



**CFA Institute®**  
CFA Program

# **DERIVATIVES, ALTERNATIVE INVESTMENTS**

CFA® Program Curriculum  
**2024 • LEVEL 1 • VOLUME 5**

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**Glossary**

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# How to Use the CFA Program Curriculum

The CFA® Program exams measure your mastery of the core knowledge, skills, and abilities required to succeed as an investment professional. These core competencies are the basis for the Candidate Body of Knowledge (CBOK™). The CBOK consists of four components:

- A broad outline that lists the major CFA Program topic areas ([www.cfainstitute.org/programs/cfa/curriculum/cbok](http://www.cfainstitute.org/programs/cfa/curriculum/cbok))
- Topic area weights that indicate the relative exam weightings of the top-level topic areas ([www.cfainstitute.org/programs/cfa/curriculum](http://www.cfainstitute.org/programs/cfa/curriculum))
- Learning outcome statements (LOS) that advise candidates about the specific knowledge, skills, and abilities they should acquire from curriculum content covering a topic area: LOS are provided in candidate study sessions and at the beginning of each block of related content and the specific lesson that covers them. We encourage you to review the information about the LOS on our website ([www.cfainstitute.org/programs/cfa/curriculum/study-sessions](http://www.cfainstitute.org/programs/cfa/curriculum/study-sessions)), including the descriptions of LOS “command words” on the candidate resources page at [www.cfainstitute.org](http://www.cfainstitute.org).
- The CFA Program curriculum that candidates receive upon exam registration

Therefore, the key to your success on the CFA exams is studying and understanding the CBOK. You can learn more about the CBOK on our website: [www.cfainstitute.org/programs/cfa/curriculum/cbok](http://www.cfainstitute.org/programs/cfa/curriculum/cbok).

*The entire curriculum, including the practice questions, is the basis for all exam questions and is selected or developed specifically to teach the knowledge, skills, and abilities reflected in the CBOK.*

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## ERRATA

The curriculum development process is rigorous and includes multiple rounds of reviews by content experts. Despite our efforts to produce a curriculum that is free of errors, there are instances where we must make corrections. Curriculum errata are periodically updated and posted by exam level and test date online on the Curriculum Errata webpage ([www.cfainstitute.org/en/programs/submit-errata](http://www.cfainstitute.org/en/programs/submit-errata)). If you believe you have found an error in the curriculum, you can submit your concerns through our curriculum errata reporting process found at the bottom of the Curriculum Errata webpage.

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## DESIGNING YOUR PERSONAL STUDY PROGRAM

An orderly, systematic approach to exam preparation is critical. You should dedicate a consistent block of time every week to reading and studying. Review the LOS both before and after you study curriculum content to ensure that you have mastered the

applicable content and can demonstrate the knowledge, skills, and abilities described by the LOS and the assigned reading. Use the LOS self-check to track your progress and highlight areas of weakness for later review.

Successful candidates report an average of more than 300 hours preparing for each exam. Your preparation time will vary based on your prior education and experience, and you will likely spend more time on some study sessions than on others.

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## CFA INSTITUTE LEARNING ECOSYSTEM (LES)

Your exam registration fee includes access to the CFA Program Learning Ecosystem (LES). This digital learning platform provides access, even offline, to all of the curriculum content and practice questions and is organized as a series of short online lessons with associated practice questions. This tool is your one-stop location for all study materials, including practice questions and mock exams, and the primary method by which CFA Institute delivers your curriculum experience. The LES offers candidates additional practice questions to test their knowledge, and some questions in the LES provide a unique interactive experience.

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## PREREQUISITE KNOWLEDGE

The CFA® Program assumes basic knowledge of Economics, Quantitative Methods, and Financial Statements as presented in introductory university-level courses in Statistics, Economics, and Accounting. CFA Level I candidates who do not have a basic understanding of these concepts or would like to review these concepts can study from any of the three pre-read volumes.

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## FEEDBACK

Please send any comments or feedback to [info@cfainstitute.org](mailto:info@cfainstitute.org), and we will review your suggestions carefully.

# Derivatives



## LEARNING MODULE

## 1

## Derivative Instrument and Derivative Market Features

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	define a derivative and describe basic features of a derivative instrument
<input type="checkbox"/>	describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets

## INTRODUCTION

1

Earlier lessons described markets for financial assets related to equities, fixed income, currencies, and commodities. These markets are known as **cash markets** or **spot markets** in which specific assets are exchanged at current prices referred to as **cash prices** or **spot prices**. Derivatives involve the future exchange of cash flows whose value is derived from or based on an underlying value. The following lessons define and describe features of derivative instruments and derivative markets.

### LEARNING MODULE OVERVIEW



- A derivative is a financial contract that derives its value from the performance of an underlying asset, which may represent a firm commitment or a contingent claim.
- Derivative markets expand the set of opportunities available to market participants beyond the cash market to create or modify exposure to an underlying.
- The most common derivative underlyings include equities, fixed income and interest rates, currencies, commodities, and credit.
- Over-the-counter (OTC) derivative markets involve the initiation of customized, flexible contracts between derivatives end users and financial intermediaries.
- Exchange-traded derivatives (ETDs) are standardized contracts traded on an organized exchange, which requires collateral on deposit to protect against counterparty default.

- For derivatives that are centrally cleared, a central counterparty (CCP) assumes the counterparty credit risk of the derivative counterparties and provides clearing and settlement services.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements does **not** provide an argument for using a derivative instrument?
  - A. Issuers may offset the financial market exposure associated with a commercial transaction.
  - B. Derivatives typically have lower transaction costs than transacting directly in the underlying.
  - C. Large exposures to an underlying can be created with derivatives for a similar cash outlay.

**Solution:**

C is correct. Derivative contracts create an exposure to the underlying with a small cash outlay, so this is the statement that does not provide an argument for using a derivative instrument. Statements A and B are statements that are valid arguments for using derivatives.

2. Which of the following words makes the following statement correct?  
Market participants use derivative agreements to exchange cash flows in the future based on a(n) \_\_\_\_\_.
  - A. Underlying
  - B. Option
  - C. Hedge

**Solution:**

A is correct. Market participants use derivative agreements to exchange cash flows in the future based on an *underlying*. B is incorrect because *option* refers to a specific derivative contract type. C is incorrect because *hedge* refers to a specific purpose of using a derivative contract.

3. Which of the following is a significant difference between exchange-traded derivative (ETD) and over the counter (OTC) derivative contracts?
  - A. ETDs create counterparty credit risk for derivative users, while OTC derivatives do not.
  - B. ETDs are standardized contracts, while OTC derivatives are customized.
  - C. ETDs have higher transaction costs compared to OTC derivatives.

**Solution:**

B is correct. Exchanges standardize contracts to facilitate trading volume. However, users often require specific customized features, and the OTC market can accommodate these needs. A is incorrect because exchanges bear the counterparty credit risk of derivatives. C is incorrect because ETDs have lower transaction costs compared to OTC derivatives.

4. If a corporate issuer enters into a centrally cleared OTC derivative contract, which of the following risks is likely of most concern to the issuer and other participants in this market?

- A. Interest rate risk
- B. Counterparty credit risk
- C. Systemic risk

**Solution:**

C is correct. Because all the credit risk is taken on by the CCP, all participants in this market are most concerned that the CCP is able to satisfy its obligations to all contracts. A is incorrect because interest rate risk is an underlying risk that can be hedged or managed with certain OTC derivative contracts. B is incorrect because the CCP assumes the credit risk from all parties to the contracts.

## DERIVATIVE FEATURES

# 2

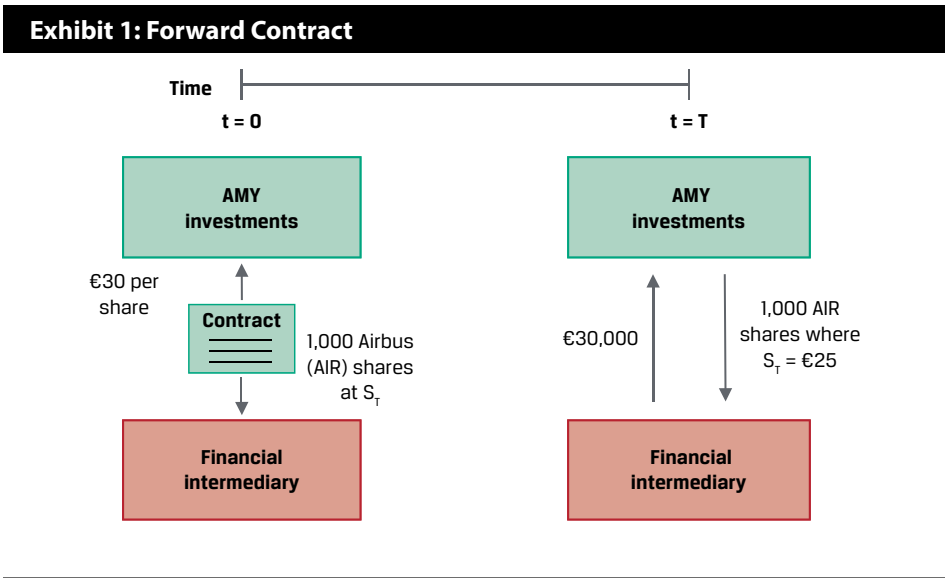


define a derivative and describe basic features of a derivative instrument

### Definition and Features of a Derivative

A **derivative** is a financial instrument that derives its value from the performance of an underlying asset. The asset in a derivative is called the **underlying**. The underlying may not be an individual asset but rather a group of standardized assets or variables, such as interest rates or a credit index.

Market participants use derivative agreements to exchange cash flows in the future based on an underlying value. For example, Exhibit 1 shows the one-time future exchange of publicly traded shares of stock at a fixed price in a derivative known as a **forward contract**.



A derivative does not directly pass through the returns of the underlying but transforms the performance of the underlying. In Exhibit 1, AMY Investments agrees today ( $t = 0$ ) to deliver 1,000 shares of Airbus (AIR) at a fixed price of €30 per share on a future date ( $t = T$ ), which in our example is in six months. The forward contract allows AMY to transfer the price risk of underlying AIR shares to a second party, or a **counterparty**, by entering into this derivative contract. If the spot price of AIR ( $S_T$ ) is €25 per share at time  $T$  in six months, AMY will either receive €30,000 from its counterparty, a financial intermediary, for 1,000 AIR shares now worth just €25,000, or simply settle with the intermediary the €5,000 difference in cash. Derivative transactions usually involve at least one financial intermediary as a counterparty. As we will see later, **counterparty credit risk**, or the likelihood that a counterparty is unable to meet its financial obligations under the contract, is an important consideration for these instruments.

A **derivative contract** is a legal agreement between counterparties with a specific **maturity**, or length of time until the closing of the transaction, or **settlement**. The buyer of a derivative enters a contract whose value changes in a way similar to a **long** position in the underlying, and the seller has exposure similar to a **short** position. The **contract size** (sometimes referred to as notional principal or amount) is agreed upon at the outset and may remain constant or change over time.

Exhibit 1 is an example of a **stand-alone derivative**, a distinct derivative contract, such as a derivative on a stock or bond. An **embedded derivative** is a derivative within an underlying, such as a callable, puttable, or convertible bond. Exhibit 2 provides a sample term sheet that includes key features of AMY Investment’s stand-alone forward contract with a financial intermediary.

Exhibit 2: Sample Forward Contract Term Sheet

<b>Contract Type:</b>	<b>Forward Transaction Term Sheet</b>	
Firm commitment or contingent right to exchange future cash flows		
<b>Maturity:</b>	<b>Start Date:</b>	[Spot start]
Final date upon which payment or settlement occurs	<b>Maturity Date:</b>	[Six months from Start Date]



**Derivative Features**

<b>Counterparties:</b> Legal entities entering the derivative contract	<b>Forward Purchaser:</b>	[Financial Intermediary]
	<b>Forward Seller:</b>	AMY Investments
<b>Underlying:</b> Reference asset or variable used as source for contract value	<b>Forward Delivery:</b>	1,000 shares of Airbus (AIR) common stock traded on the Frankfurt Stock Exchange
<b>Contract Size:</b> Amount(s) used for calculation to price and value the derivative		
<b>Underlying Price:</b> Pre-agreed price for commitment or contingent claim settlement	<b>Forward Price:</b>	€30 per share
<b>Contract Details</b>	<b>Business Days:</b>	Frankfurt
	<b>Documentation:</b>	ISDA Agreement and credit terms acceptable to both parties

The derivative between AMY and the financial intermediary is a **firm commitment**, in which a pre-determined amount is agreed to be exchanged at settlement. Firm commitments include forward contracts, futures contracts, and **swaps** involving a periodic exchange of cash flows. Another type of derivative is a **contingent claim**, in which one of the counterparties determines whether and when the trade will settle. An **option** is the primary contingent claim.

Derivative markets expand the set of opportunities available to market participants to create or modify exposure to an underlying in several ways:

- Investors can sell short to benefit from an expected decline in the value of the underlying.
- Investors may use derivatives as a tool for portfolio diversification.
- Issuers may offset the financial market exposure associated with a commercial transaction.
- Market participants may create large exposures to an underlying with a relatively small cash outlay.
- Derivatives typically have lower transaction costs and are often more liquid than underlying spot market transactions.

Issuers and investors use derivatives to increase or decrease financial market exposures. For example, use of a derivative to offset or neutralize existing or anticipated exposure to an underlying is referred to as **hedging**, with the derivative itself commonly described as a **hedge** of the underlying transaction.

**QUESTION SET****Derivative Features**

1. Identify one reason why an issuer may use a derivative instrument.

**Solution:**

An issuer may use a derivative to offset the financial market exposure associated with a commercial transaction. An issuer may also use a derivative to offset or neutralize existing or anticipated exposure to an underlying.

2. Identify which example corresponds to each of the following stand-alone or embedded derivative contract types:

A. Firm commitment	1. Callable bond
B. Contingent claim	2. Fixed-price natural gas delivery contract
C. Neither a firm commitment nor a contingent claim exchange-traded fund (ETF)	3. Purchase of a FTSE 100 Index

**Solution:**

1. B is correct. A callable bond is an example of an embedded derivative within an underlying, which is a contingent claim.
2. A is correct. A fixed-price gas delivery contract is an example of a contract, which is a firm commitment with natural gas as the underlying.
3. C is correct. A FTSE 100 Index exchange-traded fund (ETF) is neither a firm commitment nor a contingent claim but rather an example of a cash or spot market transaction.

3. Determine the correct answers to fill in the blanks: Equities are an example of a derivative \_\_\_\_\_, and a \_\_\_\_\_ is a legal entity entering a derivative contract.

**Solution:**

Equities are an example of a derivative *underlying*, and a *counterparty* is a legal entity entering a derivative contract.

4. Describe the use of a derivative for hedging purposes.

**Solution:**

Use of a derivative for hedging purposes involves offsetting or neutralizing an existing or anticipated exposure to an underlying, referred to as hedging.

5. Explain the settlement of a forward contract.

**Solution:**

A forward contract is a firm commitment. This contract results in a settlement payment on the maturity date equal to the difference between the current market price and a pre-agreed forward price.

### 3

## DERIVATIVE UNDERLYINGS



define a derivative and describe basic features of a derivative instrument

Derivatives are typically grouped by the underlying from which their value is derived. A derivative contract may reference more than one underlying. The most common derivative underlyings include equities, fixed income and interest rates, currencies, commodities, and credit.

## Equities

Equity derivatives usually reference an individual stock, a group of stocks, or a stock index, such as the FTSE 100. Options are the most common derivatives on individual stocks. Index derivatives are commonly traded as options, forwards, futures, and swaps.

Index swaps, or equity swaps, allow the investor to pay the return on one stock index and receive the return on another index or interest rate. An investment manager can use index swaps to increase or reduce exposure to an equity market or sector without trading the individual shares. These swaps are widely used in top-down asset allocation strategies. Finally, options, futures, and swaps are available based upon the realized *volatility* of equity index prices over a certain period. These contracts allow market participants to manage the risk, or dispersion, of price changes separately from the direction of equity price changes.

Options on individual stocks are purchased and sold by investors and frequently used by issuers as compensation for their executives and employees. Stock options are granted to provide incentives to work toward stronger corporate performance in the expectation of higher stock prices. Stock options can result in companies paying lower cash compensation. Companies may also issue warrants, which are options granted to employees or sold to the public that allow holders to purchase shares at a fixed price in the future directly from the issuer.

## Fixed-Income Instruments

Bonds are a widely used underlying, and related derivatives include options, forwards, futures, and swaps. Government issuers, such as the US Treasury or Japanese Ministry of Finance, usually have many bond issues outstanding. A single standardized futures contract associated with such bonds therefore often specifies parameters that allow more than one bond issue to be delivered to settle the contract.

An interest rate is not an asset but rather a fixed-income underlying used in many interest rate derivatives, such as forwards, futures, and options. Interest rate swaps are a type of firm commitment frequently used by market participants to convert from fixed to floating interest rate exposure over a certain period. For example, an investment manager can use interest rate swaps to increase or reduce portfolio duration without trading bonds. An issuer, on the other hand, might use an interest rate swap to alter the interest rate exposure profile of its liabilities.

A **market reference rate (MRR)** is the most common interest rate underlying used in interest rate swaps. These rates typically match those of loans or other short-term obligations. Survey-based Libor rates used as reference rates in the past have been replaced by rates based on a daily average of observed market transaction rates. For example, the Secured Overnight Financing Rate (SOFR) is an overnight cash borrowing rate collateralized by US Treasuries. Other MRRs include the euro short-term rate (€STR) and the Sterling Overnight Index Average (SONIA).

## Currencies

Market participants frequently use derivatives to hedge the exposure of commercial and financial transactions that arise due to foreign exchange risk. For example, exporters often enter into forward contracts to sell foreign currency and purchase domestic currency under terms matching those of a delivery contract for goods or services in a foreign country. Alternatively, an investor might sell futures on a particular currency while retaining a securities portfolio denominated in that currency to benefit from a temporary decline in the value of that currency. Options, forwards, futures, and swaps based upon sovereign bonds and exchange rates are used to manage currency risk.

## Commodities

Cash or spot markets for soft and hard commodities involve the physical delivery of the underlying upon settlement. **Soft commodities** are agricultural products, such as cattle and corn, and **hard commodities** are natural resources, such as crude oil and metals. Commodity derivatives are widely used to manage either the price risk of an individual commodity or a commodity index separate from physical delivery. For example, an airline, shipping, or freight company might purchase oil futures as a hedge against rising operating expenses due to higher fuel costs. An investor might purchase a commodity index futures contract to increase exposure to commodity prices without taking physical delivery of the underlying.

## Credit

Credit derivative contracts are based upon the default risk of a single issuer or a group of issuers in an index. **Credit default swaps (CDS)** allow an investor to manage the risk of loss from borrower default separately from the bond market. CDS contracts trade on a spread that represents the likelihood of default. For example, an investor might buy or sell a CDS contract on a high-yield index to change its portfolio exposure to high-yield credit without buying or selling the underlying bonds. Alternatively, a bank may purchase a CDS contract to offset existing credit exposure to an issuer's potential default.

## Other

Other derivative underlyings include weather, cryptocurrencies, and longevity, all of which can influence the financial performance of various market participants. For example, longevity risk is important to insurance companies and defined benefit pension plans that face exposure to increased life expectancy. Derivatives based upon these underlyings are less common and more difficult to price. Exhibit 3 provides a summary of common underlyings.

**Exhibit 3: Common Derivative Underlyings**

Asset Class	Examples	Sample Uses
<b>Equities</b>	Individual stocks	Change exposure profile (Investors)
	Equity indexes	Employee compensation (Issuers)
	Equity price volatility	
<b>Interest Rates</b>	Sovereign bonds (domestic)	Change duration exposure (Investors)
	Market reference rates	Alter debt exposure profile (Issuers)
<b>Foreign Exchange</b>	Sovereign bonds (foreign)	Manage global portfolio risks (Investors)
	Market exchange rates	Manage global trade risks (Issuers)
<b>Commodities</b>	Soft and hard commodities	Manage operating risks (Consumers/Producers)
	Commodity indexes	Portfolio diversification (Investors)

Asset Class	Examples	Sample Uses
Credit	Individual reference entities	Portfolio diversification (Investors)
	Credit indexes	Manage credit risk (Financial Intermediaries)
Other	Weather	Manage operating risks (Issuers)
	Cryptocurrencies	Manage portfolio risks (Investors)
	Longevity	

### RARE EARTH FUTURES AND THE LME LITHIUM CONTRACT

Derivative underlyings continue to adapt to the growing importance of environmental, social, and governance (ESG) factors affecting commercial and financial markets. For example, as the automotive industry shifts from internal combustion engine technology to electric vehicle (EV) production due to environmental concerns, demand for rare earth metals, such as lithium, as inputs into the EV battery production process are of increasing importance.

In response to growing demand from commodity producers and end users as well as investors, the London Metal Exchange (LME) introduced a lithium futures contract in 2021. The LME lithium contract is cash settled in USD against a weekly published spot price for battery-grade lithium hydroxide monohydrate deliverable in China, Japan, and Korea based upon a lot size of one metric ton per contract.

## Investor Scenarios

The following scenarios consider the specific goals of two parties and review the most appropriate derivative contract for each.

### Scenario 1: Hightest Capital

Hightest Capital is a US-based investment fund with a well-diversified domestic equity portfolio. Hightest's senior portfolio manager believes that health care stocks will significantly outperform the overall index over the next six months. Ace Limited is a financial intermediary and member of the Chicago Board Options Exchange (CBOE).

Hightest purchases an option based upon a standardized contract on the S&P 500 Health Care Select Sector Index (SIXV) with Ace as the financial intermediary and the spot SIXV price as the underlying. SIXV is comprised of approximately 60 health care equities included in the S&P 500 Index. The contract is a contingent claim, which grants Hightest the right to purchase SIXV at a 5% premium to the current market price (spot SIXV  $\times$  1.05) in six months.

### Scenario 2: Esterr Inc.

Esterr Inc. is a Toronto-based public company with a CAD250 million floating-rate term loan. The loan has a remaining maturity of three and a half years and is priced at three-month MRR (which is CORRA, or the Canadian Overnight Reference Rate Average) plus 150 bps. Esterr's treasurer is concerned about higher Canadian interest rates over the remaining life of the loan and would like to fix Esterr's interest expense.

Esterr enters into a CAD250 million interest rate swap contract with a financial intermediary with MRR as the underlying. Under the swap, Esterr agrees to pay a fixed interest rate and receive three-month MRR on a notional principal of CAD250 million for three and a half years based upon payment dates that match the term loan. The swap contract is a firm commitment.

## QUESTION SET



## Derivative Underlyings

1. Describe how and why an underlying may be used in employee compensation.

**Solution:**

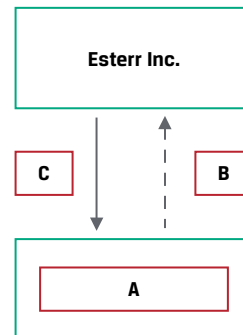
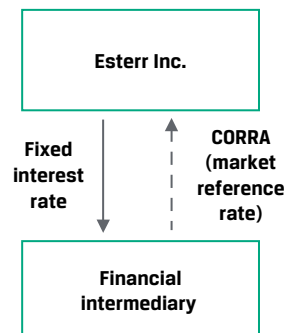
Derivatives with an equity underlying, in particular the stock of a particular issuer, may be included in the compensation of that company's employees. Stock options are granted to provide incentives to work toward stronger corporate performance in the expectation of a higher stock price, which will cause the options to increase in value.

2. Explain how a UK-based importer of goods from the euro zone might use a derivative with a currency underlying to mitigate risk.

**Solution:**

A UK-based importer of goods from the euro zone will likely pay EUR for goods that she intends to sell for GBP. To address this currency mismatch, she may consider entering a firm commitment to purchase EUR in exchange for GBP at a pre-determined price in the future based upon terms matching the import contract to offset risk to changes in the underlying spot exchange rate (i.e., GBP depreciation against EUR).

3. Identify A, B, and C in the following diagram, as in Exhibit 1, for the interest rate swap in Scenario 2 for Esterr Inc.

**Solution:**

4. Identify and describe the derivative features for the Esterr Inc. interest rate swap using the following term sheet, as in Exhibit 2.

#### Interest Rate Swap Term Sheet

<b>Start Date:</b>	[Spot start]
<b>Maturity Date:</b>	[Three years and six months from Start Date]
<b>Notional Principal:</b>	CAD250,000,000
<b>Fixed-Rate Payer:</b>	Esterr Inc.
<b>Fixed Rate:</b>	2.05% on a semiannual, Act/365 basis
<b>Floating-Rate Payer:</b>	[Financial Intermediary]
<b>Floating Rate:</b>	Three-month Canadian Overnight Repo Rate Average (CORRA) as published each Business Day by the Bank of Canada
<b>Payment Dates:</b>	Semiannual exchange on a net basis
<b>Business Days:</b>	Toronto
<b>Documentation:</b>	ISDA Agreement and credit terms to match Esterr Inc. Term Loan

A. Underlying: \_\_\_\_\_

B. Counterparties: \_\_\_\_\_ and \_\_\_\_\_

C. Contract size: \_\_\_\_\_

D. Contract type: \_\_\_\_\_

**Solution:**

- A. Underlying: Interest rate (Canadian market reference rate, CORRA)
- B. Counterparties: Esterr Inc. and Financial Intermediary
- C. Contract size: CAD250,000,000
- D. Contract type: Firm commitment (interest rate swap)

5. Identify which example corresponds to each derivative underlying type.

A. Soft commodities	1. Aluminum futures
B. Hard commodities	2. SOFR futures
C. Neither soft nor hard commodities	3. Soybean options

**Solution:**

- 1. B is correct. Aluminum futures are an example of a metals contract, which is a derivative with a hard commodity underlying.
- 2. C is correct. SOFR futures are an example of an interest rate contract, not a commodity-based derivative contract.
- 3. A is correct. Soybean options are an example of a derivative contract with an agricultural, or soft, commodity underlying.

## 4

## DERIVATIVE MARKETS



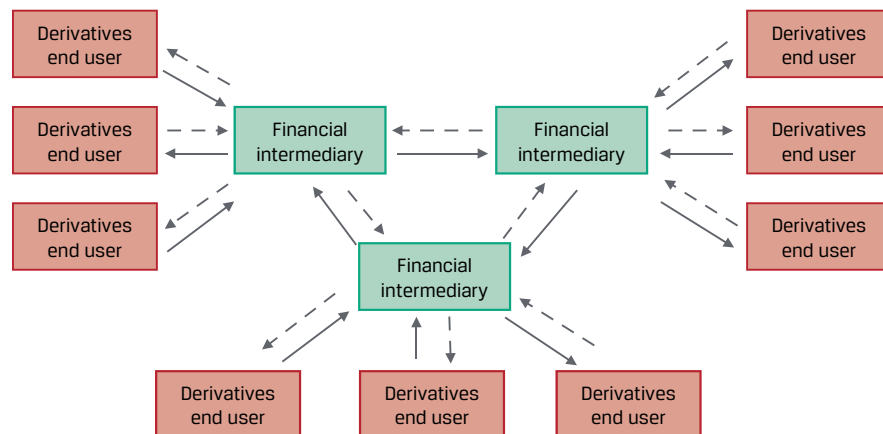
describe the basic features of derivative markets, and contrast over-the-counter and exchange-traded derivative markets

Derivatives usage was historically dominated by exchange-traded futures markets in soft and hard commodities. Derivatives were expanded to **over-the-counter (OTC)** financial derivatives in interest rates and currencies in the 1980s, then credit derivatives in the 1990s.

### Over-the-Counter (OTC) Derivative Markets

OTC markets can be formal organizations, such as NASDAQ, or informal networks of parties that buy from and sell to one another, as in the US fixed-income markets. OTC derivative markets involve contracts entered between derivatives end users and **dealers**, or financial intermediaries, such as commercial banks or investment banks. OTC dealers, known as **market makers**, typically enter into offsetting bilateral transactions with one another to transfer risk to other parties. The terms of OTC contracts can be customized to match a desired risk exposure profile. This flexibility is important to end users seeking to hedge a specific existing or anticipated underlying exposure based upon non-standard terms. The structure of the OTC derivative markets is shown in Exhibit 4.

**Exhibit 4: Over-the-Counter Derivative Markets**



### Exchange-Traded Derivative (ETD) Markets

An **exchange-traded derivative (ETD)** includes futures, options, and other financial contracts available on exchanges, such as the National Stock Exchange (NSE) in India or the Brasil, Bolsa, Balcão (B3) exchange in Brazil. ETD contracts are more formal and standardized, which facilitates a more liquid and transparent market. Terms and conditions—such as the size of each contract, type, quality, and location of underlying for commodities and maturity date—are set by the exchange. Exhibit 5 shows the key terms of the London Metals Exchange (LME) lithium futures contract described earlier.



