-value model results discussed earlier are another key driver of the credit spread term structure. Under traditional credit analysis, the specific industry or industries within which an issuer operates are considered, as well as key financial ratios, such as cash flow, leverage, and profitability versus sector and ratings peers. This company-specific analysis based on fundamental data has been complemented by more probabilistic, forward-looking structural models for company valuation. These models take stock market valuation, equity volatility, and balance sheet information into account to derive the implied default probability for a company. Holding other factors constant, any microeconomic factor that increases the implied default probability, such as greater equity volatility, will tend to drive a steeper credit spread curve, and the reverse is true with a decline in equity volatility.

Practitioners will frequently employ these tools when analyzing the term structure of credit spreads to determine fair value. For example, the Bloomberg default risk screen (DRSK) shown in Exhibit 29 combines the company-value analysis with fundamental credit ratios for a composite analysis of TransCanada Corporation, a Canadian natural gas transmission and power services company.

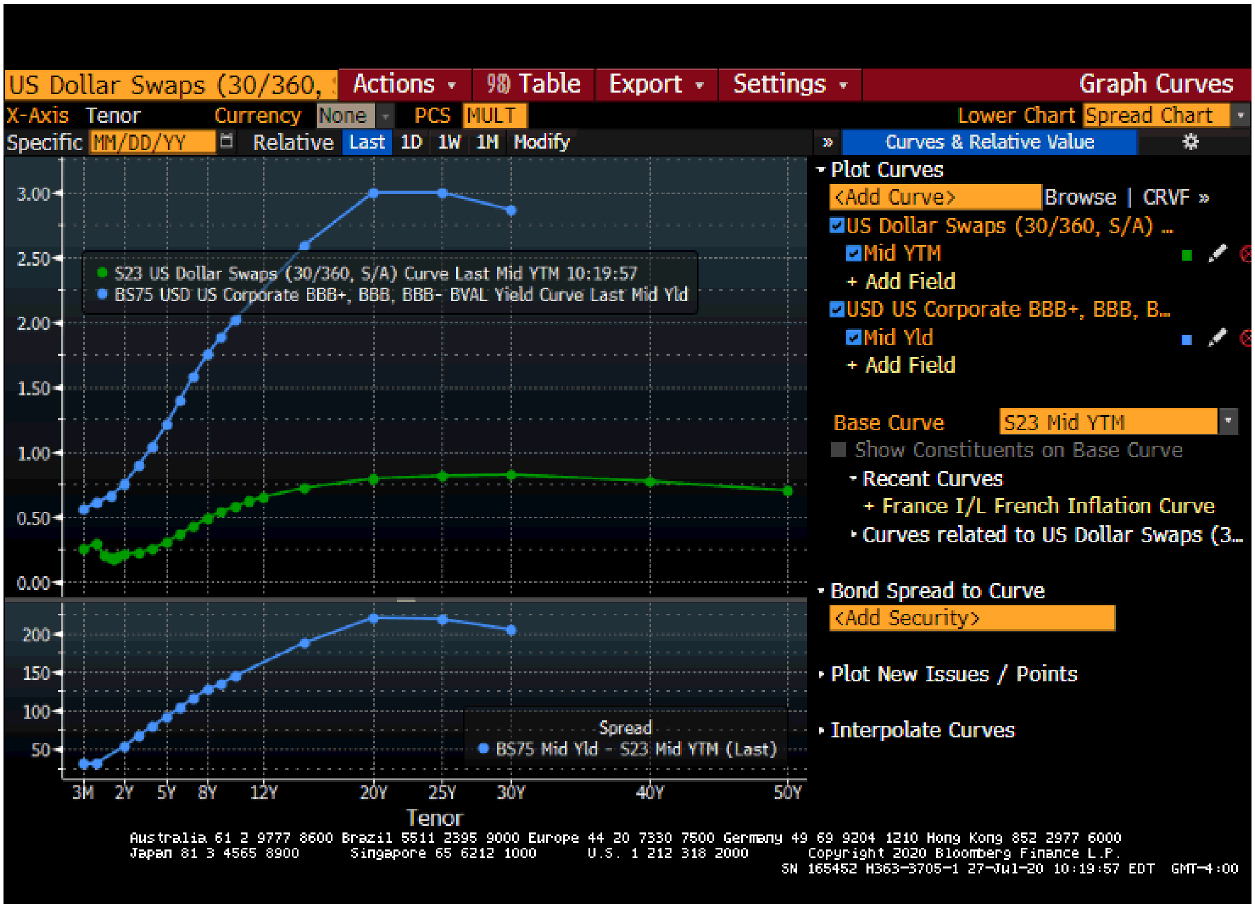
Exhibit 29: Default Risk Screen



Source: Bloomberg.

Two further considerations are important when analyzing the term structure of credit spreads. The first concerns the appropriate risk-free or benchmark rates used to determine spreads. A frequently traded government security with the nearest maturity to an outstanding corporate bond generally represents the lowest default risk for developed markets, so this is a logical benchmark choice. However, the duration and maturity of the most liquid or on-the-run government bonds rarely match those of corporate bonds trading in the secondary market, so it is often necessary to interpolate between yields of the two government securities with the closest maturity. Because the interpolation may impact the analysis for less liquid maturities, the benchmark swap curve based on interbank rates is often substituted for the government benchmark because of greater swap market liquidity for off-the-run maturities. For example, Exhibit 30 demonstrates the latter methodology on a Bloomberg screen for a composite of BBB rated US industrial corporate issuers versus the benchmark US dollar swap curve, showing a positive-sloped credit spread term structure across maturities.

Exhibit 30: Credit Spreads over Swap Rates



Source: Bloomberg.

The second consideration concerns the all-in spread over the benchmark itself. Term structure analysis should include only bonds with similar credit characteristics, which are typically senior unsecured general obligations of the issuer. Any bonds of the issuer with embedded options, first or second lien provisions, or other unique provisions should be excluded from the analysis. It is also important to note that such securities typically include cross-default provisions so that all securities across the maturity spectrum of a single issuer will be subject to recovery in the event of bankruptcy.

Using the models presented in prior sections, we can demonstrate that the *change* in market expectations of default over time is a key determinant of the shape of the credit curve term structure. This may be shown using a simple extension of the zero-coupon corporate bond example in Exhibit 2 by changing the probability of default. Using a recovery rate of 40% and changing the probability of default from 1.25% to 1.50% raises the credit spread from 77 bps in the original example to 92 bps. These calculations are shown in Exhibit 31.

Exhibit 31: Raising the Default Probability of the Five-Year, Zero-Coupon Corporate Bond

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	88.8487	35.5395	53.3092	1.5000%	98.5000%	0.7996	0.970874	0.7763
2	91.5142	36.6057	54.9085	1.4775%	97.0225%	0.8113	0.942596	0.7647
3	94.2596	37.7038	56.5558	1.4553%	95.5672%	0.8231	0.915142	0.7532
4	97.0874	38.8350	58.2524	1.4335%	94.1337%	0.8351	0.888487	0.7419
5	100.0000	40.0000	60.0000	1.4120%	92.7217%	0.8472	0.862609	0.7308
				7.2783%			CVA =	3.7670

Fair value = 86.2609 – 3.7670 = 82.4939.
Yield to maturity = 3.9240%.
Credit spread = 3.9240% – 3.00% = 0.9240%.

Flat credit spread curves imply a relatively stable expectation of default over time, whereas an upward-sloping credit curve implies that investors seek greater compensation for assuming issuer default risk over longer periods. For example, we can illustrate this in terms of a credit spread curve by holding the benchmark rate constant at 3.00% across 3-year, 5-year, and 10-year maturities while increasing the default probability over time. Although one could consider an increase in default probability each year, the following example in Exhibit 32 assumes a 1.00% default probability for Years 1, 2, and 3, a 2.00% probability of default in Years 4 and 5, and a 3.00% default probability in Years 6 through 10, with the recovery rate at a constant 40%. (Note that this is another example of the annual default probability changing over the lifetime of the bonds.) As shown in Exhibit 32, the credit spread rises from 62 bps to 86 bps to 132 bps.

Exhibit 32: Increasing the Default Probability for Longer Times to Maturity

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	94.2596	37.7038	56.5558	1.0000%	99.0000%	0.5656	0.970874	0.5491
2	97.0874	38.8350	58.2524	0.9900%	98.0100%	0.5767	0.942596	0.5436
3	100.0000	40.0000	60.0000	0.9801%	97.0299%	0.5881	0.915142	0.5382
				2.9701%			CVA =	1.6308

Fair value = 91.5142 – 1.6308 = 89.8833.
Yield to maturity = 3.6192%.
Credit spread = 3.6192% – 3.00% = 0.6192%.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	88.8487	35.5395	53.3092	1.0000%	99.0000%	0.5331	0.970874	0.5176
2	91.5142	36.6057	54.9085	0.9900%	98.0100%	0.5436	0.942596	0.5124
3	94.2596	37.7038	56.5558	0.9801%	97.0299%	0.5543	0.915142	0.5073
4	97.0874	38.8350	58.2524	1.9406%	95.0893%	1.1304	0.888487	1.0044

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
5	100.0000	40.0000	60.0000	1.9018%	93.1875%	1.1411	0.862609	0.9843
				6.8125%			CVA =	3.5259

Fair value = 86.2609 – 3.5259 = 82.7350.
Yield to maturity = 3.8633%.
Credit spread = 3.8633% – 3.00% = 0.8633%.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	76.647	30.6567	45.9850	1.0000%	99.0000%	0.4599	0.970874	0.4465
2	78.9409	31.5764	47.3646	0.9900%	98.0100%	0.4689	0.942596	0.4420
3	81.3092	32.5237	48.7855	0.9801%	97.0299%	0.4781	0.915142	0.4376
4	83.7484	33.4994	50.2491	1.9406%	95.0893%	0.9751	0.888487	0.8664
5	86.2609	34.5044	51.7565	1.9018%	93.1875%	0.9843	0.862609	0.8491
6	88.8487	35.5395	53.3092	2.7956%	90.3919%	1.4903	0.837484	1.2481
7	91.5142	36.6057	54.9085	2.7118%	87.6801%	1.4890	0.813092	1.2107
8	94.2596	37.7038	56.5558	2.6304%	85.0497%	1.4876	0.789409	1.1744
9	97.0874	38.8350	58.2524	2.5515%	82.4982%	1.4863	0.766417	1.1391
10	100.0000	40.0000	60.0000	2.4749%	80.0233%	1.4850	0.744094	1.1050
				19.9767%			CVA =	8.9187

Fair value = 74.4094 – 8.9187 = 65.4907.
Yield to maturity = 4.3235%.
Credit spread = 4.3235% – 3.00% = 1.3235%.

Positive-sloped credit spread curves are likely when a high-quality issuer with a strong competitive position in a stable industry has low leverage, strong cash flow, and a high profit margin. This type of issuer tends to exhibit very low short-term credit spreads rising with increasing maturity given greater uncertainty due to the macroeconomic environment, potential adverse changes in the competitive landscape, technological change, or other factors that drive a higher implied probability of default over time. Empirical academic studies also tend to support the view that the credit spread term structure is upward-sloping for investment-grade bond portfolios (Bedendo, Cathcart, and El-Jahel 2007).

Alternatively, high-yield issuers in cyclical industries sometimes face a downward-sloping credit term structure because of issuer- or industry-specific reasons. For example, an ownership change resulting from a leveraged buyout or private equity acquisition may often be accompanied by a significant increase in leverage. In such a case, an inverted credit curve may indicate investor expectations that the new owners will create efficiencies in the restructured organization, leading to improved future cash flow and profitability that will benefit debt investors. Another example of an inverted credit term structure might result when issuers in a historically cyclical industry (such as oil and gas exploration or retail) find themselves at the bottom of an economic cycle, with investor expectations of a recovery in the industry tied to improving credit spreads over time.

That said, it is important to distinguish between scenarios where the contractual cash flows of a risky bond are likely to occur and distressed debt scenarios where investors expect to receive only the recovery rate in a likely bankruptcy scenario. Bonds with a very high likelihood of default tend to trade on a price basis that converges toward the recovery rate rather than on a spread to benchmark rates. This scenario leads to credit spread term structures that may be considered more of an "optical" phenomenon rather than a true reflection of the relative risks and rewards of long-term versus short-term bonds from a single issuer, as illustrated in the following discussion.

To demonstrate this using our zero-coupon bond example, let us shift to a scenario where bondholders with 5-year and 10-year bonds outstanding anticipate an imminent default scenario and both bonds trade at a recovery rate of 40%.

Note that if we solve for the fair value and resulting credit spread over the benchmark yield as in the instances where default probability was 1.25%, we end up with the same VNDs for the 5-year and 10-year bonds, respectively. However, when deriving a credit spread value for both securities assuming recovery in a bankruptcy scenario and cross-default provisions across maturities, the credit valuation adjustment representing the sum of expected losses is simply the difference between the VND and the recovery rate.

For the five-year example, we can thus calculate a VND of 86.2609, a CVA of 46.2609, and a fair value with recovery at 40. This results in a yield of 20.1124% and a credit spread over the government bond of 17.1124%. In the 10-year case, the VND may be shown as 74.4094, a CVA of 34.4094, and a fair value at 40. That gives a yield of 9.5958% and a credit spread of 6.5958%. We end up with a steep and inverted "credit spread" curve.

The interpretation of the credit spread term structure is important for investors seeking to capitalize on a market view that differs from that reflected in the credit curve. For example, if a portfolio manager disagrees with the market's expectation of a high near-term default probability that declines over time, she could sell short-term protection in the credit default swap market and buy longer-term protection. In a scenario where the issuer does not default, the investor retains the premium on protection sold and may either retain or choose to sell back the longer-term credit default swap to realize a gain.

8 CREDIT ANALYSIS FOR SECURITIZED DEBT

☐ compare the credit analysis required for securitized debt to the credit analysis of corporate debt

Unlike the general obligation nature of most private or sovereign fixed-income securities, securitized debt allows issuers to finance a specific set of assets or receivables (e.g., mortgages, automobile loans, or credit card receivables) rather than an entire balance sheet. Issuers in securitized debt markets are frequently motivated to undertake financing using these more structured securities given their ability to increase debt capacity and reduce the originator's need to maintain regulatory capital or retain residual risk. The isolation of securitized assets generally decreases the relative financing cost for these assets on a stand-alone basis as compared to a general obligation financing of the debt originator. By freeing up capital, an originator is also able to continue to generate income from further originations. Investors, however, seek to benefit from greater diversification, more stable and predictable underlying cash flows, and a return that is greater than that of securities with similar ratings, which provide a reward for accepting the greater complexity associated with collateralized debt. That said, the credit analysis of such structured finance instruments requires a fundamentally different approach compared with other risky bonds given the underlying collateral, the parties associated with the origination or servicing of the portfolio over the life of the security, and the issuing entity, as well as any structural and credit enhancement features typically present in these transactions.

It is important to distinguish first and foremost among the types of securitized debt issued globally, as well as the various forms. In its summary of structured finance asset types shown in Exhibit 33, the German-based rating agency Scope Ratings AG provides its general approach to credit assessment based not only on the underlying time horizon and collateral but also on asset characteristics referred to as granularity and homogeneity.

Exhibit 33: Summary of Asset Types and Characteristics of Core Structured Finance Asset Classes

Deal Type	Underlying Collateral	Risk Horizon	Granularity	Homogeneity	Credit Analysis Approach
Asset-backed CP	Commercial discount credits or credit advances	Short-term	Granular	Homogeneous	Book
Auto ABS	Auto loans or leases	Medium-term	Granular	Homogeneous	Portfolio
CMBS	Commercial mortgages	Typically long-term	Non-granular	Heterogeneous	Loan by loan
Consumer ABS	Consumer loans	Medium-term	Granular	Homogeneous	Portfolio
CRE loans	Commercial real estate loans	Long-term	Non-granular	Heterogeneous	Loan by loan
Credit cards	Credit card balances	Short-term	Granular	Homogeneous	Book
Credit-linked notes/ repackaging	Any financial assets	Typically medium-term	Typically single asset	NA	Pass-through rating/asset by asset
LL CLOs	Leveraged corporate loans	Medium-term	Non-granular	Heterogeneous	Loan by loan
PF CLOs	Project finance debt	Long-term	Non-granular	Heterogeneous	Loan by loan
RMBS	Residential mortgages	Long-term	Granular	Homogeneous	Loan by loan or portfolio
SME ABS	Loans to small- and medium-sized businesses	Typically medium-term	Granular	Mixed	Loan by loan or portfolio
Trade receivables	Commercial credit	Short-term	Typically granular	Homogeneous	Book

Source: Adapted from Scope Ratings AG (2016b, pp. 7–8).

The concept of homogeneity refers to the degree to which underlying debt characteristics within a structured finance instrument are similar across individual obligations. On the one hand, an investor or credit analyst might draw general conclusions about the nature of homogeneous credit card or auto loan obligations given that an individual obligation faces strict eligibility criteria to be included in a specific asset pool. On the other hand, heterogeneous leveraged loan, project finance, or real estate transactions require scrutiny on a loan-by-loan basis given their different characteristics. The granularity of the portfolio refers to the actual number of obligations that make up the overall structured finance instrument. A highly granular portfolio may have hundreds of underlying debtors, suggesting it is appropriate to draw conclusions about creditworthiness based on portfolio summary statistics rather than investigating each borrower. Alternatively, an asset pool with fewer more-discrete or non-granular investments would warrant analysis of each individual obligation.

The combination of asset type and tenor as well as the relative granularity and homogeneity of the underlying obligations drive the approach to credit analysis for a given instrument type. For example, short-term structured finance vehicles with granular, homogeneous assets tend to be evaluated using a statistics-based approach to the existing book of loans. This changes to a portfolio-based approach for medium-term granular and homogeneous obligations because the portfolio is not static but changes over time. For discrete or non-granular heterogeneous portfolios, a loan-by-loan approach to credit analysis is more appropriate. The following example of a credit card securitization will provide further insight into the process.

Exhibit 34 provides a summary from the prospectus of the Synchrony Credit Card Master Note Trust $750,000,000 Series 2016-1 Asset Backed Notes issued in March 2016. As is spelled out in the prospectus, the Synchrony transaction is backed by credit card receivables having the given credit score distribution presented in the exhibit.

Exhibit 34: A Structured Debt Example, Composition by FICO Credit Score Range

FICO Credit Score Range	Receivables Outstanding	Percentage of Outstanding
Less than or equal to 599	$995,522,016	6.6%
600 to 659	$2,825,520,245	18.7%
660 to 719	$6,037,695,923	39.9%
720 and above	$5,193,614,599	34.4%
No score	$64,390,707	0.4%
Total	**$15,116,743,490**	**100%**

Source: Synchrony Credit Card Master Note Trust $750,000,000 Series 2016-1 Asset Backed Notes Prospectus (p. 93; available at investors.synchronyfinancial.com).

Investors in this type of ABS will base their probability of default on the mean default probability, recovery rate, and variance of a portfolio of borrowers reflecting the distribution of FICO scores within the pool rather than conducting an analysis of individual borrowers. The prospectus provides a broad set of details beyond the FICO scores of borrowers for further in-depth portfolio analysis, including age of the receivables, average outstanding balances, and delinquency rates.

A heterogeneous portfolio of fewer loans, however, requires a fundamentally different approach. In this instance, each obligation within the asset pool may warrant its own analysis to determine whether an individual commercial property or leveraged company is able to meet its financial obligations under the ABS contract. Here the expected default probability and recovery rate on an asset-by-asset basis is the best gauge of how the investment will perform under various scenarios.