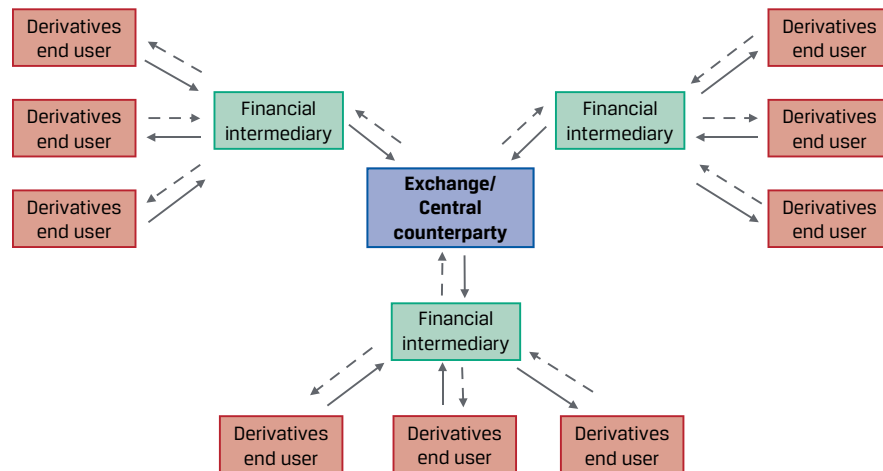
### LME Lithium Futures Contract Specifications

<b>Contract Maturities:</b>	Monthly [from 1 month to 15 months]
<b>Contract Size:</b>	One metric ton
<b>Delivery Type:</b>	Cash settled
<b>Price Quotation:</b>	USD per metric ton
<b>Final Maturity:</b>	Last LME business day of contract month
<b>Daily Settlement:</b>	LME Trading Operations calculates daily settlement values based on its published procedures
<b>Final Settlement:</b>	Based on the reported arithmetic monthly average of Fastmarkets' lithium hydroxide monohydrate 56.5% LiOH. H <sub>2</sub> O min, battery grade, spot price cif China, Japan, and Korea, USD/kg price, which is available from Fastmarkets from 16.30 London time on the last trading day

Exchange memberships are held by market makers (or dealers) that stand ready to buy at one price and sell at a higher price. With standard terms and an active market, they are often able to buy and sell simultaneously, earning a small bid–offer spread. When dealers cannot find a counterparty, risk takers (sometimes referred to as speculators) are often willing to take on exposure to changes in the underlying price.

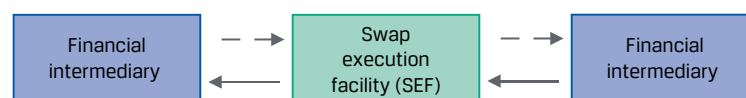
Standardization also leads to an efficient clearing and settlement process. **Clearing** is the exchange's process of verifying the execution of a transaction, exchange of payments, and recording the participants. Settlement involves the payment of final amounts and/or delivery of securities or physical commodities between the counterparties based upon exchange rules. Derivative exchanges require collateral on deposit upon inception and during the life of a trade in order to minimize counterparty credit risk. This deposit is paid by each counterparty via a financial intermediary to the exchange, which then provides a guarantee against counterparty default. Finally, ETD markets have transparency, which means that full information on all transactions is disclosed to exchanges and national regulators.

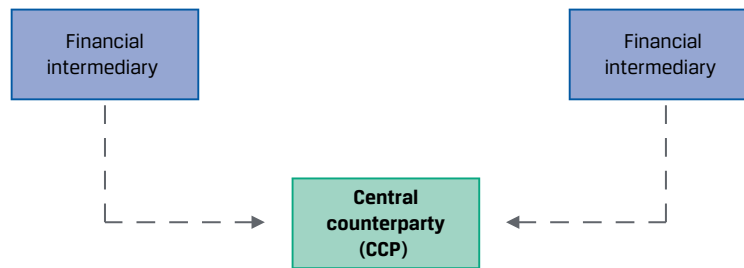
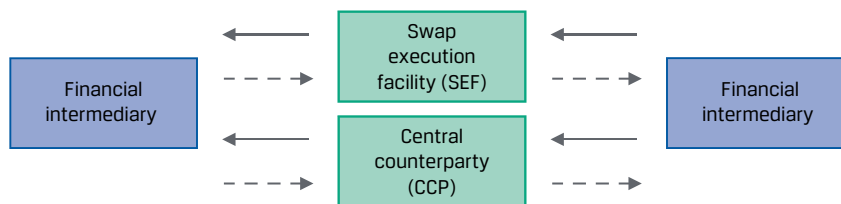
OTC and ETD markets differ in several ways. OTC derivatives offer greater flexibility and customization than ETD. However, OTC instruments have less transparency, usually involve more counterparty risk, and may be less liquid. ETD contracts are more standardized, have lower trading and transaction costs, and may be more liquid than those in OTC markets, but their greater transparency and reduced flexibility may be a disadvantage to some market participants. The structure of the ETD markets is shown in Exhibit 5.

**Exhibit 5: Exchange-Traded Derivative Markets****Central Clearing**

Following the 2008 global financial crisis, global regulatory authorities instituted a **central clearing mandate** for most OTC derivatives. This mandate requires that a **central counterparty (CCP)** assume the credit risk between derivative counterparties, one of which is typically a financial intermediary. CCPs provide clearing and settlement for most derivative contracts. Issuers and investors are able to maintain the flexibility and customization available in the OTC markets when facing a financial intermediary, while the management of credit risk, clearing, and settlement of transactions between financial intermediaries occurs in a way similar to ETD markets. This arrangement seeks to benefit from the transparency, standardization, and risk reduction features of ETD markets. However, the systemic credit risk transfer from financial intermediaries to CCPs also leads to centralization and concentration of risks. Proper safeguards must be in place to avoid excessive risk being held in CCPs.

Exhibit 6 shows the central clearing process for interest rate swaps which also applies to other swaps and derivative instruments. Under central clearing, a derivatives trade is executed in Step 1 on a **swap execution facility (SEF)**, a swap trading platform accessed by multiple dealers. The original SEF transaction details are shared with the CCP in Step 2, and the CCP replaces the existing trade in Step 3. This **novation process** substitutes the initial SEF contract with identical trades facing the CCP. The CCP serves as counterparty for both financial intermediaries, eliminating bilateral counterparty credit risk and providing clearing and settlement services.

**Exhibit 6: Central Clearing for Interest Rate Swaps****Step 1: Trade executed on an SEF**

**Step 2: SEF trade information submitted to CCP****Step 3: CCP replaces (novates) existing trade, acting as new counterparty to both financial intermediaries****Investor Scenarios**

In this section, we assess the most appropriate derivative markets for the scenarios presented in the previous lesson.

**Scenario 1. Hightest Capital.**

Hightest's index option contract would most likely be traded on the ETD derivative market. The trade has a standard size, exercise price, and maturity date.

**Scenario 2. Esterr Inc.**

Esterr's interest rate swap is likely to be traded in the OTC market. The swap contract terms are tailored to match the payment dates and remaining maturity of Esterr's term loan. Esterr's counterparty will be a financial intermediary that executes the offsetting hedge on an SEF and then novates the original SEF trade to face a CCP, which serves as the credit risk intermediary between dealers.

**QUESTION SET****Derivative Markets**

1. Describe the risk transfer process in OTC derivative markets.

**Solution:**

OTC dealers, known as market makers, typically enter into offsetting transactions with one another to transfer the risk of derivative contracts entered with end users.

2. Identify which of the following derivative markets corresponds to the following characteristics.

A. ETD	1. Standardized contracts
B. OTC	2. Includes market makers
C. Both ETD and OTC	3. Greater confidentiality

**Solution:**

1. A—ETD markets use standardized contracts.
2. C—Both ETD and OTC markets use market makers.
3. B—OTC markets have greater privacy.

3. Determine the correct answers to fill in the blanks: \_\_\_\_\_ involves the payment of final amounts and/or delivery of securities or physical commodities, while \_\_\_\_\_ is the process of verifying the execution of a transaction, exchange of payments, and recording the participants.

**Solution:**

Settlement involves the payment of final amounts and/or delivery of securities or physical commodities, while clearing is the process of verifying the execution of a transaction, exchange of payments, and recording the participants.

4. Identify one potential risk concern about the central clearing of derivatives.

**Solution:**

The central clearing mandate transfers the systemic risk of derivatives transactions from the counterparties, typically financial intermediaries, to the CCPs. One concern is the centralization and concentration of risks in CCPs. Careful oversight must occur to ensure that these risks are properly managed.

5. Describe the steps for clearing a credit default swap.

**Solution:**

The counterparties are financial intermediaries that first execute the trade on an SEF (swap execution facility). Then, trade details are shared with a CCP; the novation process substitutes the original contract with another where the CCP steps into the trade and acts as the new counterparty for each original party. The CCP clears and settles the trade.

## PRACTICE PROBLEMS

### The following information relates to questions 1-5

Montau AG is a German capital goods producer that manufactures its products domestically and delivers its products to clients globally. Montau's global sales manager shares the following draft commercial contract with his Treasury team:

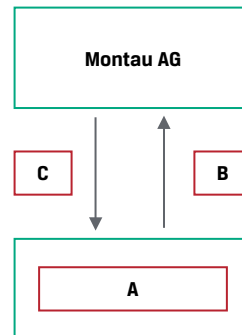
#### Montau AG Commercial Export Contract

<b>Contract Date:</b>	[Today]
<b>Goods Seller:</b>	Montau AG, Frankfurt, Germany
<b>Goods Buyer:</b>	Jeon Inc., Seoul, Korea
<b>Description of Goods:</b>	A-Series Laser Cutting Machine
<b>Quantity:</b>	One
<b>Delivery Terms:</b>	Freight on Board (FOB), Busan Korea with all shipping, tax and delivery costs payable by Goods Buyer
<b>Delivery Date:</b>	[75 Days from Contract Date]
<b>Payment Terms:</b>	100% of Contract Price payable by Goods Buyer to Good Seller on Delivery Date
<b>Contract Price:</b>	KRW650,000,000

Montau AG's Treasury manager is tasked with addressing the financial risk of this prospective transaction.

- Which of the following statements best describes why Montau AG should consider a derivative rather than a spot market transaction to manage the financial risk of this commercial contract?
  - Montau AG is selling a machine at a contract price in KRW and incurs costs based in EUR.
  - Montau AG faces a 75-day timing difference between the commercial contract date and the delivery date when Montau AG is paid for the machine in KRW.
  - Montau AG is unable to sell KRW today in order to offset the contract price of machinery delivered to Jeon Inc.
- Which of the following types of derivative and underlyings are best suited to hedge Montau's financial risk under the commercial transaction?
  - Montau AG should consider a firm commitment derivative with currency as an underlying, specifically the sale of KRW at a fixed EUR price.
  - Montau AG should consider a contingent claim derivative with the price of the machine as its underlying, specifically an A-series laser cutting machine.

- C. Montau AG should consider a contingent claim derivative with currency as an underlying, specifically the sale of EUR at a fixed KRW price.
3. Identify A, B, and C in the correct order in the following diagram, as in Exhibit 1, for the derivative to hedge Montau's financial risk under the commercial transaction.

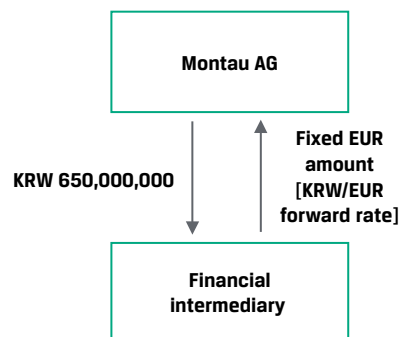
**Exhibit 1**

- A. A: Financial intermediary, B: KRW650,000,000, C: Fixed EUR amount
- B. A: Jeon Inc., B: KRW650,000,000, C: Fixed EUR amount
- C. A: Financial intermediary, B: Fixed EUR amount, C: KRW650,000,000.
4. Which of the following statements about the most appropriate derivative market to hedge Montau AG's financial risk under the commercial contract is most accurate?
- A. The OTC market is most appropriate for Montau, as it is able to customize the contract to match its desired risk exposure profile.
- B. The ETD market is most appropriate for Montau, as it offers a standardized and transparent contract to match its desired risk exposure profile.
- C. Both the ETD and OTC markets are appropriate for Montau AG to hedge its financial risk under the transaction, so it should choose the market with the best price.
5. If Montau enters into a centrally cleared derivative contract on the OTC market, which of the following statements about credit risk associated with the derivative is most likely correct?
- A. Montau faces credit risk associated with the possibility that its counterparty to the contract may not fulfill its contractual obligation.
- B. Montau poses a credit risk to its counterparty because it may fail to fulfill its contractual obligation.
- C. Montau poses a credit risk to a derivative contract end user holding a contract with the opposite features of Montau's.

## SOLUTIONS

1. B is correct. A 75-day timing difference exists between the commercial contract date and the delivery date when Montau AG is paid for the machine in KRW. A is true but does not explain why the use of a derivative is preferable to a spot market transaction. If as in C Montau were to sell the KRW it receives and buy EUR in a spot market transaction on the delivery date, it would be exposed to unfavorable changes in the KRW/EUR exchange rate over the 75-day period. A derivative contract in which the underlying KRW/EUR forward rate is agreed today and exchanged on the delivery date allows Montau to hedge or offset the EUR value of the future KRW payment. The derivative is therefore a more suitable contract to address the financial risk of the commercial transaction than a spot market sale of KRW.
2. A is correct. The derivative best suited to hedge Montau's financial risk is a firm commitment derivative in which a pre-determined amount is exchanged at settlement. The derivative underlying should be currencies, specifically the sale of KRW at a fixed EUR price in the future to offset or hedge the financial risk of the commercial contract. The machine price referenced under B is not considered an underlying, and C hedges the opposite of Montau's underlying exposure.
3. C is correct as per the following diagram:

**Exhibit 2**



4. A is correct. The OTC market is most appropriate for Montau, as OTC contracts may be customized to match Montau's desired risk exposure profile. This is important to end users seeking to hedge a specific underlying exposure based upon non-standard terms. Montau would be unlikely to find an ETD contract under B that matches the exact size and maturity date of its desired hedge, which also makes C incorrect.
5. B is correct. In a centrally cleared OTC derivative contract, the central counterparty becomes the counterparty in all contracts and assumes the credit risk associated with individual derivative contracts. A is likely incorrect because the CCP takes actions to ensure that it can fulfill its obligations to its counterparties. C is incorrect because the CCP inserts itself between parties with opposite positions.





## LEARNING MODULE

## 2

## Forward Commitment and Contingent Claim Features and Instruments

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics
<input type="checkbox"/>	determine the value at expiration and profit from a long or a short position in a call or put option
<input type="checkbox"/>	contrast forward commitments with contingent claims

## INTRODUCTION

1

An earlier lesson established a derivative as a financial instrument that derives its performance from an underlying asset, index, or other financial variable, such as equity price volatility. Primary derivative types include a firm commitment in which a predetermined amount is agreed to be exchanged between counterparties at settlement and a contingent claim in which one of the counterparties determines whether and when the trade will settle. The following lessons define and compare the basic features of forward commitments and contingent claims and explain how to calculate their values at maturity.

### LEARNING MODULE OVERVIEW



- Forwards, futures, and swaps represent firm commitments, or derivative contracts that require counterparties to exchange an underlying in the future based on an agreed-on price.
- Forwards are a flexible over-the-counter (OTC) derivative instrument, while futures are standardized and traded on an exchange with a daily settlement of contract gains and losses.
- Swap contracts are a firm commitment to exchange a series of cash flows in the future. Interest rate swaps are the most common type and involve the exchange of fixed interest payments for floating interest payments.

CFA Institute would like to thank Don Chance, PhD, CFA, for his contribution to this section, which includes material derived from material that appeared in *Derivative Markets and Instruments*, featured in the 2022 CFA® Program curriculum.

- Option contracts are contingent claims in which one of the counter-parties determines whether and when a trade will settle. The option buyer pays a premium to the seller for the right to transact the underlying in the future at a pre-agreed exercise price.
- Option contract payoff and profit profiles are non-linear as the underlying price changes, as opposed to firm commitments, such as forwards, futures, and swaps, which are linear in underlying price changes.
- Market participants often create similar exposures to an underlying using firm commitments and contingent claims, although these derivative instrument types involve different payoff and profit profiles.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements correctly describes a difference between a forward contract and a futures contract?
  - A. A forward contract sets an agreed-on price for buyer and seller, while a futures contract does not.
  - B. A forward contract sets an agreed-on transaction date for the seller to deliver the underlying to the buyer, while a futures contract does not.
  - C. A forward contract does not require daily settlement of gains and losses, while a futures contract does.

**Solution:**

C is correct. Futures contracts require daily settlement through the exchange clearinghouse mark-to-market process. Forward contracts are settled at their maturity date, although the two parties to the contract may customize alternative settlement procedures. A is incorrect because both forward and futures contracts set an agreed-on price for a future transaction. B is incorrect because both forwards and futures contracts include a maturity date when the underlying will be exchanged.

2. Identify which example fits each of the following firm commitments:

A. Futures contract purchaser	1. Agrees to make a single exchange in the future at a pre-agreed price under an OTC contract
B. Forward contract seller	2. Agrees to a single exchange in the future based on standardized terms set by an exchange
C. Fixed-rate payer on an interest rate swap	3. Agrees to a series of exchanges of interest fixed for floating interest payments

**Solution:**

1. B is correct. A forward contract seller agrees to make a single exchange in the future at a pre-agreed price under an OTC contract.
2. A is correct. A futures contract purchaser agrees to a single exchange in the future based on standardized terms set by an exchange.

3. C is correct. A fixed-rate payer on an interest rate swap agrees to a series of exchanges of fixed for floating interest payments.

3. Identify which example fits each of the following contingent claims:

- |  |  |
|--|--|
| A. Put option purchaser                                    | 1. Seeks to gain from an increase in the underlying price                                      |
| B. Call option purchaser                                   | 2. Allows the option to expire at maturity of the underlying price is above the exercise price |
| C. Both a put option purchaser and a call option purchaser | 3. Pays an option premium to the option seller when the contract is agreed on                  |

**Solution:**

1. B is correct. A call option purchaser seeks to gain from an increase in the underlying price.
2. A is correct. A put option purchaser will allow an option to expire at maturity without exercise if the underlying price is above the exercise price.
3. C is correct. Both a put option purchaser and a call option purchaser will pay a premium to the option seller when the option contract is executed.

4. An option to buy an underlying security at an exercise price of USD45 in three months trades at a premium of USD6. After three months, the underlying trades at USD50. Which of the following responses correctly describes the profit/loss position of the option buyer and seller?

- A. Option buyer earns USD5 profit, and option seller earns USD5 loss.
- B. Option buyer earns USD1 loss, and option seller earns USD1 profit.
- C. Option buyer earns USD5 profit, and option seller earns USD0.

**Solution:**

B is correct. The option buyer's position generates a payoff of USD5, equal to  $\max(0, 50 - 45)$ . The option buyer paid USD6 to buy the option position, and this cash flow more than offsets the positive payoff. Thus, the option buyer's overall profit is a loss of USD1 (i.e.,  $5 - 6$ ). For the option seller, the option position creates a negative payoff of  $-\text{USD}5$ , equal to  $-\max(0, 50 - 45)$ . However, the option seller received the option premium of USD6, so the overall profit is USD1 (i.e.,  $6 - 5$ ). A is incorrect because the USD5 amount reflects the option payoff only, not profits and losses accounting for the option premium. C is incorrect because the buyer's profit incorrectly states the payoff only to the option position, not the profit. The seller's profit would be correct only if the underlying traded at 51, not 50.

5. A put option buyer earns a positive profit in which of the following conditions?

- A. The price of the underlying at option expiration is less than the option's exercise price.
- B. The price of the underlying at option expiration is greater than the option's exercise price.

- C. The price of the underlying is less than the option's exercise price minus the option's premium.

**Solution:**

C is correct. For a put option buyer to earn a positive profit, the underlying price must be sufficiently below the put option's exercise price such that (1) the put option can be exercised with a positive payoff and (2) the positive payoff is greater than the option premium paid. Thus, only if the underlying price falls below the exercise price minus the premium can this occur. A is incorrect because this condition only implies a positive payoff on the option but would include prices at which the payoff is not greater than the premium. B is incorrect because the put option would be out of the money and would generate zero payoff.

6. Which of the following positions on the same underlying benefit from opposite price movements in an underlying?

- A. Long forward contract, short put option
- B. Short forward contract, long put option
- C. Short forward contract, short put option

**Solution:**

C is correct. A short forward position benefits as the underlying price declines, while a short put benefits only when the underlying price increases. A is incorrect because both a long forward and a short put benefit from underlying price increases. B is incorrect because both a short forward and a long put option benefit from underlying price decreases.

## 2

## FORWARDS, FUTURES, AND SWAPS



define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics

Forwards, futures, and swaps are the most common derivative contracts which represent a firm commitment. This firm commitment is an obligation of both counterparties to perform under the terms of the derivative contract. Key common features of this type of derivative include the following:

- A specific contract size
- A specific underlying
- One or more exchanges of cash flows or underlying on a specific future date or dates
- Exchange(s) based on a pre-agreed price

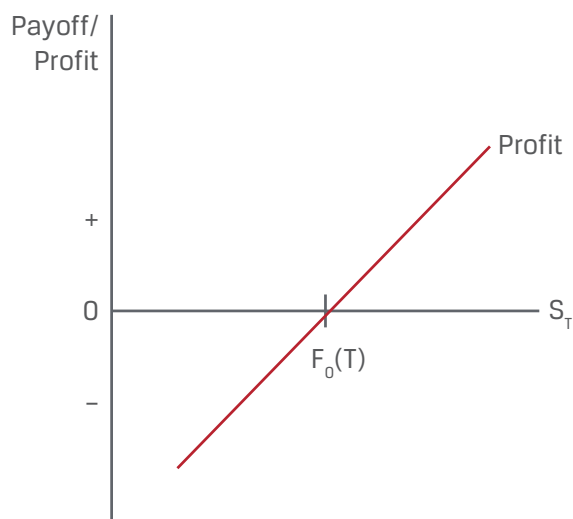
Despite their similarities, forwards, futures, and swaps each have different features, which are the subject of this lesson.

A forward contract is an over-the-counter (OTC) derivative in which two counterparties agree that one counterparty, the buyer, will purchase an underlying from the other counterparty, the seller, in the future at a pre-agreed fixed price. As noted earlier, OTC derivatives offer greater flexibility and customization than exchange-traded derivatives (ETD), but also usually involve more counterparty risk. Forward contracts

are advantageous for derivative end users seeking to hedge an existing or forecasted underlying exposure based on specific terms. For example, an importer may enter a forward contract to buy the foreign currency needed to satisfy the commercial terms of a future goods delivery contract. Forward contracts are more flexible as to the size, underlying details, maturity, and/or credit terms than a similar ETD. A forward contract buyer has a long position and will therefore benefit from price appreciation of the underlying over the life of the contract.

To gain a better understanding of forwards, we must examine their payoff profile. Assume a forward contract is agreed at time  $t = 0$  and matures at time  $T$ . At time  $t = 0$ , the counterparties do not exchange a payment upfront but, rather, agree on delivery of the underlying at time  $T$  for a **forward price** of  $F_0(T)$ . The subscript refers to the date on which the underlying price in the future is set ( $t = 0$ ), and the  $T$  in parentheses refers to the date of exchange ( $t = T$ ). The spot price of the underlying at time  $T$  is  $S_T$ . Exhibit 1 shows the payoff from the forward buyer's perspective, which is a long forward position. Note that the payoff equals the profit, as no upfront payment is made.

**Exhibit 1: Long Forward (Forward Buyer) Payoff Profile**



Outcome	Buyer Payoff	Seller Payoff
$S_T > F_0(T)$	$[S_T - F_0(T)] > 0$	$[F_0(T) - S_T] < 0$
$S_T < F_0(T)$	$[S_T - F_0(T)] < 0$	$[F_0(T) - S_T] > 0$

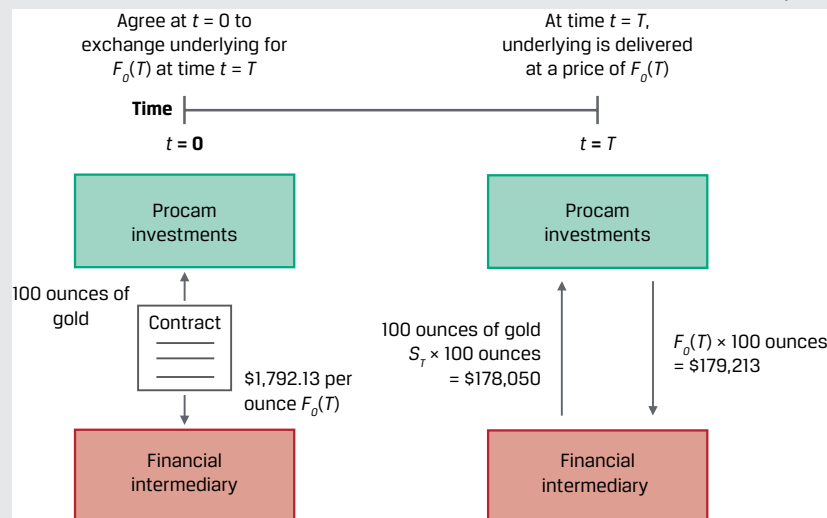
The symmetric payoff profile shown in Exhibit 1 is a common feature of firm commitments. Since the derivative price is a linear function of the underlying, firm commitments are also referred to as **linear derivatives**. At time  $T$ , the transaction is settled based on the difference between the forward price,  $F_0(T)$ , and the underlying price of  $S_T$ , or  $[S_T - F_0(T)]$  from the buyer's perspective. That is, the buyer realizes a gain if she is able to take delivery of the underlying at a market value,  $S_T$ , that exceeds the pre-agreed price,  $F_0(T)$ . If the forward price exceeds the current market value [ $F_0(T) > S_T$ ], the buyer realizes a loss and must either take delivery of an asset at a loss of  $[F_0(T) - S_T]$  or pay the seller this amount in cash. Forward contracts usually involve a single exchange in the future, as in Example 1.

## EXAMPLE 1

## Forward Gold Purchase

An investor, Procam Investments, enters a cash-settled forward contract with a financial intermediary to buy 100 ounces of gold at a forward price,  $F_0(T)$ , of \$1,792.13 per ounce in three months.

1. Today's spot gold price ( $S_0$ ) is \$1,770 per ounce.
2. At contract maturity, the gold price ( $S_T$ ) is \$1,780.50 per ounce.
3. The payoff,  $S_T - F_0(T)$ , is  $-\$11.63 = \$1,780.50 - \$1,792.13$  per ounce.
4. Procam (the buyer) must pay the financial intermediary (the seller) \$1,163 ( $= 100 \times \$11.63$ ) to settle the forward contract at maturity.



The contract may specify either the actual delivery of the underlying or a cash settlement. The settlement amount is equal to  $[S_T - F_0(T)]$  from a buyer's perspective and  $-[S_T - F_0(T)] = [F_0(T) - S_T]$  from a seller's perspective. Note that a buyer would have to pay  $S_0$  at  $t = 0$  and realize a return of  $(S_T - S_0)$  at time  $T$  in order to create a similar exposure to the long forward position in the cash market.

## QUESTION SET



## Forwards

1. Describe a scenario in which a forward contract has cash settlement of zero at maturity and neither counterparty has defaulted.

**Solution:**

A forward contract will have a cash settlement of zero at maturity if  $S_T = F_0(T)$  or the payoff from the buyer's perspective is  $[S_T - F_0(T)] = 0$ . This is often referred to as the **breakeven point** for the forward contract for both buyer and seller in the absence of transaction costs and is visually represented by the  $x$ -axis intercept of the profit line in Exhibit 1.

2. Determine the correct answers to fill in the blanks: An oil producer enters a derivative contract with an investor to sell 1,000 barrels of oil in two months at a forward price of \$64 per barrel. If the spot oil price at maturity

is \$58.50 per barrel, the investor realizes a \_\_\_\_\_ at maturity equal to \_\_\_\_\_.

**Solution:**

An oil producer enters a derivative contract with an investor to sell 1,000 barrels of oil in two months at a forward price of \$64 per barrel. If the spot oil price at maturity is \$58.50 per barrel, the investor realizes a *loss* at maturity equal to \$5,500.

The oil forward price,  $F_0(T)$ , under the contract equals \$64 per barrel.

At contract maturity, the spot oil price ( $S_T$ ) is \$58.50 per barrel.

• Investor payoff per barrel:

$$[S_T - F_0(T)] = \$58.50 - \$64.00 = -\$5.50 \text{ per barrel.}$$

• Total amount the investor pays the oil producer to settle the forward contract for 1,000 barrels at maturity:

$$1,000 \times \$5.50 = \$5,500.$$

3. Identify the most likely forward contract participants that correspond to the following statements:

- |  |   |
|--|---|
| A. forward contract purchaser                        | 1. Seeks to benefit from underlying price depreciation  |
| B. Forward contract seller                           | 2. Realizes a gain if the initial spot price of the underlying, $S_0$ , exceeds the forward price of $F_0(T)$ |
| C. Neither a forward contract purchaser nor a seller | 3. Receives a positive payoff at maturity if the spot price, $S_T$ , exceeds the forward price of $F_0(T)$    |

**Solution:**

1. The correct answer is B. A forward seller pays  $[F_0(T) - S_T]$  to the forward contract buyer at maturity and therefore benefits as the underlying spot price  $S_T$  declines over time.

2. The correct answer is C. Neither the buyer nor the seller of a forward contract realizes a gain if the initial spot price,  $S_0$ , exceeds the forward price of  $F_0(T)$ , as settlement is based on the future spot price,  $S_T$ .

3. The correct answer is A. The forward contract buyer realizes a gain at maturity if  $S_T > F_0(T)$ .

## FUTURES

# 3



define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics

**Futures contracts** are forward contracts with standardized sizes, dates, and underlyings that trade on futures exchanges. Futures markets offer both greater liquidity and protection against loss by default by combining contract uniformity with an organized market with rules, regulations, and a central clearing facility.

The futures contract buyer creates a long exposure to the underlying by agreeing to purchase the underlying at a later date at a pre-agreed price. The seller makes the opposite commitment, creating a short exposure to the underlying by agreeing to sell

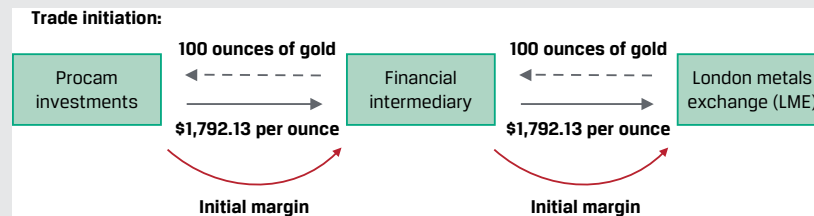
the underlying asset in the future at an agreed-on price. This agreed-on price is called the **futures price**,  $f_0(T)$ . The frequency of futures contract maturities, contract sizes, and other details are established by the exchange based on buyer and seller interest.

The most important feature of futures contracts is the daily settlement of gains and losses and the associated credit guarantee provided by the exchange through its clearinghouse. At the end of each day, the clearinghouse engages in a practice called **mark to market (MTM)**, also known as the **daily settlement**. The clearinghouse determines an average of the final futures trading price of the day and designates that price as the end-of-day **settlement price**. All contracts are then said to be marked to the end-of-day settlement price.

As with forward contracts, no cash is exchanged when a futures contract is initiated by a buyer or seller. However, each counterparty must deposit a required minimum sum (or **initial margin**) into a **futures margin account** held at the exchange that is used by the clearinghouse to settle the daily mark to market. Futures contracts must be executed with specialized financial intermediaries that clear and settle payments at the exchange on behalf of counterparties, as shown in Example 2.

### EXAMPLE 2

#### Purchase of a Gold Futures Contract



As in Example 1, Procam Investments enters a cash-settled contract to buy 100 ounces of gold at a price of \$1,792.13 per ounce in three months. Instead of the forward in Example 1, Procam purchases a futures contract [ $f_0(T) = \$1,792.13$ ] on the exchange via a financial intermediary. London Metals Exchange rules require an initial cash margin of \$4,950 per gold contract (100 ounces) sold or purchased:

- Procam deposits \$4,950 in required initial margin with the exchange.
- Today's spot gold price ( $S_0$ ) is \$1,770 per ounce, and the opening gold futures price,  $f_0(T)$ , is \$1,792.13 per ounce.
- At today's close, the gold futures price,  $f_1(T)$ , settles at \$1,797.13 per ounce.
- Procam realizes a \$500 MTM gain, or \$5 per ounce  $\times$  100 ounces. It receives a \$500 futures margin account deposit from the clearinghouse.
- Procam's futures margin account has an ending balance for the day of \$5,450, or \$4,950 initial margin plus the \$500 MTM gain.

Each futures contract specifies a **maintenance margin**, or minimum balance set below the initial margin, that each contract buyer and seller must hold in the futures margin account from trade initiation until final settlement at maturity. The clearinghouse moves funds daily between the buyer and seller margin accounts, crediting the accounts of those with mark to market gains and charging those with mark-to-market losses.

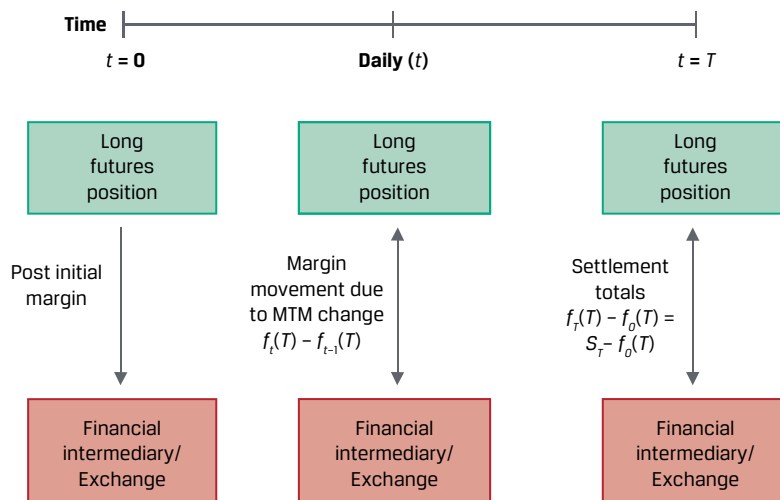


For example, London Metals Exchange rules require a maintenance margin of \$4,500 per 100-ounce gold contract sold or purchased. Now consider the *seller* of a futures contract with a position that offsets that of Procam in Example 2.

- Seller deposits \$4,950 in required initial margin with the exchange.
- Today's opening gold futures price,  $f_0(T)$ , is \$1,792.13 per ounce.
- At the close, the gold futures price,  $f_1(T)$ , settles at \$1,797.13 per ounce.
- Seller realizes a \$500 MTM loss, or \$5 per ounce  $\times$  100 ounces; \$500 is deducted from its futures margin account by the clearinghouse.
- Seller's futures margin account has an ending balance of \$4,450, or \$4,950 initial margin less the \$500 MTM loss.
- Seller's margin account is \$50 below the required maintenance margin ( $\$4,450 - \$4,500$ ) =  $-\$50$

The seller receives a **margin call**, or request to immediately deposit funds to return the account balance to the *initial margin*. The seller must deposit \$500 in order to bring the margin account back to the \$4,950 initial margin. The amount required to replenish the futures margin account is sometimes referred to as **variation margin**. If a counterparty fails to meet the margin call, it must close out the contract as soon as possible and cover any additional losses. If the counterparty cannot meet its obligations, the clearinghouse provides a guarantee that it will cover the loss itself by maintaining an insurance fund. Exhibit 2 shows the futures margining and settlement process, where  $f_0(T)$  is the futures price at inception and  $f_t(T)$  represents the futures price on day  $t$ .

### Exhibit 2: Futures Margin and Settlement Process



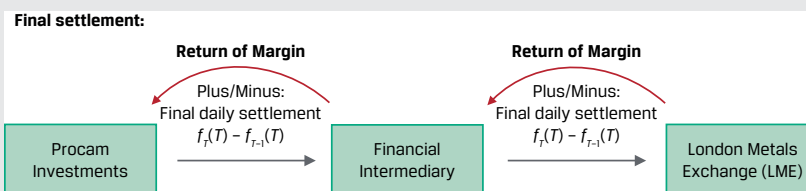
Exchanges reserve the right to impose more strict requirements than standard futures margin account rules to limit potential losses from counterparty default. For example, for large positions or a significant increase in price volatility of the underlying, an exchange may increase required margins and/or make margin calls on an intraday basis. Some futures contracts also limit daily price changes. These rules, called **price limits**, establish a band relative to the previous day's settlement price within which all trades must occur. If market participants wish to trade at a price outside these bands,

trading stops until two parties agree on a trade at a price within the prescribed range. In other cases, exchanges use what is called a **circuit breaker** to pause intraday trading for a brief period if a price limit is reached.

Similar to forward contracts, final settlement at maturity for futures contracts is based on the difference between the futures price,  $f_0(T)$ , and the underlying price of  $S_T$ , or  $[S_T - f_0(T)]$  from the buyer's perspective. Because Procum has agreed to purchase gold now (at  $t = T$ ) valued at \$1,780.50 per ounce at a price of \$1,792.13 per ounce, it owes \$11.63 per ounce, or \$1,163  $[(\$1,780.50 - \$1,792.13) \times 100 \text{ oz.}]$  under both the forward from Example 1 and this futures contract. The *net* payoff profile shown in Exhibit 1 is the same for a futures contract as for a forward assuming they have the same maturity date, with the difference being the *timing* of the cash flows due to the daily futures contract mark-to-market settlement, as shown in Example 3.

### EXAMPLE 3

#### Final Settlement of a Gold Futures Contract



For purposes of exposition, we compress the three months of gold futures price changes from Example 2 into six days of trading in the following spreadsheet:

#### Exhibit 3

Procum's Futures Margin Account		
Gold Contract	100	ounces
# of Contracts	1	
Initial Futures Price $f_0(T)$	\$1,792.13	per ounce
Initial Position Value	\$179,213	
Initial Margin	\$4,950	
Maintenance Margin	\$4,500	

Day	Futures Price	Day Gain (Loss)	Total Gain (Loss)	Margin Balance	Margin Call
$T - 6$	\$1,792.13			\$4,950	
$T - 5$	\$1,797.13	\$500	\$500	\$5,450	—
$T - 4$	\$1,786.25	(\$1,088)	(\$588)	\$4,362	\$588
$T - 3$	\$1,782.19	(\$406)	(\$994)	\$4,544	—
$T - 2$	\$1,777.45	(\$474)	(\$1,468)	\$4,070	\$880
$T - 1$	\$1,779.50	\$205	(\$1,263)	\$5,155	—
$T$	\$1,780.50	\$100	<b>(\$1,163)</b>	\$5,255	\$1,468

### Procam's Results from the Futures Contract

Gold Contract	100
# of Contracts	1
Initial Futures Price $f_0(T)$	\$1,792.13
Initial Position Value	\$179,213
Total Gain (Loss)	(\$1,163)
Final Futures Price $F_T(T)$	\$1,780.50
Final Position Value	\$178,050
Sum of Margin Calls	\$1,468
Beginning less Ending Margin Balance	(\$305)
Total Payments to Margin Account	\$1,163

As the gold futures price,  $f_0(t)$ , falls and the margin balance drops below the \$4,500 maintenance margin minimum over time, Procam must immediately replenish its balance to the \$4,950 initial margin each time this occurs:

- On Day  $T - 4$ , gold futures fall \$10.88 per ounce (\$1,797.13 – \$1,786.25). Procam's balance falls \$1,088 ( $100 \times \$10.88$ ) to \$4,362 (\$5,450 – \$1,088). Procam faces a margin call of \$588 (\$4,950 – \$4,362).
- On Day  $T - 2$ , gold futures fall \$4.74 per ounce (\$1,782.19 – \$1,777.45). Procam's balance falls \$474 ( $100 \times \$4.74$ ) to \$4,070 (\$4,544 – \$474). Procam faces a margin call of \$880 (\$4,950 – \$4,070).

On the final trading day, Procam has paid a total of \$1,468 (\$588 + \$880) in margin calls and its futures margin account balance is \$5,155, or \$205 in excess of the \$4,950 initial margin. Procam has a cumulative MTM loss of \$1,263 (\$1,468 – \$205) at the start of the last trading day.

- The prior day's gold futures settlement price,  $f_{T-1}(T)$ , is \$1,779.50.
- Gold futures rise \$1 per ounce on the final trading day to settle at  $f_T(T) = \$1,780.50$  per ounce, the same as for the forward in Example 1.
- The daily change in Procam's margin account is an increase of \$100 (\$1 per ounce  $\times$  100 ounces), bringing the margin account to \$5,255.
- Procam's futures margin balance of \$5,255 is returned at settlement for a net return of \$305 (\$5,255 – \$4,950) in margin.
- Procam receives a net return of \$305 in margin at settlement for a cumulative loss upon settlement of \$1,163 (\$305 – \$1,468).

Both the forward and futures contracts involve a \$1,163 settlement loss, but the forward is fully settled at maturity while the futures contract is settled based on the daily MTM. The time value of money principle suggests that these forward and futures settlements are not equivalent amounts of money, but the differences are small for shorter maturities and low interest rates. Also note that under the forward contract in Example 1, the financial intermediary bears counterparty risk to Procam for the forward settlement. In practice, financial intermediaries often use collateral arrangements similar to futures margining for forwards or other derivatives to reduce counterparty credit risk.

At maturity, the number of outstanding contracts, or **open interest**, is settled via cash or physical delivery. A counterparty may instead choose to enter an offsetting futures contract before expiration to close out a position; for example, a futures

contract buyer may simply sell the open contract. The clearinghouse marks the contract to the current price relative to the previous settlement price and closes out the participant's position.

Futures contracts specify whether physical delivery of an underlying or cash settlement occurs at expiration. For example, a commodity futures contract with physical delivery obligates the seller to deliver an underlying asset of a specific type, amount, and quality to a designated location. The buyer must accept and pay for delivery, which ensures that the futures price converges with the spot price at expiration.

## QUESTION SET



### Futures

1. Determine the correct answers to fill in the blanks: If a futures contract buyer's margin account falls below the \_\_\_\_\_, or minimum balance that each contract buyer and seller must hold in the account from trade initiation until final settlement, the buyer must immediately deposit funds to return the account balance to the \_\_\_\_\_.

#### Solution:

If a futures contract buyer's margin account falls below the *maintenance margin*, or minimum balance that each contract buyer and seller must hold in the account from trade initiation until final settlement, the buyer must immediately deposit funds to return the account balance to the *initial margin*.

2. Describe the mark-to-market process for a futures contract.

#### Solution:

The exchange clearinghouse determines an average of the final futures prices of the day and designates that price as the end-of-day settlement price. The daily settlement of gains and losses takes place via each counterparty's futures margin account.

3. Identify these futures contract participants that correspond to the following statements:

A. Futures contract purchaser	1. Must make a margin deposit at contract initiation and maintain a minimum balance until maturity
B. Futures contract seller	2. Receives a margin account deposit if the futures price increases on any trading day
C. Both a futures contract purchaser and a seller	3. Receives a positive payoff if the spot price $S_T$ is below the futures price, $f_0(T)$ , at maturity

#### Solution:

1. The correct answer is C. Both futures contract buyers and sellers must deposit margin and maintain a minimum margin balance (maintenance margin) over the life of a contract.
2. The correct answer is A. The futures contract buyer realizes a mark-to-market gain on any trading day when the futures price increases and receives a corresponding margin account deposit.

3. The correct answer is B. The futures contract seller realizes a positive payoff at maturity if  $S_T < f_0(T)$ .

## SWAPS

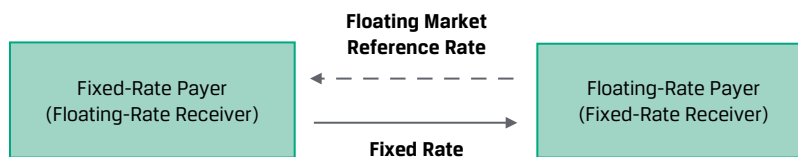
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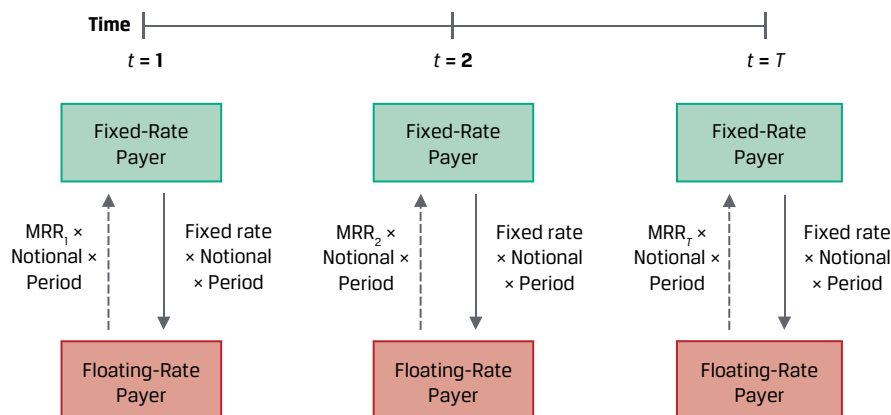
define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics

A swap is a firm commitment under which two counterparties exchange a series of cash flows in the future. One set of cash flows is typically variable, or floating, and determined by a market reference rate that resets each period. The other cash flow stream is usually fixed or may vary based on a different underlying asset or rate. In this case, we refer to the counterparty paying the variable cash flows as the **floating-rate payer** (or fixed-rate receiver) and the counterparty paying fixed cash flows as the **fixed-rate payer** (floating-rate receiver), as shown in Exhibit 4.

### Exhibit 4: Swap Mechanics



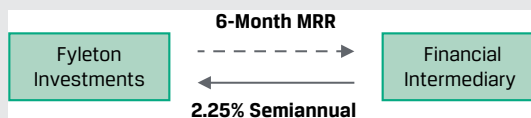
Swaps and forwards have similar features, such as a start date, a maturity date, and an underlying that are negotiated between counterparties and specified in a contract. Interest rate swaps in which a fixed rate is exchanged for a floating rate are the most common swap contract. For each period in the future, the market reference rate (MRR) paid by the floating-rate payer resets, while the fixed rate (referred to as the **swap rate**) is constant, as shown in Exhibit 5.

**Exhibit 5: Swap as a Series of Forward Exchanges**

Counterparties usually exchange a net payment on fixed- and floating-rate payments on the swap as in Example 4.

**EXAMPLE 4****Fyleton Investments Swap**

Fyleton Investments has entered a five-year, receive-fixed GBP200 million interest rate swap with a financial intermediary to increase the duration of its fixed-income portfolio. Under terms of the swap, Fyleton has agreed to receive a semiannual GBP fixed rate of 2.25% and pay six-month MRR.



Calculate the first swap cash flow exchange if six-month MRR is set at 1.95%.

- The financial intermediary owes Fyleton a fixed cash flow payment of GBP2,250,000 (= GBP200 million  $\times$  0.0225/2).
- Fyleton owes the financial intermediary a floating cash flow payment of GBP1,950,000 (= GBP 200 million  $\times$  0.0195/2).
- The fixed and floating payments are netted against one another, and the net result is that the financial intermediary pays Fyleton GBP300,000 (= GBP2,250,000 – GBP1,950,000).

The notional principal is usually not exchanged but, rather, is used for fixed and floating interest payment calculations, as in Example 4. The example demonstrates how an investment manager might use an interest rate swap to change portfolio duration without trading bonds. Issuers often use swaps to alter the exposure profile of a liability, such as a term loan.

As with futures and forward contracts, no money is exchanged when a swap contract is initiated. The value of a swap at inception is therefore effectively zero, ignoring transaction costs. In an earlier fixed-income lesson, it was shown that implied forward rates may be derived from spot rates. Forward MRRs may be used to determine the expected future cash flows for the floating leg of an interest rate swap. The swap rate for the fixed leg payments is determined by solving for a constant fixed yield that equates the present value of the fixed and floating leg payments.

As market conditions change and time passes, the mark-to-market value of a swap will deviate from zero, resulting in a positive MTM to one counterparty and an offsetting negative MTM to the other. Swap credit terms are privately negotiated between counterparties in an over-the-counter agreement and may range from uncollateralized exposure, where each counterparty bears the full default risk of the other, to terms similar to futures margining for one or both counterparties. An event of counterparty default usually triggers swap termination and MTM settlement as for any other debt claim. Centrally cleared swaps between financial intermediaries and a central counterparty (CCP) include margin provisions similar to futures in order to standardize and reduce counterparty risk.

## QUESTION SET



### Swaps

1. Describe a similarity of and a difference between forward and swap contracts.

**Solution:**

**Similarities:** Both forwards and swaps represent firm commitments with an initial value of zero where cash flows are exchanged in the future at a pre-agreed price.

**Difference:** Forwards usually involve one future exchange of cash flows, while a swap contract involves more than one exchange of future cash flows.

2. Determine the correct answers to fill in the blanks: Under a swap contract, we refer to the counterparty paying the variable cash flows as the \_\_\_\_-\_\_\_\_ \_\_\_\_ and the counterparty paying fixed cash flows as the \_\_\_\_-\_\_\_\_ \_\_\_\_.

**Solution:**

Under a swap contract, we refer to the counterparty paying the variable cash flows as the *floating-rate payer* (or *fixed-rate receiver*) and the counterparty paying fixed cash flows as the *fixed-rate payer* (or *floating-rate receiver*).

3. Identify the interest rate swap participants that correspond to the following statements:

A. Fixed-rate payer

1. Makes a payment each interest period based on a market reference rate

B. Floating-rate payer

2. May face a positive or a negative mark to market over the life of an interest rate swap contract

C. Both a fixed-rate payer and a floating-rate payer

3. Receives a net payment on the swap for any interest period for which the market reference rate exceeds the fixed rate

**Solution:**

1. The correct answer is B. A floating-rate payer on a swap makes a payment each period based on a market reference rate.

2. The correct answer is C. Both a fixed-rate payer and a floating-rate payer may face a positive MTM or negative MTM on a swap contract.

3. The correct answer is A. A fixed-rate payer (also known as the float-ing-rate receiver) receives a net payment if the market reference rate exceeds the fixed rate for a given period.

## 5

### OPTIONS

- ☐ define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics
- ☐ determine the value at expiration and profit from a long or a short position in a call or put option
- ☐ contrast forward commitments with contingent claims

Contingent claims are a type of ETD or OTC derivative contract in which one of the counterparties has the right to determine whether a trade will settle based on the underlying value. Option contracts are the most common contingent claim. Similar to forwards and futures, options are derivative contracts between a buyer and a seller that specify an underlying, contract size, a pre-agreed execution price, and a maturity date. The option buyer has the right but not the obligation to transact the trade, and the option seller has the obligation to fulfill the transaction as chosen by the option buyer. As a consequence, the payoff to an option buyer is always zero or positive. It can never be negative.

Assume an option buyer pays a premium ( $c_0$ ) of \$5 at  $t = 0$  for the right—but not the obligation—to buy stock  $S$  at time  $T$  at a pre-agreed price ( $X$ ) of \$30. The option buyer's decision at maturity depends on the stock price at maturity ( $S_T$ ), as shown in the following two scenarios:

#### Scenario 1: Transact ( $S_T > X$ )

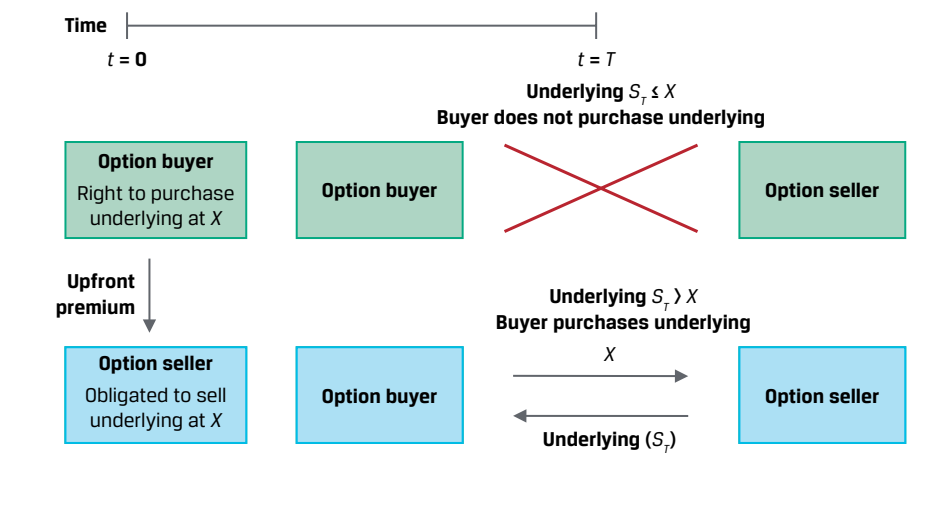
- If  $S_T = \$40$ , the option buyer chooses to exercise the option and buy the stock for  $X = \$30$ .
- The option buyer gains \$10 [ $(S_T - X) = \$40 - \$30$ ] on the transaction.
- The option buyer realizes a \$5 profit [ $(S_T - X) - c_0 = (\$40 - \$30) - \$5$ ].

#### Scenario 2: Do Not Transact ( $S_T < X$ )

- If  $S_T = \$25$ , the option buyer chooses not to exercise the option and buy the stock for  $X = \$30$ .
- The option buyer realizes a \$5 loss ( $c_0 = \$5$ ).

Exhibit 6 shows the option mechanics for the case where the option buyer pays for the right to purchase an underlying stock  $S$  at a pre-agreed execution price of  $X$  in the future ( $t = T$ ).



**Exhibit 6: Call Option Mechanics**

The decision to transact the underlying is referred to as an **exercise**, and the pre-agreed execution price is called the **exercise price** (or strike price). Option buyers may transact the underlying in the future at their sole discretion at the exercise price, a pre-agreed future spot price that may be above, at, or below the forward price, as shown in an earlier lesson. This right to exercise in the future has a value that is paid upfront to the option seller in the form of an **option premium**.

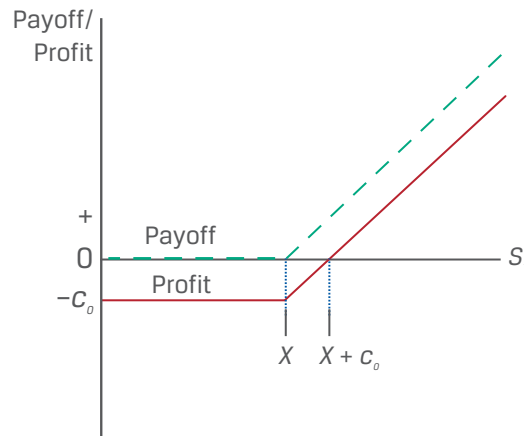
Option contract terms, such as the right to buy or sell, exercise price, maturity, and size, may either be agreed on between the counterparties in an over-the-counter transaction or executed on an exchange based on standardized terms. This lesson focuses on **European options**, or options that may be exercised only at maturity, although other option styles exist. **American options**, for example, may be exercised at any time from contract inception until maturity. Note the labels “European” and “American” refer not to where these options are used but, rather, to the difference in when they can be exercised.

Two primary option types exist—namely, (1) the right to buy an underlying known as a **call option** and (2) the right to sell the underlying, or a **put option**. An option buyer will exercise a call or put option only if it returns a positive payoff. If not exercised, the option expires worthless, and the option buyer’s loss equals the premium paid.

One factor in an option’s value prior to maturity ( $t < T$ ) is the option’s exercise value at time  $t$ , which is referred to as an option’s **intrinsic value**.

We can say a call option is **in-the-money** at time  $t$  if the spot price,  $S_t$ , exceeds  $X$ , with an intrinsic value equal to  $(S_t - X)$ . Both **out-of-the-money** options (where  $S_t < X$ ) and **at-the-money** options ( $S_t = X$ ) have zero intrinsic value, so their price,  $c_t$ , consists solely of time value.

Call option buyers will gain from a rise in the price of the underlying. Exhibit 7 shows the payoff and profit at maturity for a call option buyer.

**Exhibit 7: Long Call Payoff Profile**

The option buyer pays a call option premium,  $c_0$ , at time  $t = 0$  to the option seller and has the right to purchase the underlying,  $S_T$ , at an exercise price of  $X$  at time  $t = T$ . The exercise *payoff* ( $S_T - X$ ) is positive if  $S_T > X$  and zero if  $S_T \leq X$ . The call option value at maturity,  $c_T$ , may be expressed as follows:

$$c_T = \max(0, S_T - X). \quad (1)$$

The call option buyer's *profit* equals the *payoff* minus the call premium,  $c_0$  (ignoring the time value of money in this lesson):

$$\Pi = \max(0, S_T - X) - c_0. \quad (2)$$

This asymmetric payoff profile is a common feature of contingent claims, which are sometimes referred to as **non-linear derivatives**.

**EXAMPLE 5****Highest Capital—Call Option Purchase**

Highest Capital purchases a call option on the S&P 500 Health Care Select Sector Index (SIXV). This six-month exchange-traded option contract has a size of 100 index units and an exercise price of \$1,240 per unit versus the initial SIXV spot price of \$1,180.95. The option premium paid upfront is \$24.85 per unit, or \$2,485 ( $= \$24.85 \times 100$ ). As the option nears maturity, a Highest analyst is asked to determine the expected option payoff and profit per unit at maturity under different scenarios for the SIXV spot price on the exercise date.

She compiles the following table:

$c_0$  = Call option premium = \$24.85 per unit.

= Exercise price at time  $T$  = \$1,240 per unit.

$S_T$  = Spot price per unit at time  $T$ .