A second critical aspect of the credit exposure associated with ABS relates to the origination and servicing of assets over the life of the transaction. The prospectus and other related documents determine the roles and responsibilities of these related parties over the life of an ABS transaction. Upon inception of the transaction, investors rely on the originator/servicer to establish and enforce loan eligibility criteria, secure and maintain proper documentation and records, and maximize timely repayment and contract enforceability in cases of delinquency. Once the asset pool has been identified, investors are also exposed to operational and counterparty risk over the life of an ABS transaction. That is, they remain exposed to the ability of the servicer to effectively manage and service the portfolio over the life of the transaction. For an auto ABS transaction, this may involve the ability to repossess and sell a vehicle at a price close to the residual value in a timely manner in the event that a borrower is unable to pay, while in a commercial real estate transaction, it may involve identifying and replacing a non-performing tenant. Investors in an asset portfolio whose composition changes over time also face exposure to the replacement of obligors over time. In all such instances, not only is the creditworthiness of the servicer important but also of importance is its track record in meeting these servicing obligations, which are frequently gauged by analyzing the performance of more seasoned transactions handled by the same servicer over the credit cycle.

For example, in the case of the Synchrony Credit Card Master Note Trust transaction, Synchrony Financial acts as servicer of the trust and Synchrony Bank, as sub-servicer, is primarily responsible for receiving and processing collections on the receivables. A potential investor might therefore evaluate not only the performance of other debt backed by credit card receivables but also how outstanding notes serviced by Synchrony have performed over time versus its servicing competitors.

Finally, the structure of a collateralized or secured debt transaction is a critical factor in analyzing this type of investment. These structural aspects include both the nature of the obligor itself, which is often a special purpose entity (SPE) whose sole purpose is to acquire a specified pool of assets and issue ABS to finance the SPE, and any structural enhancements of the transaction, which may include overcollateralization, credit tranching (i.e., tiering the claim priorities of ownership or interest), or other characteristics.

A key question related to the issuer is its relationship to the originator—namely, the degree to which the bankruptcy of the obligor is related to that of the originator. The bankruptcy remoteness is typically determined by whether the transfer of the assets from the originator to the SPE may be deemed a true sale, which otherwise allows for the ability to separate risk between the originator and SPE at a later date.

Second, additional credit enhancements are a key structural element to be evaluated in the context of credit risk. Credit enhancements for ABS take on several forms beyond the bankruptcy remoteness of the SPE. For example, ABS transactions frequently have payout or performance triggers that protect investors in the case of adverse credit events. Certain events related to the servicer or seller—such as failure to make deposits or payments or other adverse events—may trigger early repayment ("amortization") of the security. For consumer transactions such as credit card or automotive ABS, the primary protection against a decline in asset quality for investors is additional return built into the transaction that is greater than the expected or historical loss of the asset pool. This additional return is often called the excess spread. Issuers create subordinated tranches of debt that provide added protection to those rated higher and benefit from a greater excess spread cushion over the life of the financing.

Covered bonds, which originated in Germany in the 18th century but have since been adopted by issuers across Europe, Asia, and Australia, have some similarities with these structured finance investments but also have fundamental differences that warrant special consideration. A covered bond is a senior debt obligation of a financial institution that gives recourse to both the originator/issuer and a predetermined underlying collateral pool. Each country or jurisdiction specifies the eligible collateral types and the specific structures permissible in its covered bond market. Covered bonds most frequently have either commercial or residential mortgages meeting specific criteria or public sector debt as underlying collateral.

The dual recourse to the issuing financial institution and the underlying asset pool has been a hallmark of covered bonds since their inception, but it was also reinforced under the European Union Bank Recovery and Resolution Directive (BRRD; see Scope

Ratings AG 2016a). Under the BRRD, covered bonds enjoy unique protection among bank liabilities in the event of restructuring or regulatory intervention. Additionally, the financial institution has the ongoing obligation to maintain sufficient assets in the cover pool to satisfy the claims of covered bondholders at all times, and the obligations of the financial institution with respect to the cover pool are supervised by public or other independent bodies.

Another aspect of covered bonds that needs to be considered in credit analysis is the dynamic nature of the cover pool. In contrast to a static pool of mortgage loans (which expose investors to prepayment risk in the case of US mortgage-backed securities), cover pool sponsors must replace any prepaid or non-performing assets in the cover pool to ensure sufficient cash flows to the maturity of the covered bond.

Analysts should also be aware of various redemption regimes that exist to align the covered bond's cash flows as closely as possible to the original maturity schedule in the event of default of a covered bond's financial sponsor. These include hard-bullet covered bonds; if payments do not occur according to the original schedule, a bond default is triggered and bond payments are accelerated. Another type is soft-bullet covered bonds, which delay the bond default and payment acceleration of bond cash flows until a new final maturity date, which is usually up to a year after the original maturity date. Conditional pass-through covered bonds, in contrast, convert to pass-through securities after the original maturity date if all bond payments have not yet been made.

Credit analysis for covered bonds follows traditional credit analysis in evaluating both the issuer and the cover pool. Given the additional credit enhancements, recovery rates tend to be high and default probabilities low, making covered bonds a relatively safe credit asset. As a result, rating agencies often assign a credit rating to covered bonds that is several notches above that of the issuing financial institution.

SUMMARY

We have covered several important topics in credit analysis. Among the points made are the following:

- Three factors important to modeling credit risk are the expected exposure to default, the recovery rate, and the loss given default.
- These factors permit the calculation of a credit valuation adjustment that is subtracted from the (hypothetical) value of the bond, if it were default risk free, to get the bond's fair value given its credit risk. The credit valuation adjustment is calculated as the sum of the present values of the expected loss for each period in the remaining life of the bond. Expected values are computed using risk-neutral probabilities, and discounting is done at the risk-free rates for the relevant maturities.
- The CVA captures investors' compensation for bearing default risk. The compensation can also be expressed in terms of a credit spread.
- Credit scores and credit ratings are third-party evaluations of creditworthiness used in distinct markets.
- Analysts may use credit ratings and a transition matrix of probabilities to adjust a bond's yield to maturity to reflect the probabilities of credit migration. Credit spread migration typically reduces expected return.
- Credit analysis models fall into two broad categories: structural models and reduced-form models.

- Structural models are based on an option perspective of the positions of the stakeholders of the company. Bondholders are viewed as owning the assets of the company; shareholders have call options on those assets.
- Reduced-form models seek to predict *when* a default may occur, but they do not explain the *why* as structural models do. Reduced-form models, unlike structural models, are based only on observable variables.
- When interest rates are assumed to be volatile, the credit risk of a bond can be estimated in an arbitrage-free valuation framework.
- The discount margin for floating-rate notes is similar to the credit spread for fixed-coupon bonds. The discount margin can also be calculated using an arbitrage-free valuation framework.
- Arbitrage-free valuation can be applied to judge the sensitivity of the credit spread to changes in credit risk parameters.
- The term structure of credit spreads depends on macro and micro factors.
- As it concerns macro factors, the credit spread curve tends to become steeper and to widen in conditions of weak economic activity. Market supply and demand dynamics are important. The most frequently traded securities tend to determine the shape of this curve.
- Issuer- or industry-specific factors, such as the chance of a future leverage-decreasing event, can cause the credit spread curve to flatten or invert.
- When a bond is very likely to default, it often trades close to its recovery value at various maturities; moreover, the credit spread curve is less informative about the relationship between credit risk and maturity.
- For securitized debt, the characteristics of the asset portfolio themselves suggest the best approach for a credit analyst to take when deciding among investments. Important considerations include the relative concentration of assets and their similarity or heterogeneity as it concerns credit risk.

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PRACTICE PROBLEMS

The following information relates to questions 1-8

Lena Liecken is a senior bond analyst at Taurus Investment Management. Kristel Kreming, a junior analyst, works for Liecken in helping conduct fixed-income research for the firm's portfolio managers. Liecken and Kreming meet to discuss several bond positions held in the firm's portfolios.

Bonds I and II both have a maturity of one year, an annual coupon rate of 5%, and a market price equal to par value. The risk-free rate is 3%. Historical default experiences of bonds comparable to Bonds I and II are presented in Exhibit 1.

Exhibit 1: Credit Risk Information for Comparable Bonds

Bond	Recovery Rate	Percentage of Bonds That Survive and Make Full Payment
I	40%	98%
II	35%	99%

Bond III is a zero-coupon bond with three years to maturity. Liecken evaluates similar bonds and estimates a recovery rate of 38% and a risk-neutral default probability of 2%, assuming conditional probabilities of default. Kreming creates Exhibit 2 to compute Bond III's credit valuation adjustment. She assumes a flat yield curve at 3%, with exposure, recovery, and loss given default values expressed per 100 of par value.

Exhibit 2: Analysis of Bond III

Date	Exposure	Recovery	Loss Given Default	Probability of Default	Probability of Survival	Expected Loss	Present Value of Expected Loss
0							
1	94.2596	35.8186	58.4410	2.0000%	98.0000%	1.1688	1.1348
2	97.0874	36.8932	60.1942	1.9600%	96.0400%	1.1798	1.1121
3	100.0000	38.0000	62.0000	1.9208%	94.1192%	1.1909	1.0898
Sum				5.8808%		3.5395	3.3367

Bond IV is an AA rated bond that matures in five years, has a coupon rate of 6%, and a modified duration of 4.2. Liecken is concerned about whether this bond will be downgraded to an A rating, but she does not expect the bond to default during the next year. Kreming constructs a partial transition matrix, which is presented in Exhibit 3, and suggests using a model to predict the rating change of Bond IV using leverage ratios, return on assets, and macroeconomic variables.

Exhibit 3: Partial One-Year Corporate Transition Matrix (entries in %)

From/To	AAA	AA	A
AAA	92.00	6.00	1.00
AA	2.00	89.00	8.00
A	0.05	1.00	85.00
Credit Spread (%)	0.50	1.00	1.75

Kreming calculates the risk-neutral probabilities, compares them with the actual default probabilities of bonds evaluated over the past 10 years, and observes that the actual and risk-neutral probabilities differ. She makes two observations regarding the comparison of these probabilities:

Observation 1 Actual default probabilities include the default risk premium associated with the uncertainty in the timing of the possible default loss.

Observation 2 The observed spread over the yield on a risk-free bond in practice includes liquidity and tax considerations, in addition to credit risk.

1. The expected exposure to default loss for Bond I is:
   A. less than the expected exposure for Bond II.
   B. the same as the expected exposure for Bond II.
   C. greater than the expected exposure for Bond II.

2. Based on Exhibit 1, the loss given default for Bond II is:
   A. less than that for Bond I.
   B. the same as that for Bond I.
   C. greater than that for Bond I.

3. Based on Exhibit 1, the expected future value of Bond I at maturity is *closest* to:
   A. 98.80.
   B. 103.74.
   C. 105.00.

4. Based on Exhibit 1, the risk-neutral default probability for Bond I is *closest* to:
   A. 2.000%.
   B. 3.175%.
   C. 4.762%.

5. Based on Exhibit 2, the credit valuation adjustment for Bond III is *closest* to:
   A. 3.3367.

B. 3.5395.

C. 5.8808.

6. Based on Exhibit 3, if Bond IV's credit rating changes during the next year to an A rating, its expected price change would be *closest* to:

   A. –8.00%.

   B. –7.35%.

   C. –3.15%.

7. Kreming's suggested model for Bond IV is a:

   A. structural model.

   B. reduced-form model.

   C. term structure model.

8. Which of Kreming's observations regarding actual and risk-neutral default probabilities is correct?

   A. Only Observation 1

   B. Only Observation 2

   C. Both Observation 1 and Observation 2

The following information relates to questions 9-23

Daniela Ibarra is a senior analyst in the fixed-income department of a large wealth management firm. Marten Koning is a junior analyst in the same department, and David Lok is a member of the credit research team.

The firm invests in a variety of bonds. Ibarra is presently analyzing a set of bonds with some similar characteristics, such as four years until maturity and a par value of €1,000. Exhibit 1 includes details of these bonds.

Exhibit 1: A Brief Description of the Bonds Being Analyzed

Bond	Description
B1	A zero-coupon, four-year corporate bond with a par value of €1,000. The wealth management firm's research team has estimated that the risk-neutral probability of default for each date for the bond is 1.50%, and the recovery rate is 30%.
B2	A bond similar to B1, except that it has a fixed annual coupon rate of 6% paid annually.
B3	A bond similar to B2 but rated AA.
B4	A bond similar to B2 but the coupon rate is the one-year benchmark rate plus 4%.

Ibarra asks Koning to assist her with analyzing the bonds. She wants him to perform the analysis with the assumptions that there is no interest rate volatility and that the government bond yield curve is flat at 3%.

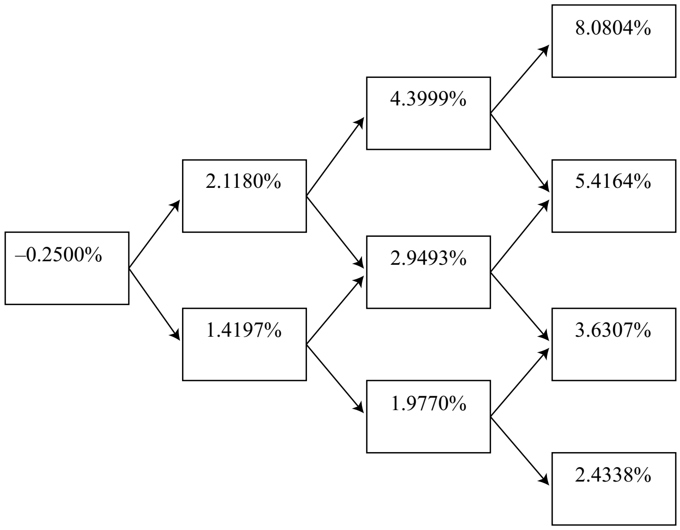
Ibarra performs the analysis assuming an upward-sloping yield curve and volatile interest rates. Exhibit 2 provides the data on annual payment benchmark government bonds.

She uses these data to construct a binomial interest rate tree, shown in Exhibit 3, based on an assumption of future interest rate volatility of 20%.

Exhibit 2: Par Curve for Annual Payment Benchmark Government Bonds

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	–0.25%	€100	1.002506	–0.2500%	
2	0.75%	€100	0.985093	0.7538%	1.7677%
3	1.50%	€100	0.955848	1.5166%	3.0596%
4	2.25%	€100	0.913225	2.2953%	4.6674%

Exhibit 3: Interest Rate Tree



Answer the first five questions (1–5) based on the assumptions made by Marten Koning, the junior analyst. Answer Questions 8–12 based on the assumptions made by Daniela Ibarra, the senior analyst.

Note: All calculations in this problem set are carried out on spreadsheets to preserve precision. The rounded results are reported in the solutions.

9. The market price of Bond B1 is €875. The bond is:

   A. fairly valued.

   B. overvalued.

   C. undervalued.

10. Koning realizes that an increase in the recovery rate would lead to an increase in the bond's fair value, whereas an increase in the probability of default would lead to a decrease in the bond's fair value. He is not sure, however, which effect would be greater. So, he increases both the recovery rate and the probability of default by 25% of their existing estimates and recomputes the bond's fair value. The recomputed fair value is closest to:

    A. €843.14.

    B. €848.00.

    C. €855.91.

11. The fair value of Bond B2 is closest to:

    A. €1,069.34.

    B. €1,111.51.

    C. €1,153.68.

12. The market price of Bond B2 is €1,090. If the bond is purchased at this price and there is a default on Date 3, the rate of return to the bond buyer would be closest to:

    A. –28.38%.

    B. –41.72%.

    C. –69.49%.

13. Bond B3 will have a modified duration of 2.75 at the end of the year. Based on the representative one-year corporate transition matrix in Exhibit 4 and assuming no default, how should the analyst adjust the bond's yield to maturity to assess the expected return on the bond over the next year?

Exhibit 4: Representative One-Year Corporate Transition Matrix (entries are in %)

From/To	AAA	AA	A	BBB	BB	B	CCC, CC, C	D
AAA	**90.00**	9.00	0.60	0.15	0.10	0.10	0.05	0.00
AA	1.50	**88.00**	9.50	0.75	0.15	0.05	0.03	0.02
A	0.05	2.50	**87.50**	8.40	0.75	0.60	0.12	0.08
BBB	0.02	0.30	4.80	**85.50**	6.95	1.75	0.45	0.23
BB	0.01	0.06	0.30	7.75	**79.50**	8.75	2.38	1.25
B	0.00	0.05	0.15	1.40	9.15	**76.60**	8.45	4.20
CCC, CC, C	0.00	0.01	0.12	0.87	1.65	18.50	**49.25**	29.60
Credit Spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%	

A. Add 7.7 bps to YTM.

B. Subtract 7.7 bps from YTM.

C. Subtract 9.0 bps from YTM.

14. David Lok has estimated the probability of default of Bond B1 to be 1.50%. He is presenting the approach the research team used to estimate the probability of default. Which of the following statements is Lok likely to make in his presentation if the team used a reduced-form credit model?

    A. Option pricing methodologies were used, with the volatility of the underlying asset estimated based on historical data on the firm's stock price.

    B. Regression analysis was used, with the independent variables including both firm-specific variables, such as the debt ratio and return on assets, and macroeconomic variables, such as the rate of inflation and the unemployment rate.

    C. The default barrier was first estimated, followed by the estimation of the probability of default as the portion of the probability distribution that lies below the default barrier.

15. In the presentation, Lok is asked why the research team chose to use a reduced-form credit model instead of a structural model. Which statement is he likely to make in reply?

    A. Structural models are outdated, having been developed in the 1970s; reduced-form models are more modern, having been developed in the 1990s.

    B. Structural models are overly complex because they require the use of option pricing models, whereas reduced-form models use regression analysis.

    C. Structural models require "inside" information known to company management, whereas reduced-form models can use publicly available data on the firm.

16. As previously mentioned, Ibarra is considering a future interest rate volatility of 20% and an upward-sloping yield curve, as shown in Exhibit 2. Based on her analysis, the fair value of Bond B2 is closest to:

    A. €1,101.24.

    B. €1,141.76.

    C. €1,144.63.

17. Ibarra wants to know the credit spread of Bond B2 over a theoretical comparable-maturity government bond with the same coupon rate as this bond. The foregoing credit spread is closest to:

    A. 108 bps.

    B. 101 bps.

    C. 225 bps.

18. Ibarra is interested in analyzing how a simultaneous decrease in the recovery rate and the probability of default would affect the fair value of Bond B2. She decreases both the recovery rate and the probability of default by 25% of their existing estimates and recomputes the bond's fair value. The recomputed fair value is closest to:

    A. €1,096.59.

B. €1,108.40.

C. €1,111.91.

19. The wealth management firm has an existing position in Bond B4. The market price of B4, a floating-rate note, is €1,070. Senior management has asked Ibarra to make a recommendation regarding the existing position. Based on the assumptions used to calculate the estimated fair value only, her recommendation should be to:

A. add to the existing position.

B. hold the existing position.

C. reduce the existing position.

20. The issuer of the floating-rate note, B4, is in the energy industry. Ibarra believes that oil prices are likely to increase significantly in the next year, which will lead to an improvement in the firm's financial health and a decline in the probability of default from 1.50% in Year 1 to 0.50% in Years 2, 3, and 4. Based on these expectations, which of the following statements is correct?

A. The CVA will decrease to €22.99.

B. The note's fair value will increase to €1,177.26.

C. The value of the FRN, assuming no default, will increase to €1,173.55.

21. The floating-rate note, B4, is currently rated BBB by Standard & Poor's and Fitch Ratings (and Baa by Moody's Investors Service). Based on the research department assumption about the probability of default in Question 18 and her own assumption in Question 19, which action does Ibarra *most likely* expect from the credit rating agencies?

A. Downgrade from BBB to BB.

B. Upgrade from BBB to AAA.

C. Place the issuer on watch with a positive outlook.

22. During the presentation about how the research team estimates the probability of default for a particular bond issuer, Lok is asked for his thoughts on the shape of the term structure of credit spreads. Which statement is he most likely to include in his response?