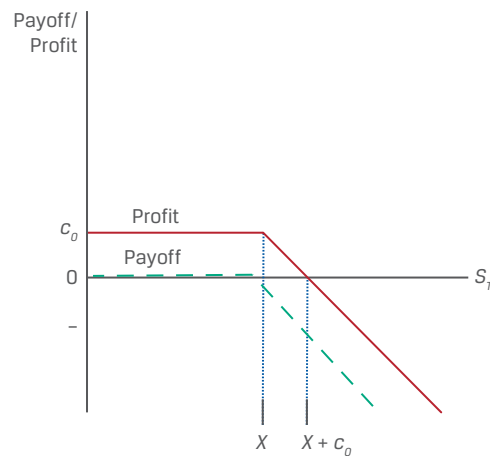
SIXV Spot Price ( $S_T$ )	Exercise Price ( $X$ )	Payoff $\max(0, S_T - X)$	Profit $\max(0, S_T - X) - c_0$
\$1,280	\$1,240	\$40	\$15.15
\$1,260	\$1,240	\$20	-\$4.85

SIXV Spot Price ( $S_T$ )	Exercise Price ( $X$ )	Payoff $\max(0, S_T - X)$	Profit $\max(0, S_T - X) - c_0$
\$1,240	\$1,240	\$0	-\$24.85
\$1,220	\$1,240	\$0	-\$24.85

Example 5 raises another question regarding an option's value prior to maturity ( $t < T$ ). The longer the time to option maturity, the more likely it is that a favorable change in the underlying price will increase both the likelihood and profitability of exercise. This **time value** of an option is always positive and declines to zero as an option reaches maturity.

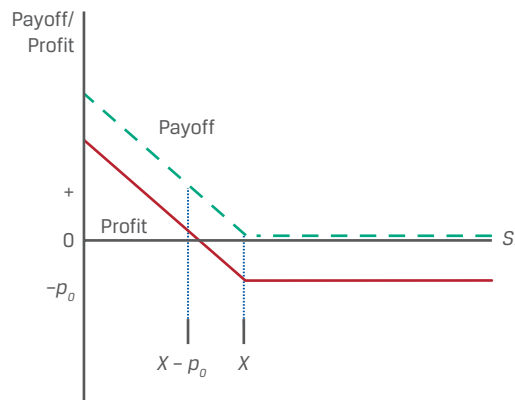
In contrast to the call option buyer with unlimited upside potential and a loss limited to the premium paid, the call option seller receives a maximum of the premium and faces unlimited downside risk as the underlying appreciates above the exercise price. The short call payoff profile in Exhibit 8 is a mirror image of Exhibit 7.

**Exhibit 8: Short Call Payoff Profile**



The option buyer and seller payoff profiles demonstrate the one-sided nature of counterparty credit risk for contingent claims. That is, the option seller has no credit exposure to the option buyer once the premium is paid. However, the option buyer faces the counterparty credit risk of the option seller equal to the option payoff at maturity.

Put option buyers benefit from a lower underlying price by selling the underlying at a pre-agreed exercise price. A put option buyer exercises when the underlying price is below the exercise price ( $S_T < X$ ), as shown in Exhibit 9.

**Exhibit 9: Long Put Payoff Profile**

The put option buyer pays a premium of  $p_0$  at inception to the option seller and will exercise the option only if  $(X - S_T) > 0$ . As in Equations 1 and 2, we may show the long put option *payoff* and *profit*,  $\Pi$ , as follows:

$$p_T = \max(0, X - S_T). \quad (3)$$

$$\Pi = \max(0, X - S_T) - p_0. \quad (4)$$

The payoff or profit from a put option seller's perspective is the opposite of the put option buyer's gain or loss for a given underlying price at expiration. As in the case of the call option seller, the put option seller has a maximum gain equal to the premium. However, although a call option seller faces unlimited potential loss as the underlying appreciates beyond the exercise price, the put option seller's loss is usually limited because the underlying price cannot fall below zero. The short put option *payoff* and *profit* are as follows:

$$-p_T = -\max(0, X - S_T). \quad (5)$$

$$\Pi = -\max(0, X - S_T) + p_0. \quad (6)$$

**QUESTION SET****Options**

1. Calculate the SIXV spot price at maturity from Example 5 at which Highest Capital will reach a breakeven point and earn zero profit.

**Solution:**

The call purchaser has a profit equal to  $\max(0, S_T - X) - c_0$ . In the case of Highest in Example 5, the SIXV exercise price is \$1,240 and the initial call premium is \$24.85. The breakeven or zero profit point is therefore equal to  $\$1,240 + \$24.85$ , or  $\$1,264.85$ .

2. A put option seller receives a \$5 premium for a put option sold on an underlying with an exercise price of \$30. What is the option seller's maximum profit under the contract? What is the maximum loss under the contract?

**Solution:**

A put option seller receives a \$5 premium ( $p_0$ ) for a put option sold on an underlying with an exercise price ( $X$ ) of \$30. The put option seller's profit is  $\Pi = -\max(0, X - S_T) + p_0$ . If the option is unexercised,  $-\max(0, X - S_T) = 0$  and the put seller earns  $p_0 = \$5$ . If the option is exercised and  $S_T = 0$ , then  $\Pi = -\max(0, 30 - 0) + 5 = -\$25$ . Therefore, the option seller's maximum profit under the contract is \$5 and the maximum loss under the contract is \$25.

3. Identify the option contract participants that correspond to the following statements:

- |  |  |
|--|--|
| A. Put option seller                                 | 1. Has no counterparty credit risk to the option buyer once the upfront premium has been paid              |
| B. Call option seller                                | 2. Earns a profit equal to the premium if the underlying price at maturity is less than the exercise price |
| C. Both a put option seller and a call option seller | 3. Earns a profit equal to the premium if the underlying price at maturity exceeds the exercise price      |

**Solution:**

1. The correct answer is C. An option seller has no counterparty credit risk to the option buyer once the upfront premium has been paid.
2. The correct answer is B. A call option seller earns a profit equal to the premium if the underlying price at maturity is less than the exercise price.
3. The correct answer is A. A put option seller earns a profit equal to the premium if the underlying price at maturity is greater than the exercise price.

## CREDIT DERIVATIVES

# 6



define forward contracts, futures contracts, swaps, options (calls and puts), and credit derivatives and compare their basic characteristics

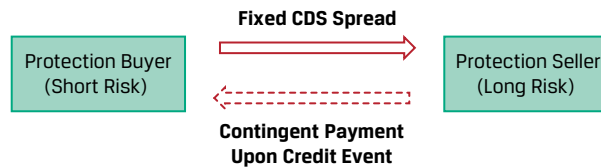
Credit derivative contracts are based on a credit underlying, or the default risk of a single debt issuer or a group of debt issuers in an index. The most common credit derivative contract is a credit default swap. CDS contracts allow an investor to manage the risk of loss from issuer default separately from a cash bond. CDS contracts trade based on a credit spread (**CDS credit spread**) similar to that of a cash bond. Credit spreads depend on the probability of default (POD) and the loss given default (LGD), as shown in an earlier lesson. A *higher* credit spread (or higher likelihood of issuer financial distress) corresponds to a *lower* cash bond price, and vice versa.

Despite their name, CDS contracts are contingent claims that share some features of firm commitments. Unlike the call and put options discussed earlier, both the timing of exercise and payment upon exercise under a CDS contract vary depending

on the underlying issuer(s). As in the case of a standard interest rate swap, a CDS contract priced at a par spread has a zero net present value, and the notional amount is not exchanged but, rather, serves as a basis for spread and settlement calculations.

In a CDS contract, the credit protection buyer pays the credit protection seller to assume the risk of loss from the default of an underlying third-party issuer. If an issuer **credit event** occurs—usually defined as bankruptcy, failure to pay an obligation, or an involuntary debt restructuring—the credit protection seller pays the credit protection buyer to settle the contract. This contingent payment equals the issuer loss given default for the CDS contract notional amount. Exhibit 10 shows the periodic cash flows under a CDS contract.

#### Exhibit 10: Periodic Payments under a Credit Default Swap



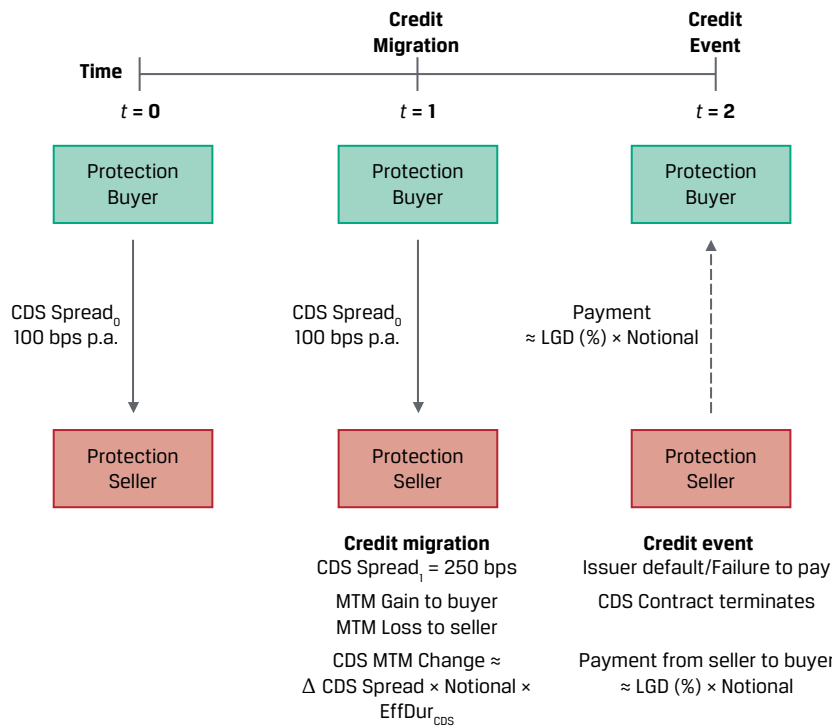
The underlying may be a corporate or sovereign issuer, an index of issuers, or a special purpose entity with a portfolio of loans, mortgages, or bonds.

A buyer can use a CDS contract as a hedge of existing credit exposure to the underlying issuer. The credit protection afforded by a CDS is similar to insurance for a buyer with an existing fixed-income exposure to the third-party issuer. The buyer may suffer a loss in value on its fixed-income exposure from the credit event but will receive a payment from the CDS contract that will offset that loss.

A credit protection buyer without the corresponding fixed-income exposure who buys a CDS is seeking to gain from higher credit spreads (which correspond to lower cash bond prices) for an underlying issuer and is therefore short credit risk.

The credit protection seller receives a periodic fixed spread payment in exchange for assuming the contingent risk of paying the credit protection buyer to offset the loss under a credit event. The contract structure is similar to insurance, with the periodic premium over the life of the contract agreed to upfront and with the timing and size of the loss under the credit event being unknown. The seller's position is therefore similar to that of a long risk position in the issuer's underlying bond.

For example, Exhibit 11 shows the CDS contract for an underlying issuer that experiences credit migration ( $t = 1$ ) followed by a credit event ( $t = 2$ ).

**Exhibit 11: CDS Contract with Credit Migration and Credit Event**

The protection buyer agrees to pay a fixed spread of 100 bps p.a. at  $t = 0$  for the contract term. As the issuer's CDS spread widens to 250 bps p.a. at  $t = 1$ , the buyer gains on the CDS contract due to the low fixed spread paid while the seller loses due to the low fixed spread received relative to the current higher CDS market spread. As for any fixed-income instrument, the effective duration of the remaining contract may be used to approximate the MTM change. An issuer credit event at  $t = 2$  causes the contract to terminate, and the seller must make a payment to the buyer equal to the percentage of loss (LGD) multiplied by the CDS contract notional.

**QUESTION SET****Credit Derivatives**

1. Determine the correct answer to fill in the blanks: The contingent payment under a credit default swap equals the \_\_\_\_\_ for the CDS notional amount specified in the contract.

**Solution:**

The contingent payment under a credit default swap equals the *loss given default* for the CDS notional amount specified in the contract.

2. Describe how a credit protection seller's position is similar to that of an underlying cash bond investment.

**Solution:**

A credit protection seller receives a periodic CDS spread payment in exchange for the contingent risk of payment to the buyer under an issuer credit event. A cash bond investor receives a periodic coupon that incorporates

an issuer's credit spread in exchange for a potential loss if the issuer defaults. Under the CDS contract and the cash bond, this potential payment or loss equals the LGD. The credit protection seller's position is therefore similar to that of a long risk position in the issuer's underlying bond.

3. Identify the CDS contract participants that correspond to the following statements:

- |  |  |
|--|--|
| A. Credit protection buyer                                       | 1. Seeks to gain from higher issuer credit spreads   |
| B. Credit protection seller                                      | 2. Enters into a derivative contract that transfers the risk of loss from a credit event of an underlying third-party issuer |
| C. Both a credit protection buyer and a credit protection seller | 3. Faces an MTM gain on the CDS contract if the CDS spread of the underlying issuer falls                                    |

**Solution:**

1. The correct answer is A. A credit protection buyer seeks to gain from higher issuer credit spreads.
2. The correct answer is C. Both the credit protection buyer and credit protection seller enter into a derivative contract that transfers the risk of loss from a credit event of an underlying third-party issuer.
3. The correct answer is B. A credit protection seller faces an MTM gain on the CDS contract if the CDS spread of the underlying issuer falls. The decline in the issuer's CDS spread versus the original fixed spread on the CDS contract means that the protection seller is receiving an above-market spread. This above-market CDS spread more than compensates the seller for the new, lower level of credit risk and results in an MTM gain.

## 7

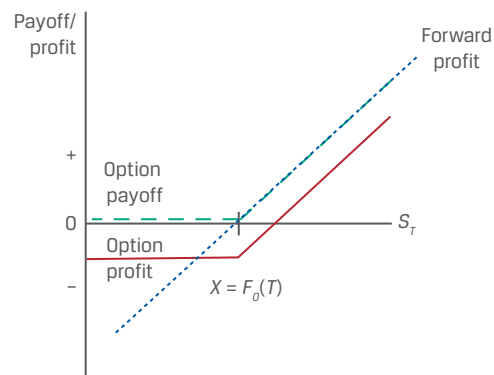
## FORWARD COMMITMENTS VS. CONTINGENT CLAIMS



contrast forward commitments with contingent claims

A firm commitment requires both counterparties to perform under a derivative contract, while an option buyer can decide whether to perform under the contract at maturity depending on the underlying price relative to the exercise price. Market participants often create similar exposures to an underlying using these different derivative instrument types. For example, both a long forward position and a long call option position will gain from an increase in the underlying price. Exhibit 12 contrasts the payoff and profit of these two derivative contracts, where the exercise price,  $X$ , equals the forward price,  $F_0(T)$ .



**Exhibit 12: Long Forward and Long Call Option Payoff Profiles**

As shown earlier, both the long forward and the call option payoffs increase as  $S_T$  rises. In the case of a forward, this linear relationship is equal to  $[S_T - F_0(T)]$ , with the payoff equal to profit because no cash is exchanged at inception. For the buyer of a call option with an exercise price of  $F_0(T)$ , Equation 2 changes to

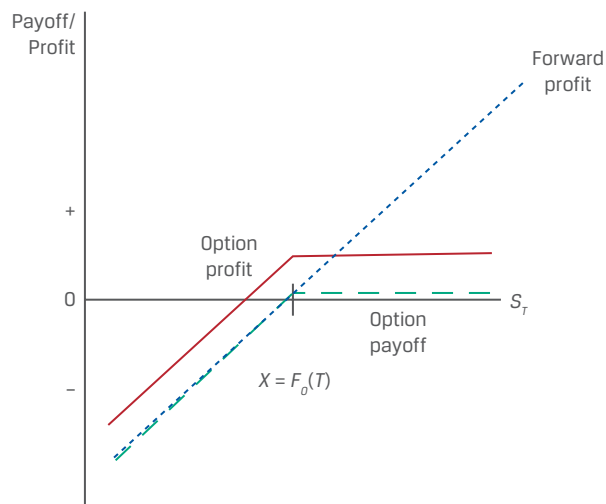
$$\Pi = \max[0, S_T - F_0(T)] - c_0.$$

Setting the forward payoff/profit  $[S_T - F_0(T)]$  equal to the call option profit,  $\Pi$ , gives us the following relative profit profile between the forward and option:

- $S_T - F_0(T) > -c_0$  Forward profit exceeds option profit
- $S_T - F_0(T) = -c_0$  Forward profit equals call option profit
- $S_T - F_0(T) < -c_0$  Option profit exceeds forward profit

The side-by-side comparison in Exhibit 12 between the forward and call option profit diagrams shows the long call option's similarity to a long position in the underlying with downside protection in exchange for paying a premium.

Another contingent claim that benefits from a rise in the underlying price is the sold put option. While the long call option and long forward payoffs both rise when the underlying price is above the exercise price, the put option seller's profit is limited to the upfront premium. Exhibit 13 contrasts the short put payoff and profit with a long forward if the exercise price,  $X$ , equals  $F_0(T)$ .

**Exhibit 13: Long Forward and Short Put Option Payoff Profile**

We can compare the long forward payoff/profit of  $[S_T - F_0(T)]$  to a modified version of Equation 6:

$$\Pi = -\max[0, F_0(T) - S_T] + p_0.$$

Setting the forward profit  $[S_T - F_0(T)]$  equal to the put option profit,  $\Pi$ , gives us the following relative profit profile between the forward and option:

- $S_T - F_0(T) > p_0$  Forward profit exceeds option profit
- $S_T - F_0(T) = p_0$  Forward profit equals option profit
- $S_T - F_0(T) < p_0$  Option profit exceeds forward profit

The side-by-side comparison in Exhibit 13 of the forward and sold put option profit diagrams shows the sold put option's similarity to a long position in the underlying, with gains from price appreciation forgone in exchange for receiving a premium. The apparent symmetry between long call and short put positions and the long forward position will be examined in greater detail in a later lesson.

### QUESTION SET



### Firm Commitments and Contingent Claims

1. Determine the correct answers to fill in the blanks: A \_\_\_\_\_ forward position, a \_\_\_\_\_ call option position, and a \_\_\_\_\_ put option position will gain from a decrease in the underlying price.

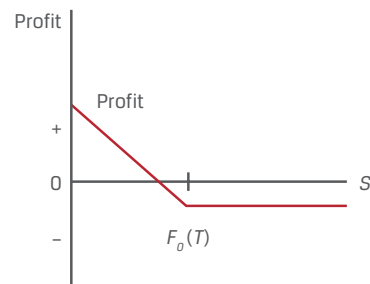
#### Solution:

A *short* forward position, a *short* call option position, and a *long* put option position will gain from a decrease in the underlying price.

2. Identify the derivative positions that correspond to the following profit profiles at maturity assuming that the exercise price ( $X$ ) equals  $F_0(T)$ :

A. Short forward position

1.



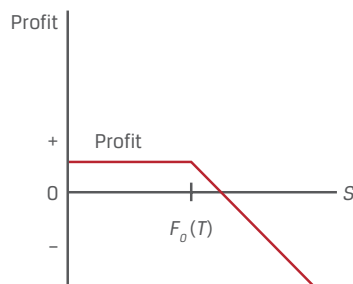
B. Long put position

2.



C. Short call position

3.



**Solution:**

1. The correct answer is B.
2. The correct answer is A.
3. The correct answer is C.

3. Describe the point at which a short forward and a long put with an exercise price ( $X$ ) equal to the forward price,  $F_0(T)$  have the same profit.

**Solution:**

The short forward and long put positions will have the same profit when  $F_0(T) - S_T = -p_0$ . A short forward position gains from a decline in the underlying price with a payoff/profit of  $[F_0(T) - S_T]$ , and a long put position has a profit of  $\Pi = \max[0, F_0(T) - S_T] - p_0$ , from Equation 4, with an exercise price equal to the forward price,  $X = F_0(T)$ .

## PRACTICE PROBLEMS

### The following information relates to questions 1-4

Biomian Limited is a Mumbai-based biotech company with common stock and listed futures and options on the National Stock Exchange (NSE). The Viswan Family Office (VFO) currently owns 10,000 Biomian common shares. VFO would like to reduce its long Biomian position and diversify its equity market exposure but will delay a cash sale of shares for tax reasons for six months.

1. Which of the following derivative contracts available to VFO's chief investment officer is best suited to reduce exposure to a decline in Biomian's stock price in the next six months?
  - A. A short put position on Biomian stock that expires in six months
  - B. A long call position on Biomian stock that expires in six months
  - C. A short futures position in Biomian stock that settles in six months
2. VFO's market strategist believes that Biomian's share price will rise over the next six months but would like to protect against a decline in Biomian's share price over the period. Which of the following positions is best suited for VFO to manage its existing Biomian exposure based on this view?
  - A. A long put position on Biomian stock that expires in six months
  - B. A short call position on Biomian stock that expires in six months
  - C. A long futures position in Biomian stock that settles in six months
3. Assume that Biomian shares rise over the next six months. Which of the following statements about VFO's derivative strategies under this scenario is most accurate?
  - A. A forward sale of Biomian shares in six months would be more profitable than purchasing the right to sell Biomian shares in six months.
  - B. Purchasing the right to sell Biomian shares in six months would be more profitable than a forward sale of Biomian shares in six months.
  - C. We do not have enough information to determine whether a forward sale or the right to sell Biomian shares will be more profitable in six months.
4. VFO's market strategist is considering a six-month call option strategy on the NIFTY 50 benchmark Indian stock market index to increase broad market equity exposure. The NIFTY 50 price today is INR15,200, and the strategist observes that a call option with a INR16,000 exercise price ( $X$ ) is trading at a premium of INR1,500. Which of the following represents the payoff and profit of this strategy just prior to maturity if the NIFTY 50 is trading at INR16,500?
  - A. Payoff is INR500; profit is -INR1,000.
  - B. Payoff is INR1,300; profit is INR800.

- C. Payoff is INR1,300; profit is INR500.
-

## SOLUTIONS

1. C is correct. VFO may consider either a short futures position in (or a forward sale of) Biomian shares in six months to achieve its objective. This firm commitment allows VFO to offset its long position with a short position in six months at a pre-agreed price. The futures contract is an exchange-traded derivative with standardized terms set by the exchange and requires initial margin and daily settlement. Answers A and B are contingent claims that can both potentially increase, not decrease, VFO's exposure to Biomian stock in six months.
2. A is correct. VFO should purchase a six-month put option on Biomian shares to manage its exposure based on the market strategist's view. This contingent claim grants VFO the right but not the obligation to sell Biomian shares at a pre-agreed exercise price in exchange for a premium. A put option buyer exercises the option at maturity when the underlying price is below the exercise price. This allows VFO to continue to benefit from a rise in Biomian's share price over the next six months with a limited downside. Neither B nor C provides VFO with downside protection if Biomian stock declines in six months.
3. C is correct. Under a forward sale of Biomian shares, the profit is  $[F_0(T) - S_T]$ . If the shares rise significantly over the next six months—that is,  $S_T > F_0(T)$ —then VFO's loss on the derivative is the difference between the Biomian forward price,  $F_0(T)$ , and the spot price,  $S_T$ . Under the long put option on Biomian shares, VFO's profit is  $\max(0, X - S_T) - p_0$ . If Biomian shares rise significantly over the next six months (i.e.,  $S_T > X$ ), then the option expires worthless and VFO's loss is limited to the put premium paid,  $p_0$ . If  $[F_0(T) - S_T] > -p_0$ , then VFO's loss would be greater under the firm commitment than under the contingent claim.
4. A is correct. The profit is equal to  $\Pi = \max(0, S_T - X) - c_0$ , and the payoff is equal to  $\max(0, S_T - X)$ . The exercise price is INR16,000, and the spot price just prior to maturity is INR16,500, so  $\Pi = -1,000 [= (16,500 - 16,000) - 1,500]$ , and the payoff is equal to INR500  $[= (16,500 - 16,000)]$ .

## LEARNING MODULE

## 3

## Derivative Benefits, Risks, and Issuer and Investor Uses

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	describe benefits and risks of derivative instruments
<input type="checkbox"/>	compare the use of derivatives among issuers and investors

## INTRODUCTION

1

Earlier lessons described how derivatives expand the set of opportunities available to market participants to create or modify exposure or to hedge the price of an underlying. This learning module describes the benefits and risks of using derivatives and compares their use among issuers and investors.

### LEARNING MODULE OVERVIEW



- Derivatives allow market participants to allocate, manage, or trade exposure without exchanging an underlying in the cash market.
- Derivatives also offer greater operational and market efficiency than cash markets and allow users to create exposures unavailable in cash markets.
- Derivative instruments can involve risks such as a high degree of implicit leverage and less transparency in some cases than cash instruments, as well as basis, liquidity, and counterparty credit risks. Excessive risk taking in the past by market participants through the use of derivatives has contributed to market destabilization and systemic risk.
- Issuers typically use derivative instruments to offset or hedge market-based underlying exposures that impact their assets, liabilities, and earnings.
- Issuers usually seek hedge accounting treatment for derivatives to minimize income statement and cash flow volatility.

- Investors use derivatives to modify investment portfolio cash flows, replicate investment strategy returns in cash markets, and/or create exposures unavailable to cash market participants.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

- Which of the following statements does **not** describe a likely operational advantage of a futures market transaction as compared to a cash market transaction?
  - It is easier to take a short position in the futures market than in the cash market.
  - There is greater liquidity in the futures market than in the cash market.
  - Cash requirements to buy in the cash market are lower than margin requirements to buy in the futures market.

#### Solution:

C is correct. The opposite is true: Margin requirements of a futures contract are typically only a small percentage of the cash requirement to buy the same amount of underlying in the cash market. A and B are both incorrect because both of these statements describe operational advantages of futures markets over cash markets.

- Identify which derivative risk fits each of the following statements:

A. Basis risk	1. Potential divergence between the cash flow timing of a derivative versus an underlying or hedged transaction
B. Liquidity risk	2. Potential divergence between the expected value of a derivative versus an underlying or hedged transaction
C. Counterparty credit risk	3. Potential for a derivatives contract participant to fail to meet their obligations under an agreement

#### Solution

- B is correct. Liquidity risk is the potential divergence between the cash flow timing of a derivative versus an underlying or hedged transaction.
- A is correct. Basis risk is the potential divergence between the expected value of a derivative versus an underlying or hedged transaction.
- C is correct. Counterparty credit risk is the potential for a derivatives contract participant to fail to meet their obligations under an agreement.



3. Identify which benefit of derivatives use fits each of the following examples:

- |   |  |
|---|--|
| A. Price discovery function                       | 1. Equity market participants monitor index futures prior to the market open for an indication of the direction of cash market prices in early trading.                        |
| B. Operational advantages                         | 2. An issuer may wish to lock in its future debt costs in advance of the maturity of an outstanding debt issuance.   |
| C. Ability to allocate, transfer, and manage risk | 3. Futures contracts in physical commodities eliminate the need to directly transport, insure, and store a physical asset in order to take a position in its underlying price. |

#### Solution

1. A is correct. Equity market participants monitoring index futures prior to the market open for an indication of the direction of cash market prices in early trading is an example of the derivatives price discovery function.
2. C is correct. An issuer locking in its future debt costs in advance of the maturity of an outstanding debt issuance is an example of the ability to allocate, transfer, and manage risk.
3. B is correct. Futures contracts in physical commodities eliminating the need to directly transport, insure, and store a physical asset in order to take a position in its underlying price is an example of the operational advantages of a derivative.

4. Which of the following hedge accounting designations is appropriate for categorizing a corporate issuer's use of an interest swap converting a floating-rate debt into a fixed-rate debt?

- A. Fair value hedge
- B. Cash flow hedge
- C. Net investment hedge

#### Solution:

B is correct. Cash flow hedge treatment is appropriate for instances in which a variable cash flow is converted to a fixed cash flow through the use of a derivative. A is incorrect because a fair value hedge is appropriate accounting treatment for derivative contracts that offset fluctuations in the fair value of the underlying. C is incorrect because a net investment hedge offsets the foreign currency risk of the value of a foreign subsidiary.

## DERIVATIVE BENEFITS

# 2



describe benefits and risks of derivative instruments

Earlier lessons demonstrated how market participants use derivative instruments as an alternative to cash markets to hedge or offset commercial risk as well as create or modify exposure to the price of an underlying. We now take a more detailed look at these and other benefits of the use of derivatives, while also considering several risks unique to derivative instruments.

Derivative instruments provide users the opportunity to allocate, transfer, and/or manage risk without trading an underlying. Cash or spot market prices for financial instruments and commercial goods and services are a critical source of information for the decision to buy or sell. However, in many instances, issuers and investors face a timing difference between an economic decision and the ability to transact in a cash market.

For example, issuers face the following timing differences when making operational and financing decisions:

- A manufacturer may need to order commodity inputs for its production process in advance of receiving finished-goods orders.
- A retailer may await a shipment of goods priced in a foreign currency before selling domestic currency to make payment.
- An issuer may wish to lock in its future debt costs in advance of the maturity of an outstanding debt issuance.

Investors may face similar timing issues when making portfolio decisions that are separate from cash market transactions, as in the following cases:

- An investor may seek to capitalize on a market view but lack the necessary cash on hand to transact in the spot market.
- In anticipation of a future stock dividend, debt coupon, or principal repayment, an investor may decide today how it will reinvest the proceeds in the future.

The ability to buy or sell a derivative instrument today at a pre-agreed price at a future date can bridge the timing gap between an economic decision and the ability to transact in underlying price risk under these scenarios. The use of forward commitments or contingent claims to allocate or transfer risk across time and among market participants able and willing to accept those exposures is a consistent theme in derivative markets. Example 1 builds on an earlier illustration of how an issuer may benefit from the use of a derivative associated with a commercial contract.

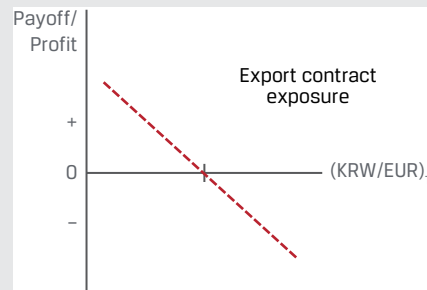
#### EXAMPLE 1

##### Foreign Exchange Risk Transfer of an Export Contract

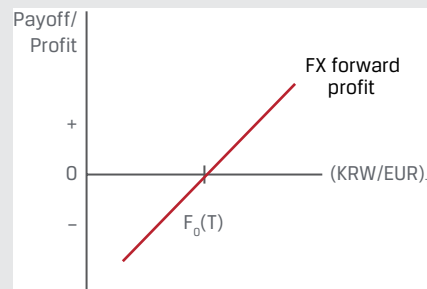
Recall Montau AG, the German capital goods producer introduced earlier, which signs a commercial contract with Jeon, Inc., a Korean manufacturer, to deliver a laser cutting machine at a price of KRW650,000,000 in 75 days. Montau has fixed domestic currency (EUR) costs and therefore faces a timing mismatch between EUR costs incurred and EUR revenue realized upon the delivery of the machine and sale of KRW received in exchange for EUR in the spot FX market.

Describe Montau's currency exposure and how an FX forward contract may be used to mitigate its FX price risk.

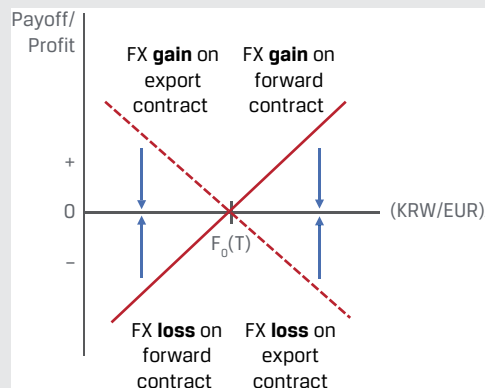
Montau will receive KRW in 75 days, which it must sell to cover its EUR costs. This exposes the firm to KRW/EUR exchange rate changes for 75 days. Specifically, if the KRW depreciates versus the EUR, Montau will be able to purchase fewer EUR than expected with the KRW proceeds, resulting in a loss due to the FX timing mismatch. Montau's exposure profile due to the mismatch is as follows:



In order to mitigate its export contract exposure, Montau enters an FX forward to sell KRW and purchase EUR at a fixed price  $[F_0(T)]$  in 75 days to eliminate the KRW/EUR exchange rate mismatch arising from the export contract. The FX forward payoff and profit profile is as follows:



The FX forward payoff and profit profile offsets Montau's export contract exposure as a hedge, as shown in the following combined diagram:



The ability to trade and/or manage risk using derivatives extends to the creation of exposure profiles, which are unavailable in cash markets. The following example combines a long cash position with a sold derivative to increase an investor's expected return, based on a specific market view.

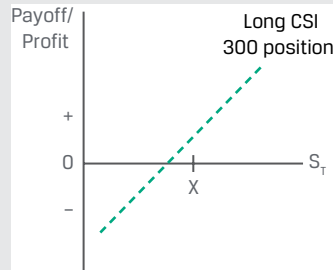
## EXAMPLE 2

### Covered Call Option Strategy

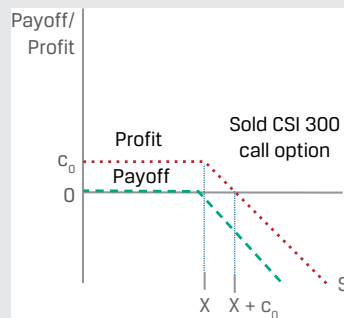
South China Sprintwyck Investments (SCSI) has a Chinese equity portfolio that has outperformed in the first half of the year due to an overweight position in health care industry shares. SCSI holds a long position in the Shenzhen China Securities Index (CSI) 300 Health Care Index (CSI 300) traded on the China Financial Futures Exchange (CFFEX). SCSI's CIO expects volatility in the CSI 300 to decline and the CSI 300 price at year-end to be at or slightly above the current spot price. Rather than sell the CSI 300 position today in the cash market, she decides to sell a CSI 300 call option at an exercise price 5% above the current spot market price for the remaining six months of the year.

Describe the *difference* in SCSI's CSI 300 payoff profile between 1.) the long cash position and 2.) the sold call option plus long cash position (referred to as a **covered call** strategy) at the end of the year.

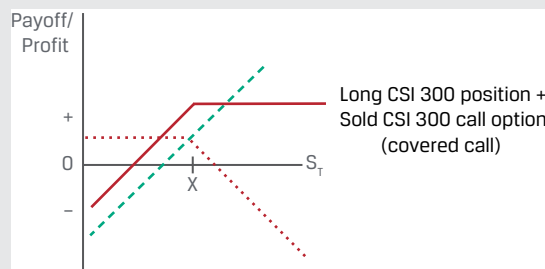
1. SCSI's long cash position will rise or fall in value as the CSI 300 spot price changes until the end of the year.



The *difference* between 1.) and 2.) is the sale of a CSI 300 call option. SCSI sells the call and receives an upfront premium. SCSI assumes unlimited downside risk as the CSI 300 price rises above the exercise price:



2. The combination of the long CSI 300 cash position and the short CSI 300 call option results in the profit profile represented by the solid line below:



The covered call strategy payoff may be described as follows:

- **CSI 300 appreciates by less than 5%:** The call option expires unexercised. SCSI's profit equals the long CSI 300 position plus the call premium.
- **CSI 300 appreciates by more than 5%:** The call option is exercised. SCSI must pay the option buyer an amount equal to the gain on its long CSI 300 position above the exercise price. The option payoff is said to be "covered" by the long cash position. SCSI retains the call premium.

Derivative instrument prices serve a price discovery function beyond the underlying cash or spot market. For example, futures prices are often seen as revealing information about the direction of cash markets in the future, although they cannot be strictly considered an unbiased forecast of future spot prices.

Market participants often use futures prices to gauge the direction of cash markets in the future in the following ways:

- Equity market participants frequently monitor equity index futures prices prior to the stock market open for an indication of the direction of cash market prices in early trading.
- Analysts often use interest rate futures markets to extract investor expectations of a central bank benchmark interest rate increase or decrease at a future meeting.
- Commodity futures prices are a gauge of supply and demand dynamics between producers, consumers, and investors across maturities.

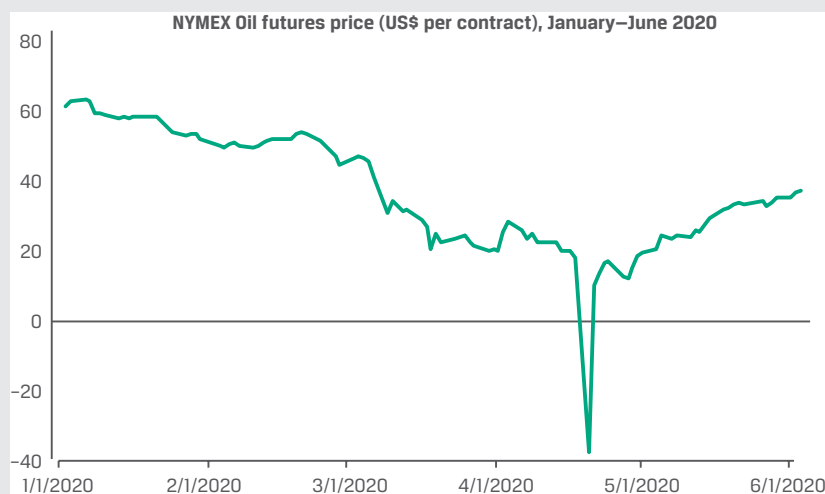
Example 3 provides a case of supply and demand effects on futures prices.

### EXAMPLE 3

#### Negative Oil Futures Prices in 2020

In April 2020, the West Texas Intermediate (WTI) crude oil futures price fell below zero for the first time ever. The New York Mercantile Exchange (NYMEX) WTI crude oil futures contract has an underlying of 1,000 barrels of crude oil delivered to Cushing, Oklahoma, where energy companies store nearly 80 million barrels of oil. Widespread lockdowns in the wake of the COVID-19 pandemic caused demand to plummet, while producers could not cut oil production quickly enough in anticipation of the severe decline. Oil inventory in Cushing skyrocketed as a result.

#### NYMEX Oil Futures Price (USD per contract), January–June 2020



Source: Bloomberg.

As the May futures contract approached expiration in late April 2020, investors assuming the financial risk of oil prices were surprised as oil refiners avoided physical delivery of oil upon contract settlement due to a sharp rise in storage costs. This forced the May futures price sharply lower, to a closing price of –USD37.63 on 20 April 2020. Oil producers and refiners used this futures pricing information to adjust supply more quickly and reduce pressure on storage facilities.

The price discovery function of derivatives extends to contingent claims. As we will see in a later lesson, option prices reflect several characteristics of the underlying, including the expected price risk of the underlying, known as **implied volatility**. The

implied volatility of an option may be derived from its current price and other option features and provides a measure of the general level of uncertainty in the price of the underlying.

Derivative transactions offer a number of operational advantages to cash or spot markets in many instances.

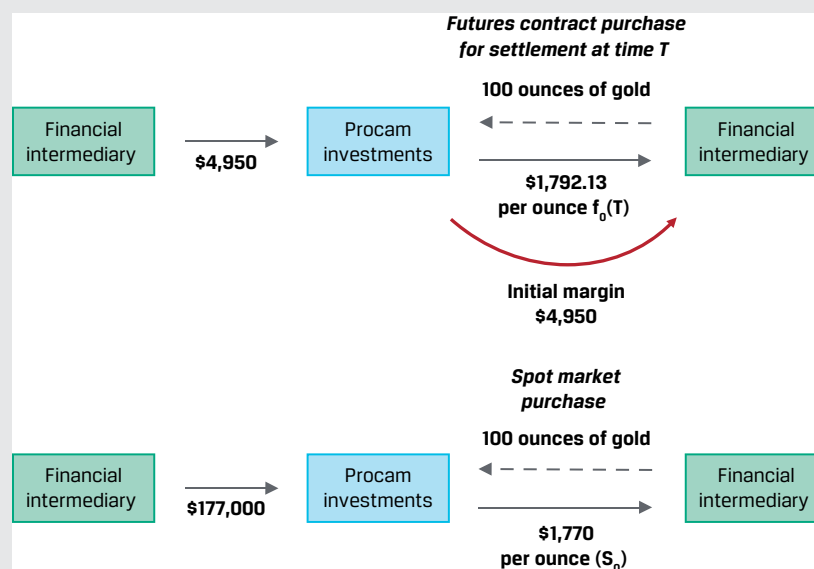
1. **Transaction Costs:** Commodity derivatives eliminate the need to transport, insure, and store a physical asset in order to take a position in its underlying price.
2. **Increased Liquidity:** Derivative markets typically have greater liquidity as a result of the reduced capital required to trade derivatives versus an equivalent cash position in an underlying.
3. **Upfront Cash Requirements:** Initial futures margin requirements and option premiums are low relative to the cost of a cash market purchase.
4. **Short Positions:** In the absence of a liquid derivative market, taking a short position involves the costly process of locating a cash owner willing to lend the underlying for a period sufficient to facilitate a short sale.

Example 4 contrasts the cash outlay of a spot trade with a futures contract based on transaction details familiar from an earlier lesson.

#### EXAMPLE 4

##### Purchase of Spot Gold versus a Gold Futures Contract

In an earlier example, Procam Investments enters a futures contract to buy 100 ounces of gold at a futures price [ $f_0(T)$ ] of USD1,792.13 per ounce that expires in three months and must post USD4,950 in initial margin as required by the exchange. The spot gold price ( $S_0$ ) at the time Procam enters the futures contract is USD1,770 per ounce. Assume that Procam is able to borrow funds from a financial intermediary at a rate of 5% per year. Contrast the expected opportunity cost of the initial margin for the three-month futures contract with a cash purchase of 100 ounces of gold for the same three-month period.



1. Procam borrows the USD4,950 initial margin from a financial intermediary at 5% for three months.

Opportunity (interest) cost of the initial margin:

$$= (\text{USD}4,950 \times .05)/4 = \text{USD}61.88$$

2. Procam borrows USD177,000 ( $\text{USD}1,770 \times 100$  ounces) for the spot gold purchase from a financial intermediary at 5% for three months.

Opportunity (interest) cost of the cash gold purchase price:

$$= (\text{USD}177,000 \times .05)/4 = \text{USD}2,212.50$$

Procam gains exposure to the underlying price of 100 ounces of gold in the futures market at a fraction of the spot market cost. This comparison ignores three points about the futures contract and spot gold transactions:

1. Procam's margin requirements change as the futures price fluctuates between  $t = 0$  and  $t = T$ . Specifically, Procam faces margin calls if a lower futures price causes its margin account balance to fall below the maintenance margin. However, as the underlying price is usually bounded by zero, it is unlikely that Procam's borrowing cost for margin will exceed that of the spot cash purchase.
2. Procam's spot gold purchase involves physical gold. It must therefore cover the cost of delivery, storage, and insurance for three months.
3. The spot gold purchase takes place at a price of  $S_0 = \text{USD}1,770$  per ounce, while the pre-agreed futures price  $[f_0(T)] = \text{USD}1,792.13$  per ounce. At time  $T$  in three months, both the futures contract and the spot purchase result in a long position for Procam at a price of  $S_T$ . The expected cost and return for these two transactions should therefore be equal, but the prices agreed at  $t = 0$  are not. In a later lesson, we will explore the relationship between the spot gold price ( $S_0$ ) and the gold futures price  $[f_0(T)]$  and the effect of both borrowing and storage costs.

The operational efficiency of derivative markets also leads to greater market efficiency. When prices deviate from fundamental values, derivative markets offer less costly ways to exploit the mispricing. As noted earlier, less capital is required, transaction costs are lower, and short selling is easier. As a result of these market features, fundamental value is often reflected in derivative markets before it is restored in the underlying cash market. The existence of derivative markets therefore often causes financial markets in general to function more effectively. Exhibit 1 summarizes derivative market benefits for market participants.

#### Exhibit 1: Benefits of Derivative Instruments

Purpose	Description
Risk Allocation, Transfer, and Management	Allocate, trade, and/or manage underlying exposure without trading the underlying Create exposures unavailable in cash markets
Information Discovery	Deliver information regarding expected price in the future and information regarding expected risk of underlying
Operational Advantages	Reduced cash outlay, lower transaction costs versus the underlying, increased liquidity and ability to "short"
Market Efficiency	Less costly to exploit arbitrage opportunities or mispricing

## QUESTION SET



## Derivative Benefits

1. Describe a scenario in which an issuer faces a timing difference between an economic decision and an ability to transact in cash markets to manage price risk.

**Solution:**

An issuer may need to 1.) order commodity inputs for its production process in advance of receiving finished-goods orders, 2.) await a shipment of goods in a foreign currency before selling domestic currency to make payment, or 3.) lock in its future debt costs in advance of the maturity of an outstanding debt issuance.

2. Determine the correct answers to fill in the blanks: Derivative markets typically have greater \_\_\_\_\_ than the underlying spot markets, a result of the reduced \_\_\_\_\_ required to trade derivatives versus an equivalent cash position in an underlying.

**Solution:**

Derivative markets typically have greater *liquidity* than the underlying spot markets, a result of the reduced *capital* required to trade derivatives versus an equivalent cash position in an underlying.

3. Identify the proper derivative market benefits that correspond to the following statements:

A. Price discovery function	1. Futures margin requirements are quite low versus the cost of a cash market purchase.
B. Operational advantages	2. The ability to buy or sell a derivative today eliminates the timing mismatch between an economic decision and the ability to transact.
C. Risk transfer	3. Investors track an equity index futures price to gauge sentiment before the market opens.

**Solution:**

1. The correct answer is B. The low level of futures margin requirements versus the cost of a cash market purchase is an example of the operational advantage of using derivatives.
2. The correct answer is C. The ability to buy or sell a derivative contract today eliminates the timing mismatch between an economic decision and the ability to transact, which is an example of the risk transfer function of derivatives.
3. The correct answer is A. Investors use derivatives in a price discovery function when tracking an equity index futures price to gauge sentiment before the market opens.



## DERIVATIVE RISKS

## 3

- ☐ describe benefits and risks of derivative instruments

While derivatives offer benefits such as the ability to efficiently hedge, allocate, and/or transfer risk as well as greater operational and market efficiency, the greater complexity of derivative instruments and positions also gives rise to greater potential risks associated with their use.

The greater operational efficiency of derivative strategies that limit an investor's initial cash outlay translates to a high degree of implicit leverage versus a similar cash market position. To illustrate this effect, we return to an earlier example to measure and compare the leverage between a futures contract and a cash market purchase with and without the use of borrowing.

**EXAMPLE 5****Implicit Leverage of Spot versus Futures Purchases**

In Example 4, we compared a spot purchase and a three-month futures contract purchase of 100 ounces of gold by Procum Investments.

- Procum's spot market purchase at  $S_0 = \text{USD}1,770$  per ounce results in a purchase price of USD177,000 ( $\text{USD}1,770 \times 100$ ) paid in cash.
- Procum's three-month futures contract purchase at  $f_0(T) = \text{USD}1,792.13$  per ounce results in a purchase price of USD179,213 ( $\text{USD}1,792.13 \times 100$ ). This purchase requires a USD4,950 initial margin deposit at the exchange.

In three months, the spot gold price ( $S_T$ ) is USD1,780.50 per ounce, and both the long cash position and the long futures contract position are valued at USD178,050 ( $\text{USD}1,780.50 \times 100$ ). We divide the value change by the initial cash outlay for each transaction to compare implicit leverages:

- Spot value change: 0.593% gain  $[(\text{USD}178,050 - \text{USD}177,000) / \text{USD}177,000]$ .
- Futures value change: 23.5% loss  $[(\text{USD}178,050 - \text{USD}179,213) / \text{USD}4,950]$ .

Note the large order-of-magnitude difference between the cash and futures transactions. As mentioned earlier, we will explore the relationship between borrowing costs, gold storage costs (ignored in this example), the spot gold price ( $S_0$ ), and the gold futures price [ $f_0(T)$ ] in a later lesson.

Implicit leverage is further magnified in an extreme case where only borrowed funds are used to enter the cash and futures transactions, as in Example 4:

- Spot value change versus borrowing cost: 47.5% gain  $[(\text{USD}178,050 - \text{USD}177,000) / \text{USD}2,212.50]$ .
- Futures value change versus borrowing cost: 1,879% loss  $[(\text{USD}178,050 - \text{USD}179,213) / \text{USD}61.88]$ .

This example emphasizes a very important point about derivatives: their inherent leverage magnifies the realized returns and risks and contributes to the severity of derivative-related losses.

Procam gains exposure to USD177,000 in underlying gold price risk with just USD61.88, implying a leverage ratio of 2,860. Procam benefits from this high leverage as gold prices rise, but its losses are rapidly magnified as gold prices decline. For example, as the futures price falls from USD179,213 to USD178,050, the modest -0.649% gold price change translates to a loss of USD1,163. Procam's actual loss on this transaction is both the interest payment (USD61.88) and the loss on the trade (USD1,163), for a total loss of USD1,224.88, nearly 20 times (USD1,224.88/ USD61.88) the cost of financing.

The same principle holds for non-linear derivatives such as sold options, where an option seller may face unlimited downside risk as underlying price changes make it more favorable for an option buyer to exercise for a gain far in excess of the premium paid.

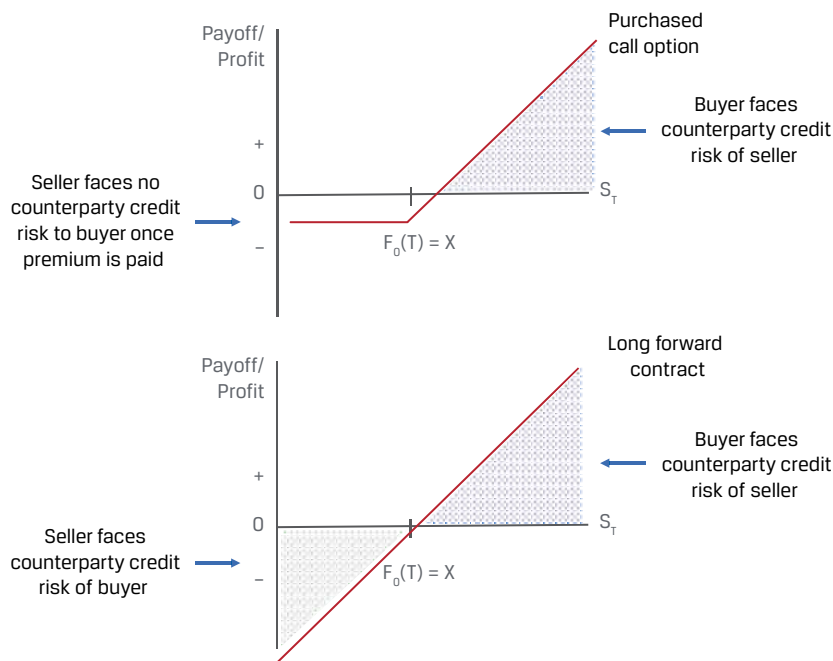
Leverage in derivatives creates significant exposures for the counterparties involved. These risks are mitigated through a combination of trading and exposure risk management, daily marking to market, the use of collateral arrangements, transaction and exposure limits, and centralized counterparties.

Derivatives offer the flexibility to create exposures beyond cash markets, which can add significant portfolio complexity and involve risks that are not well understood by stakeholders. This risk increases when a combination of derivatives and/or embedded derivatives is involved. For example, **structured notes** are a broad category of securities that incorporate the features of debt instruments and one or more embedded derivatives designed to achieve a particular issuer or investor objective. For instance, structured notes designed to create a derivative-based payoff profile for individual investors may involve greater cost, lower liquidity, and less transparency than an equivalent stand-alone derivative instrument.

Derivative users hedging commercial or financial exposure usually assume that a derivative will be highly effective in offsetting the price risk of an underlying. However, in some instances, the expected value of a derivative differs unexpectedly from that of the underlying, in what is known as **basis risk**. Basis risk may arise if a derivative instrument references a price or index that is similar to, but does not exactly match, an underlying exposure such as a different market reference rate or an issuer CDS spread versus that of an actual bond. Basis risk is affected by supply and demand dynamics in derivative markets, among other factors.

A related risk that can arise for both hedgers and risk takers is **liquidity risk**, or a divergence in the cash flow timing of a derivative versus that of an underlying transaction. The daily settlement of gains and losses in the futures market can give rise to liquidity risk. If an investor or issuer using a futures contract to hedge an underlying transaction is unable to meet a margin call due to a lack of funds, the counterparty's position is closed out and the investor or issuer must cover any losses on the derivative trade.

Counterparty credit risk is of critical importance to derivative market participants. Unlike loan and bond markets, where credit exposures are predictably based on notional outstanding plus accrued interest, daily swings in the price of an underlying, among other factors affecting derivative prices, require more frequent exposure monitoring and management. Counterparty credit exposure varies by the derivative type and market in which a derivative is transacted. Exhibit 2 contrasts the counterparty credit risk of a contingent claim with that of a forward commitment, using the example of a purchased call option and a long forward position.

**Exhibit 2: Counterparty Credit Risk of an Option versus a Forward**

As for market type, the daily settlement of MTM gains and losses, which characterizes exchange-traded derivatives, substantially reduces counterparty credit risk. Exchanges reserve the right to increase margin requirements or require intraday margining for highly volatile or concentrated positions. In over-the-counter (OTC) markets, credit terms privately arranged between counterparties vary from uncollateralized exposure to terms similar to futures margining for one or both counterparties using collateral.

Broader derivatives use among market participants has increased the focus of financial market supervisory authorities on the potential market-wide impact, or **systemic risk**, associated with these instruments. Regulators continue to specifically focus on the impact of financial innovation and financial conditions to ensure financial stability as they monitor risk taking and leverage among derivative market participants. Market reforms such as the central clearing mandate for swaps between financial intermediaries and a central counterparty (CCP), outlined earlier, include margin provisions similar to futures in order to standardize and reduce counterparty credit risk.

Exhibit 3 summarizes the key risks associated with derivative instruments and markets.

**Exhibit 3: Risks of Derivative Instruments**

Risk	Description
Greater Potential for Speculative Use	High degree of implicit leverage for some derivative strategies may increase the likelihood of financial distress.
Lack of Transparency	Derivatives add portfolio complexity and may create an exposure profile that is not well understood by stakeholders.
Basis Risk	Potential divergence between the <b>expected value</b> of a derivative instrument versus an underlying or hedged transaction

Risk	Description
Liquidity Risk	Potential divergence between the <b>cash flow timing</b> of a derivative instrument versus an underlying or hedged transaction
Counterparty Credit Risk	Derivative instruments often give rise to <b>counterparty credit exposure</b> , resulting from differences in the current price versus the expected future settlement price.
Destabilization and Systemic Risk	Excessive risk taking and use of leverage in derivative markets may contribute to market stress, as in the 2008 financial crisis.

**QUESTION SET****Derivative Risks**

- Determine the correct answers to fill in the blanks: The \_\_\_\_\_ operational efficiency of derivative strategies that limit an investor's initial cash outlay translates to a \_\_\_\_ degree of implicit leverage versus a similar cash market position.

**Solution:**

The *greater* operational efficiency of derivative strategies that limit an investor's initial cash outlay translates to a *high* degree of implicit leverage versus a similar cash market position.

- Describe the counterparty credit risk faced by the seller of a call option.

**Solution:**

The seller of a call option receives an upfront premium in exchange for the right to purchase the underlying at the exercise price at maturity. Once the seller of a call option receives the premium from the option buyer, it has no further counterparty credit risk to the option buyer.

- Match these derivative market risks to the following statements:

A. Liquidity risk	1. The risk that excessive risk taking and use of leverage in derivative markets contribute to market stress
B. Basis risk	2. The risk of a divergence in the cash flow timing of a derivative versus that of an underlying transaction
C. Systemic risk	3. The risk that the expected value of a derivative differs unexpectedly from that of the underlying

**Solution:**

- The correct answer is C. Systemic risk involves excessive risk taking and use of leverage in derivative markets that contribute to market stress.
- The correct answer is A. Liquidity risk is the divergence in the cash flow timing of a derivative versus that of an underlying transaction.
- The correct answer is B. Basis risk involves the risk that the expected value of a derivative differs unexpectedly from that of an underlying.

## ISSUER USE OF DERIVATIVES

## 4

- ☐ compare the use of derivatives among issuers and investors

Issuers, investors, and financial intermediaries use derivative instruments to increase, decrease, or modify exposure to an underlying to meet their financial objectives. Financial analysts must gain a deeper understanding of the various uses of derivatives among market participants in order to interpret and replicate the wide range of strategies encountered in practice.

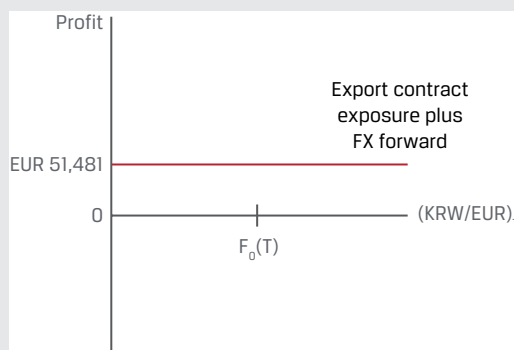
Non-financial corporate issuers often face risks to their assets, liabilities, and earnings as a result of changes in the price of an underlying. For example, a corporate issuer that uses a traded commodity in its operations will face greater earnings volatility due to input price changes. The additional earnings volatility has the potential to increase the corporate issuer's cost of borrowing as well as to create difficulties for investors to estimate its future corporate earnings. The impact of foreign exchange volatility on corporate issuer earnings is illustrated by extending an earlier example.

**EXAMPLE 6****Foreign Exchange Risk and Earnings Volatility**

Recall from Example 1 that Montau AG will deliver a laser cutting machine for KRW650,000,000 in 75 days and has hedged its FX exposure by agreeing to sell the KRW it will receive and purchase EUR upon delivery in an over-the-counter FX forward with a financial intermediary.

Montau agrees to a KRW/EUR forward exchange rate  $[F_0(T)]$  of 1,350 (i.e., 1,350 KRW = 1 EUR), at which it will sell 650,000,000 KRW and receive 481,481 EUR (650,000,000/1,350) in 75 days. The production manager estimates the machine's cost to be €430,000. Montau's Treasury manager compiles the following profit margin scenarios at different KRW/EUR spot rates ( $S_T$ ):

Spot KRW/EUR ( $S_T$ )	Unhedged EUR Proceeds	Unhedged Profit Margin	Hedged EUR Proceeds	Hedged Profit Margin
1,525	€426,230	-1%	€481,481	11%
1,400	€464,286	7%	€481,481	11%
1,280	€507,813	15%	€481,481	11%
1,225	€530,612	19%	€481,481	11%



If Montau does not hedge its KRW/EUR exposure, its profit margin will fluctuate based on the KRW/EUR spot rate when the machine is delivered. A weaker KRW versus EUR results in a lower profit margin, while KRW appreciation results in a higher profit margin.