A. The term structure of credit spreads typically is flat or slightly upward sloping for high-quality investment-grade bonds. High-yield bonds are more sensitive to the credit cycle, however, and can have a more upwardly sloped term structure of credit spreads than investment-grade bonds or even an inverted curve.

B. The term structure of credit spreads for corporate bonds is always upward sloping—more so the weaker the credit quality because probabilities of default are positively correlated with the time to maturity.

C. There is no consistent pattern for the term structure of credit spreads. The shape of the credit term structure depends entirely on industry factors.

23. The final question for Lok is about covered bonds. The person asking says, "I've heard about them but don't know what they are." Which statement is Lok most likely to make to describe a covered bond?

A. A covered bond is issued in a non-domestic currency. The currency risk is then fully hedged using a currency swap or a package of foreign exchange forward contracts.

B. A covered bond is issued with an attached credit default swap. It essentially is a "risk-free" government bond.

C. A covered bond is a senior debt obligation giving recourse to the issuer as well as a predetermined underlying collateral pool, often commercial or residential mortgages.

The following information relates to questions 24-30

Anna Lebedeva is a fixed-income portfolio manager. Paulina Kowalski, a junior analyst, and Lebedeva meet to review several positions in Lebedeva's portfolio.

Lebedeva begins the meeting by discussing credit rating migration. Kowalski asks Lebedeva about the typical impact of credit rating migration on the expected return on a bond. Lebedeva asks Kowalski to estimate the expected return over the next year on a bond issued by Entre Corp. The BBB rated bond has a yield to maturity of 5.50% and a modified duration of 7.54. Kowalski calculates the expected return on the bond over the next year given the partial credit transition and credit spread data in Exhibit 1. She assumes that market spreads and yields will remain stable over the year.

Exhibit 1: One-Year Transition Matrix for BBB Rated Bonds and Credit Spreads

	AAA	AA	A	BBB	BB	B	CCC, CC, C
Probability (%)	0.02	0.30	4.80	85.73	6.95	1.75	0.45
Credit spread	0.60%	0.90%	1.10%	1.50%	3.40%	6.50%	9.50%

Lebedeva next asks Kowalski to analyze a three-year bond, issued by VraiRive S.A., using an arbitrage-free framework. The bond's coupon rate is 5%, with interest paid annually and a par value of 100. In her analysis, she makes the following three assumptions:

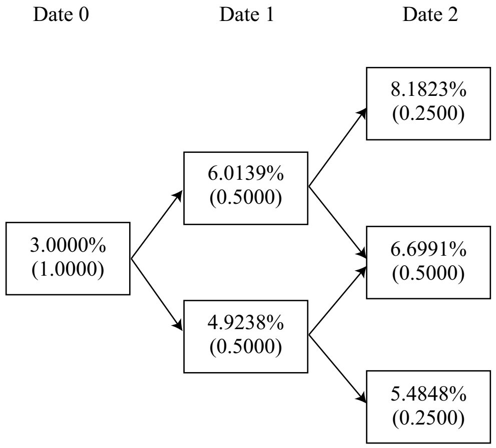
- The annual interest rate volatility is 10%.
- The recovery rate is one-third of the exposure each period.
- The annual probability of default each year is 2.00%.

Selected information on benchmark government bonds for the VraiRive bond is presented in Exhibit 2, and the relevant binomial interest rate tree is presented in Exhibit 3.

Exhibit 2: Par Curve Rates for Annual Payment Benchmark Government Bonds

Maturity	Coupon Rate	Price	Discount Factor	Spot Rate	Forward Rate
1	3.00%	100	0.970874	3.0000%	3.0000%
2	4.20%	100	0.920560	4.2255%	5.4656%
3	5.00%	100	0.862314	5.0618%	6.7547%

Exhibit 3: One-Year Binomial Interest Rate Tree for 10% Volatility (risk-neutral probabilities in parentheses)



Kowalski estimates the value of the VraiRive bond assuming no default(VND) as well as the fair value of the bond. She then estimates the bond's yield to maturity and the bond's credit spread over the benchmark in Exhibit 2. Kowalski asks Lebedeva, "What might cause the bond's credit spread to decrease?"

Lebedeva and Kowalski next discuss the drivers of the term structure of credit spreads. Kowalski tells Lebedeva the following:

Statement 1 The credit term structure for the most highly rated securities tends to be either flat or slightly upward sloping.

Statement 2 The credit term structure for lower-rated securities is often steeper, and credit spreads widen with expectations of strong economic growth.

Next, Kowalski analyzes the outstanding bonds of DLL Corporation, a high-quality issuer with a strong, competitive position. Her focus is to determine the rationale for a positive-sloped credit spread term structure.

Lebedeva ends the meeting by asking Kowalski to recommend a credit analysis approach for a securitized asset-backed security (ABS) held in the portfolio. This non-static asset pool is made up of many medium-term auto loans that are homogeneous, and each loan is small relative to the total value of the pool.

24. The *most appropriate* response to Kowalski's question regarding credit rating migration is that it has:

A. a negative impact.

B. no impact.

C. a positive impact.

25. Based on Exhibit 1, the one-year expected return on the Entre Corp. bond is *closest* to:

A. 3.73%.

B. 5.50%.

C. 7.27%.

26. Based on Kowalski's assumptions and Exhibit 2 and Exhibit 3, the credit spread on the VraiRive bond is *closest* to:

A. 0.6949%.

B. 0.9388%.

C. 1.4082%.

27. The *most appropriate* response to Kowalski's question relating to the credit spread is:

A. an increase in the probability of default.

B. an increase in the loss given default.

C. a decrease in the risk-neutral probability of default.

28. Which of Kowalski's statements regarding the term structure of credit spreads is correct?

A. Only Statement 1

B. Only Statement 2

C. Both Statement 1 and Statement 2

29. DLL's credit spread term structure is *most* consistent with the firm having:

A. low leverage.

B. weak cash flow.

C. a low profit margin.

30. Given the description of the asset pool of the ABS, Kowalski should recommend a:

A. loan-by-loan approach.

B. portfolio-based approach.

C. statistics-based approach.

SOLUTIONS

1. B is correct. The expected exposure is the projected amount of money that an investor could lose if an event of default occurs, before factoring in possible recovery. The expected exposure for both Bond I and Bond II is 100 + 5 = 105.

2. C is correct. The loss given default is a positive function of the expected exposure to default loss and a negative function of the recovery rate. Because Bond II has a lower recovery rate than Bond I and the same expected exposure to default loss (100 + 5 = 105), it will have a higher loss given default than Bond I will have. The loss given default for Bond I is 105 × (1 – 0.40) = 63.00. The loss given default for Bond II is 105 × (1 – 0.35) = 68.25.

3. B is correct. In the event of no default, the investor is expected to receive 105. In the event of a default, the investor is expected to receive 105 – [105 × (1 – 0.40)] = 42. The expected future value of the bond is, therefore, the weighted average of the no-default and default amounts, or (105 × 0.98) + (42 × 0.02) = 103.74.

4. B is correct. The risk-neutral default probability, P^*, is calculated using the current price, the expected receipt at maturity with no default (that is, 100 + 5 = 105), the expected receipt at maturity in the event of a default (that is, 0.40 × 105 = 42), and the risk-free rate of interest (0.03):

   $$100 = \frac{[105 \times (1 - P^*)] + (42 \times P^*)}{1.03}.$$

   Solving for P^* gives 0.031746, or 3.1746%.

5. A is correct. The CVA is the sum of the present value of expected losses on the bond, which from Exhibit 2 is 3.3367.

6. C is correct. The expected percentage price change is the product of the negative of the modified duration and the difference between the credit spread in the new rating and the old rating:

   Expected percentage price change = –4.2 × (0.0175 – 0.01) = –0.0315, or –3.15%.

7. B is correct. A reduced-form model in credit risk analysis uses historical variables, such as financial ratios and macroeconomic variables, to estimate the default intensity. A structural model for credit risk analysis, in contrast, uses option pricing and relies on a traded market for the issuer's equity.

8. B is correct. Observation 1 is incorrect, but Observation 2 is correct. The actual default probabilities do not include the default risk premium associated with the uncertainty in the timing of the possible default loss. The observed spread over the yield on a risk-free bond in practice does include liquidity and tax considerations, in addition to credit risk.

9. B is correct. The following table shows that the credit valuation adjustment (CVA) for the bond is €36.49, the sum of the present values of expected loss. The steps taken to complete the table are as follows.

   Step 1 Exposure at date T is $\frac{€1{,}000}{(1 + r)^{4-T}}$, where r is 3%. That is, exposure is computed by discounting the face value of the bond using the risk-free rate and the number of years until maturity.

Step 2 Recovery = Exposure × Recovery rate.

Step 3 Loss given default (LGD) = Exposure – Recovery.

Step 4 Probability of default (POD) on Date 1 is 1.50%. The probability of survival (POS) on Date 1 is 98.50%.

For subsequent dates, POD is calculated as the annual default probability multiplied by the previous date's POS.

For example, to determine the Date 2 POD (1.4775%), the annual default probability (1.50%) is multiplied by the Date 1 POS (98.50%).

Step 1 POS in Dates 2–4 = POS in the previous year – POD.

That is, POS in year T = POS in year $(T - 1)$ – POD in year T.

POS can also be determined by subtracting the annual default probability from 100% and raising it to the power of the number of years:

$(100\% - 1.5000\%)^1 = 98.5000\%$.

$(100\% - 1.5000\%)^2 = 97.0225\%$.

$(100\% - 1.5000\%)^3 = 95.5672\%$.

$(100\% - 1.5000\%)^4 = 94.1337\%$.

Step 2 Expected loss = LGD × POD.

Step 3 Discount factor (DF) for date T is $\frac{1}{(1 + r)^T}$, where r is 3%.

Step 4 PV of expected loss = Expected loss × DF.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	€915.14	€274.54	€640.60	1.5000%	98.5000%	€9.61	0.970874	€9.33
2	€942.60	€282.78	€659.82	1.4775%	97.0225%	€9.75	0.942596	€9.19
3	€970.87	€291.26	€679.61	1.4553%	95.5672%	€9.89	0.915142	€9.05
4	€1,000.00	€300.00	€700.00	1.4335%	94.1337%	€10.03	0.888487	€8.92
							CVA =	€36.49

The value of the bond if it were default free would be 1,000 × DF for Date 4 = €888.49.

Fair value of the bond considering CVA = €888.49 – CVA = €888.49 – €36.49 = €852.00.

Because the market price of the bond (€875) is greater than the fair value of €852, B is correct.

A is incorrect because the market price of the bond differs from its fair value.

C is incorrect because although the bond's value if the bond were default free is greater than the market price, the bond has a risk of default, and CVA lowers its fair value to below the market price.

10. B is correct. The recovery rate to be used now in the computation of fair value is 30% × 1.25 = 37.5%, whereas the default probability to be used is 1.50% × 1.25 =

1.875%.

Using the steps outlined in the solution to Question 1, the following table is prepared, which shows that the bond's CVA increases to 40.49. Thus, Koning concludes that a change in the probability of default has a greater effect on fair value than a similar change in the recovery rate. The steps taken to complete the table are the same as those in the previous problem. There are no changes in exposures and discount factors in this table.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	€915.14	€343.18	€571.96	1.8750%	98.1250%	€10.72	0.970874	€10.41
2	€942.60	€353.47	€589.12	1.8398%	96.2852%	€10.84	0.942596	€10.22
3	€970.87	€364.08	€606.80	1.8053%	94.4798%	€10.95	0.915142	€10.03
4	€1,000.00	€375.00	€625.00	1.7715%	92.7083%	€11.07	0.888487	€9.84
							CVA =	€40.49

Changes in the default probability and recovery rates do not affect the value of the default-free bond. So, it is the same as in the previous question: €888.49.

Fair value of the bond considering CVA = €888.49 – CVA = €888.49 – €40.49 = €848.00

11. A is correct. The following table shows that the CVA for the bond is €42.17, the sum of the present values of expected loss. The steps taken to complete the table are as follows.

Step 1 Exposure at Date 4 is €1,000 + Coupon amount = €1,000 + €60 = €1,060. Exposure at a date T prior to that is the coupon on date T + PV at date T of subsequent coupons + PV of €1,000 to be received at Date 4. For example, exposure at Date 2 is

$$€60 + \frac{€60}{1+0.03} + \frac{€60}{(1+0.03)^2} + \frac{€1,000}{(1+0.03)^2} = €60 + \frac{€60}{1+0.03} + \frac{€1,060}{(1+0.03)^2}.$$
$$= €1,117.40.$$

Steps 2 through 8 are the same as those in the solution to Question 1.

Date	Exposure	Recovery	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0								
1	€1,144.86	€343.46	€801.40	1.5000%	98.5000%	€12.02	0.970874	€11.67
2	€1,117.40	€335.22	€782.18	1.4775%	97.0225%	€11.56	0.942596	€10.89
3	€1,089.13	€326.74	€762.39	1.4553%	95.5672%	€11.10	0.915142	€10.15
4	€1,060.00	€318.00	€742.00	1.4335%	94.1337%	€10.64	0.888487	€9.45
							CVA =	€42.17

The value of the bond if it were default free would be €60 × DF_1 + €60 × DF_2 + €60 × DF_3 + €1,060 × DF_4 = €1,111.51.

Fair value of the bond considering CVA = €1,111.51 – €42.17 = €1,069.34.

12. A is correct. If default occurs on Date 3, the rate of return can be obtained by solving the following equation for internal rate of return (IRR):

$$€1,090 = \frac{€60}{1 + \text{IRR}} + \frac{€60}{(1 + \text{IRR})^2} + \frac{€326.74}{(1 + \text{IRR})^3}.$$

In this equation, €60 is the amount of coupon received at Dates 1 and 2 prior to default at Date 3. The amount €326.74 is the recovery at Time 3 (from the CVA table in the solution to the previous question). The solution to the foregoing equation can be obtained using the cash flow IRR function on your calculator.

13. B is correct. For each possible transition, the expected percentage price change, computed as the product of the modified duration and the change in the spread as shown in Exhibit 4 (relating to question 5), is calculated as follows:

    From AA to AAA: –2.75 × (0.60% – 0.90%) = +0.83%.

    From AA to A: –2.75 × (1.10% – 0.90%) = –0.55%.

    From AA to BBB: –2.75 × (1.50% – 0.90%) = –1.65%.

    From AA to BB: –2.75 × (3.40% – 0.90%) = –6.88%.

    From AA to B: –2.75 × (6.50% – 0.90%) = –15.40%.

    From AA to C: –2.75 × (9.50% – 0.90%) = –23.65%.

    The expected percentage change in the value of the AA rated bond is computed by multiplying each expected percentage price change for a possible credit transition by its respective transition probability given in Exhibit 4 and summing the products:

    (0.0150 × 0.83%) + (0.8800 × 0%) + (0.0950 × –0.55%) + (0.0075 × –1.65%) + (0.0015 × –6.88%) + (0.0005 × –15.40%) + (0.0003 × –23.65%)

    = –0.0774%.

    Therefore, the expected return on the bond over the next year is its YTM minus 0.0774%, assuming no default.

14. B is correct. Statement B is correct because a reduced-form credit model involves regression analysis using information generally available in the financial markets, such as the measures mentioned in the statement.

    Statement A is incorrect because it is consistent with the use of a structural model and not a reduced-form model. It is a structural model that is based on the premise that a firm defaults on its debt if the value of its assets falls below its liabilities and that the probability of that event has the characteristics of an option.

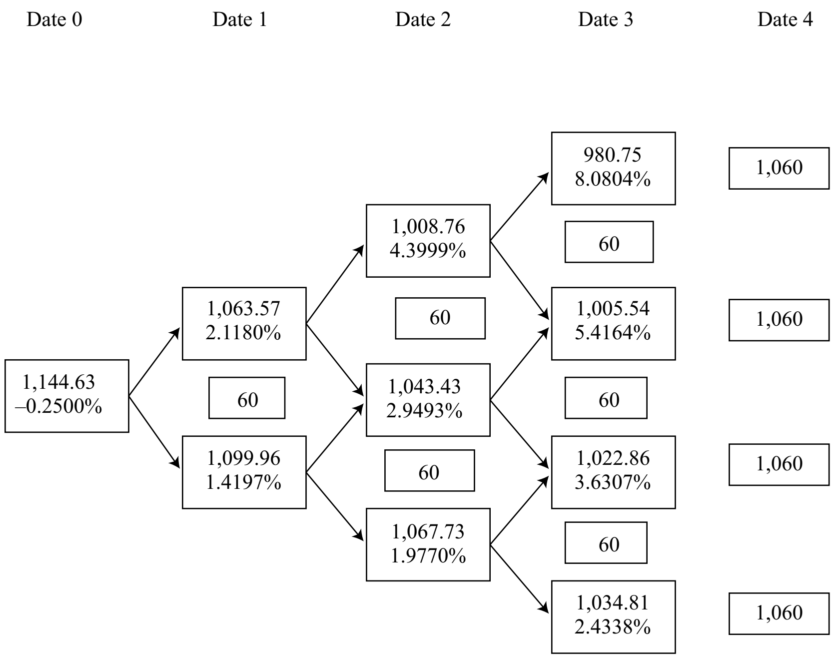
    Statement C is incorrect because it is consistent with the use of a structural model and not a reduced-form model. A structural model involves the estimation of a default barrier, and default occurs if the value of firm's assets falls below the default barrier.

15. C is correct. Structural models require information best known to the managers of the company. Reduced-form models require information only generally available in financial markets.

    A is incorrect because although it is literally true, when the models were developed is immaterial. Structural models are currently used in practice by commercial banks and credit rating agencies.

    B is incorrect because computer technology facilitates valuation using option pricing models as well as regression analysis.

16. A is correct. The following tree shows the valuation assuming no default of Bond B2, which pays a 6% annual coupon.



The scheduled year-end coupon and principal payments are placed to the right of each forward rate in the tree. For example, the Date 4 values are the principal plus the coupon of 60. The following are the four Date 3 values for the bond, shown above the interest rate at each node:

€1,060/1.080804 = €980.75.

€1,060/1.054164 = €1,005.54.

€1,060/1.036307 = €1,022.86.

€1,060/1.024338 = €1,034.81.

These are the three Date 2 values:

$$\frac{(0.5 \times €980.75) + (0.5 \times €1,005.54) + €60}{1.043999} = €1,008.76.$$

$$\frac{(0.5 \times €1,005.54) + (0.5 \times €1,022.86) + €60}{1.029493} = €1,043.43.$$

$$\frac{(0.5 \times €1,022.86) + (0.5 \times €1,034.81) + €60}{1.019770} = €1,067.73.$$

These are the two Date 1 values:

$$\frac{(0.5 \times €1,008.76) + (0.5 \times €1,043.43) + €60}{1.021180} = €1,063.57.$$

$$\frac{(0.5 \times €1,043.43) + (0.5 \times €1,067.73) + €60}{1.014197} = €1,099.96.$$

This is the Date 0 value:

$$\frac{(0.5 \times €1,063.57) + (0.5 \times €1,099.96) + €60}{0.997500} = €1,144.63.$$

So, the value of the bond assuming no default is 1,144.63. This value could also have been obtained more directly using the benchmark discount factors from Exhibit 1:

€60 × 1.002506 + €60 × 0.985093 + €60 × 0.955848 + €1,060 × 0.913225
= €1,144.63.

The benefit of using the binomial interest rate tree to obtain the VND is that the same tree is used to calculate the expected exposure to default loss.

The credit valuation adjustment table is now prepared following these steps:

Step 1 Compute the expected exposures as described in the following, using the binomial interest rate tree prepared earlier.

The expected exposure for Date 4 is €1,060.

The expected exposure for Date 3 is

(0.1250 × €980.75) + (0.3750 × €1,005.54) + (0.3750 × €1,022.86) + (0.1250 × €1,034.81) + 60
= €1,072.60.

The expected exposure for Date 2 is

(0.25 × €1,008.76) + (0.50 × €1,043.43) + (0.25 × €1,067.73) + €60 = €1,100.84.

The expected exposure for Date 1 is

(0.50 × €1,063.57) + (0.50 × €1,099.96) + 60 = €1,141.76.

Step 2 LGD = Exposure × (1 – Recovery rate).

Step 3 The initial default probability is 1.50%. For subsequent dates, POD is calculated as the default probability multiplied by the previous date's POS.

For example, to determine the Date 2 POD (1.4775%), the default probability (1.5000%) is multiplied by the Date 1 POS (98.5000%).

Step 4 POS is determined by subtracting the default probability from 100% and raising it to the power of the number of years:

$(100\% - 1.5000\%)^1 = 98.5000\%$.

$(100\% - 1.5000\%)^2 = 97.0225\%$.

$(100\% - 1.5000\%)^3 = 95.5672\%$.

$(100\% - 1.5000\%)^4 = 94.1337\%$.

Step 5 Expected loss = LGD × POD.

Step 6 Discount factors in year T are obtained from Exhibit 2.

Step 7 PV of expected loss = Expected loss × DF.

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0							
1	€1,141.76	€799.23	1.5000%	98.5000%	€11.99	1.002506	€12.02
2	€1,100.84	€770.58	1.4775%	97.0225%	€11.39	0.985093	€11.22
3	€1,072.60	€750.82	1.4553%	95.5672%	€10.93	0.955848	€10.44

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
4	€1,060.00	€742.00	1.4335%	94.1337%	€10.64	0.913225	€9.71
						CVA =	€43.39

Fair value of the bond considering CVA = €1,144.63 – CVA = €1,144.63 – €43.39 = €1,101.24.

17. A is correct. The corporate bond's fair value is computed in the solution to Question 16 as €1,101.24. The YTM can be obtained by solving the following equation for IRR:

$$€1,101.24 = \frac{€60}{1+\text{IRR}} + \frac{€60}{(1+\text{IRR})^2} + \frac{€60}{(1+\text{IRR})^3} + \frac{€1,060}{(1+\text{IRR})^4}.$$

The solution to this equation is 3.26%.

Valuation of a four-year, 6% coupon bond under no default is computed in the solution to Question 16 as 1,144.63. So, the YTM of a theoretical comparable-maturity government bond with the same coupon rate as the corporate bond, B2, can be obtained by solving the following equation for IRR:

$$€1,144.63 = \frac{€60}{1+\text{IRR}} + \frac{€60}{(1+\text{IRR})^2} + \frac{€60}{(1+\text{IRR})^3} + \frac{€1,060}{(1+\text{IRR})^4}.$$

The solution to this equation is 2.18%. So, the credit spread that the analyst wants to compute is 3.26% – 2.18% = 1.08%, or 108 bps.

B is incorrect because it is the spread over the four-year government par bond that has a YTM of 2.25% in Exhibit 2: 3.26% – 2.25% = 1.01%, or 101 bps. Although this spread is commonly used in practice, the analyst is interested in finding the spread over a theoretical 6% coupon government bond.

C is incorrect because it is the YTM of the coupon four-year government bond in Exhibit 2.

18. B is correct. The recovery rate to be used now in the computation of fair value is 30% × 0.75 = 22.500%, whereas the default probability to be used is 1.50% × 0.75 = 1.125%.

The tree that shows the valuation assuming no default of Bond B2 in the solution to Question 8 will not be affected by the foregoing changes. Accordingly, VND remains €1,144.63.

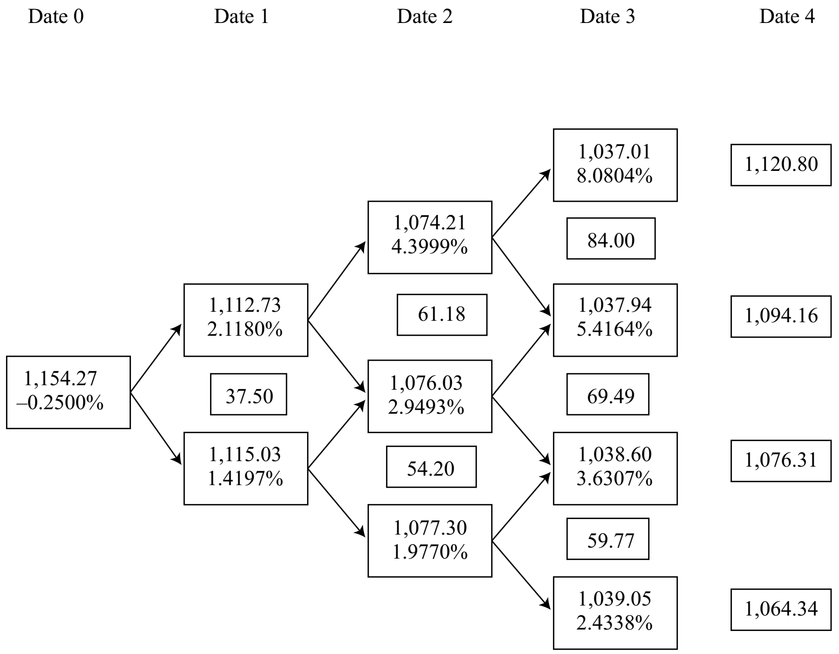
Following the steps outlined in the solution to Question 8, the following table is prepared, which shows that the CVA for the bond decreases to €36.23. Thus, Ibarra concludes that a decrease in the probability of default has a greater effect on fair value than a similar decrease in the recovery rate. The steps taken to complete the table are the same as those in Question 8. There are no changes in exposures or discount factors in this table.

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0							
1	€1,141.76	€884.87	1.1250%	98.8750%	€9.95	1.002506	€9.98
2	€1,100.84	€853.15	1.1123%	97.7627%	€9.49	0.985093	€9.35
3	€1,072.60	€831.26	1.0998%	96.6628%	€9.14	0.955848	€8.74

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
4	€1,060.00	€821.50	1.0875%	95.5754%	€8.93	0.913225	€8.16
						CVA =	€36.23

Fair value of the bond considering CVA = €1,144.63 – CVA = €1,144.63 – €36.23 = €1,108.40.

19. A is correct. The following tree shows the valuation assuming no default of the floating-rate note (FRN), B4, which has a quoted margin of 4%.



The scheduled year-end coupon and principal payments are placed to the right of each forward rate in the tree. For example, the four Date 4 values are the principal plus the coupon.

€1,000 × (1 + 0.080804 + 0.04) = €1,120.80.

€1,000 × (1 + 0.054164 + 0.04) = €1,094.16.

€1,000 × (1 + 0.036307 + 0.04) = €1,076.31.

€1,000 × (1 + 0.024338 + 0.04) = €1,064.34.

The following are the four Date 3 bond values for the note, shown above the interest rate at each node:

€1,120.80/1.080804 = €1,037.01.

€1,094.16/1.054164 = €1,037.94.

€1,076.31/1.036307 = €1,038.60.

€1,064.34/1.024338 = €1,039.05.

The three Date 3 coupon amounts are computed based on the interest rate at Date 2 plus the quoted margin of 4%:

€1,000 × (0.043999 + 0.04) = €84.00.

€1,000 × (0.029493 + 0.04) = €69.49.

€1,000 × (0.019770 + 0.04) = €59.77.

There are three Date 2 bond values:

$$\frac{(0.5 \times €1,037.01) + (0.5 \times €1,037.94) + €84.00}{1.043999} = €1,074.21.$$

$$\frac{(0.5 \times €1,037.94) + (0.5 \times €1,038.60) + €69.49}{1.029493} = €1,076.03.$$

$$\frac{(0.5 \times €1,038.60) + (0.5 \times €1,039.05) + €59.77}{1.019770} = €1,077.30.$$

The two Date 2 coupon amounts are computed based on the interest rate at Date 1 plus the quoted margin of 4%:

€1,000 × (0.021180 + 0.04) = €61.18.

€1,000 × (0.014197 + 0.04) = €54.20.

The Date 1 coupon amount is computed based on the interest rate at date 0 plus the quoted margin of 4%:

€1,000 × (–0.0025 + 0.04) = €37.50.

These are the calculations for the bond values for Date 1 and Date 0:

$$\frac{(0.5 \times €1,074.21) + (0.5 \times €1,076.03) + €61.18}{1.021180} = €1,112.73.$$

$$\frac{(0.5 \times €1,076.06) + (0.5 \times €1,077.30) + €54.20}{1.014197} = €1,115.0.$$

Then, the VND is calculated as follows:

$$\frac{(0.5 \times €1,112.73) + (0.5 \times €1,115.03) + €37.50}{0.9975} = €1,154.27.$$

The expected exposures are then computed using the binomial interest rate tree prepared earlier. For example, the expected exposure for Date 4 is computed as follows:

(0.125 × €1,120.80) + (0.375 × €1,094.16) + (0.375 × €1,076.31) + (0.125 × €1,064.34)

= €1,087.07.

Similarly, the expected exposure for Date 3 is computed as follows:

(0.125 × €1,037.01) + (0.375 × €1,037.94) + (0.375 × €1,038.60) + (0.125 × €1,039.05) + (0.250 × €84) + (0.500 × €69.49) + (0.250 × €59.77)

= €1,108.90.

The expected exposures for Dates 2 and 1 are computed similarly, and the credit valuation adjustment table is completed following Steps 2–7 outlined in the solution to Question 8.

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0							
1	€1,151.38	€805.97	1.5000%	98.5000%	€12.09	1.002506	€12.12
2	€1,133.58	€793.51	1.4775%	97.0225%	€11.72	0.985093	€11.55
3	€1,108.90	€776.23	1.4553%	95.5672%	€11.30	0.955848	€10.80

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
4	€1,087.07	€760.95	1.4335%	94.1337%	€10.91	0.913225	€9.96
						CVA =	€44.43

Fair value of the FRN considering CVA = €1,154.27 – CVA = €1,154.27 – €44.43 = €1,109.84.

Because the market price of €1,070 is less than the estimated fair value, the analyst should recommend adding to existing positions in the FRN.

B and C are incorrect because the FRN is perceived to be undervalued in the market.

20. A is correct. The changing probability of default will not affect the binomial tree prepared in the solution to Question 11. The Date 1 value remains €1,154.27, which is also the VND. The expected exposures, loss given default, and discount factors are also unaffected by the changing probability of default. The following is the completed credit valuation adjustment table.

Date	Exposure	LGD	POD	POS	Expected Loss	DF	PV of Expected Loss
0							
1	€1,151.38	€805.97	1.5000%	98.5000%	€12.09	1.002506	€12.12
2	€1,133.58	€793.51	0.4925%	98.0075%	€3.91	0.985093	€3.85
3	€1,108.90	€776.23	0.4900%	97.5175%	€3.80	0.955848	€3.64
4	€1,087.07	€760.95	0.4876%	97.0299%	€3.71	0.913225	€3.39
						CVA =	€22.99

Thus, CVA decreases to €22.99.

21. C is correct. The credit rating agencies typically make incremental changes, as seen in a transition matrix provided in Exhibit 1. Ibarra believes the bond is undervalued, because her assessment of the probability of default and the recovery rate is more optimistic than that of the agencies. Therefore, she most likely expects the credit rating agencies to put the issuer on a positive watch.

A is incorrect because the bond is perceived to be undervalued, not overvalued. Ibarra is not expecting a credit downgrade.

B is incorrect because it is not the *most likely* expectation. The rating agencies rarely change an issuer's rating from BBB all the way to AAA. In Exhibit 1 (relating to question 5) the probability of a BBB rated issuer going from BBB to AAA is 0.02%, whereas to go from BBB to A it is 4.80%.

22. A is correct.

B is incorrect because, although generally true for investment-grade bonds, the statement neglects the fact that high-yield issuers sometimes face a downward-sloping credit term structure. Credit term structures are not *always* upward sloping.

C is incorrect because there is a consistent pattern for the term structure of credit spreads: Typically, it is upwardly sloped because greater time to maturity is associated with higher projected probabilities of default and lower recovery rates.

23. C is correct. A covered bond is a senior debt obligation of a financial institution that gives recourse to the originator/issuer as well as a predetermined underlying collateral pool. Each country or jurisdiction specifies the eligible collateral

types as well as the specific structures permissible in the covered bond market. Covered bonds usually have either commercial or residential mortgages meeting specific criteria or public sector exposures as underlying collateral.

A is incorrect. The term "covered" is used in foreign exchange analysis, for instance, "covered interest rate parity." In the context of securitized debt, a covered bond is secured by specific assets in addition to the overall balance sheet of the issuer.

B is incorrect because a covered bond does not involve a credit default swap. In addition, an issuer is not likely to sell a credit default swap on its own liability.

24. A is correct. Credit spread migration typically reduces the expected return for two reasons. First, the probabilities for rating changes are not symmetrically distributed around the current rating; they are skewed toward a downgrade rather than an upgrade. Second, the increase in the credit spread is much larger for downgrades than is the decrease in the spread for upgrades.

25. A is correct. The expected return on the Entre Corp. bond over the next year is its yield to maturity plus the expected percentage price change in the bond over the next year. In the following table, for each possible transition, the expected percentage price change is the product of the bond's modified duration of 7.54, multiplied by –1, and the change in the spread, weighted by the given probability:

$$\text{Expected percentage price change} = (0.0002 \times 6.786\%) + (0.0030 \times 4.524\%) + (0.0480 \times 3.016\%) + (0.8573 \times 0.000\%) + (0.0695 \times -14.326\%) + (0.0175 \times -37.700\%) + (0.0045 \times -60.320\%) = -1.76715\%.$$

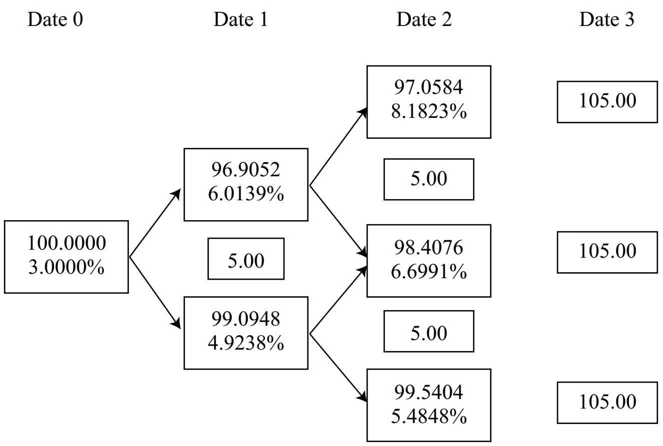
So, the expected return on the Entre Corp. bond is its yield to maturity plus the expected percentage price change due to credit migration:

$$\text{Expected return} = 5.50\% - 1.77\% = 3.73\%.$$

	Expected % Price Change (1)	**Probability (2)**	**Expected % Price Change × Probability (1 × 2)**
From BBB to AAA	–7.54 × (0.60% – 1.50%) = 6.786%	0.0002	0.00136
From BBB to AA	–7.54 × (0.90% – 1.50%) = 4.524%	0.0030	0.01357
From BBB to A	–7.54 × (1.10% – 1.50%) = 3.016%	0.0480	0.14477
From BBB to BB	–7.54 × (3.40% – 1.50%) = –14.326%	0.0695	–0.99566
From BBB to B	–7.54 × (6.50% – 1.50%) = –37.700%	0.0175	–0.65975
From BBB to CCC, CC, or C	–7.54 × (9.50% – 1.50%) = –60.320%	0.0045	–0.27144
		Total:	–1.76715

26. C is correct. The credit spread can be calculated in three steps:

Step 1 Estimate the value of the three-year VraiRive bond assuming no default. Based on Kowalski's assumptions and Exhibit 2 and Exhibit 3, the value of the three-year VraiRive bond assuming no default is 100.0000.



Supporting calculations:

The bond value in each node is the value of next period's cash flows discounted by the forward rate. For the three nodes on Date 2, the bond values are as follows:

$105/1.081823 = 97.0584.$

$105/1.066991 = 98.4076.$

$105/1.054848 = 99.5404.$

For the two nodes on Date 1, the two bond values are as follows:

$[(0.5 \times 97.0584) + (0.5 \times 98.4076) + 5.00]/1.060139 = 96.9052.$

$[(0.5 \times 98.4076) + (0.5 \times 99.5404) + 5.00]/1.049238 = 99.0948.$

Finally, for the node on Date 0, the bond value is

$[(0.5 \times 96.9052) + (0.5 \times 99.0948) + 5.00]/1.030000 = 100.0000.$

Therefore, the VND for the VraiRive bond is 100.0000.

Step 2 Calculate the credit valuation adjustment, and then subtract the CVA from the VND from Step 1 to establish the fair value of the bond. The CVA equals the sum of the present values of each year's expected loss and is calculated as follows:

Date	Expected Exposure	Loss Given Default	Probability of Default	Discount Factor	Present Value of Expected Loss
1	103.0000	68.6667	2.0000%	0.970874	1.3333
2	103.3535	68.9023	1.9600%	0.920560	1.2432
3	105.0000	70.0000	1.9208%	0.862314	1.1594
				CVA =	3.7360

Supporting calculations:

The expected exposures at each date are the bond values at each node, weighted by their risk-neutral probabilities, plus the coupon payment:

Date 1: $(0.5 \times 96.9052) + (0.5 \times 99.0948) + 5.00 = 103.0000.$

Date 2: (0.25 × 97.0584) + (0.5 × 98.4076) + (0.25 × 99.5404) + 5.00 = 103.3535.

Date 3: 105.0000

The loss given default on each date is 2/3 of the expected exposure.

The probability of default on each date is as follows:

Date 1: 2%

Date 2: 2% × (100% – 2%) = 1.96%.

Date 3: 2% × $(100\% - 2\%)^2$ = 1.9208%.

The discount factor on each date is 1/(1 + spot rate for the date) raised to the correct power.

Finally, the credit valuation adjustment each year is the product of the LGD times the POD times the discount factor, as shown in the last column of the table. The sum of the three annual CVAs is 3.7360.

So, the fair value of the VraiRive bond is the VND less the CVA, or VND – CVA = 100 – 3.7360 = 96.2640.

Step 3 Based on the fair value from Step 2, calculate the yield to maturity of the bond, and solve for the credit spread by subtracting the yield to maturity on the benchmark bond from the yield to maturity on the VraiRive bond. The credit spread is equal to the yield to maturity on the VraiRive bond minus the yield to maturity on the three-year benchmark bond (which is 5.0000%). Based on its fair value of 96.2640, the VraiRive bond's yield to maturity is

$$96.2640 = \frac{5}{(1+\text{YTM})} + \frac{5}{(1+\text{YTM})^2} + \frac{105}{(1+\text{YTM})^3}.$$

Solving for YTM, the yield to maturity is 6.4082%. Therefore, the credit spread on the VraiRive bond is 6.4082% – 5.0000% = 1.4082%.

27. C is correct. A decrease in the risk-neutral probability of default would decrease the credit valuation adjustment and decrease the credit spread. In contrast, increasing the bond's loss-given-default assumption and increasing the probability-of-default assumption would increase the credit valuation adjustment and decrease the fair value of the bond (and increase the yield to maturity and the credit spread over its benchmark).

28. A is correct. For investment-grade bonds with the highest credit ratings, credit spreads are extremely low, and credit migration is possible only in one direction given the implied lower bound of zero on credit spreads. As a result, the credit term structure for the most highly rated securities tends to be either flat or slightly upward sloping. Securities with lower credit quality, however, face greater sensitivity to the credit cycle. Credit spreads would decrease, not increase, with the expectation of economic growth. There is a countercyclical relationship between credit spreads and benchmark rates over the business cycle. A strong economic climate is associated with higher benchmark yields but lower credit spreads because the probability of issuers defaulting declines in such good times.