Example 6 illustrates one of the most common derivative strategies used by corporate issuers. The FX forward payoff offsets Montau's income statement and cash flow volatility due to currency changes.

Derivatives accounting has evolved from off-balance-sheet treatment to the reporting of these instruments on the balance sheet at their fair market value. This change aligns the recognition of derivative gains and losses with their designated risk management purpose, increasing transparency and disclosure of derivatives use. Many corporate issuers also establish risk management policies governing the objectives, guidelines, risk limits, and internal approval processes associated with derivatives use.

Derivatives accounting standards specify that any derivative purchased or sold must be marked to market through the income statement via earnings unless it is embedded in an asset or liability or qualifies for **hedge accounting**. Hedge accounting allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility. Categorizing derivatives by hedge designation type sheds further light on both their intent and their expected financial statement impact, to the benefit of financial analysts and stakeholders.

For example, derivatives designated as absorbing the variable cash flow of a floating-rate asset or liability such as foreign exchange, interest rates, or commodities are referred to as **cash flow hedges**. Cash flow hedges may be either forward commitments or contingent claims. For instance, the FX forward in Example 6 is a cash flow hedge that offsets the variability of Montau's functional currency (EUR) proceeds from its commercial transaction in KRW. A swap to a fixed rate for a floating-rate debt liability is another example of a cash flow hedge.

A **fair value hedge** designation applies when a derivative is deemed to offset the fluctuation in fair value of an asset or liability. For example, an issuer might convert a fixed-rate bond issuance to a floating-rate obligation by entering into an interest rate swap to receive a fixed rate and pay a market reference rate through the bond's maturity. Alternatively, a commodities producer might sell its inventory forward in anticipation of lower future prices.

**Net investment hedges** occur when either a foreign currency bond or a derivative such as an FX swap or forward is used to offset the exchange rate risk of the equity of a foreign operation. Exhibit 4 summarizes these hedge designations.

#### Exhibit 4: Hedge Accounting Designation Types

Hedge Type	Description	Examples
Cash Flow	Absorbs variable cash flow of floating-rate asset or liability (forecasted transaction)	Interest rate swap to a fixed rate for floating-rate debt FX forward to hedge forecasted sales
Fair Value	Offsets fluctuation in fair value of an asset or liability	Interest rate swap to a floating rate for fixed-rate debt Commodity future to hedge inventory
Net Investment	Designated as offsetting the FX risk of the equity of a foreign operation	Currency swap Currency forward

Hedge accounting treatment for derivatives is highly desirable for corporate issuers, as it allows them to recognize derivative gains and losses at the same time as the associated underlying hedged transaction. Derivative mark-to-market changes are held within an equity account (Other Comprehensive Income) and released at the same time the underlying hedged transaction is recognized in earnings.

In order to qualify for this treatment, the dates, notional amounts, and other contract features of a derivative must closely match those of the underlying transaction. For this reason, issuers are far more likely to use OTC markets to create customized hedges to meet their specific needs. For example, Montau AG would face earnings volatility from the derivative position if it were to use a standardized two-month foreign exchange futures contract rather than the 75-day over-the-counter FX forward contract with a financial intermediary, as in Example 6.

## QUESTION SET



### Issuer Use of Derivatives

1. Describe hedge accounting treatment.

**Solution:**

Hedge accounting allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to reduce financial statement volatility.

2. Match these hedge designation types to the following statements:

A. Cash flow hedge	1. A derivative used to offset the fluctuation in fair value of an asset or liability
B. Fair value hedge	2. A derivative designated as absorbing the variable cash flow of a floating-rate asset or liability
C. Net investment hedge	3. A derivative designated as offsetting the foreign exchange risk of the equity of a foreign operation

**Solution:**

1. The correct answer is B. A fair value hedge is a derivative used to offset the fluctuation in fair value of an asset or liability.
2. The correct answer is A. A cash flow hedge is a derivative designated as absorbing the variable cash flow of a floating-rate asset or liability.
3. The correct answer is C. A net investment hedge is a derivative designated as offsetting the foreign exchange risk of the equity of a foreign operation.

3. Describe an example of a fair value hedge an issuer might use.

**Solution:**

An issuer might convert a fixed-rate bond issuance to a floating-rate obligation by entering into an interest rate swap to receive a fixed rate and pay a market reference rate through the bond's maturity. Alternatively, a commodities producer might sell its inventory forward in anticipation of lower cash prices in the future.

## 5

## INVESTOR USE OF DERIVATIVES

- ☐ compare the use of derivatives among issuers and investors

Issuers predominantly use derivatives to offset or hedge market-based underlying exposures incidental to their commercial operations and financing activities. In contrast, investors use derivatives to replicate a cash market strategy, hedge a fund's value against adverse movements in underlyings, or modify or add exposures using derivatives, which in some cases are unavailable in cash markets.

The greater liquidity and reduced capital required to trade derivatives may lead an investor to replicate a desired position using a derivative rather than cash. Alternatively, derivative hedges enable investors to isolate certain underlying exposures in the investment process while retaining a position in others. One example is the use of FX hedges when investing overseas in order to minimize the volatility of return due to currency fluctuations. Finally, the flexibility to take short positions or to increase or otherwise modify exposure using derivatives beyond cash alternatives is an attractive feature for portfolio managers targeting excess returns by using a variety of strategies.

An investment fund's prospectus typically specifies which derivative instruments may be used within a fund and for which purpose.

Several examples in earlier lessons illustrated the use of derivatives from an investor perspective using both forward commitments and contingent claims.

#### Forward Commitments

- Recall an earlier example in which Procam Investments purchased a three-month gold forward or futures contract. The derivatives contract increased Procam's exposure to the underlying price of gold in the future with no initial cash outlay and no requirement to take immediate delivery of the physical asset, as in the case of a cash purchase.
- In the example of Fyleton Investments, the fund entered into a GBP interest rate swap to increase the duration of its assets with no initial cash outlay.

#### Contingent Claims

- One example had Hightest Capital purchasing a call option to benefit from an expected increase in a health care stock index price above the exercise in exchange for an upfront premium.
- The SCSI long equity and short call (or covered call) example created an exposure profile under which SCSI realized a higher return than in its original long cash position if the underlying index price at the end of the year was stable to slightly higher, in accordance with the CIO's view.

In these and other instances throughout the curriculum, we find investors to be less focused than issuers on hedge accounting treatment, as an investment fund's derivative position is typically marked to market each day and included in the daily net asset value (NAV) of the portfolio or fund. This also explains why investors tend to transact more frequently in standardized and highly liquid exchange-traded derivative markets than do issuers.



## QUESTION SET



## Investor Use of Derivatives

1. Determine the correct answer to complete the following sentence: An investment fund's \_\_\_\_\_ typically specifies which derivative instruments may be used within a fund and for which purpose.

**Solution:**

An investment fund's *prospectus* typically specifies which derivative instruments may be used within a fund and for which purpose.

2. Describe two purposes of investor derivatives use within a fund.

**Solution:**

The purpose of investor derivatives use within a fund is usually to modify the fund's exposure to increase the return of the fund under specific market conditions and/or to offset or hedge the fund's value against adverse movements in underlyings, such as exchange rates, interest rates, and securities markets.

3. Match these derivative market participants to the following statements:

A. Investors	1. They use derivatives to offset or hedge market-based underlying exposures incidental to their commercial operations and financing activities.
B. Both issuers and investors	2. They tend to transact more frequently in exchange-traded derivative markets.
C. Issuers	3. They use derivatives to change their exposure to an underlying asset price without transacting in the cash market.

**Solution:**

- The correct answer is C. Issuers use derivatives to offset or hedge market-based underlying exposures incidental to their commercial operations and financing activities.
- The correct answer is A. Investors tend to transact more frequently in exchange-traded derivative markets.
- The correct answer is B. Both issuers and investors use derivatives to change their exposure to an underlying asset price without transacting in the cash market.

## PRACTICE PROBLEMS

### The following information relates to questions 1-4

Consider the following structured note offered by Baywhite Financial:

#### Baywhite Financial LLC 80% Principal Protected Structured Note

<b>Description:</b>	The Baywhite Financial LLC 80% Principal Protected Structured Note ("the Note") is linked to the performance of the S&P 500 Health Care Select Sector Index (SIXV).
<b>Issuer:</b>	Baywhite Financial LLC
<b>Start Date:</b>	[Today]
<b>Maturity Date:</b>	[Six months from Start Date]
<b>Issuance Price:</b>	102% of Face Value
<b>Face Value:</b>	Sold in a minimum denomination of USD1,000 and multiple units thereof
<b>Payment at Maturity:</b>	At maturity, you will receive a cash payment, for each USD1,000 principal amount note, of USD800 plus the Additional Amount, which may be zero.
<b>Partial Principal Protection Percentage:</b>	80% Principal Protection (20% Principal at Risk)
<b>Additional Amount:</b>	At maturity, you will receive the greater of 100% of the returns on the S&P 500 Health Care Select Sector Index (SIXV) in excess of 5% above the current spot price of the SIXV or zero.

As a financial analyst for a wealth management advisory firm, you have been tasked with comparing the features of the Baywhite Financial LLC Structured Note with those of a similar exchange-traded, stand-alone derivative instrument alternative in order to make a recommendation to the firm's clients.

- Which of the following statements best describes the derivative instrument that is embedded in the Baywhite Financial LLC Structured Note?
  - The Structured Note has an embedded long futures contract with the S&P 500 Health Care Select Sector Index (SIXV) as an underlying.
  - The Structured Note has an embedded long call option contract with the S&P 500 Health Care Select Sector Index (SIXV) as an underlying.
  - The Structured Note has an embedded short put option contract with the S&P 500 Health Care Select Sector Index (SIXV) as an underlying.
- Which of the following statements best contrasts the credit risk of the Baywhite Financial LLC Structured Note with the counterparty credit risk of an investor

entering into the embedded exchange-traded derivative on a stand-alone basis?

- A. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 20% of the note's face value, as the remaining 80% is principal protected. An investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of a financial intermediary.
  - B. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 80% of the note's face value, as the remaining 20% is associated with the embedded derivative. An investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of a financial intermediary.
  - C. An investor in the Baywhite Structured Note assumes the credit risk of Baywhite Financial LLC for 100% of the note's face value, while an investor entering into the SIXV derivative on a stand-alone basis assumes the counterparty credit risk of an exchange and its clearinghouse.
3. Which of the following statements most accurately describes the liquidity of the Baywhite Structured Note versus that of the embedded exchange-traded derivative?
- A. The Baywhite Structured Note is likely to be more liquid than the stand-alone SIXV call option, as the Note has 80% principal protection while an investor in the stand-alone derivative may lose the entire option premium if it expires worthless at maturity.
  - B. The Baywhite Structured Note is likely to be more liquid than the stand-alone SIXV call option, as the Note is priced at a stated 2% premium above par while an investor in the stand-alone derivative faces the lack of transparency as well as basis, liquidity, and counterparty credit risks associated with derivative transactions.
  - C. Structured notes such as the Baywhite Financial LLC Structured Note often involve greater cost, lower liquidity, and less transparency than an equivalent stand-alone derivative instrument, while the exchange-traded SIXV derivative contract is standardized and trades in a liquid, transparent market.
4. Which of the following statements best describes how an investor should evaluate the terms of the Baywhite Financial LLC Structured Note as compared with the stand-alone derivative price in order to make a recommendation?
- A. The Baywhite Financial LLC Structured Note issuance price of 2% above par value should be compared with the upfront premium for a six-month SIXV call option with an exercise price at 5% above the current SIXV spot price.
  - B. The Baywhite Financial LLC Structured Note 20% Principal at Risk should be compared with the upfront premium for a six-month SIXV call option with an exercise price at 5% above the current SIXV spot price.
  - C. The Baywhite Financial LLC Structured Note issuance price of 2% above par value *plus* the 20% Principal at Risk should be compared with the upfront premium for a six-month SIXV call option with an exercise price at 5% above the current SIXV spot price.
-

## SOLUTIONS

1. B is correct. The Structured Note is linked to the performance of the S&P 500 Health Care Select Sector Index (SIXV). Note that the SIXV derivative is similar to that in the earlier SCSi CSI 300 example. The “Additional Amount” paid at maturity is equal to the greater of 100% of the returns on the S&P 500 Health Care Select Sector Index (SIXV) in excess of 5% above the current spot price of the SIXV or zero. This payoff profile  $[\text{Max}(0, S_T - X)]$  is identical to that of a purchased six-month SIXV call option with an exercise price (X) at 5% above today’s SIXV spot price.
2. C is correct. The investor assumes the credit risk of Baywhite Financial LLC for the full value of the structured note as the structured note issuer. Under the purchased exchange-traded SIXV call option, the investor faces the risk of the exchange and its clearinghouse, which provides a guarantee of contract settlement backed by the exchange insurance fund.
3. C is correct. The Structured Note is likely to be far less liquid than the stand-alone SIXV call option, which is traded on a derivatives exchange. Recall from an earlier lesson that exchange-traded contracts are more formal and standardized, which facilitates a more liquid and transparent market. Note also that the Baywhite Financial LLC Structured Note is issued at 102% of face value, suggesting that an investor will likely forgo this premium if selling the note prior to maturity.
4. C is correct. The 20% Principal at Risk, or USD200 of Face Value for each USD1,000 (the minimum denomination), combined with the 2% (or USD20) issue premium, should be compared with the upfront premium for a six-month SIXV call option with an exercise price at 5% above the current SIXV spot price. The comparison should also consider the additional credit risk and liquidity risk of the Structured Note versus the exchange-traded option.

## LEARNING MODULE

## 4

## Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain how the concepts of arbitrage and replication are used in pricing derivatives
<input type="checkbox"/>	explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset

### INTRODUCTION

1

Earlier derivative lessons established the features of derivative instruments and markets and addressed both the benefits and risks associated with their use. Forward commitments and contingent claims were distinguished by their different payoff profiles and other characteristics. We now turn our attention to the pricing and valuation of these instruments. As a first step, we explore how the price of a forward commitment is related to the spot price of an underlying asset in a way that does not allow for arbitrage opportunities. Specifically, the strategy of replication shows that identical payoffs to a forward commitment can be achieved from spot market transactions combined with borrowing or lending at the risk-free rate. Finally, the second lesson demonstrates how costs or benefits associated with owning an underlying asset affect the forward commitment price.

#### LEARNING MODULE OVERVIEW



- Forward commitments are an alternative means of taking a long or short position in an underlying asset. A link between forward prices and spot prices exists to prevent investors from taking advantage of arbitrage opportunities across cash and derivative instruments.
- A forward commitment may be replicated with a long or short spot position in the underlying asset and borrowing or lending at a risk-free rate. Investors can recreate a variety of positions by using appropriate combinations of spot, forward, and risk-free positions.

CFA Institute would like to thank Don Chance, PhD, CFA, for his contribution to this section, which includes material derived from material that appeared in *Derivative Markets and Instruments*, featured in the 2022 CFA® Program curriculum.

- The risk-free rate provides a fundamental link between spot and forward prices for underlying assets with no additional costs or benefits of ownership.
- The cost of carry is the net of the costs and benefits related to owning an underlying asset for a specific period and must be factored into the difference between the spot price and a forward price of a specific underlying asset.
- The cost of carry may include costs, such as storage and insurance for physical commodities, or benefits of ownership, such as dividends for stocks and interest for bonds. Foreign exchange represents a special case in which the cost of carry is the interest rate differential between two currencies.
- Forward prices may be greater than or less than the underlying spot price, depending on the specific cost of carry associated with owning the underlying asset.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements correctly describes how to replicate a long forward position?
  - A. Sell a risk-free bond, and buy a cash market position in the underlying.
  - B. Buy a risk-free bond, and buy a cash market position in the underlying.
  - C. Buy a risk-free bond, and sell a cash market position in the underlying.

**Solution:**

A is correct. Selling a risk-free bond provides the necessary cash to buy the underlying in the cash market. At the bond's maturity, the underlying is sold at the future spot price, and the proceeds are used to pay off the bond. The profit on this transaction is dependent on the future spot price of the underlying compared to the underlying cash market price multiplied by one plus the risk-free rate, and this profit position is identical to that of a long forward position in the underlying. B is incorrect because buying the risk-free bond creates a need for cash in addition to buying the underlying in the cash market. C is incorrect because this combination would replicate a short forward position.

2. Which of the following is closest to the arbitrage profit available to an investor who is able to buy an asset for a spot price of GBP50 at  $t = 0$  and simultaneously sell a six-month forward commitment on the same asset at a forward price of GBP52.50? The risk-free rate of interest is 4%, and the asset has no additional costs or benefits.
  - A. GBP0.99
  - B. GBP0.48

**C.** GBP1.51

**Solution:**

C is correct. The investor borrows at 4% for six months to buy the asset today for GBP50. After six months, the investor pays the lender  $S_0(1+r)^T$ , or GBP50.99 [= GBP50(1.04)<sup>0.5</sup>] in principal and interest and delivers the asset to satisfy the forward commitment to sell at GBP52.50. The investor's arbitrage profit is GBP1.51 (= GBP52.50 – GBP50.99). A is incorrect because this answer reflects the difference between the no-arbitrage forward price and the current spot price. B is incorrect because this answer reflects the difference between the forward price discounted back one year (rather than six months) and the current spot price.

3. Which of the following statements correctly describes the relationship between a forward commitment price compared to the underlying spot price when the benefits of owning the underlying are greater than the costs of owning the underlying (including the opportunity interest costs)?

- A.** Forward commitment price > spot price.
- B.** Forward commitment price < spot price.
- C.** Forward commitment price = spot price.

**Solution:**

B is correct. Greater benefits associated with the underlying will be associated with a higher spot price relative to the forward commitment price, and if these benefits are greater than the costs of owning the underlying, then the spot price will be greater than the forward commitment price. A is incorrect because this inequality would be true in the case of the costs exceeding the benefits. C is incorrect because an equality between the two prices describes the rare circumstance in which costs and benefits are exactly equal.

4. Which of the following statements best defines a convenience yield?

- A.** Convenience yield reflects the preference that market participants exhibit for buying forward contracts to avoid having to pay cash up front.
- B.** Convenience yield reflects the preference that market participants exhibit for buying in the spot market to avoid having to pay for storage.
- C.** Convenience yield reflects the preference that market participants exhibit for buying in the spot market for non-cash reasons, including low inventories in the underlying cash market.

**Solution:**

C is correct. The convenience yield is a non-cash benefit associated with owning an underlying physical commodity that arises under certain economic conditions, including low inventories of the underlying. A is incorrect because the statement suggests convenience yield is a cost of owning the underlying. B is incorrect because the statement is contradictory in that it states that convenience yield causes market participants to prefer spot markets but incorrectly attributes this to a cost of owning the underlying.

## 2

## ARBITRAGE



explain how the concepts of arbitrage and replication are used in pricing derivatives

An earlier lesson on market efficiency established that market prices should not allow for the possibility of riskless profit or arbitrage in the absence of transaction costs. In its simplest form, an arbitrage opportunity arises if the “law of one price” does not hold, or an identical asset trades at the same time at different prices in different places.

In the case of a derivative contract whose value is derived from future cash flows associated with the price of an underlying asset, arbitrage opportunities arise either if two assets with identical *future cash flows* trade at different prices or if an asset with a known future price does not trade at the *present value* of its future price determined using an appropriate discount rate.

The first case of assets with identical future cash flows trading at different prices is illustrated in Exhibit 1.

### Exhibit 1: Assets with Identical Future Cash Flows Trade at Different Prices



For example, assume the two assets are zero-coupon bonds with identical features and the same issuer. Both bonds mature on the same future date with a payoff of par and have the same risk of default between now and the maturity date.

1. Bond A has a price of EUR99 at time  $t = 0$  ( $S_0^A = EUR99$ ).
2. Bond B has a price of EUR99.15 at time  $t = 0$  ( $S_0^B = EUR99.15$ ).
3. Both bonds have an expected future price of EUR100 ( $S_T^A = S_T^B = EUR100$ ).

This scenario represents an arbitrage opportunity for an investor.

- At time  $t = 0$ , the investor can:
  - Sell Bond B short to receive proceeds of EUR99.15 and purchase Bond A for EUR99;
  - Realize a net cash inflow at  $t = 0$  of EUR0.15.
- At time  $t = T$ , when both bonds mature, the investor:
  - Receives EUR100 for Bond A and uses this to buy Bond B for EUR100 to cover the short position.
  - The offsetting cash flows at time  $T$  leave the investor with a riskless profit of the EUR0.15 price difference between Bonds A and B at time 0.



Other investors taking note of this discrepancy will also seek to earn a riskless profit at time  $t = 0$  by selling Bond B, driving its price down, and buying Bond A, driving its price up, until the prices converge. The arbitrage opportunity disappears once the bonds have the same price ( $S_0^A = S_0^B$ ).

A second type of derivative-related arbitrage opportunity arises when an asset with a known future price does not trade at the present value (PV) of its future price. An earlier time-value-of-money lesson distinguished between discrete and continuous compounding in calculating present versus future value. The future value of a single cash flow based on a *discrete* number of uniform periods follows the general formula in Equation 1:

$$FV_N = PV(1 + r)^N, \quad (1)$$

where  $r$  is the stated interest rate per *period* and  $N$  is the number of compounding periods. *Continuous compounding* is the case in which the length of the uniform periods approaches zero, so the number of periods per year approaches infinity and is calculated using the natural logarithm, as shown in Equation 2:

$$FV_T = PVe^{rT}. \quad (2)$$

While derivatives may be priced with either approach, for purposes of this and later lessons, we will use the discrete compounding method for individual underlying assets. However, for underlying assets that represent a portfolio, such as an equity, fixed-income, commodity, or credit index, or where the underlying involves foreign exchange where interest rates are denominated in two currencies, continuous compounding will be the preferred method. Ignoring additional costs or benefits associated with asset ownership, the appropriate discount rate,  $r$ , is the risk-free rate, as demonstrated in Example 1.

### EXAMPLE 1

#### Spot vs. Discounted Known Future Price of Gold

In an earlier lesson, Procam Investments entered into a contract to buy 100 ounces of gold at an agreed-upon price of USD1,792.13 per ounce in three months. In this example, Procam does the opposite trade, given a discrepancy between spot and discounted known future gold prices. Assume that today's spot gold price ( $S_0$ ) is USD1,770 per ounce, and the annualized risk-free interest rate ( $r$ ) is 2%. For purposes of this example, we assume that Procam can borrow at the risk-free rate and gold may be stored at no cost. Under these conditions, we demonstrate how Procam can generate a riskless profit:

- At time  $t = 0$ :
- Procam borrows USD177,000 at 2.0% interest for three months and purchases 100 ounces of gold at today's spot price.
- Procam enters into a forward contract today to sell 100 ounces of gold at a price of USD1,792.13 per ounce in three months.

$S_0 < S_T(1+r)^{-T}$	$S_0(1+r)^T < S_T$
Borrow $S_0$ at $r$ and Buy $S_0$ . Agree to sell at $S_T$	Sell at $S_T$ and Repay $S_0(1+r)^T$
Borrow USD1,770 at 2% and Buy gold ( $S_0$ )	Sell at USD1,792.13 ( $S_T$ ) and Repay $S_0(1+r)^T = \text{USD1,778.78}$
Contract to sell at $S_T = \text{USD1,792.13}$ per ounce	Riskless profit of USD13.35 per ounce
$CF_0 = 0$	$CF_T = (\text{USD1,792.13} - \text{USD1,778.78}) > 0$
0	Time T

- At time  $t = T$  (in three months):

- Procam delivers 100 ounces of gold under the forward contract and receives USD179,213 ( $= 100 \times \text{USD1,792.13}$ ).
- Procam repays the loan principal with interest:

$$\text{USD177,878.44} = \text{USD177,000}(1.02)^{0.25}.$$

- Procam's riskless profit at time  $T$  is equal to the difference between the forward sale proceeds and the loan principal and interest:  
 $\text{USD1,334.56} = \text{USD179,213} - \text{USD177,878.44}.$

In Example 1, the spot price of gold,  $S_0$ , is below the present value of the known future price of gold in three months' time ( $S_0 < S_T(1 + r)^{-T}$ , since  $\text{USD1,770} < \text{USD1,783.28}$ ). Procam earns a riskless profit of USD13.35 per ounce by borrowing and purchasing gold in the cash market and simultaneously selling gold in the forward market at an agreed-upon price. We would expect that as other investors recognize and pursue this opportunity, the spot price will increase (and the forward price will fall) until the spot price is equal to the discounted value of the known future price ( $S_0 = S_T(1 + r)^{-T}$ ) to eliminate this arbitrage opportunity.

The two key arbitrage concepts used to price derivatives for an underlying with no additional cash flows may be summarized as follows:

- Identical assets or assets with identical cash flows traded at the same time must have the same price ( $S_0^A = S_0^B$ ).
- Assets with a known future price must have a spot price that equals the future price discounted at the risk-free rate ( $S_0 = S_T(1 + r)^{-T}$ ).

These arbitrage conditions establish the relationship between spot prices, forward commitment prices, and the risk-free rate shown in Exhibit 2.

**Exhibit 2: Spot Prices, Forward Commitment Prices, and the Risk-Free Rate**

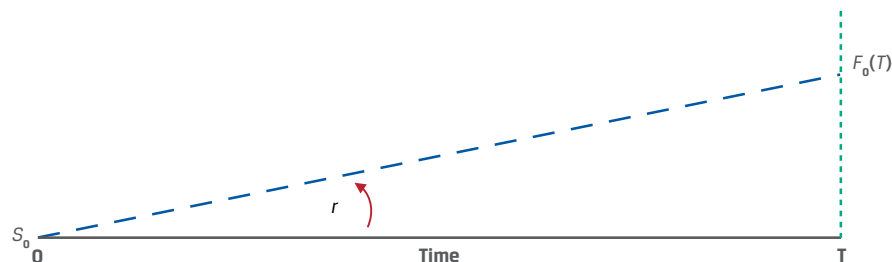
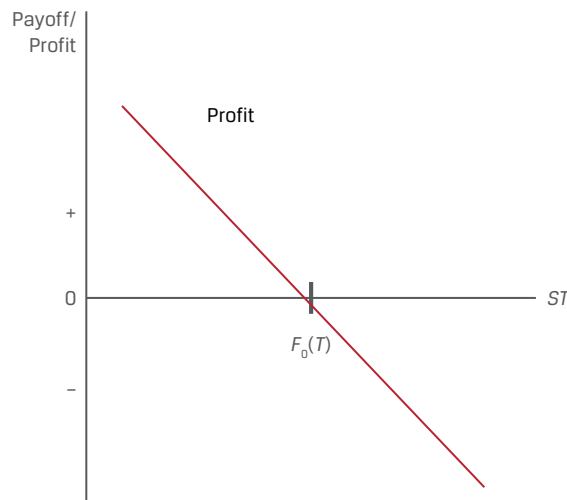


Exhibit 2 shows the case of  $r > 0$  for an asset with no additional income or costs under discrete compounding. Note that for a given time  $T$ , the forward price will be higher relative to the spot price with a higher risk-free rate  $r$ . Also, for a given risk-free rate  $r$ , as  $T$  increases, the forward price will increase relative to the spot price. It is important to note that the relevant risk-free rate for most market participants is the repo rate, introduced in an earlier lesson, where borrowed funds are collateralized by highly liquid securities.

Recall from an earlier lesson that a forward commitment has a symmetric payoff profile. That is, at time  $T$ , the transaction is settled on the basis of the difference between the forward price  $F_0(T)$  and the underlying price of  $S_T$ , or  $F_0(T) - S_T$  from the seller's perspective, as in Exhibit 3.

**Exhibit 3: Forward Commitment Seller Payoff Profile**

The forward seller realizes a gain if the seller is able to deliver the underlying at a market value ( $S_T$ ) below the pre-agreed price,  $F_0(T)$ . In Example 1, Procam borrows to buy the underlying at a spot price below the present value of  $S_T$  to lock in a riskless gain. So far, we have taken the forward price,  $F_0(T)$ , as given. In what follows, we demonstrate how no-arbitrage conditions may be used to establish the relationship between spot and forward prices by replicating or recreating an exact offsetting position for a forward commitment.

**REPLICATION****3**

explain how the concepts of arbitrage and replication are used in pricing derivatives

**Replication** is a strategy in which a derivative's cash flow stream may be recreated using a combination of long or short positions in an underlying asset and borrowing or lending cash. In contrast to the earlier arbitrage examples, replication is typically used to mirror or offset a derivative position when the law of one price holds and no riskless arbitrage profit opportunities exist. For example, Exhibit 4 compares a long forward commitment to the alternative of borrowing funds at the risk-free rate,  $r$ , to buy the underlying asset at today's spot price,  $S_0$ .

**Exhibit 4: Forward Commitment Replication****Borrow  $S_0$  and purchase at spot ( $S_0$ )**Borrow USD1,783.28 ( $S_0$ ) today at  $t = 0$ Buy at  $S_0 = \text{USD1,783.28}$ 

$$CF_0 = (\text{USD1,783.28} - \text{USD1,783.28}) = 0$$

**Repay loan ( $S_0(1+r)^T$ ) and sell at spot ( $S_T$ )**Repay USD1,792.13 ( $S_0(1+r)^T$ ) at  $t = T$ See at spot ( $S_T$ )

$$CF_T = (S_T - \$1,792.13)$$

**Enter long forward contract ( $F_0(T)$ )**Agree to buy at USD1,792.13 ( $F_0(T)$ ) at  $t = T$ 

$$CF_0 = 0$$

**Settle long forward contract ( $S_T - F_0(T)$ )**Settle contract ( $S_T - F_0(T)$ ) at  $t = T$ 

$$CF_T = (S_T - \$1,792.13)$$

**EXAMPLE 2****Replication of a Forward Commitment**

In Example 1, we reintroduced Procam Investments, which borrowed and purchased gold in the spot market and simultaneously sold gold in the forward market to earn a riskless profit. Here we change the assumption about today's spot gold price ( $S_0$ ). Specifically, the spot gold price has risen to USD1,783.28 ( $= \text{USD1,792.13}(1.02)^{-0.25}$ ) to eliminate the earlier arbitrage opportunity when the spot price was USD1,770. Assume again that the risk-free interest rate ( $r$ ) is 2% and gold can be stored at no cost.

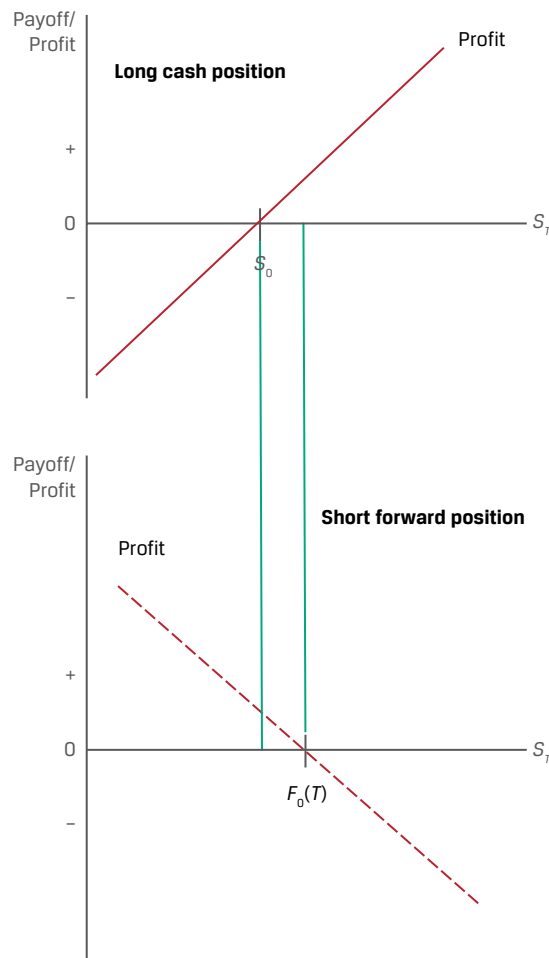
1. Long forward commitment (agree to buy at  $F_0(T)$  at  $t = T$ )
  - Procam enters a forward commitment to buy 100 ounces of gold in three months at a forward price,  $F_0(T)$ , of USD1,792.13 per ounce.
  - If  $S_T = \text{USD1,900}$  per ounce,
    - Profit = USD10,787 =  $100 \times (\text{USD1,900} - \text{USD1,792.13})$ .
  - If  $S_T = \text{USD1,700}$  per ounce,
    - Profit = -USD9,213 =  $100 \times (\text{USD1,700} - \text{USD1,792.13})$ .
2. Borrow and purchase (borrow  $S_0$  and buy asset at  $S_0$  at  $t = 0$ ):
  - Procam borrows USD178,328 at 2% and buys 100 ounces of gold at today's spot price,  $S_0$ .
  - Procam sells the gold in three months at spot price  $S_T$ .
  - Procam repays the loan principal and interest:  $\text{USD179,213} = \text{USD178,328}(1.02)^{0.25}$ .
  - If  $S_T = \text{USD1,900}$ ,
    - Profit = USD10,787 =  $100 \times (\text{USD1,900} - \text{USD1,792.13})$ .
  - If  $S_T = \text{USD1,700}$ ,
    - Profit = -USD9,213 =  $100 \times (\text{USD1,700} - \text{USD1,792.13})$ .

Note that the same profits are observed under Scenarios 1 and 2, which are equal to  $S_T - F_0(T)$ . If we set the forward price,  $F_0(T)$ , equal to the future value of the spot rate using the risk-free rate ( $F_0(T) = S_0(1 + r)^T$ ), the no-arbitrage condition demonstrates that Procam generates the same cash flow at time  $T$  regardless of the direction of gold prices whether the company

1. enters into a long forward commitment settled at time  $T$  or
2. borrows at the risk-free rate, buys the underlying asset, and holds it until time  $T$ .

A separate but related form of replication pairing a long asset with a short forward is shown in Exhibit 5. The top half shows the profit/loss profile of the long asset, and the bottom half shows that of the short forward position. The long asset position produces a profit (loss) when  $S_T > S_0$  ( $S_T < S_0$ ). The short forward position produces a profit (loss) when  $S_T < F_0$  ( $S_T > F_0$ ).

**Exhibit 5: Payoffs for Long Asset and Short Forward Positions**



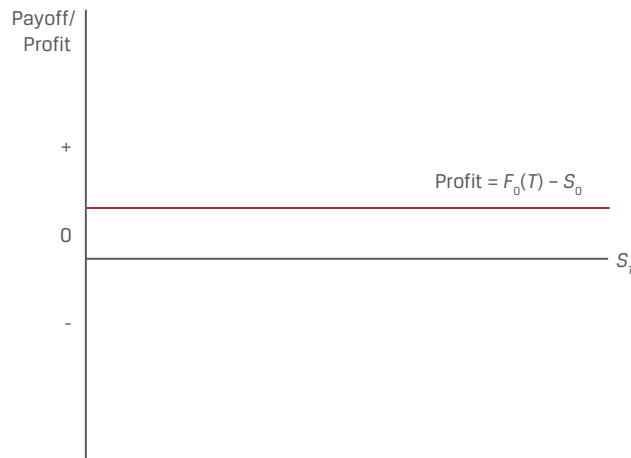
In contrast to Example 2, these positions appear to offset one another. The following example evaluates the return generated by these combined positions.

**EXAMPLE 3****Risk-Free Trade Replication: Long Asset, Short Forward**

Procam Investments buys 100 ounces of gold at today's spot price ( $S_0$ ) of USD1,783.28 and simultaneously enters a forward commitment to sell gold at the forward price,  $F_0(T)$ , of USD1,792.13. Again, we assume that gold can be stored at no cost, and here we solve for Procam's rate of return.

- At  $t = 0$ , Procam's cash flow is  $-S_0 = -\text{USD}178,328$ .
- At  $t = T$ , Procam's cash flow is  $F_0(T) = \text{USD}179,213$ .
- Solve for the rate of return on Procam's strategy, as follows:
  - $\text{USD}179,213 = \text{USD}178,328(1 + r)^{0.25}$ .
  - $r = 2.0\%$ , which is equal to the risk-free rate.

This example demonstrates that Procam can hedge its long gold cash position with a short derivative (i.e., selling the forward contract) and earn the risk-free rate of return as long as the no-arbitrage condition ( $F_0(T) = S_0(1 + r)^T$ ) holds. The combined return is shown in Exhibit 6. Note that if Procam borrows at the risk-free rate to purchase the underlying asset at  $S_0$ , it will earn zero.

**Exhibit 6: Combined Long Asset and Short Forward Profit**

Because the forward price is assumed to be greater than the spot price in this example and  $r$  is positive, the risk-free profit is a positive amount and is the same (i.e., risk free) regardless of the price of the underlying (i.e.,  $S_T$ ). As we will see in the next lesson, there may be benefits or costs to owning an underlying asset that cause the risk-free return in Exhibit 6 to differ from  $F_0(T) - S_0$  and possibly even be negative.

## QUESTION SET



## Arbitrage and Replication

1. Determine the correct answers to fill in the blanks: If the law of one price does not hold, a(n) \_\_\_\_\_ asset trades at the same time at \_\_\_\_\_ prices in different places.

**Solution:**

If the law of one price does not hold, a(n) *identical* asset trades at the same time at *different* prices in different places.

2. An investor observes that the spot price,  $S_0$ , of an underlying asset with no additional costs or benefits exceeds its known future price discounted at the risk-free rate,  $S_T(1 + r)^{-T}$ . Describe and justify an arbitrage strategy that generates a riskless profit for the investor.

**Solution:**

Since the spot price of the underlying asset exceeds the known future price discounted at the risk-free rate ( $S_0 > F_0(T)(1 + r)^{-T}$ ), at  $t = 0$ , the investor:

- Sells the underlying asset short in the spot market at  $S_0$
- Simultaneously enters a long forward contract at  $F_0(T)$
- Lends  $S_0$  at the risk-free rate  $r$  to receive  $S_0(1 + r)^T$  at time  $T$ .

At time  $t = T$ , the investor:

- Settles the long forward position and receives  $S_T - F_0(T)$
- Offsets the short underlying asset position at  $S_T$ , and
- Retains  $S_0(1 + r)^T - F_0(T)$  as a riskless profit regardless of the underlying spot price at time  $T$ .

3. Formulate a replication strategy for a three-month short forward commitment for 1,000 shares of a non-dividend-paying stock.

**Solution:**

The replication strategy for a three-month short forward commitment on a non-dividend-paying stock involves the short sale of 1,000 shares of stock at  $t = 0$  and investment of proceeds at the risk-free rate,  $r$ . At time  $t = T$ , the short sale is covered at  $S_T$ , and under the no-arbitrage condition of  $F_0(T) = S_0(1 + r)^T$ , the return is equal to  $F_0(T) - S_T$  for both the short forward and the replication strategy.

4. Calculate the arbitrage profit if a spot asset with no additional costs or benefits trades at a spot price of 100, the three-month forward price for the underlying asset is 102, and the risk-free rate is 5%.

**Solution:**

The forward price,  $F_0(T) = S_0(1 + r)^T$ , at which no-arbitrage opportunities would exist is 101.23 ( $= 100(1.05)^{0.25}$ ). With an observed forward price of 102, the arbitrage opportunity would be to sell the forward contract and buy the underlying, borrowing at the risk-free rate to fund the purchase. The arbitrage profit is the difference between the observed forward price and the no-arbitrage forward price of 0.77 ( $= 102 - 101.23$ ).

5. Describe the relationship between the spot and forward price for an underlying asset if the risk-free rate is negative.

**Solution:**

The relationship between the spot and forward rate for an asset with no additional costs or benefits of ownership other than the opportunity cost (risk-free rate) is equal to  $F_0(T) = S_0(1 + r)^T$ . In a case where  $r < 0$ ,  $(1 + r)^T < 1$  and therefore the forward price,  $F_0(T)$ , is below the spot price,  $S_0$ , if the risk-free rate is negative.

## 4

### COSTS AND BENEFITS ASSOCIATED WITH OWNING THE UNDERLYING



explain the difference between the spot and expected future price of an underlying and the cost of carry associated with holding the underlying asset

In the prior lesson, replication was used to illustrate the basic relationship between entering into a spot transaction versus a forward commitment. The linkage between the spot price of an asset with no associated cash flows and a forward commitment on the same asset was shown to be the risk-free rate of interest,  $r$ . In this lesson, we discuss **cost of carry** as the net of the costs and benefits related to owning an underlying asset for a specific period.

In the forward commitment example from the prior lesson, where no costs or benefits were associated with the underlying asset, the following relationship between the spot and forward prices was established:

$$F_0(T) = S_0(1 + r)^T. \quad (3)$$

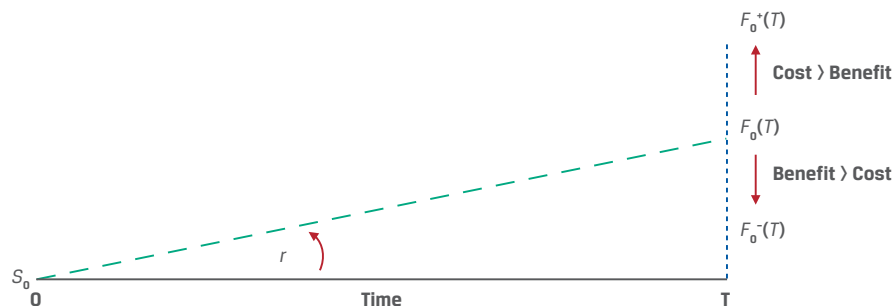
This relationship is shown under continuous compounding in Equation 4:

$$F_0(T) = S_0e^{rT}. \quad (4)$$

The risk-free rate,  $r$ , denotes the **opportunity cost** of holding (“**carrying**”) the asset, whether or not the long investor borrows to finance the asset. This opportunity cost is present for all asset classes discussed below.

Equations 3 and 4 represent the special case of an underlying asset with no additional associated cash flows. However, many assets have additional costs or benefits of ownership that must be reflected in the forward commitment price in order to prevent riskless arbitrage opportunities from arising between underlying spot and derivative prices. Exhibit 7 demonstrates the effect of costs and benefits (usually dividend or interest income) on the spot price, forward commitment price, and risk-free rate relationships.



**Exhibit 7: Spot Prices, Forward Commitment Prices, and the Risk-Free Rate with Underlying Asset Costs and Benefits**


If an underlying asset owner incurs costs in addition to the opportunity cost, she should expect to be compensated for these added costs through a higher forward price,  $F_0^+(T)$ . Income or other benefits accrue to the underlying asset owner and therefore should reduce the forward price to  $F_0^-(T)$ .

For underlying assets with ownership benefits or income ( $I$ ) or costs ( $C$ ) expressed as a known amount in present value terms at  $t = 0$ —shown as  $PV_0()$ —the relationship between spot and forward prices in discrete compounding terms can be shown as

$$F_0(T) = [S_0 - PV_0(I) + PV_0(C)](1 + r)^T. \quad (5)$$

In other instances, the additional costs or benefits are expressed as a *rate of return* over the life of the contract. For income ( $i$ ) and cost ( $c$ ) expressed as rates of return, the relationship between spot and forward prices under continuous compounding is

$$F_0(T) = S_0 e^{(r+c-i)T}. \quad (6)$$

Whether expressed as a known amount in present value terms or as a rate of return, the forward price must incorporate the net effect of all costs and benefits associated with owning the underlying asset, including the following.

- **Opportunity cost (risk-free interest rate,  $r$ ):** A positive risk-free rate causes a forward price to be greater than the underlying spot price, all else equal, and the higher the risk-free rate, the greater the positive difference between the two. This opportunity cost applies to any asset.
- **Other costs of ownership ( $C$ ,  $c$ ):** Owners of some underlying assets, such as physical commodities, must incur storage, transportation, insurance, and/or spoilage costs. An owner entering a contract for future delivery will expect to be compensated for these costs, resulting in a forward price that is therefore greater than the underlying spot price, all else equal.
- **Benefits of ownership ( $I$ ,  $i$ ):** Alternatively, the owners of some underlying assets enjoy cash flow or other benefits associated with owning the underlying asset as opposed to a derivative on the asset. A counterparty entering a derivative contract for future delivery of an underlying asset forgoes these benefits and will therefore reduce the forward price by this amount. Stock dividends or bond coupons are examples of cash flow benefits.

Exhibit 8 illustrates the relationship between forward and spot prices in the presence of costs and benefits. For example, when the opportunity cost and other costs of ownership exceed the benefits, the forward price will be above the underlying spot asset price.

**Exhibit 8: Forward vs. Spot Price**

<b>Cost vs. Benefit</b>	<b>Forward vs. Spot Price</b>
Opportunity and Other Cost > Benefit	$F_0(T) > S_0$
Opportunity and Other Cost < Benefit	$F_0(T) < S_0$
Opportunity and Other Cost = Benefit	$F_0(T) = S_0$

Each underlying asset type has different costs and benefits, which may vary over time and across markets. For example, owners of some individual equity securities receive the benefit of regular stock dividends, as in Example 4, while others do not. For equity indexes, the benefit is usually expressed as a rate of return, as shown in Example 5.

**EXAMPLE 4****Highest Equity Forward with Dividend**

Assume Highest Capital agrees to deliver 1,000 Unilever (UL) shares at an agreed-upon price to a financial intermediary in six months under a forward contract. Assume that UL has a spot price ( $S_0$ ) of EUR50 and pays no dividend ( $I = 0$ ), and assume a risk-free rate ( $r$ ) of 5%. We may use Equation 3 to solve for the forward price,  $F_0(T)$ , in six months:

$$F_0(T) = S_0(1 + r)^T.$$

$$\text{EUR}51.23 = \text{EUR}50(1.05)^{0.5}.$$

Now assume instead that Unilever pays a quarterly dividend of EUR0.30, which occurs in exactly three months and again at time  $T$ , with other details unchanged. Use Equation 5 with  $PV_0(C) = 0$  to solve for  $F_0(T)$  in six months:

$$F_0(T) = [S_0 - PV_0(I)](1 + r)^T.$$

First, solve for the present value of the dividend per share,  $PV_0(I)$ , as follows:

$$PV_0(I) = \text{EUR}0.30(1.05)^{-0.25} + \text{EUR}0.30(1.05)^{-0.5}.$$

$$\text{EUR}0.5892 = 0.2964 + 0.2928.$$

Substitute  $PV_0(I) = \text{EUR}0.5892$  into Equation 5 to solve for  $F_0(T)$ :

$$\begin{aligned} F_0(T) &= (\text{EUR}50 - \text{EUR}0.5892)(1.05)^{0.5} \\ &= \text{EUR}50.6310. \end{aligned}$$

Highest's forward contract for 1,000 UL shares would therefore be priced at EUR50,631.00 ( $= 1,000 \times \text{EUR}50.6310$ ). The opportunity cost of borrowing at the risk-free rate is EUR1.23 per share for six months and equals  $F_0(T) - S_0$  when Unilever pays no dividend. A EUR0.30 quarterly dividend reduces the difference between the forward and spot price to approximately EUR0.63.

**EXAMPLE 5****Stock Index Futures with Dividend Yield**

The Viswan Family Office (VFO) would like to enter into a three-month forward commitment contract to purchase the NIFTY 50 benchmark Indian stock market index traded on the National Stock Exchange. The spot NIFTY 50 index price is INR15,200, the index dividend yield is 2.2%, and the Indian rupee risk-free rate is 4%. Use Equation 6 (with  $c = 0$ ) to solve for the forward price:

$$\begin{aligned} F_0(T) &= S_0 e^{(r+c-i)T} \\ &= 15,200 e^{(0.04 - 0.022)0.25} \\ &= \text{INR}15,268.55. \end{aligned}$$

Foreign exchange requires some adjustments to establish the no-arbitrage condition between spot and forward prices, as shown in the prior lesson. First, it is important to distinguish the “price” of the underlying asset, which for equities, fixed income, or commodities refers to units of currency for each asset—for example, one share of stock or the principal amount for a bond. In contrast, the foreign exchange rate is expressed as a spot rate ( $S_{0,f/d}$ ) specifying the number of units of a *price* currency (here denoted as *f* or foreign currency) in the numerator per *single* unit of a *base* currency (here shown as *d* or domestic currency) in the denominator. For example, for a USD/EUR spot rate ( $S_{0,f/d}$ ) of 1.20, the US dollar is the price currency (*f*), and the euro is the base currency (*d*), with USD1.20 equal to EUR1.

An FX (foreign exchange) forward contract involves the sale of one currency and purchase of the other on a future date at a forward price ( $F_{0,f/d}$ ) agreed on at inception. A *long* FX forward position involves the *purchase* of the base currency and the *sale* of the price currency. For example, a long USD/EUR FX forward position is the sale of US dollars and purchase of euros at a forward rate.

In the prior replication example, the derivative cash flow stream was recreated by combining a long or short position in an underlying asset and borrowing or lending cash. Here the foreign and domestic currency *each* has an opportunity cost—namely, the foreign risk-free rate ( $r_f$ ) and the domestic risk-free rate ( $r_d$ ), respectively.

**EXAMPLE 6****AUD/USD Foreign Exchange Forward Replication**

In order to replicate one currency’s return in terms of the other for a given spot price today of  $S_{0,f/d}$ , we may solve for a forward rate  $F_{0,f/d}(T)$  using the earlier arbitrage concept that assets with a known future price must have a spot price ( $S_{0,f/d}$ ) equal to the future price discounted at the risk-free rate.

Assume the current AUD/USD spot price is 1.3335. The Australian dollar is the price currency or foreign currency, and the US dollar is the base or domestic currency (AUD1.3335 = USD1). The six-month Australian dollar risk-free rate is 0.05%, and the six-month US dollar risk-free rate is 0.20%.

- At time  $t = 0$ :
  1. Borrow USD1,000 at the 0.20% US dollar risk-free rate for six months.
  2. Purchase AUD1,333.50 at the AUD/USD spot rate ( $S_{0,f/d} = 1.3335$ ).
  3. Lend the AUD1,333.50 received at the 0.05% Australian dollar risk-free rate for six months.

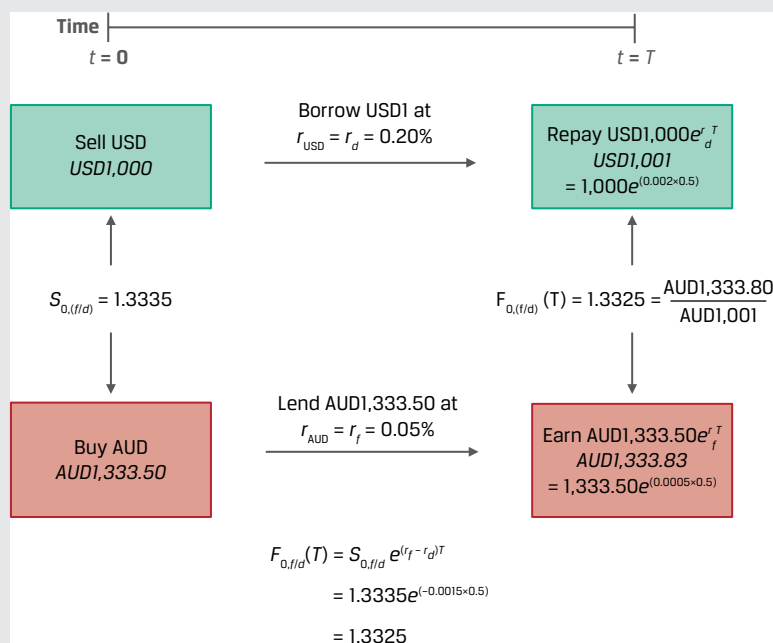
- At time  $t = T$  in six months:

- Receive Australian dollar loan proceeds of 1,333.83 ( $= 1,333.50e^{(0.0005 \times 0.5)}$ ).
- Exchange Australian dollar proceeds for US dollars at  $S_{T,USD}$  to repay the US dollar loan.
- Repay the US dollar loan (with interest) of 1,001 ( $= 1,000e^{(0.002 \times 0.5)}$ ).

If the exchange in Step 5 at time  $T$  is at a spot price,  $S_{T,USD}$ , at which the Australian dollar loan proceeds (Step 4) exactly offset the US dollar loan (Step 6), no riskless arbitrage opportunity exists. Solve for  $S_{T,USD}$  by dividing the Australian dollars in Step 4 by the US dollars in Step 6:

$$S_{T,USD} = 1.3325 (= \text{AUD}1,333.83/\text{USD}1,001).$$

The following diagram summarizes the cash flows at time  $t = 0$  and time  $T$ :



For the no-arbitrage condition to hold between the FX spot and forward price, the amount of Australian dollars necessary to purchase USD1 at time  $T$  ( $F_{0,USD/AUD}(T)$ ) must have a spot price ( $S_{0,USD/AUD}$ ) equal to the discounted future price. The relevant discount rate here involves the *difference* between the foreign and domestic risk-free rates, as shown in the following modified version of Equation 4:

$$F_{0,USD/AUD}(T) = S_{0,USD/AUD} e^{(r_f - r_d)T}. \quad (7)$$

Solve for  $F_{0,AUD/USD}(T)$  in Example 6 as follows:

$$F_{0,USD/AUD}(T) = 1.3325 (= 1.3335e^{(-0.0015 \times 0.5)}).$$

From Equation 7, we see that it is the risk-free interest rate *differential* ( $r_f - r_d$ ), rather than the absolute level of interest rates, that determines the spot versus forward FX price relationship. For example, in Example 6, the Australian dollar risk-free rate is 0.15% below the US dollar rate ( $r_f - r_d < 0$ ). Borrowing at the higher US dollar rate and lending at the lower Australian dollar rate results in a no-arbitrage forward price at which *fewer* Australian dollars are required to purchase USD1 in the future, so the Australian dollar is said to trade at a *premium* in the forward market versus the US dollar. Exhibit 9 summarizes the relationship between forward and spot FX prices.

**Exhibit 9: FX Forward vs. Spot Price Relationship**

Interest Rate Differential	Forward vs. Spot Price	Foreign Currency Forward	FX Forward Premium/Discount
$(r_f - r_d) > 0$	$F_{0,ffd}(T) > S_{0,ffd}$	Discount	Premium
$(r_f - r_d) < 0$	$F_{0,ffd}(T) < S_{0,ffd}$	Premium	Discount
$(r_f - r_d) = 0$	$F_{0,ffd}(T) = S_{0,ffd}$	Neither a premium nor a discount	Neither a premium nor a discount

We now examine this FX spot versus forward relationship in the case of the Montau AG example from an earlier lesson.

**EXAMPLE 7****Montau AG's FX Forward Rate**

An earlier lesson introduced Montau AG, a German capital goods producer. Montau signs a commercial contract with Jeon, Inc., a South Korean manufacturer, to deliver a laser cutting machine at a price of KRW650 million in 75 days. Montau faces a timing mismatch between domestic euro costs incurred and euro revenue realized upon the delivery of the machine and sale of South Korean won received in exchange for euros in the spot FX market. Montau enters a long KRW/EUR FX forward contract. That is, Montau agrees to sell South Korean won and purchase euros at a fixed price,  $F_0(T)$ , in 75 days to eliminate the KRW/EUR exchange rate mismatch arising from the export contract.

In this version of the Montau AG example, we use today's spot exchange rate and the domestic ( $r_d$ ) and foreign ( $r_f$ ) risk-free rates to solve for the KRW/EUR forward rate. As seen in Equation 7, the difference between spot and forward FX rates involves the *difference* in risk-free rates. In the KRW/EUR case, the South Korean won is the price currency or foreign currency and  $r_f$  is therefore the South Korean won interest rate. The euro is the base or domestic currency. We may therefore rewrite Equation 7 as follows:

$$F_{0,KRW/EUR}(T) = S_{0,KRW/EUR} e^{(r_{KRW} - r_{EUR})T}.$$

Assume a spot KRW/EUR rate ( $S_0$ ) of 1,300 (that is, KRW1,300 = EUR1), a South Korean won risk-free rate of 0.75%, and a euro risk-free rate of  $-0.25\%$  ( $r_d$ ). Calculate the KRW/EUR forward rate in 75 days consistent with no arbitrage.

$$F_{0,KRW/EUR}(T) = 1,300 \times e^{(0.0075 + 0.0025) \times (75/365)}.$$

Solving for the KRW/EUR forward rate gives us  $F_0(T) = 1,302.67$ . Notice that the 1% difference between South Korean won and euro interest rates leads to a forward price that is above the spot price ( $F_{0,ffd}(T) > S_{0,ffd}$ ). That is, in six months' time, more South Korean won will be required to purchase a euro, and the South Korean won is said to trade at a forward discount versus the euro.

To demonstrate the no-arbitrage condition between the forward and spot rates, assume that Montau converts the KRW650 million into euros at the 1,300 KRW/EUR spot rate to receive EUR500,000 (= KRW650,000,000/1,300) and invests this for 75 days at a continuously compounded  $r_f = -0.25\%$ :

$$\text{EUR}499,743.22 = \text{EUR}500,000 \times e^{[-0.0025 \times (75/365)]}.$$

Assume that Jeon Inc. invests the KRW650 million at the South Korean won risk-free rate ( $r_d = 0.75\%$ ) for 75 days to receive

$$\text{KRW}651,002,484.59 = \text{KRW}650,000,000 \times e^{[0.0075 \times (75/365)]}.$$

An arbitrage-free forward commitment price should therefore allow Montau to convert KRW651,002,484.59 into EUR499,743.22 after 75 days.

$$F_0(T) = 651,002,484.59/499,743.22 = 1,302.67.$$

This confirms that the two prices (spot and forward exchange rates) are consistent with the risk-free interest rate differential in the two different currencies.

In contrast to securities or cash stored electronically, commodities usually involve known costs associated with the storage, insurance, transportation, and potential spoilage (in the case of soft commodities) of these physical assets. A non-cash benefit of holding a physical commodity versus a derivative is known as a **convenience yield**. In physical goods markets, economic conditions may arise that cause market participants to prefer to own the physical commodity. As a simple example, if crude oil inventories are very low, refineries may bid up the spot oil price so that forward prices do not fully reflect storage costs and interest rates. The following example illustrates the impact of these carry costs on the relationship between spot and forward commitment prices for a commodity, as well as the possibility of a convenience yield.