29. A is correct. Positive-sloped credit spread curves may arise when a high-quality issuer with a strong competitive position in a stable industry has low leverage, strong cash flow, and a high profit margin. This type of issuer tends to exhibit very low short-term credit spreads that rise with increasing maturity given great-

er uncertainty due to the macroeconomic environment, potential adverse changes in the competitive landscape, technological change, or other factors that drive a higher implied probability of default over time. Empirical academic studies also tend to support the view that the credit spread term structure is upward sloping for investment-grade bond portfolios.

30. B is correct. The auto ABS is granular, with many small loans relative to the size of the total portfolio. The auto loans are also homogeneous. These characteristics support using the portfolio-based approach. A loan-by-loan approach would be inefficient because of the large number of basically similar loans; this approach is best for a portfolio of discrete, large loans that are heterogeneous. A statistics-based approach would work for a static book of loans, whereas the auto loan portfolio would be dynamic and would change over time.

LEARNING MODULE

5

Credit Default Swaps

by Brian Rose, and Don M. Chance, PhD, CFA.

Brian Rose (USA). Don M. Chance, PhD, CFA, is at Louisiana State University (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
□	describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product
□	describe credit events and settlement protocols with respect to CDS
□	explain the principles underlying and factors that influence the market's pricing of CDS
□	describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve
□	describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments

INTRODUCTION

1

Derivative instruments in which the underlying is a measure of a borrower's credit quality are widely used and well established in a number of countries. We explore basic definitions of such instruments, explain the main concepts, cover elements of valuation and pricing, and discuss applications.

BASIC DEFINITIONS AND CONCEPTS

2

□	describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product

A **credit derivative** is a derivative instrument in which the underlying is a measure of a borrower's credit quality. Four types of credit derivatives are (1) total return swaps, (2) credit spread options, (3) credit-linked notes, and (4) credit default swaps, or CDS. CDS are the most liquid of the four and, as such, are the topic we focus on. In a CDS, one party makes payments to the other and receives in return the promise of compensation if a third party defaults.

In any derivative, the payoff is based on (derived from) the performance of an underlying instrument, rate, or asset that we call the "underlying." For a CDS, the underlying is the credit quality of a borrower. At its most fundamental level, a CDS provides compensation equal to expected recovery when a credit event occurs, but it also changes in value to reflect changes in the market's perception of a borrower's credit quality well in advance of default. The value of a CDS will rise and fall as opinions change about the likelihood and severity of a potential default. The actual event of default might never occur, but a decline in the price of a bond when investors perceive an increase in the likelihood of default is a mark-to-market loss to the bondholder. The most common credit events include bankruptcy, failure to pay, and restructuring. Another type of credit event which may be encountered in sovereign and municipal government bond markets is a moratorium or, more drastically, a repudiation of debt in which the governmental authority declares a moratorium on payments due under the terms of the obligation or challenges the validity of the entire debt obligation. (Other, less common credit events are also defined in the International Swaps and Derivatives Association's Credit Derivatives Definitions, but we will not consider them here.) Credit default swaps are designed to protect creditors against credit events such as these. The industry has expended great effort to provide clear guidance on what credit events are covered by a CDS contract. As with all efforts to write a perfect contract, however, no such device exists and disputes do occasionally arise. We will take a look at these issues later.

In addition to hedging credit risk, investors use CDS to

- leverage their portfolios,
- access maturity exposures not available in the cash market,
- access credit risk while limiting interest rate risk, and
- improve the liquidity of their portfolios given the illiquidity in the corporate bond market.

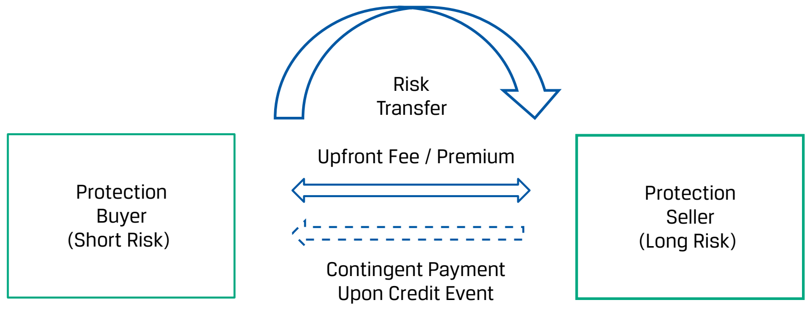
In addition, the CDS market has increased transparency and insight into the actual cost of credit risk. The higher relative liquidity and relative sophistication of CDS investors allow for more accurate price discovery and facilitate trading during liquidity events when the cash market for bonds becomes illiquid. While many of the applications listed above are beyond the scope of this reading, a basic understanding of this important fixed-income tool is necessary for all investment professionals.

Let's now define a **credit default swap**:

> *A credit default swap is a derivative contract between two parties, a credit protection buyer and credit protection seller, in which the buyer makes a series of cash payments to the seller and receives a promise of compensation for credit losses resulting from a credit event in an underlying.*

In a CDS contract there are two counterparties, the **credit protection buyer** and the **credit protection seller**. The buyer agrees to make a series of periodic payments to the seller over the life of the contract (which are determined and fixed at contract initiation) and receives in return a promise that if default occurs, the protection seller will compensate the protection buyer. If default occurs, the periodic payments made by the protection buyer to the protection seller terminate. Exhibit 1 shows the structure of payment flows.

Exhibit 1: Payment Structure of a CDS



Credit default swaps are somewhat similar to put options. Put options effectively enable the option holder to sell (put) the underlying security to the option seller if the underlying performs poorly relative to the exercise price. Similarly, in the event of a credit event on the underlying security, the buyer of credit protection receives a payment from the credit protection seller equal to the par or notional value of the security less the expected recovery value. If the credit quality of the underlying deteriorates but there is no outright credit event, the credit protection buyer is compensated only if the contract is unwound. How that compensation occurs and how much protection it provides are some points we will discuss.

A CDS does not eliminate credit risk. The definition of a default in the swap contract may not perfectly align with a traditional default event, so the magnitude of the change in value of the contract may differ from the change in value of the underlying. In addition, the credit protection buyer assumes counterparty risk with respect to the credit protection seller. Although there are no guarantees that the credit protection seller will not default, as was seen with several large financial institutions in the financial crisis that started in 2007, most credit protection sellers are relatively high-quality borrowers. If they were not, they could not be active sellers of credit protection.

The majority of CDS are written on debt issued by corporate borrowers, which will be our focus in this reading. But note that CDS can also be written on the debt of sovereign governments and state and local governments. In addition, CDS can be written on portfolios of loans, mortgages, or debt securities.

Types of CDS

There are three types of CDS: single-name CDS, index CDS, and tranche CDS. Other CDS-related instruments, such as options on CDS (or CDS swaptions) are beyond the scope of this discussion. A CDS on one specific borrower is called a **single-name CDS**. The borrower is called the **reference entity**, and the contract specifies a **reference obligation**, a particular debt instrument issued by the borrower. Only a small subset of issuers, typically with large outstanding liquid debt, have single-name CDS. The designated instrument is usually a senior unsecured obligation, but the reference obligation is not the only instrument covered by the CDS. Any debt obligation issued by the borrower that is ranked equal to or higher than the reference obligation with respect to the priority of claims is covered. The payoff of the CDS is determined by the **cheapest-to-deliver** obligation, which is the debt instrument that can be purchased and delivered at the lowest cost but has the same seniority as the reference obligation.

EXAMPLE 1

Cheapest-to-Deliver Obligation

1. Assume that a company with several debt issues trading in the market files for bankruptcy (i.e., a credit event takes place). What is the cheapest-to-deliver obligation for a CDS contract where the reference bond is a five-year senior unsecured bond?

   A. A subordinated unsecured bond trading at 20% of par

   B. A five-year senior unsecured bond trading at 50% of par

   C. A two-year senior unsecured bond trading at 45% of par

Solution:

C is correct. The cheapest-to-deliver, or lowest-priced, instrument is the two-year senior unsecured bond trading at 45% of par. Although the bond in A trades at a lower dollar price, it is subordinated and, therefore, does not qualify for coverage under the CDS. Note that even though the CDS holder holds the five-year bonds, he will receive payment on the CDS based on the cheapest-to-deliver obligation, not the specific obligation he holds.

A second type of credit default swap, an **index CDS**, involves a portfolio of single-name CDS. This type of instrument allows participants to take positions on the credit risk of a combination of companies, in much the same way that investors can trade index or exchange-traded funds that are combinations of the equities of companies. The two most commonly traded CDS index products are the North American indexes (CDX) and the European, Asian, and Australian indexes (iTraxx). Correlation of defaults is a strong determinant of a portfolio's behavior. For index CDS, this concept takes the form of a factor called **credit correlation**, and it is a key determinant of the value of an index CDS. Analyzing the effects of those correlations is a highly specialized subject, but be aware that much effort is placed on modeling how defaults by certain companies are connected to defaults by other companies. The more correlated the defaults, the more costly it is to purchase protection for a combination of the companies. In contrast, for a diverse combination of companies whose defaults have low correlations, it will be much less expensive to purchase protection.

A third type of CDS is the **tranche CDS**, which covers a combination of borrowers but only up to pre-specified levels of losses—much in the same manner that asset-backed securities are divided into tranches, each covering particular levels of losses. Coverage of tranche CDS is beyond the scope of this reading.

3 IMPORTANT FEATURES OF CDS MARKETS

☐ describe credit events and settlement protocols with respect to CDS

As we will describe in more detail later, the CDS market is large, global, and well organized. The unofficial industry governing body is the International Swaps and Derivatives Association (ISDA), which publishes industry-supported conventions that facilitate the functioning of the market. Parties to CDS contracts generally agree that their contracts will conform to ISDA specifications. These terms are specified in a document called the **ISDA Master Agreement**, which the parties to a CDS sign.

In Europe, the standard CDS contract is called the Standard Europe Contract, and in the United States and Canada, it is called the Standard North American Contract. Other standardized contracts exist for Asia, Australia, Latin America, and a few other specific countries.

Each CDS contract specifies a **notional amount**, or "notional" for short, which is the amount of protection being purchased. The notional amount can be thought of as the *size* of the contract. It is important to understand that the total notional amount of CDS can exceed the amount of debt outstanding of the reference entity. As we will discuss later, the credit protection buyer does not have to be an actual creditor holding exposure (i.e., owning a loan, bond, or other debt instrument). It can be simply a party that believes that there will be a change in the credit quality of the reference entity.

As with all derivatives, the CDS contract has an expiration or maturity date, and coverage is provided up to that date. The typical maturity range is 1 to 10 years, with 5 years being the most common and actively traded maturity, but the two parties can negotiate any maturity. Maturity dates are typically the 20th day of March, June, September, or December. The March and September maturity dates are the most liquid, as these are when the index CDS contracts roll.

The buyer of a CDS pays a periodic premium to the seller, referred to as the **CDS spread**, which is a return over a market reference rate required to protect against credit risk. It is sometimes referred to as a credit spread. Conceptually, it is the same as the credit spread on a bond, the compensation for bearing credit risk.

An important advancement in the development of CDS has been in establishing standard annual coupon rates on CDS contracts. (Note that the reference bond will make payments that are referred to collectively as the coupon while a CDS on the reference bond will have its own coupon rate.) Formerly, the coupon rate on the CDS was set at the credit spread. If a CDS required a rate of 4% to compensate the protection seller for the assumption of credit risk, the protection buyer made quarterly payments amounting to 4% annually. Now CDS coupon rates are standardized, with the most common coupons being either 1% or 5%. The 1% rate typically is used for a CDS on an investment-grade company or index, and the 5% rate is used for a CDS on a high-yield company or index. Obviously, either standardized rate might not be the appropriate rate to compensate the seller. Clearly, not all investment-grade companies have equivalent credit risk, and not all high-yield companies have equivalent credit risk. In effect, the standard rate may be too high or too low. This discrepancy is accounted for by an **upfront payment**, commonly called the **upfront premium**. The differential between the credit spread and the standard rate is converted to a present value basis. Thus, a credit spread greater than the standard rate would result in a cash payment from the protection buyer to the protection seller. Similarly, a credit spread less than the standard rate would result in a cash payment from the protection seller to the protection buyer.

Regardless of whether either party makes an upfront payment, the reference entity's credit quality could change during the life of the contract, thereby resulting in changes in the value of the CDS. These changes are reflected in the price of the CDS in the market. Consider a high-yield company with a 5% credit spread and a CDS coupon of 5%. Therefore, there is no upfront payment. The protection buyer simply agrees to make payments equal to 5% of the notional over the life of the CDS. Now suppose that at some later date, the reference entity experiences a decrease in its credit quality. The credit protection buyer is thus paying 5% for risk that now merits a rate higher than 5%. The coverage and cost of protection are the same, but the risk being covered is greater. The value of the CDS to the credit protection buyer has, therefore, increased, and if desired, she could unwind the position to capture the gain. The credit protection seller has experienced a loss in value of the instrument because he is receiving 5% to cover a risk that is higher than it was when the contract was initiated. It should be apparent that absent any other exposure to the reference entity, if the credit quality of the reference entity decreases, the credit protection buyer gains and the credit protection seller loses. The market value of the CDS reflects these gains and losses.

The terminology in CDS markets can be confusing. In equity and fixed-income markets, we think of buyers as being long and sellers as being short. In the CDS market, however, that is not always true. In single-name CDS, the *buyer* of credit protection is *short credit exposure* and the *seller* of credit protection is *long credit exposure*. This is consistent with the fact that in the financial world, "shorts" are said to benefit when things go badly. When credit quality deteriorates, the credit protection buyer benefits, and when it improves, the credit protection seller benefits. To make things even more confusing, though, the opposite is true in CDS index positions: The *buyer* of a CDX is *long* credit exposure and the seller of a CDX is *short* credit exposure. To minimize the confusion, we use the terms *credit protection seller* and *credit protection buyer* throughout our discussion. .

Credit and Succession Events

The **credit event** is what defines default by the reference entity—that is, the event that triggers a payment from the credit protection seller to the credit protection buyer. This event must be unambiguous: Did it occur, or did it not? For the market to function well, the answer to this question must be clear.

As previously mentioned, the most common credit events include bankruptcy, failure to pay, and restructuring. **Bankruptcy** is a declaration provided for by a country's laws that typically involves the establishment of a legal procedure that forces creditors to defer their claims. Bankruptcy essentially creates a temporary fence around the company through which the creditors cannot pass. During the bankruptcy process, the defaulting party works with its creditors and the court to attempt to establish a plan for repaying the debt. If that plan fails, there is likely to be a full liquidation of the company, at which time the court determines the payouts to the various creditors. Until liquidation occurs, the company normally continues to operate. Many companies do not liquidate and are able to emerge from bankruptcy. A bankruptcy filing by the reference entity is universally regarded as a credit event in CDS contracts.

Another credit event recognized in standard CDS contracts is **failure to pay**, which occurs when a borrower does not make a scheduled payment of principal or interest on an outstanding obligation after a grace period, without a formal bankruptcy filing. (Failure to pay credit events are defined in the CDS contract. ISDA contracts define failure to pay events uniformly, but the same is not true for bespoke CDS.) The third type of event, **restructuring**, refers to a number of possible events, including reduction or deferral of principal or interest, change in seniority or priority of an obligation, or change in the currency in which principal or interest is scheduled to be paid. To qualify as a credit event, the restructuring must be either involuntary or coercive. An involuntary credit event is one that is forced on the borrower by the creditors. A coercive credit event is one that is forced on the creditors by the borrower. Debt restructuring is not a credit event in the United States; issuers generally restructure under *bankruptcy*, which *is* a credit event. Restructuring is a credit event in other countries where the use of bankruptcy court to reorganize is less common. The Greek debt crisis is a good example of a restructuring that triggered a credit event.

Determination of whether a credit event occurs is done by a 15-member group within the ISDA called the Determinations Committee (DC). Each region of the world has a Determinations Committee, which consists of 10 CDS dealer (sell-side) banks and 5 non-bank (buy-side) end users. To declare a credit event, there must be a supermajority vote of 12 members.

The Determinations Committees also play a role in determining whether a **succession event** occurred. A succession event arises when there is a change in the corporate structure of the reference entity, such as through a merger, a divestiture, a spinoff, or any similar action in which ultimate responsibility for the debt in question becomes unclear. For example, if a company acquires all of the shares of a target company, it ordinarily assumes the target company's debt as well. Many mergers, however, are more complicated and can involve only partial acquisition of shares. Spinoffs and divestitures can also involve some uncertainty about who is responsible for certain debts. When such a question arises, it becomes critical for CDS holders. The question is ordinarily submitted to a Determinations Committee, and its resolution often involves complex legal interpretations of contract provisions and country laws. If a succession event is declared, the CDS contract is modified to reflect the DC's interpretation of whoever it believes becomes the obligor for the original debt. Ultimately, the CDS contract could be split among multiple entities.

Settlement Protocols

If the DC declares that a credit event has occurred, the two parties to a CDS have the right, but not the obligation, to settle. **Settlement** typically occurs 30 days after declaration of the credit event by the DC. CDS can be settled by **physical settlement** or by **cash settlement**. The former is less common and involves actual delivery of the debt instrument in exchange for a payment by the credit protection seller of the notional amount of the contract. In cash settlement, the credit protection seller pays cash to the credit protection buyer. Determining the amount of that payment is a critical factor because opinions can differ about how much money has actually been lost. The payment should essentially be the loss that the credit protection buyer has incurred, but determining that amount is not straightforward. Default on a debt does not mean that the creditor will lose the entire amount owed. A portion of the loss could be recovered. The percentage of the loss recovered is called the **recovery rate** (RR). (In most models, the recovery rate applies only to the principal.) The complement is called the **loss given default** (LGD), which is essentially an estimate of the expected credit loss. The **payout amount** is determined as the loss given default multiplied by the notional.

$$\text{Loss given default} = 1 - \text{Recovery rate (\%)}.$$

$$\text{Payout amount} = \text{LGD} \times \text{Notional}.$$

Actual recovery can be a very long process, however, and can occur much later than the payoff date of the CDS. To determine an appropriate LGD, the industry conducts an auction in which major banks and dealers submit bids and offers for the cheapest-to-deliver defaulted debt. This process identifies the market's expectation for the recovery rate and the complementary LGD, and the CDS parties agree to accept the outcome of the auction, even though the actual recovery rate can ultimately be quite different, which is an important point if the CDS protection buyer also holds the underlying debt.

EXAMPLE 2

Settlement Preference

A French company files for bankruptcy, triggering various CDS contracts. It has two series of senior bonds outstanding: Bond A trades at 30% of par, and Bond B trades at 40% of par. Investor X owns €10 million of Bond A and owns €10 million of CDS protection. Investor Y owns €10 million of Bond B and owns €10 million of CDS protection.

1. Determine the recovery rate for both CDS contracts.

Solution:

Bond A is the cheapest-to-deliver obligation, trading at 30% of par, so the recovery rate for both CDS contracts is 30%.

2. Explain whether Investor X would prefer to cash settle or physically settle her CDS contract or whether she is indifferent.

Solution:

Investor X has no preference between settlement methods. She can cash settle for €7 million [(1 – 30%) × €10 million] and sell her bond for €3 million, for total proceeds of €10 million. Alternatively, she can physically deliver her entire €10 million face amount of bonds to the counterparty in exchange for €10 million in cash.

3. Explain whether Investor Y would prefer to cash settle or physically settle his CDS contract or whether he is indifferent.

Solution:

Investor Y would prefer a cash settlement because he owns Bond B, which is worth more than the cheapest-to-deliver obligation. He will receive the same €7 million payout on his CDS contract but can sell Bond B for €4 million, for total proceeds of €11 million. If he were to physically settle his contract, he would receive only €10 million, the face amount of his bond.

CDS Index Products

So far, we have mostly been focusing on single-name CDS. As noted, there are also index CDS products. A company called Markit has been instrumental in producing CDS indexes. Of course, a CDS index is not in itself a traded instrument any more than a stock index is a traded product. As with the major stock indexes, however, the industry has created traded instruments based on the Markit indexes. These instruments are CDS that generate a payoff based on any default that occurs on any entity covered by the index.

The Markit indexes are classified by region and further classified (or divided) by credit quality. The two most commonly traded regions are North America and Europe. North American indexes are identified by the symbol CDX, and European, Asian, and Australian indexes are identified as iTraxx. Within each geographic category are investment-grade and high-yield indexes. The former are identified as CDX IG and iTraxx Main, each comprising 125 entities. The latter are identified as CDX HY, consisting of 100 entities, and iTraxx Crossover, consisting of up to 75 high-yield entities. Investment-grade index CDS are typically quoted in terms of spreads, whereas high-yield index CDS are quoted in terms of prices. Both types of products use standardized coupons. All CDS indexes are equally weighted. Thus, if there are 125 entities, the settlement on one entity is 1/125 of the notional. (Note that some confusion might arise from quoting certain CDS as prices and some as spreads, but keep in mind that the bond market quotes bonds often as prices and sometimes as yields. For example, a Treasury bond can be described as having a price of 120 or a yield of 2.68%. Both terms, combined with the other characteristics of the bond, imply the same concept.)

Markit updates the components of each index every six months by creating new series while retaining the old series. The latest-created series is called the **on-the-run** series, whereas the older series are called **off-the-run** series. When an investor moves from one series to a new one, the move is called a **roll**. When an entity within an index defaults, that entity is removed from the index and settled as a single-name CDS based on its relative proportion in the index. The index then moves forward with a smaller notional.

Index CDS are typically used to take positions on the credit risk of the sectors covered by the indexes as well as to protect bond portfolios that consist of or are similar to the components of the indexes. (An important reminder: When you *buy* a CDS index position, you are *long the credit exposure*, but when you *buy* a single-name CDS position, you have *bought credit protection*. To avoid confusion, we do not talk about buying and selling CDS herein but focus on the desired exposure, using the terms *buy protection* and *sell protection*.)

Standardization is generally undertaken to increase trading volume, which is somewhat limited in the single-name market with so many highly diverse entities. With CDS indexes on standardized portfolios based on the credit risk of well-identified companies, market participants have responded by trading them in large volumes. Indeed, index CDS are typically more liquid than single-name CDS, with average daily trading volume several times that of single-name CDS.

EXAMPLE 3

Hedging and Exposure Using Index CDS

Assume that an investor sells $500 million of protection using the CDX IG index, which has 125 reference entities. Concerned about the creditworthiness of a few of the components, the investor hedges a portion of the credit risk in each. For Company A, he purchases $3 million of single-name CDS protection, and Company A subsequently defaults.

1. What is the investor's net notional exposure to Company A?

Solution:

The investor is long $4 million notional credit exposure ($500 million/125) through the index CDS and is short $3 million notional credit exposure through the single-name CDS. His net notional credit exposure is $1 million.

2. What proportion of his exposure to Company A has he hedged?

Solution:

He has hedged 75% of his exposure ($3 million out of $4 million).

3. What is the remaining notional on his index CDS trade?

Solution:

His index CDS has $496 million remaining notional credit exposure ($500 million original notional minus the $4 million notional related to Company A, which is no longer in the index).

Market Characteristics

Credit default swaps trade in the over-the-counter market. To better understand this market, we will first review how credit derivatives and specifically CDS were started.

As financial intermediaries, banks draw funds from savings-surplus sectors, primarily consumers, and channel them to savings-deficit sectors, primarily businesses. Corporate lending is a core element of banking. When a bank makes a corporate loan, it assumes two primary risks. One is that the borrower will not repay principal and interest, and the other is that interest rates will change such that the return the bank is earning on its outstanding loans is less than the rate available on comparable instruments in the marketplace. The former is called **credit risk** or **default risk**, and the latter is called **interest rate risk**. There are many ways to manage interest rate risk. Until around the mid-1990s, credit risk was largely managed using traditional methods—such as analysis of the borrower, its industry, and the macroeconomy—as well as control methods, such as credit limits, monitoring, and collateral. In effect, the only defenses against credit risk were to not make a loan, to lend but require collateral (the value of which is also at risk), or to lend and closely monitor the borrower, hoping that any problems could be foreseen and dealt with before a default occurred.

Around 1995, credit derivatives were created to provide a new and potentially more effective method of managing credit risk. They allow credit risk to be transferred from the lender to another party. In so doing, they facilitate the separation of interest rate risk from credit risk. Banks can then provide their most important service—lending—knowing that the credit risk can be transferred to another party if so desired. This ability to easily transfer credit risk allows banks to greatly expand their loan business. Given that lending is such a large and vital component of any economy, credit derivatives facilitate economic growth and have expanded to cover, and indeed are primarily focused on, the short-, intermediate-, and long-term bond markets. In fact, credit derivatives are more effective in the bond market, in which terms and conditions are far more standard, than in the bank loan market. Of the four types of credit derivatives, credit default swaps have clearly established themselves as the most widely used instrument. Indeed, in today's markets CDS are nearly the only credit derivative used to any great extent. CDS transactions are executed in the over-the-counter market by phone, instant message, or the Bloomberg message service. Trade information is reported to the **Depository Trust and Clearinghouse Corporation**, which is a US-headquartered entity providing post-trade clearing, settlement, and information services for many kinds of securities. Regulations require the central clearing of many CDS contracts, meaning that parties will send their contracts through clearinghouses that collect and distribute payments and impose margin requirements, as well as mark positions to market. Central clearing of CDS has risen dramatically since 2010. Currently, slightly more than half of all CDS are centrally cleared, up from just 10% in 2010.

The CDS market today is considerably smaller than it was prior to the 2008 financial crisis. The Bank for International Settlements reported that as of December 2019, the gross notional amount of CDS was about $7.6 trillion with a market value of $199 billion. (For comparison, the notional amounts for interest rate contracts—forward rate agreements, swaps, options—as of December 2019 was about $449 trillion.) As of December 2007, CDS gross notional was $57.9 trillion, nearly 8 times larger.

More than 90% of all CDS market activity is now derived from trading in five major CDS indexes: iTraxx Europe, iTraxx Europe Crossover, iTraxx Europe Senior Financials, CDX IG, and CDS HY.

BASICS OF VALUATION AND PRICING

4

☐ explain the principles underlying and factors that influence the market's pricing of CDS

Derivatives are typically priced by solving for the cost of a position that fully offsets the underlying exposure and earns the risk-free rate. In the context of CDS, this "price" is the CDS spread or upfront payment for a particular coupon rate under the contract. Although CDS are referred to as "swaps," they in fact resemble options because of the contingent nature of the payment made by the protection seller to the protection buyer if a credit event occurs as established by the ISDA Determinations Committee as outlined above.

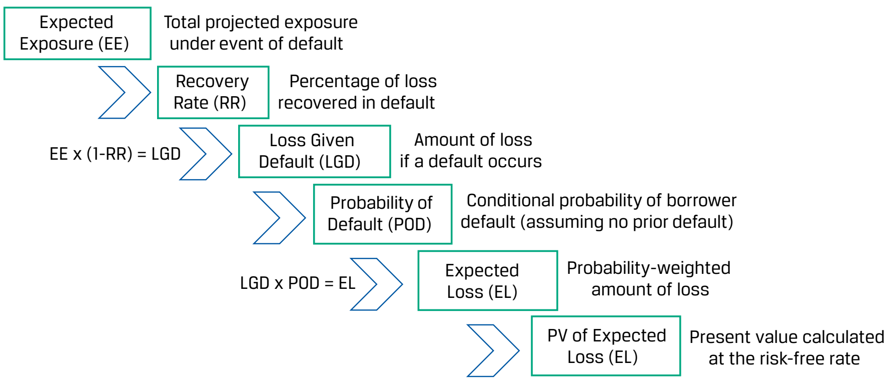
Unlike conventional derivative instruments, the CDS settlement amount under a credit event as declared by the ISDA Determinations Committee is far less clear than for derivatives whose underlying involves actively traded assets, such as equities, interest rates, or currencies. Credit does not "trade" in the traditional sense but, rather, exists implicitly within the bond and loan market. The unique debt structure and composition of each CDS reference entity adds to the complexity of establishing the basis between a CDS contract and a specific outstanding bond or loan.

The details of credit derivative models are beyond the scope of this reading, but it is important for investment industry analysts to have a thorough understanding of the factors that determine CDS pricing.

Basic Pricing Concepts

In our earlier coverage of credit strategies, we established that the credit valuation adjustment (CVA) may be thought of as the present value of credit risk for a loan, bond, or derivative obligation. In principle, the CVA should, therefore, be a reasonable approximation for the CDS hedge position outlined previously that would leave an investor with a risk-free rate of return. Exhibit 2 summarizes the CVA calculation for a financial exposure.

Exhibit 2: Credit Valuation Adjustment



CVA = Σ (PV of Expected Loss)

CVA is a function of expected exposure (EE), recovery rate, loss given default, the **probability of default** (POD) to arrive at an expected loss (EL), and a discount factor to arrive at the present value of expected loss.

Considering each of these CVA components in turn, the expected exposure reflects the notional value of the underlying CDS contract. Recall that the recovery rate is the percentage of loss recovered from a bond in default, whereas the loss given default is a function of the loss severity multiplied by the exposure amount.

The probability of default is a key element of CDS pricing that may be illustrated using a simple example. Consider a one-period CDS swap with no upfront payment where we ignore the time value of money and assume that default is possible only at maturity. The fair price of CDS protection for this period for a given borrower may be estimated as

$$\text{CDS spread} \approx (1 - \text{RR}) \times \text{POD}.$$

For example, if the probability of default is 2% and the recovery rate is 60%, the estimated CDS spread for the period would be 80 bps for the period. Assuming a $100 notional contract value and a period of a year, the CDS contract fair value would be (the present value of) $0.80.

It is important to note that the POD is a conditional probability over time. That is, assuming a two-period case, the probability of default in Period 2 is contingent on "surviving" to (i.e., not defaulting by) the end of Period 1. Note that we simplify the analysis by assuming discrete times of potential default versus the continuous time assumption common in CDS pricing models.

For example, consider a two-year, 5%, $1,000 loan with one interest payment of $50 due in one year and final interest and principal of $1,050 due in two years. Assume further that we estimate a 2% chance of defaulting on the first payment and a 4% chance of defaulting on the second payment. To calculate the POD over the life of the loan, we first determine the **probability of survival** (POS) for Period 1. The POS is 0.98 (100% minus the 2% POD at T_1) multiplied by 0.96 (100% minus the 4% POD at T_2), approximately 94.08%. Thus, the POD over the life of the loan is 100% − 94.08% = 5.92%.

This conditional probability of default is also known as the **hazard rate**, as described in an earlier reading. The hazard rate is the probability that an event will occur *given that it has not already occurred*.

Now consider another possibility, a 10-year bond with an equivalent hazard rate of 2% each year. Suppose we want to know the probability that the borrower will not default during the entire 10-year period. The probability that a default will occur at some point during the 10 years is one minus the probability of no default in 10 years. The probability of no default in 10 years is $0.98 \times 0.98 \ldots 0.98 = 0.98^{10} = 0.817$. Thus, the probability of default is $1 - 0.817 = 0.183$, or 18.3%. This somewhat simplified example illustrates how a low probability of default in any one period can turn into a surprisingly high probability of default over a longer period of time. Note that we have simplified the analysis by assuming a constant hazard rate, which may not be the case in practice.

EXAMPLE 4

Hazard Rate and Probability of Survival

Assume that a company's hazard rate is a constant 8% per year, or 2% per quarter. An investor sells five-year CDS protection on the company with the premiums paid quarterly over the next five years.

1. What is the probability of survival for the first quarter?

Solution:

The probability of survival for the first quarter is 98% (100% minus the 2% hazard rate).

2. What is the conditional probability of survival for the second quarter?

Solution:

The conditional probability of survival for the second quarter is also 98%, because the hazard rate is constant at 2%. In other words, *conditional on the company having survived the first quarter*, there is a 2% probability of default in the second quarter.

3. What is the probability of survival through the second quarter?

Solution:

The probability of survival through the second quarter is 96.04%. The probability of survival through the first quarter is 98%, and the conditional probability of survival through the second quarter is also 98%. The probability of survival through the second quarter is thus 98% × 98% = 96.04%. Alternatively, 1 – 96.04% = 3.96% is the probability of default sometime during the first two quarters.

Understanding the concept of pricing a CDS is facilitated by recognizing that there are essentially two sides, or legs, of a contract. There is the **protection leg**, which is the contingent payment that the credit protection seller may have to make to the credit protection buyer, and the **premium leg**, which is the series of payments the credit protection buyer promises to make to the credit protection seller. Exhibit 3 provides an illustration of the process.

Exhibit 3: Determination of CDS Protection vs. Premium Legs

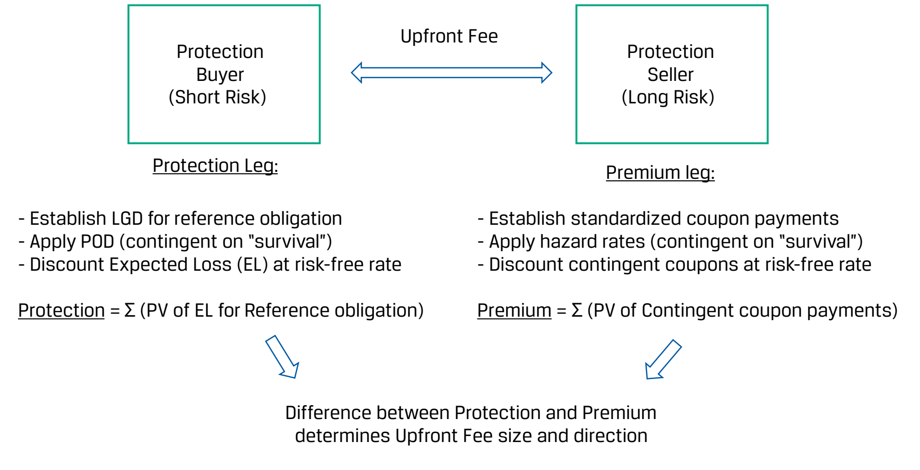


Exhibit 3 shows the upfront payment as the difference in value of the protection and premium legs. The party with a claim on the greater present value must pay the difference at the initiation date of the contract:

Upfront payment = PV (Protection leg) – PV (Premium leg).

If the result is greater (less) than zero, the protection buyer (seller) pays the protection seller (buyer). Actual CDS pricing and valuation models are more mathematically complex but are based on this conceptual framework.

The Credit Curve and CDS Pricing Conventions

The credit spread of a debt instrument is the rate in excess of a market reference rate that investors expect to receive to justify holding the instrument. The reference rate may itself contain some credit risk, as it reflects the rate at which commercial banks lend to one another. The credit spread can be expressed roughly as the probability of default multiplied by the loss given default, with LGD in terms of a percentage. The credit spreads for a range of maturities of a company's debt make up its **credit curve**. The credit curve is somewhat analogous to the term structure of interest rates, which is the set of rates on default-free debt over a range of maturities, but the credit curve applies to non-government borrowers and incorporates credit risk into each rate.

The CDS market for a given borrower is integrated with the credit curve of that borrower. In fact, given the evolution and high degree of efficiency of the CDS market, the credit curve is essentially determined by the CDS rates. The curve is affected by a number of factors, a key one of which is the set of aforementioned hazard rates. A constant hazard rate will tend to flatten the credit curve. Upward-sloping credit curves imply a greater likelihood of default in later years, whereas downward-sloping credit curves imply a greater probability of default in the earlier years. Downward-sloping curves are less common and often a result of severe near-term stress in the financial markets. The credit curve would not be completely flat even if the hazard rates are constant, because of discounting. For example, a company issuing 5- and 10-year zero-coupon bonds could have equally likely probabilities of default and hence equal expected payoffs. The present values of the payoffs are not the same, however, and so the discount rates that equate the present value to the expected payoffs will not be the same.

EXAMPLE 5

Change in Credit Curve

A company's 5-year CDS trades at a credit spread of 300 bps, and its 10-year CDS trades at a credit spread of 500 bps.

1. The company's 5-year spread is unchanged, but the 10-year spread widens by 100 bps. Describe the implication of this change in the credit curve.

Solution:

This change implies that although the company is not any riskier in the short term, its longer-term creditworthiness is less attractive. Perhaps the company has adequate liquidity for the time being, but after five years it must begin repaying debt or it will be expected to have cash flow difficulties.

2. The company's 10-year spread is unchanged, but the 5-year spread widens by 500 bps. Describe the implication of this change in the credit curve.

Solution:

This change implies that the company's near-term credit risk is now much greater. In fact, the probability of default will decrease if the company can survive for the next five years. Perhaps the company has run into liquidity issues that must be resolved soon, and if not resolved, the company will default.

CDS Pricing Conventions

With corporate bonds, we typically refer to their values in terms of prices or spreads. The spread is a more informative measure than price. A high-yield bond can be offered with a coupon equal to its yield and, therefore, a price of par value. An investment-grade bond with the same maturity can likewise be offered with a coupon equal to its yield, and therefore, its price is at par. These two bonds would have identical prices at the offering date, and their prices might even be close through much of their lives, but they are quite different bonds. Focusing on their prices would, therefore, provide little information. Their spreads are much more informative. With a market reference rate or the risk-free rate as a benchmark, investors can get a sense for the amount of credit risk implied by their prices, maturities, and coupons. The same is true for CDS. Although CDS have their own prices, their spreads are far more informative.

The reference entity will not necessarily have outstanding debt with credit spreads matching the 1% or 5% standardized coupons conventionally used in CDS contracts. Therefore, the present value of the promised payments from the credit protection buyer to the credit protection seller will most likely be different than the present value of the coupons on the reference entity's debt. The present value difference is the upfront premium paid from one party to the other.

Present value of credit spread = Upfront premium + Present value of fixed coupon.

A good rough approximation used in the industry is that the upfront premium is

Upfront premium ≈ (Credit spread – Fixed coupon) × Duration.

The upfront premium must ultimately be converted to a price, which is done by subtracting the percentage premium from 100.

Upfront premium % = 100 – Price of CDS in currency per 100 par.

Note that the duration used here is effective duration, since the cash flows arising from the coupon leg of the CDS are uncertain because they are contingent on the reference entity not defaulting.

EXAMPLE 6

Premiums and Credit Spreads

1. Assume a high-yield company's 10-year credit spread is 600 bps and the duration of the CDS is 8 years. What is the approximate upfront premium required to buy 10-year CDS protection? Assume high-yield companies have 5% coupons on their CDS.

Solution:

To buy 10-year CDS protection, an investor would have to pay a 500 bp coupon plus the present value of the difference between that coupon and the current market spread (600 bps). In this case, the upfront premium would be approximately 100 bps × 8 (duration), or 8% of the notional.

2. Imagine an investor sold five-year protection on an investment-grade company and had to pay a 2% upfront premium to the buyer of protection. Assume the duration of the CDS to be four years. What are the company's credit spreads and the price of the CDS per 100 par?

Solution:

The value of the upfront premium is equal to the premium (–2%) divided by the duration (4), or –50 bps. The sign of the upfront premium is negative because the seller is paying the premium rather than receiving it. The credit spread is equal to the fixed coupon (100 bps) plus the upfront premium, amortized over the duration of the CDS (–50 bps), or 50 bps. As a reminder, because the company's credit spread is less than the fixed coupon, the protection seller must pay the upfront premium to the protection buyer. The price in currency would be 100 minus the upfront premium, but the latter is negative, so the price is 100 – (–2) = 102.

Valuation Changes in CDS during Their Lives

As with any traded financial instrument, a CDS has a value that fluctuates during its lifetime. That value is determined in the competitive marketplace. Market participants constantly assess the current credit quality of the reference entity to determine its current value and (implied) credit spread. Clearly, many factors can change over the life of the CDS. By definition, the duration shortens through time. Likewise, the probability of default, the expected loss given default, and the shape of the credit curve will all change as new information is received. The exact valuation procedure of the CDS is precisely the same as it is when the CDS is first issued and simply incorporates the new inputs. The new market value of the CDS reflects gains and losses to the two parties.

Consider the following example of a five-year CDS with a fixed 1% coupon. The credit spread on the reference entity is 2.5%. In promising to pay 1% coupons to receive coverage on a company whose risk justifies 2.5% coupons, the present value of the protection leg exceeds the present value of the payment leg. The difference is the upfront premium, which will be paid by the credit protection buyer to the credit protection seller. During the life of the CDS, assume that the credit quality of the reference entity improves, such that the credit spread is now 2.1%. Now, consider a newly created CDS with the same remaining maturity and 1% coupon. The present value of the payment leg would still be less than the present value of the protection leg, but the difference would be less than it was when the original CDS was created because the risk is now less. Logically, it should be apparent that for the original transaction, the seller has gained and the buyer has lost. The difference between the original upfront premium and the new value is the seller's gain and buyer's loss. A rough approximation of the change in value of the CDS for a given change in spread is as follows:

Profit or loss for the buyer of protection ≈ Change in spread in bps × Duration × Notional.

Alternatively, we might be interested in the CDS percentage price change, which is obtained as

% Change in CDS price = Change in spread in bps × Duration.

The percentage change in the price of a bond is approximately the change in its yield multiplied by its modified duration. For the CDS, the change in yield is analogous to the change in spread, measured in basis points. The duration of the CDS is analogous to the duration of the bond on which the CDS is written.

EXAMPLE 7

Profit and Loss from Change in Credit Spread

An investor buys $10 million of five-year protection, and the CDS contract has a duration of four years. The company's credit spread was originally 500 bps and widens to 800 bps.

1. Does the investor (credit protection buyer) benefit or lose from the change in credit spread?

Solution:

The investor owns protection and is therefore short the credit exposure. As the credit spread widens (the credit quality of the underlying deteriorates), the value of the credit protection she owns increases.

2. Estimate the CDS price change and estimated profit to the investor.

Solution:

The percentage price change is estimated as the change in spread (300 bps) multiplied by the duration (4), or 12%. The profit to the investor is 12% times the notional ($10 million), or $1.2 million.

Monetizing Gains and Losses

As with any financial instrument, changes in the price of a CDS give rise to opportunities to unwind the position and either capture a gain or realize a loss. This process is called **monetizing** a gain or loss. Keep in mind that the protection seller is effectively long the reference entity. He has entered into a contract to insure the debt of the reference entity, for which he receives a series of promised payments and possibly an upfront premium. He clearly benefits if the reference entity's credit quality improves because he continues to receive the same compensation but bears less risk. Using the opposite argument, the credit protection buyer benefits from a deterioration of the reference entity's credit quality. Thus, the credit protection seller is more or less long the company's bonds and the credit protection buyer is more or less short the company's bonds. As the company's credit quality changes through time, the market value of the CDS changes, giving rise to gains and losses for the CDS counterparties. The counterparties can realize those gains and losses by entering into new offsetting contracts, effectively selling their CDS positions to other parties.

Going back to the example in the previous section where the credit quality of the reference entity improved—the credit spread on the reference entity declined from 2.5% to 2.1%. The implied upfront premium on a new CDS that matches the terms of the original CDS with adjusted maturity is now the market value of the original CDS. The premium on the new CDS is smaller than that on the original CDS.

Now, suppose that the protection buyer in the original transaction wants to unwind her position. She would then enter into a new CDS as a protection seller and receive the newly calculated upfront premium. As we noted, this value is less than what he paid originally. Likewise, the protection seller in the original transaction could offset his position by entering into a new CDS as a protection buyer. He would pay an upfront premium that is less than what he originally received. The original protection buyer monetizes a loss, and the seller monetizes a gain. The transaction to unwind the CDS does not need to be done with the same original party, although doing so offers some advantages. Central clearing of CDS transactions facilitates the unwind transaction.

At this point, we have identified two ways of realizing a profit or loss on a CDS. One is to effectively exercise the CDS in response to a default. The other is to unwind the position by entering into a new offsetting CDS in the market. A third, less common method occurs if there is no default. A party can simply hold the position until expiration, at which time the credit protection seller has captured all of the premiums and has not been forced to make any payments, and the seller's obligation for any further payments is terminated. The spread of the CDS will go to zero, in much the same manner as a bond converges toward par as it approaches maturity.

The CDS seller clearly gains, having been paid to bear the risk of default that is becoming increasingly unlikely, and the CDS buyer loses. The buyer loses on the CDS because it paid premiums to receive protection in the event of a default, which did not occur. Although the CDS position itself is a loss, the buyer's overall position is not necessarily a loss. If the buyer is a creditor of the reference entity, the premium "loss" is no different than a homeowner's insurance premium payment on his house; he wouldn't consider that payment a loss simply because his house did not burn down.

5 APPLICATIONS OF CDS

☐ describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve

Credit default swaps, as demonstrated, facilitate the transfer of credit risk. As simple as that concept seems, there are many different circumstances under which CDS are used. In this section, we consider some applications of this instrument.

Any derivative instrument has two general uses. One is to exploit an expected movement in the underlying. The derivative typically requires less capital and is usually an easier instrument in which to create a short economic exposure as compared with the underlying. The derivatives market can also be more efficient, meaning that it can react to information more rapidly and have more liquidity than the market for the underlying. Thus, information or an expectation of movement in the underlying can often be exploited much more efficiently with the derivative than with the underlying directly.

The other trading opportunity facilitated by derivatives is in valuation differences between the derivative and the underlying. If the derivative is mispriced relative to the underlying, one can take the appropriate position in the derivative and an offsetting position in the underlying. If the valuation assessment is correct and other investors come to the same conclusion, the values of the derivative and underlying will converge, and the investor will earn a return that is essentially free of risk because the risk of the underlying has been hedged away by holding offsetting positions in the derivative and the underlying. Whether this happens as planned depends on both the efficiency of the market and the quality of the valuation model. Differences can also exist between the derivative and other derivatives on the same underlying.

These two general types of uses are also the major applications of CDS. We will refer to them as managing credit exposures, meaning the taking on or shedding of credit risk in light of changing expectations and/or valuation disparities. With valuation disparities, the focus is on differences in the pricing of credit risk in the CDS market relative to that of the underlying bonds.

Managing Credit Exposures

The most basic application of a CDS is to increase or decrease credit exposure. The most obvious such application is for a lender to buy protection to reduce its credit exposure to a borrower. For the seller of protection, the trade adds credit exposure. A lender's justification for using a CDS seems obvious. The lender may have assumed too much credit risk but does not want to sell the bond or loan because there can be significant transaction costs, because later it may want the bond or loan back, or because the market for the bond or loan is relatively illiquid. If the risk is temporary, it is almost always easier to temporarily reduce risk by using a CDS. Beyond financial institutions, any organization exposed to credit risk is potentially a candidate for using CDS.

The justification for selling credit protection is somewhat less obvious. The seller can be a CDS dealer, whose objective is to profit from making markets in CDS. A dealer typically attempts to manage its exposure by either diversifying its credit risks or hedging the risk by entering into a transaction with yet another party, such as by shorting the debt or equity of the reference entity, often accompanied by investment of the funds in a repurchase agreement, or repo. If the dealer manages the risk effectively, the risk assumed in selling the CDS is essentially offset when the payment for assuming the risk exceeds the cost of removing the risk. Achieving this outcome successfully requires sophisticated credit risk modeling.

Although dealers make up a large percentage of protection sellers, not all sellers are dealers. Consider that any bondholder is a buyer of credit and interest rate risk. If the bondholder wants only credit risk, it can obtain it by selling protection, which would require far less capital and incur potentially lower overall transaction costs than buying the bond. Moreover, the CDS can be more liquid than the bond, so the position can be unwound much more easily.

As noted, it is apparent why a party making a loan might want credit protection. Consider, however, that a party with no exposure to the reference entity might also purchase credit protection. Such a position is called a **naked credit default swap**, and it has resulted in some controversy in regulatory and political circles. In buying protection without owning the underlying, the investor is taking a position that the entity's credit quality will improve.

Some regulators and politicians believe it is inappropriate for a party with no exposure to a borrower to speculate that the borrower's financial condition will deteriorate. This controversy accelerated during the financial crisis of 2008–2009 because many investors bought protection without owning the underlying and benefited from the crisis.

The counterargument, however, is that elsewhere in the financial markets, such bets are made all of the time in the form of long puts, short futures, and short sales of stocks and bonds. These instruments are generally accepted as a means of protecting oneself against poor performance in the financial markets. Credit protection is also a means of protecting oneself against poor performance. In addition, proponents of naked CDS argue that they bring liquidity to the credit market, potentially providing more stability, not less. Nonetheless, naked CDS trading is banned in Europe for sovereign debt, although it is generally permitted otherwise.

CDS trading strategies, with or without naked exposure, can take several forms. An investor can choose to be long or short the credit exposure, as we have previously discussed. Alternatively, the party can be a credit protection seller on one reference entity and a credit protection buyer on a different entity. This is called a **long/short credit trade**. This transaction is a bet that the credit position of one entity will improve relative to that of another. The two entities might be related in some way or might produce substitute goods. For example, one might take a position that because of competition and changes in the luxury car industry, the credit quality of Daimler will improve and that of BMW will weaken, so selling protection on Daimler and buying protection on BMW would be appropriate. Similarly, an investor may undertake a long/short trade based on other factors, such as environmental, social, and governance (ESG) considerations. For instance, an investor may be concerned about a company's poor ESG-related practices and policies relative to another company. In this case, the investor could buy protection using the CDS of a company with weak ESG practices and policies and sell protection using the CDS of a company with strong ESG practices and policies. Example 8 provides a case study of ESG considerations in a long/short ESG trade.

EXAMPLE 8

Long/Short Trade with ESG Considerations

Overview

An analyst is evaluating two US apparel companies: Atelier and Trapp. Atelier is a large company that focuses on high-end apparel brands. It is profitable despite a high cost structure. Trapp is smaller and less profitable than Atelier. Trapp focuses on less expensive brands and strives to keep costs low. Both companies purchase their merchandise from suppliers all over the world. The analyst recognizes that apparel companies must maintain adequate oversight over their suppliers to control the risks of reputational damage and inventory disruptions. Supplier issues are particularly relevant for Atelier and Trapp following a recent fire that occurred at the factory of Global Textiles, a major supplier to both companies. The fire resulted in multiple casualties and unfavorable news headlines.

The analyst notices a significant difference in the way Atelier and Trapp approach ESG considerations. After the fire at its supplier, Atelier signed an "Accord on Fire and Building Safety," which is a legally binding agreement between global apparel manufacturers, retailers, and trade unions in the country where the fire occurred. After signing the accord, Atelier made a concerted effort to fix and enhance machinery in factories of its suppliers. Its objective was to improve workplace safety—notably, to reduce lost employee time due to factory incidents and the rate of factory accidents and fatalities.

Investors view Atelier's corporate governance system favorably because management interests and stakeholder interests are strongly aligned. Atelier's board of directors includes a high percentage of independent directors and is notably diverse. In contrast, Trapp's founder is the majority owner of the company and serves as CEO and chairman of the board of directors. Furthermore, Trapp's board is composed mainly of individuals who have minimal industry expertise. As a consequence, Trapp's board was unprepared to adequately respond to the Global Textiles fire. Given the lack of independence and expertise of Trapp's board, investors consider Trapp's corporate governance system to be poor. Because of its emphasis on low costs and reflecting its less experienced board, Trapp chose not to sign the accord.

Implications for CDS

Single-name CDS on both Atelier and Trapp are actively traded in the market, although Trapp's CDS is less liquid. Before the Global Textiles fire, five-year CDS for Trapp traded at a spread of 250 bps, compared to a spread of 150 bps for the five-year CDS for Atelier. The difference in spreads reflects Trapp's lower trading liquidity, perceived lower creditworthiness (primarily reflecting its smaller size and lower profitability), and hence higher default risk relative to Atelier.

After the Global Textiles fire, spreads on the CDS for all companies in the apparel sector widened considerably. Credit spreads for the five-year CDS on Atelier widened by 60 bps (to 210 bps), and credit spreads for the five-year CDS on Trapp widened by 75 bps (to 325 bps). The analyst believes that over the longer term, the implications of the fire at Global Textiles will be even more adverse for Trapp relative to Atelier. The analyst's view largely reflects Trapp's higher ESG-related risks, especially the perceived weaker safety in its factories and its weaker corporate governance system. In particular, the analyst believes that spreads of Trapp's CDS will remain wider than their pre-fire level of 250 bps, but Atelier's CDS spreads will return to their pre-fire level of 150 bps.

1. Describe how the analyst can use CDS to exploit the potential opportunity.

Solution

The analyst can try to exploit the potential opportunity by buying protection (shorting the credit) on Trapp using five-year CDS and selling protection (going long the credit) on Atelier using five-year CDS. This trade would reflect both the anticipated continuing adverse spreads for Trapp relative to the pre-fire level and the return of spreads for Atelier to their lower pre-fire levels. For example, assume Atelier's five-year CDS spread returns to 150 bps from 210 bps, but Trapp's five-year CDS spread narrows to just 300 bps from 325 bps. The difference in spreads between the two companies' CDS would have widened from 115 bps (325 bps − 210 bps) right after the factory fire occurred to 150 bps (300 bps − 150 bps). This 35 bp difference in spread would represent profit (excluding trading costs) to the analyst from the long/short trade.

Similar to a long/short trade involving individual entities (companies), an investor might also create a long/short trade using CDS indexes. For example, if the investor anticipates a weakening economy, she could buy protection using a high-yield CDS index and sell protection using an investment-grade CDS index. As high-yield spreads widen relative to investment-grade spreads, the trade would realize a profit. As another example, a trader expecting a strengthening in the Asian economy relative to the European economy could buy protection using a European CDS index and sell protection using an Asian CDS index. As Asia spreads narrow relative to European spreads, the trade would realize a profit.

Another type of long/short trade, called a **curve trade**, involves buying single-name or index protection at one maturity and selling protection on the same reference entity at a different maturity. Consider two CDS maturities, which we will call the short-term and the long-term to keep things simple. We will assume the more common situation of an upward-sloping credit curve, meaning that long-term CDS rates (and credit spreads) are higher than short-term rates. If the curve changes shape, it becomes either steeper or flatter. A steeper (flatter) curve means that long-term credit risk increases (decreases) relative to short-term credit risk. An investor who believes that long-term credit risk will increase relative to short-term credit risk (credit curve steepening) can buy protection by buying a long-term single-name CDS or selling a long-term CDS index and sell protection by selling a short-term single-name CDS or buying a short-term CDS index. In the short run, a curve-steepening trade is bullish. It implies that the short-term outlook for the reference entity is better than the long-term outlook. In the short run, a curve-flattening trade is bearish. It implies that the short-run outlook for the reference entity looks worse than the long-run outlook and reflects the expectation of near-term problems for the reference entity.

EXAMPLE 9

Curve Trading

An investor owns some intermediate-term bonds issued by a company and has become concerned about the risk of a near-term default, although he is not very concerned about a default in the long term. The company's two-year CDS currently trades at 350 bps, and the four-year CDS is at 600 bps.

1. Describe a potential curve trade that the investor could use to hedge the default risk.

Solution:

The investor anticipates a flattening credit curve for the reference company, with spreads rising at the shorter end of the curve. Thus, he would buy credit protection on the two year (buy the two-year single-name CDS) while selling credit protection further out on the curve (sell the four-year single-name CDS).

2. Explain why an investor may prefer to use a curve trade as a hedge against the company's default risk rather than simply buying protection on the reference entity.

Solution:

The long/short trade reduces the cost of buying near-term credit protection, with the cost of the credit protection offset by the premium received from selling protection further out on the curve. This works only as long as the investor's expectations about the relative risk of near- and longer-term default hold true.

Of course, there can be changes to the credit curve that take the form of simple shifts in the general level of the curve, whereby all spreads go up or down by roughly equal amounts. As with long-duration bonds relative to short-duration bonds, the values of longer-term CDS will move more than those of shorter-term CDS. As an example, a trader who believes that all spreads will go up will want to be a buyer of credit protection but will realize that longer-term CDS will move more than short-term CDS. Thus, she might want to buy protection at the longer part of the curve and hedge by selling protection at the shorter part of the curve. She will balance the sizes of the positions so that the volatility of the position she believes will gain in value will be more than that of the other position. If more risk is desired, she might choose to trade only the more volatile leg.

VALUATION DIFFERENCES AND BASIS TRADING

6

☐ describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments

Different investors will have different assessments of the price of credit risk. Such differences of opinion will lead to valuation disparities. Clearly, there can be only one appropriate price at which credit risk can be eliminated, but that price is not easy to determine. The party that has the best estimate of the appropriate price of credit risk can capitalize on its knowledge or ability at the expense of another party. Any such comparative advantage can be captured by trading the CDS against either the reference entity's debt or equity or derivatives on its debt or equity, but such trading is critically dependent on the accuracy of models that isolate the credit risk component of the return. The details of those models are left to CDS specialists, but it is important for candidates to understand the basic ideas.

The yield on the bond issued by the reference entity to a CDS contains a factor that reflects the credit risk. In principle, the amount of yield attributable to credit risk on the bond should be the same as the credit spread on a CDS. It is, after all, the compensation paid to the party assuming the credit risk, regardless of whether that risk is borne by a bondholder or a CDS seller. But there may be a difference in the credit risk compensation in the bond market and CDS market. This differential pricing can arise from mere differences of opinions, differences in models used by participants in the two markets, differences in liquidity in the two markets, and supply and demand conditions in the repo market, which is a primary source of financing for bond purchases. A difference in the credit spreads in these two markets is the foundation of a strategy known as a **basis trade**.

The general idea behind most basis trades is that any such mispricing is likely to be temporary and the spreads should return to equivalence when the market recognizes the disparity. For example, suppose the bond market implies a 5% credit risk premium whereas the CDS market implies a 4% credit risk premium. The trader does not know which is correct but believes these two rates will eventually converge. From the perspective of the CDS, its risk premium is too low relative to the bond credit risk premium. From the perspective of the bond, its risk premium is too high relative to the CDS market, which means its price is too low. So, the CDS market could be pricing in too little credit risk, and/or the bond market could be pricing in too much credit risk. Either market could be correct; it does not matter. The investor would buy the bond at a price that appears to overestimate its credit risk and, at the same time, buy credit protection at what appears to be an unjustifiably low premium, simultaneously hedging interest rate risk exposure with a duration strategy or interest rate derivatives. The risk is balanced because the default potential on the bond is protected by the CDS. If convergence occurs, the trade would capture the 1% differential in the two markets.

To determine the profit potential of such a trade, it is necessary to decompose the bond yield into the risk-free rate plus the funding spread plus the credit spread. The risk-free rate plus the funding spread is essentially the market reference rate. The credit spread is then the excess of the yield over the market reference rate and can be compared with the credit spread in the CDS market. If the spread is higher in the bond market than in the CDS market, it is said to be a negative basis. If the spread is higher in the CDS market than in the bond market, it is said to be a positive basis. Note that in practice, the above decomposition can be complicated by the existence of embedded options, such as with callable and convertible bonds or when the bond is not selling near par. Those factors would need to be accounted for in the calculations.

EXAMPLE 10

Bonds vs. Credit Default Swaps

An investor wants to be long the credit risk of a given company. The company's bond currently yields 6% and matures in five years. A comparable five-year CDS contract has a credit spread of 3.25%. The investor can borrow at MRR, which is currently 2.5%.

1. Calculate the bond's credit spread.

Solution:

The bond's credit spread is equal to the yield (6%) minus the market reference rate (2.5%). Therefore, the bond's credit spread is currently 3.5%.

2. Identify a basis trade that would exploit the current situation.

Solution:

The bond and CDS markets imply different credit spreads. Credit risk is cheap in the CDS market (3.25%) relative to the bond market (3.5%). The investor should buy protection in the CDS market at 3.25% and go long the bond, with its 3.5% credit spread, netting 25 bps.

Another type of trade using CDS can occur within the instruments issued by a single entity. Credit risk is an element of virtually every unsecured debt instrument or the capital leases issued by a company. Each of these instruments is priced to reflect the appropriate credit risk. Investors can use the CDS market to first determine whether any of these instruments is incorrectly priced relative to the CDS and then buy the cheaper one and sell the more expensive one. Again, there is the assumption that the market will adjust. This type of trading is much more complex, however, because priority of claims means that not all of the instruments pay off equally if default occurs.

EXAMPLE 11

Using CDS to Trade on a Leveraged Buyout

An investor believes that a company will undergo a leveraged buyout (LBO) transaction, whereby it will issue large amounts of debt and use the proceeds to repurchase all of the publicly traded equity, leaving the company owned by management and a few insiders.

1. Why might the CDS spread change?

Solution:

Taking on the additional debt will almost surely increase the probability of default, thereby increasing the CDS spread.

2. What equity-versus-credit trade might an investor execute in anticipation of such a corporate action?

Solution:

The investor might consider buying the stock and buying credit protection. Both legs will profit if the LBO occurs because the stock price will rise as the company repurchases all outstanding equity and the CDS price will rise as its spread widens to reflect the increased probability of default.

CDS indexes also create an opportunity for a type of arbitrage trade. If the cost of the index is not equivalent to the aggregate cost of the index components, an investor might go long the cheaper instrument and short the more expensive instrument. There is the implicit assumption that convergence will occur. If it does, the investor gains the benefit while basically having neutralized the risk. Transaction costs in this type of arbitrage trade can be quite significant and nullify the profit potential for all but the largest investors.

SUMMARY

- A credit default swap (CDS) is a contract between two parties in which one party purchases protection from another party against losses from the default of a borrower for a defined period of time.
- A CDS is written on the debt of a third party, called the reference entity, whose relevant debt is called the reference obligation, typically a senior unsecured bond.
- A CDS written on a particular reference obligation normally provides coverage for all obligations of the reference entity that have equal or higher seniority.
- The two parties to the CDS are the credit protection buyer, who is said to be short the reference entity's credit, and the credit protection seller, who is said to be long the reference entity's credit.
- The CDS pays off upon occurrence of a credit event, which includes bankruptcy, failure to pay, and, in some countries, involuntary restructuring.
- Settlement of a CDS can occur through a cash payment from the credit protection seller to the credit protection buyer as determined by the cheapest-to-deliver obligation of the reference entity or by physical delivery of the reference obligation from the protection buyer to the protection seller in exchange for the CDS notional.
- A cash settlement payoff is determined by an auction of the reference entity's debt, which gives the market's assessment of the likely recovery rate. The credit protection buyer must accept the outcome of the auction even though the ultimate recovery rate could differ.
- CDS can be constructed on a single entity or as indexes containing multiple entities. Bespoke CDS or baskets of CDS are also common.
- The fixed payments made from CDS buyer to CDS seller are customarily set at a fixed annual rate of 1% for investment-grade debt or 5% for high-yield debt.

- Valuation of a CDS is determined by estimating the present value of the payment leg, which is the series of payments made from the protection buyer to the protection seller, and the present value of the protection leg, which is the payment from the protection seller to the protection buyer in event of default. If the present value of the payment leg is greater than the present value of the protection leg, the protection buyer pays an upfront premium to the seller. If the present value of the protection leg is greater than the present value of the payment leg, the seller pays an upfront premium to the buyer.
- An important determinant of the value of the expected payments is the hazard rate, the probability of default given that default has not already occurred.
- CDS prices are often quoted in terms of credit spreads, the implied number of basis points that the credit protection seller receives from the credit protection buyer to justify providing the protection.
- Credit spreads are often expressed in terms of a credit curve, which expresses the relationship between the credit spreads on bonds of different maturities for the same borrower.
- CDS change in value over their lives as the credit quality of the reference entity changes, which leads to gains and losses for the counterparties, even though default may not have occurred or may never occur. CDS spreads approach zero as the CDS approaches maturity.
- Either party can monetize an accumulated gain or loss by entering into an offsetting position that matches the terms of the original CDS.
- CDS are used to increase or decrease credit exposures or to capitalize on different assessments of the cost of credit among different instruments tied to the reference entity, such as debt, equity, and derivatives of debt and equity.

PRACTICE PROBLEMS

The following information relates to questions 1-6

UNAB Corporation

On 1 January 20X2, Deem Advisors purchased a $10 million six-year senior unsecured bond issued by UNAB Corporation. Six months later (1 July 20X2), concerned about the portfolio's credit exposure to UNAB, Doris Morrison, the chief investment officer at Deem Advisors, buys $10 million protection on UNAB with a standardized coupon rate of 5%. The reference obligation of the CDS is the UNAB bond owned by Deem Advisors. UNAB adheres to the ISDA CDS protocols.

On 1 January 20X3, Morrison asks Bill Watt, a derivatives analyst, to assess the current credit quality of UNAB bonds and the value of Deem Advisors' CDS on UNAB debt. Watt gathers the following information on UNAB's debt issues currently trading in the market:

Bond 1: A two-year senior unsecured bond trading at 40% of par

Bond 2: A five-year senior unsecured bond trading at 50% of par

Bond 3: A five-year subordinated unsecured bond trading at 20% of par

With respect to the credit quality of UNAB, Watt makes the following statement:

"There is severe near-term stress in the financial markets, andUNAB's credit curve clearly reflects the difficult environment."

On 1 July 20X3, UNAB fails to make a scheduled interest payment on the outstanding subordinated unsecured obligation after a grace period; however, the company does not file for bankruptcy. Morrison asks Watt to determine if UNAB experienced a credit event and, if so, to recommend a settlement preference.

Kand Corporation

Morrison is considering purchasing protection on Kand Corporation debt to hedge the portfolio's position in Kand. She instructs Watt to determine if an upfront payment would be required and, if so, the amount of the premium. Watt presents the information for the CDS in Exhibit 1.

Exhibit 1: Summary Data for 10-year CDS on Kand Corporation

Credit spread	700 bps
Duration	7 years
Coupon rate	5%

Morrison purchases 10-year protection on Kand Corporation debt. Two months later the credit spread for Kand Corporation has increased by 200 bps. Morrison asks Watt to close out the firm's CDS position on Kand Corporation by entering into a new, offsetting contract.

Tollunt Corporation

Deem Advisors' chief credit analyst recently reported that Tollunt Corporation's five-year bond is currently yielding 7% and a comparable CDS contract has a credit spread of 4.25%. Since the current market reference rate is 2.5%, Watt has recommended executing a basis trade to take advantage of the pricing of Tollunt's bonds and CDS. The basis trade would consist of purchasing both the bond and the CDS contract.

1. If UNAB experienced a credit event on 1 July, Watt should recommend that Deem Advisors:

   A. prefer a cash settlement.

   B. prefer a physical settlement.

   C. be indifferent between a cash or a physical settlement.

2. According to Watt's statement, the shape of UNAB's credit curve is *most likely*:

   A. flat.

   B. upward-sloping.

   C. downward-sloping.

3. Should Watt conclude that UNAB experienced a credit event?

   A. Yes

   B. No, because UNAB did not file for bankruptcy

   C. No, because the failure to pay occurred on a subordinated unsecured bond

4. Based on Exhibit 1, the upfront premium as a percent of the notional for the CDS protection on Kand Corporation would be *closest* to:

   A. 2.0%.

   B. 9.8%.

   C. 14.0%.

5. If Deem Advisors enters into a new offsetting contract two months after purchasing protection on Kand Corporation, this action will *most likely* result in:

   A. a loss on the CDS position.

   B. a profit on the CDS position.

   C. neither a loss nor a profit on the CDS position.

6. If convergence occurs in the bond and CDS markets for Tollunt Corporation, a basis trade will capture a profit *closest* to:

   A. 0.25%.

   B. 1.75%.

   C. 2.75%.

The following information relates to questions 7-14

John Smith, a fixed-income portfolio manager at a €10 billion sovereign wealth fund (the Fund), meets with Sofia Chan, a derivatives strategist with Shire Gate Securities (SGS), to discuss investment opportunities for the Fund. Chan notes that SGS adheres to ISDA (International Swaps and Derivatives Association) protocols for credit default swap (CDS) transactions and that any contract must conform to ISDA specifications. Before the Fund can engage in trading CDS products with SGS, the Fund must satisfy compliance requirements.

Smith explains to Chan that fixed-income derivatives strategies are being contemplated for both hedging and trading purposes. Given the size and diversified nature of the Fund, Smith asks Chan to recommend a type of CDS that would allow the Fund to simultaneously fully hedge multiple fixed-income exposures.

Smith and Chan discuss opportunities to add trading profits to the Fund. Smith asks Chan to determine the probability of default associated with a five-year investment-grade bond issued by Orion Industrial. Selected data on the Orion Industrial bond are presented in Exhibit 1.

Exhibit 1: Selected Data on Orion Industrial Five-Year Bond

Year	Hazard Rate
1	0.22%
2	0.35%
3	0.50%
4	0.65%
5	0.80%

Chan explains that a single-name CDS can also be used to add profit to the Fund over time. Chan describes a hypothetical trade in which the Fund sells £6 million of five-year CDS protection on Orion, where the CDS contract has a duration of 3.9 years. Chan assumes that the Fund closes the position six months later, after Orion's credit spread narrowed from 150 bps to 100 bps.

Chan discusses the mechanics of a long/short trade. In order to structure a number of potential trades, Chan and Smith exchange their respective views on individual companies and global economies. Chan and Smith agree on the following outlooks.

Outlook 1: The European economy will weaken.

Outlook 2: The US economy will strengthen relative to that of Canada.

Outlook 3: The credit quality of electric car manufacturers will improve relative to that of traditional car manufacturers.

Chan believes US macroeconomic data are improving and that the general economy will strengthen in the short term. Chan suggests that a curve trade could be used by the Fund to capitalize on her short-term view of a steepening of the US credit curve.

Another short-term trading opportunity that Smith and Chan discuss involves the merger and acquisition market. SGS believes that Delta Corporation may make an unsolicited bid at a premium to the market price for all of the publicly traded shares of Zega, Inc. Zega's market capitalization and capital structure are comparable to Delta's; both firms are highly levered. It is anticipated that Delta will issue new equity along with 5- and 10-year senior unsecured debt to fund the acquisition, which will significantly increase its debt ratio.

7. To satisfy the compliance requirements referenced by Chan, the Fund is *most likely* required to:

   A. set a notional amount.

   B. post an upfront payment.

   C. sign an ISDA master agreement.

8. Which type of CDS should Chan recommend to Smith?

   A. CDS index

   B. Tranche CDS

   C. Single-name CDS

9. Based on Exhibit 1, the probability of Orion defaulting on the bond during the first three years is *closest* to:

   A. 1.07%.

   B. 2.50%.

   C. 3.85%.

10. To close the position on the hypothetical Orion trade, the Fund:

    A. sells protection at a higher premium than it paid at the start of the trade.

    B. buys protection at a lower premium than it received at the start of the trade.

    C. buys protection at a higher premium than it received at the start of the trade.

11. The hypothetical Orion trade generated an approximate:

    A. loss of £117,000.

    B. gain of £117,000.

    C. gain of £234,000.

12. Based on the three economic outlook statements, a profitable long/short trade would be to:

    A. sell protection using a Canadian CDX IG and buy protection using a US CDX IG.

    B. buy protection using an iTraxx Crossover and sell protection using an iTraxx Main.

    C. buy protection using an electric car CDS and sell protection using a traditional car CDS.

13. The curve trade that would *best* capitalize on Chan's view of the US credit curve

is to:

A. buy protection using a 20-year CDX and buy protection using a 2-year CDX.

B. buy protection using a 20-year CDX and sell protection using a 2-year CDX.

C. sell protection using a 20-year CDX and buy protection using a 2-year CDX.

14. A profitable equity-versus-credit trade involving Delta and Zega is to:

A. short Zega shares and buy protection on Delta using the 10-year CDS.

B. go long Zega shares and buy protection on Delta using 5-year CDS.

C. go long Delta shares and buy protection on Delta using 5-year CDS.

SOLUTIONS

1. A is correct. Deem Advisors would prefer a cash settlement. Deem Advisors owns Bond 2 (trading at 50% of par), which is worth more than the cheapest-to-deliver obligation (Bond 1, also a senior secured bond, trading at 40% of par). Based on the price of this cheapest-to-deliver security, the estimated recovery rate is 40%. Thus, Deem Advisors can cash settle for $6 million [= (1 – 40%) × $10 million] on its CDS contract and sell the bond it owns, Bond 2, for $5 million, for total proceeds of $11 million. If Deem Advisors were to physically settle the contract, only $10 million would be received, the face amount of the bonds, and it would deliver Bond 2.

   B is incorrect because if Deem Advisors were to physically settle the contract, it would receive only $10 million, which is less than the $11 million that could be obtained from a cash settlement. C is incorrect because Deem Advisors would not be indifferent between settlement protocols as the firm would receive $1 million more with a cash settlement in comparison to a physical settlement.

2. C is correct. A downward-sloping credit curve implies a greater probability of default in the earlier years than in the later years. Downward-sloping curves are less common and often are the result of severe near-term stress in the financial markets.

   A is incorrect because a flat credit curve implies a constant hazard rate (conditional probability of default). B is incorrect because an upward-sloping credit curve implies a greater probability of default in later years.

3. A is correct. UNAB experienced a credit event when it failed to make the scheduled coupon payment on the outstanding subordinated unsecured obligation. Failure to pay, a credit event, occurs when a borrower does not make a scheduled payment of principal or interest on outstanding obligations after a grace period, even without a formal bankruptcy filing.

   B is incorrect because a credit event can occur without filing for bankruptcy. The three most common credit events are bankruptcy, failure to pay, and restructuring.

   C is incorrect because a credit event (failure to pay) occurs when a borrower does not make a scheduled payment of principal or interest on *any* outstanding obligations after a grace period, even without a formal bankruptcy filing.

4. C is correct. An approximation for the upfront premium is (Credit spread – Fixed coupon rate) × Duration of the CDS. To buy 10-year CDS protection, Deem Advisors would have to pay an approximate upfront premium of 1,400 bps [(700 – 500) × 7], or 14% of the notional.

   A is incorrect because 200 bps, or 2%, is derived by taking the simple difference between the credit spread and the fixed coupon rate (700 – 500), ignoring the duration component of the calculation. B is incorrect because 980 bps, or 9.8%, is the result of dividing the credit spread by the fixed coupon rate and multiplying by the duration of the CDS [(700/500) × 7].

5. B is correct. Deem Advisors purchased protection and therefore is economically short and benefits from an increase in the company's spread. Since putting on the protection, the credit spread increased by 200 bps, and Deem Advisors realizes the profit by entering into a new, offsetting contract (sells protection to another party at a higher premium).

   A is incorrect because a decrease (not increase) in the spread would result in a loss for the credit protection buyer. C is incorrect because Deem Advisors, the credit protection buyer, would profit from an increase in the company's credit spread, not break even.

6. A is correct. A difference in credit spreads in the bond market and CDS market is the foundation of the basis trade strategy. If the spread is higher in the bond market than in the CDS market, it is said to be a negative basis. In this case, the bond credit spread is currently 4.50% (bond yield minus MRR) and the comparable CDS contract has a credit spread of 4.25%. The credit risk is cheap in the CDS market relative to the bond market. Since the protection and the bond were both purchased, if convergence occurs, the trade will capture the 0.25% differential in the two markets (4.50% – 4.25%).

   B is incorrect because the bond market implies a 4.50% credit risk premium (bond yield minus the market reference rate) and the CDS market implies a 4.25% credit risk premium. Convergence of the bond market credit risk premium and the CDS credit risk premium would result in capturing the differential, 0.25%. The 1.75% is derived by incorrectly subtracting MRR from the credit spread on the CDS (= 4.25% – 2.50%).

   C is incorrect because convergence of the bond market credit risk premium and the CDS credit risk premium would result in capturing the differential, 0.25%. The 2.75% is derived incorrectly by subtracting the credit spread on the CDS from the current bond yield (= 7.00% – 4.25%).

7. C is correct. Parties to CDS contracts generally agree that their contracts will conform to ISDA specifications. These terms are specified in the ISDA master agreement, which the parties to a CDS sign before any transactions are made. Therefore, to satisfy the compliance requirements referenced by Chan, the sovereign wealth fund must sign an ISDA master agreement with SGS.

8. A is correct. A CDS index (e.g., CDX and iTraxx) would allow the Fund to simultaneously fully hedge multiple fixed-income exposures. A tranche CDS will also hedge multiple exposures, but it would only partially hedge those exposures.

9. A is correct. Based on Exhibit 1, the probability of survival for the first year is 99.78% (100% minus the 0.22% hazard rate). Similarly, the probability of survival for the second and third years is 99.65% (100% minus the 0.35% hazard rate) and 99.50% (100% minus the 0.50% hazard rate), respectively. Therefore, the probability of survival of the Orion bond through the first three years is equal to $0.9978 \times 0.9965 \times 0.9950 = 0.9893$, and the probability of default sometime during the first three years is $1 - 0.9893$, or 1.07%.

10. B is correct. The trade assumes that £6 million of five-year CDS protection on Orion is initially sold, so the Fund received the premium. Because the credit spread of the Orion CDS narrowed from 150 bps to 100 bps, the CDS position will realize a financial gain. This financial gain is equal to the difference between the upfront premium received on the original CDS position and the upfront premium to be paid on a new, offsetting CDS position. To close the position and monetize this gain, the Fund should unwind the position by buying protection for a lower premium (relative to the original premium collected).

11. B is correct. The gain on the hypothetical Orion trade is £117,000, calculated as

follows.

Approximate profit = Change in credit spread (in bps) × Duration × Notional amount.

Approximate profit = (150 bps – 100 bps) × 3.9 × £6 million.

Approximate profit = 0.005 × 3.9 × £6 million.

= £117,000.

The Fund gains because it sold protection at a spread of 150 bps and closed out the position by buying protection at a lower spread of 100 bps.

12. B is correct. Based on Outlook 1, Chan and Smith anticipate that Europe's economy will weaken. In order to profit from this forecast, one would buy protection using a high-yield CDS index (e.g., iTraxx Crossover) and sell protection using an investment-grade CDS index (e.g., iTraxx Main).

13. B is correct. To take advantage of Chan's view of the US credit curve steepening in the short term, a curve trade will entail shorting (buying protection using) a long-term (20-year) CDX and going long (selling protection using) a short-term (2-year) CDX. A steeper curve means that long-term credit risk increases relative to short-term credit risk.

14. B is correct. The shares of Zega can be sold at a higher price as a result of the unsolicited bid in the market. If Delta Corporation issues significantly more debt, there is a higher probability that it may default. If the Fund sells protection on Delta now, the trade will realize a profit as credit spreads widen. An equity-versus-credit trade would be to go long (buy) the Zega shares and buy protection on Delta.