#### EXAMPLE 8

##### Procam's Gold Forward Contract with Storage Costs

Recall from earlier examples that Procam borrowed and purchased gold in the spot market and simultaneously sold gold in the forward market for three months to earn a riskless profit. Under the assumption of a 2% risk-free rate, we demonstrated that the spot gold price ( $S_0$ ) would need to rise to USD1,783.28 (= USD1,792.13(1.02)<sup>-0.25</sup>) in order to eliminate the earlier arbitrage opportunity for a given USD1,792.13 forward price where gold may be stored at no cost.

Given the spot price of USD1,783.28, how would the forward gold price change to satisfy the no-arbitrage condition if we were to introduce a USD2 per ounce cost of gold storage and insurance payable at the end of the contract?

The forward commitment price for a commodity with known storage cost amounts may be determined using Equation 5, where  $PV_0(I) = 0$ :

$$F_0(T) = [S_0 + PV_0(C)](1 + r)^T.$$

First, solve for the present value of the storage cost per ounce  $PV_0(C)$  as follows:

$$PV_0(C) = \text{USD}2(1.02)^{-0.25}$$

$$= \text{USD}1.99.$$

Substitute  $PV_0(C) = \text{USD}1.99$  into Equation 5 to solve for  $F_0(T)$ :

$$F_0(T) = (\text{USD}1,783.28 + \text{USD}1.99)(1.02)^{0.25}$$

$$= \text{USD}1,794.13.$$

Note that the addition of storage and insurance costs increases the difference between the spot and forward price. Finally, note that a forward price,  $F_0(T)$ , significantly *below* the no-arbitrage price may indicate the presence of a convenience yield.

The additional costs and benefits of underlying asset ownership are summarized in Exhibit 10.

**Exhibit 10: Cost of Carry for Underlying Assets**

Asset Class	Examples	Benefits ( <i>i</i> )	Costs ( <i>r</i> , <i>c</i> )
Asset without Cash Flows	Non-dividend-paying stock	None	Risk-free rate
Equities	Dividend-paying stocks Equity indexes	Dividend Dividend yield	Risk-free rate
Foreign Exchange	Sovereign bonds (foreign)  Market exchange rates	None	Difference between foreign and domestic risk-free rates ( $r_f - r_d$ )
Commodities	Soft and hard commodities  Commodity indexes	Convenience yield	Risk-free rate  Storage cost
Interest Rates	Sovereign bonds (domestic) Market reference rates	Interest income	Risk-free rate
Credit	Single reference entity Credit indexes	Credit spread	Risk-free rate

Interest rates and credit have a term structure—that is, different prices or rates for different maturities. These forward contracts are addressed in a later lesson.

**QUESTION SET****Cost of Carry**

- Describe the relationship between the spot and forward price for an underlying asset whose benefits exceed the opportunity and other costs of ownership.

**Solution:**

If the benefits for an owner of an underlying asset exceed the opportunity and other costs of owning the underlying asset, the spot price will be greater than the forward price.

- Identify which example corresponds to each of the following relationships between the spot and forward rate:

1.  $F_0(T) > S_0$

A. A fixed-coupon bond priced at par whose coupon is above the risk-free rate

2.  $F_0(T) < S_0$

B. A foreign currency forward where the domestic risk-free rate is greater than the foreign risk-free rate

3. Not enough information to determine the relationship between  $F_0(T)$  and  $S_0$

C. A commodity with a convenience yield as well as storage and insurance costs

**Solution:**

1. B is correct. The FX forward rate is greater than the spot rate if the domestic risk-free rate is greater than the foreign risk-free rate.



2. A is correct. A fixed-coupon bond priced at par has an income that exceeds the opportunity cost of the risk-free rate, so  $F_0(T) < S_0$ .
3. C is correct. We do not have enough information to fully evaluate the benefit (convenience yield) versus the cost (risk-free rate, storage, and insurance) of holding the physical commodity asset in this example.

3. Determine the correct answers to fill in the blanks: A positive risk-free rate causes a forward price to be \_\_\_\_\_ than the underlying spot price, all else equal, and the higher the risk-free rate, the \_\_\_\_\_ the difference between the two.

**Solution:**

A positive risk-free rate causes a forward price to be *greater* than the underlying spot price, all else equal, and the higher the risk-free rate, the *greater* the difference between the two.

4. An analyst observes that the current spot MXN/USD exchange rate is 19.50, the Mexican peso six-month risk free rate is 4%, and the six-month US dollar risk free rate is 0.25%. Describe the relationship between the MXN/USD spot and six-month forward rate, and justify your answer.

**Solution:**

The relationship between the MXN/USD FX spot and forward price depends on the risk-free interest rate differential between the Mexican peso and the US dollar ( $r_f - r_d > 0$ , since  $3.75\% = 4.0\% - 0.25\%$ ). Since the Mexican peso rate is 3.75% above the US dollar rate, we would expect that borrowing at a higher Mexican peso rate and lending at a lower US dollar rate would result in a no-arbitrage forward price at which more Mexican pesos are required to purchase USD1 in the future, so the Mexican peso is said to trade at a discount in the forward market versus the US dollar ( $F_{0,MXN/USD}(T) > S_{0,MXN/USD}$ ).

We can show this using Equation 7 for the case of MXN/USD as follows:

$$F_{0,MXN/USD}(T) = S_{0,MXN/USD} e^{(r_{MXN} - r_{USD})T}.$$

Based on the current  $S_{0,fxd}$  of 19.50,  $r_f = 4\%$ ,  $r_d = 0.25\%$ , and  $T$  of 0.5, we may solve for the no-arbitrage forward price,  $F_{0,fxd}$ , as

$$F_{0,MXN/USD}(T) = 19.50 e^{[(0.04 - 0.0025) \times 0.5]}$$

$$= 19.8691.$$

5. Assume that new, stricter environmental regulations associated with oil storage and insurance cause the cost of these services to increase sharply. Describe the anticipated effect of these increased costs on the relationship between oil spot and forward commitment prices.

**Solution:**

Owners of physical commodities who must incur storage and insurance costs over time expect to be compensated by higher forward commitment prices for future delivery of the underlying assets. Assuming other factors are constant, higher storage and insurance costs therefore lead to higher forward commitment prices and a greater difference between spot and forward prices. As shown in Equation 5, an increase in  $C$  increases  $F_0(T)$  for a given  $S_0$ :



$$F_0(T) = [S_0 - PV_0(I) + PV_0(C)] (1 + r)^T. \quad (5)$$



## LEARNING MODULE

## 5

## Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration
<input type="checkbox"/>	explain how forward rates are determined for interest rate forward contracts and describe the uses of these forward rates.

### INTRODUCTION

1

Earlier lessons introduced forward commitment features, payoff profiles, and concepts used in pricing these derivative instruments. In particular, the relationship between spot and forward commitment prices was established as the opportunity cost of owning the underlying asset (represented by the risk-free rate) as well as any additional cost or benefit associated with holding the underlying asset. This price relationship both prevents arbitrage and allows a forward commitment to be replicated using spot market transactions and risk-free borrowing or lending.

In the first lesson, we explore the pricing and valuation of forward commitments on a mark-to-market basis from inception through maturity. This analysis is essential for issuers, investors, and financial intermediaries alike to assess the value of any asset or liability portfolio that includes these instruments. The second lesson addresses forward pricing for the special case of underlying assets with different maturities such as interest rates, credit spreads, and volatility. The prices of these forward commitments across the so-called term structure are an important building block for pricing swaps and related instruments in later lessons.

#### LEARNING MODULE OVERVIEW



- A forward commitment price agreed upon at contract inception remains fixed and establishes the basis on which the underlying asset (or cash) will be exchanged in the future versus the spot price at maturity.

- For an underlying asset that does not generate cash flows, the value of a long forward commitment prior to expiration equals the current spot price of the underlying asset minus the present value of the forward price discounted at the risk-free rate. The reverse is true for a short forward commitment. Foreign exchange represents a special case in which the spot versus forward price is a function of the *difference* between risk-free rates across currencies.
- For an underlying asset with additional costs and benefits, the forward contract mark-to-market (MTM) value is adjusted by the sum of the present values of all additional cash flows through maturity.
- Underlying assets with a term structure, such as interest rates, have different rates or prices for different times-to-maturity. These zero or spot and forward rates are derived from coupon bonds and market reference rates and establish the building blocks of interest rate derivatives pricing.
- Implied forward rates represent a breakeven reinvestment rate linking short-dated and long-dated zero-coupon bonds over a specific period.
- A forward rate agreement (FRA) is a contract in which counterparties agree to apply a specific interest rate to a future period.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Match the following situations with their corresponding forward contract valuation for an asset with no additional costs or benefits.

1. At time $t = 0$ , the spot price of the underlying asset rises instantaneously and other market parameters remain unchanged.	A. The forward contract buyer has an MTM gain.
2. At time $t$ , the present value of the forward price discounted at the risk-free rate ( $r$ ) equals the current spot price ( $S_t$ ).	B. The forward contract seller has an MTM gain.
3. At time $T$ , the forward contract price, $F_0(T)$ , is greater than the current spot price, $S_T$ .	C. The MTM value of the forward contract is zero.

#### Solution:

1. A is correct. In order to satisfy the no-arbitrage condition, the original spot price,  $S_0$  at  $t = 0$ , must equal the present value of the forward price discounted at the risk-free rate,  $r$ . An immediate increase in the spot price to  $S_0^+ > S_0$  results in an MTM gain for the forward buyer.
2. C is correct. At any time  $t$ , the MTM value,  $V_t(T)$ , is equal to the difference between the current spot price,  $S_t$ , and the present value of the forward price discounted at the risk-free rate,  $r$ , or  $F_0(T)(1 + r)^{-(T-t)}$ . When  $S_t = F_0(T)(1 + r)^{-(T-t)}$ , then  $V_t(T) = 0$ .
3. B is correct. The MTM value to the forward contract seller upon settlement at time  $T$  is equal to the settlement value of  $F_0(T) - S_T$ .

2. An increase in the risk-free rate,  $r$ , following the inception of a forward contract will cause which of the following to the forward contract's MTM value to the forward seller if other parameters remain unchanged.

- A. The forward contract's MTM value to the forward seller will be unchanged.
- B. The forward contract's MTM value to the forward seller will increase.
- C. The forward contract's MTM value to the forward seller will decrease.

**Solution:**

C is correct. The mark-to-market value from the forward seller's perspective is equal to  $V_t(T)$  in the following equation:

$$V_t(T) = F_0(T)(1 + r)^{-(T-t)} - S_t.$$

An increase in the risk-free rate,  $r$ , following the inception of a forward contract will cause the present value of the forward price,  $F_0(T)$ , to fall, and this will reduce the MTM value from the contract seller's perspective.

3. Which of the following is closest to the two-year zero rate given a 3% annual coupon bond priced at 99 per 100 face value if a one-year annual coupon bond from the same issuer has a yield-to-maturity of 2.50%?

- A. 3.5266%
- B. 3.5000%
- C. 3.5422%

**Solution:**

C is correct. The yield-to-maturity and the zero rate for a bond with a single cash flow at maturity in one period are identical, so the one-year zero rate,  $z_1$ , equals 2.50%. Solve for the two-year zero rate ( $z_2$ ) in the following equation:

$$99 = 3/1.025 + 103/(1 + z_2)^2.$$

Solve for  $z_2$  to get 3.5422%. A is incorrect because 3.5266% is the internal rate of return (IRR) solved for cash flows of  $-99$  at  $t = 0$ ,  $3$  at  $t = 1$ , and  $103$  at  $t = 2$ . This response assumes a flat term structure, which is not a correct assumption given the question. B is incorrect because this response implies that we can find the correct answer by assuming the coupon rate is the simple average of the one- and two-year zero rates.

4. Which of the following is a correct description of a 2y3y forward rate?

- A. The implied two-year rate beginning three years in the future.
- B. The implied three-year rate beginning two years in the future.
- C. The implied one-year rate beginning two years in the future.

**Solution:**

B is correct. In the terminology of forward rates, the first number reflects the point in time when a forward rate begins; thus, the forward rate stated above reflects a rate starting two years in the future. The second number reflects the maturity of the rate. Thus, the 2y3y forward rate reflects a three-year rate starting in two years. A is incorrect because this is the description of the 3y2y forward rate. C is incorrect because this is the description of the 2y1y rate.

## 2

## PRICING AND VALUATION OF FORWARD CONTRACTS



explain how the value and price of a forward contract are determined at initiation, during the life of the contract, and at expiration

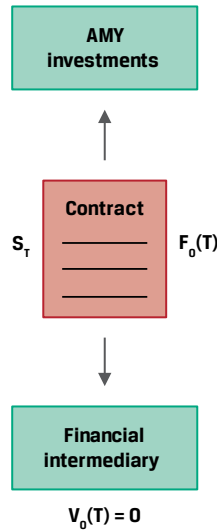
**Pricing versus Valuation of Forward Contracts**

When counterparties enter into forward, futures, or swap contracts with one another, these contracts have an initial value of zero (ignoring trading and transaction costs as well as counterparty credit exposure). While forward commitments require no cash outlay at inception, their price incorporates the opportunity cost of a long cash position as measured by the risk-free rate. The forward price or forward rate established at inception remains fixed and determines the basis on which the underlying asset (or cash) will be exchanged in the future versus the spot price at maturity.

As time passes and/or the underlying asset spot price and other parameters change, the value of a forward contract changes. This mark-to-market value of a contract reflects the change in the underlying price and other factors that would result in a gain or loss to a counterparty if the forward contract were to be settled immediately. The MTM gain of the forward seller will equal the MTM loss of the forward buyer and vice versa. Recall that a key difference between exchange-traded futures and over-the-counter forwards is that the futures clearinghouse settles these MTM changes in cash on a daily basis, while forward contract settlement typically occurs at maturity.

***Pricing and Valuation of Forward Contracts at Initiation***

The prior learning module established that a forward contract agreed at time  $t = 0$  occurs at a forward price,  $F_0(T)$ , that satisfies no-arbitrage conditions for the underlying spot price ( $S_0$ ), the risk-free rate of return ( $r$ ), and any additional costs or benefits associated with underlying asset ownership until the forward contract matures at time  $T$ . In an earlier example, AMY Investments agreed to purchase 1,000 Airbus (AIR) shares trading at the spot price ( $S_T$ ) at maturity at an agreed upon forward price,  $F_0(T)$ , of EUR30 per share, as shown in Exhibit 1.

**Exhibit 1: Forward Contract Value at Initiation**Time:  $t = 0$ 

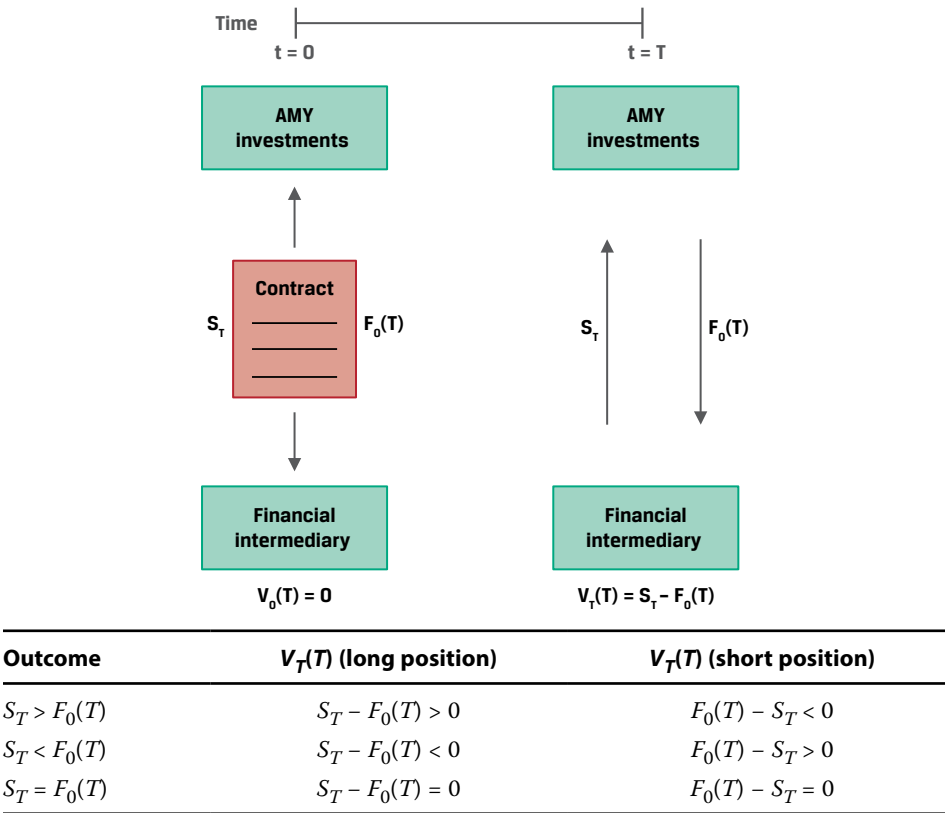
If we assume  $S_0 = \text{EUR}29.70$ ,  $r = 1.00\%$  and  $T$  is one year,  $F_0(T)$  of EUR30 satisfies the no-arbitrage condition at  $t = 0$  (ignoring transaction costs). The forward contract is neither an asset nor a liability to AMY Investments (the buyer) or the financial intermediary (the seller) and therefore has a value of zero to both parties:

$$V_0(T) = 0 \quad (1)$$

**Pricing and Valuation of Forward Contracts at Maturity**

Recall from an earlier lesson that a forward commitment has a symmetric payoff profile. That is, at time  $T$  a forward contract is settled based on the difference between the forward price,  $F_0(T)$ , and the underlying spot price,  $S_T$ , or  $S_T - F_0(T)$ , from the buyer's perspective, as shown in Exhibit 2.

Exhibit 2: Forward Contract Value at Maturity



AMY must pay the agreed upon price of  $F_0(T)$  in exchange for the underlying asset at the current spot price of  $S_T$ . Since the contract settles at maturity, its *value* at maturity is equal to the settlement amount from each counterparty's perspective. For example, at time  $t = T$ , the value of the forward contract at maturity,  $V_T(T)$ , from the perspective of AMY (the forward buyer) equals:

$$V_T(T) = S_T - F_0(T) \tag{2}$$

EXAMPLE 1

Value of Biomian Forward Positions at Maturity

The Viswan Family Office (VFO) currently owns 10,000 common non-dividend-paying shares of Biomian Limited, a Mumbai-based biotech company, at a spot price of INR 295 per share. VFO agrees to sell forward 1,000 shares of Biomian stock to a financial intermediary for INR300.84 per share in six months. Calculate the contract value at maturity,  $V_T(T)$ , from both the buyer's and the seller's perspective if the spot price at maturity ( $S_T$ ) is:

- (1)  $S_T = \text{INR } 287$
- (2)  $S_T = \text{INR } 312$

Solution:

At contract inception, VFO enters into a forward contract with a financial intermediary to sell Biomian for  $F_0(T) = \text{INR } 300.84$ . The forward contract value at initiation for both VFO and the financial intermediary,  $V_0(T)$ , is zero.



(1)  $S_T = \text{INR}287$  and  $F_0(T) = \text{INR}300.84$ . The contract value per share at maturity equals its settlement value from the perspective of both the financial intermediary (buyer) and VFO (seller), as follows:

- Buyer (long forward position):  $V_T(T) = S_T - F_0(T)$

$$V_T(T) = -\text{INR}13.84 = 287 - 300.84$$

- Viswan Family Office (short forward position):  $V_T(T) = F_0(T) - S_T$

$$V_T(T) = \text{INR}13.84 = 300.84 - 287$$

(2)  $S_T = \text{INR}312$  and  $F_0(T) = \text{INR}300.84$ . The contract value per share at maturity equals its settlement value from the perspective of both the financial intermediary (buyer) and VFO (seller), as follows:

- Buyer (long forward position):  $V_T(T) = S_T - F_0(T)$

$$V_T(T) = \text{INR}11.16 = 312 - 300.84$$

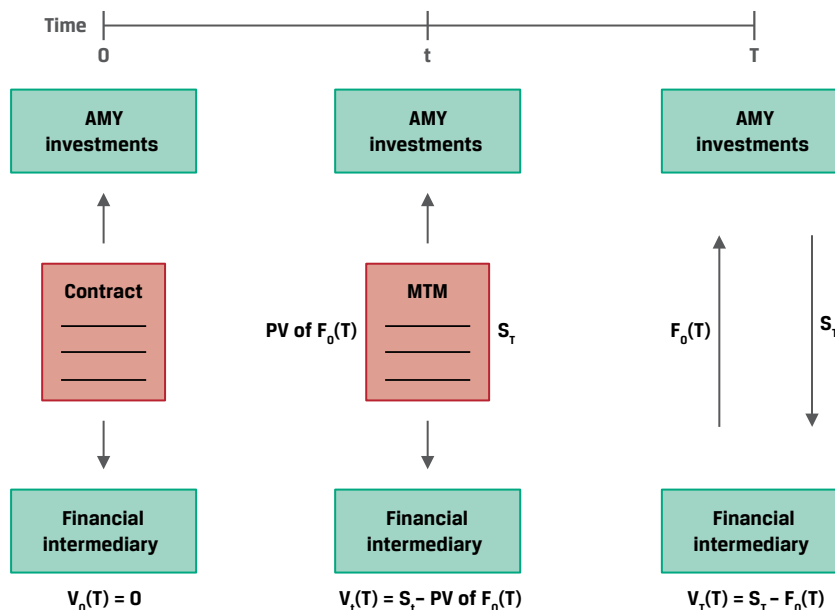
- Viswan Family Office (short forward position):  $V_T(T) = F_0(T) - S_T$

$$V_T(T) = -\text{INR}11.16 = 300.84 - 312$$

### Pricing and Valuation of Forward Contracts during the Life of the Contract

Once a forward contract is initiated between two counterparties, the passage of time and changes in the underlying asset's spot price, among other factors, will cause the forward contract value to change. The mark-to-market value of a contract at any point in time from inception to maturity,  $V_t(T)$ , reflects the relationship between the current spot price at time  $t$  ( $S_t$ ) and the present value of the forward price at time  $t$  discounted at the current risk-free rate. Exhibit 3 shows this relationship for the Airbus equity forward example from AMY Investments' perspective (the long forward position).

**Exhibit 3: Forward Contract Value at Inception, over Time, and at Maturity**



Recall that under no-arbitrage conditions, the forward price of an underlying asset with no additional cost or benefit of ownership equals the future value of the spot price at the risk-free rate,  $r$ :

$$F_0(T) = S_0(1 + r)^T \quad (3)$$

At any time  $t$  over the life of the contract where  $t < T$ , we can show the present value of the forward price,  $F_0(T)$ , at time  $t$  as follows:

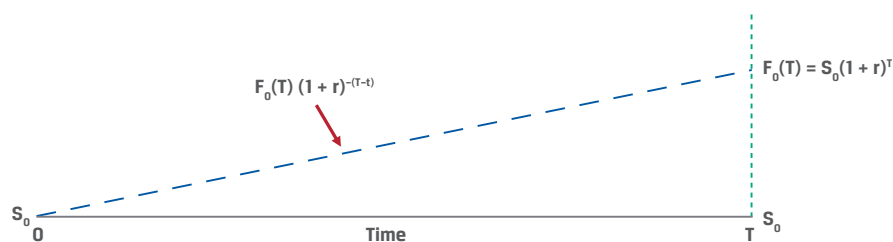
$$PV_t \text{ of } F_0(T) = F_0(T)(1 + r)^{-(T-t)} \quad (4)$$

If  $S_t$  is the spot price of the underlying asset at time  $t$ , Equation 5 shows the forward contract MTM value at time  $t$ ,  $V_t(T)$ , from the long forward position's perspective:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)} \quad (5)$$

Exhibit 4 shows the relationship between the spot price, the forward price, and the present value of the forward price,  $F_0(T)(1 + r)^{-(T-t)}$ , over time as represented by the dashed line. The slope of the dashed line is equal to  $r$ .

**Exhibit 4: Present Value of the Forward Price over Time**



We explore these relationships, as well as the MTM impact of an instantaneous change in the spot price ( $S_0$ ), in the following example.

#### EXAMPLE 2

##### Implied Risk-Free Rate and Biomian Forward Contract MTM

As in Example 1, VFO enters into a six-month forward contract with a financial intermediary to sell Biomian shares for  $F_0(T) = \text{INR}300.84$  per share. The current spot price is  $\text{INR}295$  per share.

1. Calculate the risk-free rate implied by the spot and forward prices.
2. Calculate the forward contract MTM from VFO's perspective if Biomian's share price rises instantaneously to  $\text{INR}325$  at contract inception ( $t = 0$ ).

##### Solution:

1. We can use Equation 3—with  $S_0 = 295$ ,  $F_0(T) = 300.84$ , and  $T = 0.5$ —to solve for the risk-free rate,  $r$ :

$$F_0(T) = S_0(1 + r)^T$$

$$300.84 = 295(1 + r)^{0.5}$$

$$r = 0.04 \text{ or } 4\%$$

2. As the forward contract seller, we must rearrange Equation 5 to solve for the contract MTM value from VFO's perspective as follows:

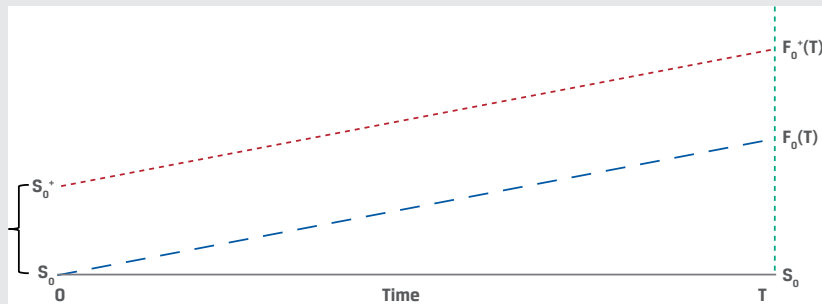
$$V_t(T) = F_0(T)(1+r)^{-(T-t)} - S_t$$

Note that if  $t = 0$ ,  $F_0(T)(1+r)^{-(T-t)}$  simplifies to  $F_0(T)(1+r)^{-T} = S_0$ , so the contract value from VFO's perspective can be shown simply as:

$$V_t(T) = S_0 - S_t$$

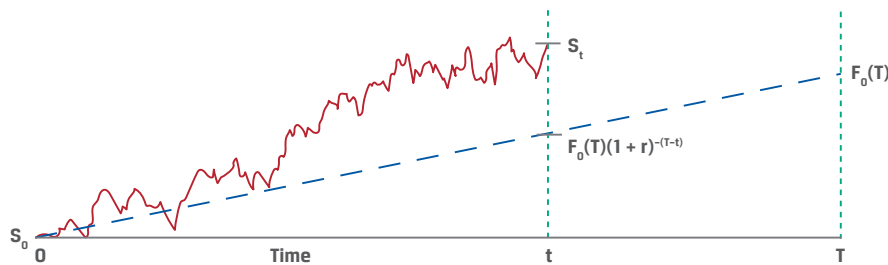
$$-\text{INR}30 = \text{INR}295 - \text{INR}325$$

If we consider the new higher Biomian price as  $S_0^+ = \text{INR}325$ , we can show this MTM change in the following diagram:



The combination of spot price changes over the life of a forward contract and the passage of time causes the MTM value of a forward contract to fluctuate over time, representing a gain or loss to contract participants so long as  $S_t \neq F_0(T)(1+r)^{-(T-t)}$ . Exhibit 5 shows this relationship, assuming a constant risk-free rate over the life of a contract for long and short forward contract positions.

**Exhibit 5: Forward Contract MTM**



Outcome	$V_t(T)$ (long position)	$V_t(T)$ (short position)
$S_t > F_0(T)(1+r)^{-(T-t)}$	MTM gain	MTM loss
$S_t < F_0(T)(1+r)^{-(T-t)}$	MTM loss	MTM gain
$S_t = F_0(T)(1+r)^{-(T-t)}$	No MTM gain or loss	No MTM gain or loss

Example 3 illustrates these combined effects for VFO's Biomian forward contract and examines the effect of a change in the risk-free rate.

**EXAMPLE 3****Biomian Forward Contract MTM over Time and Changes in Risk-Free Rate**

As in earlier examples, VFO enters into a six-month forward contract with a financial intermediary to sell Biomian shares at  $F_0(T) = \text{INR}300.84$  per share. The spot price at  $t = 0$  is  $\text{INR}295$  per share and the risk-free rate is 4%.

1. Calculate the forward contract MTM from VFO's perspective in three months ( $t = 0.25$ ) if Biomian's spot price ( $S_t$ ) falls to  $\text{INR}285$  per share.
2. Show the forward contract MTM from VFO's perspective in Question (1) if the risk-free rate doubles from 4% to 8%, and interpret the results.

**Solution:**

(1) From VFO's perspective as the forward contract seller, rearrange Equation 5 to solve for the contract MTM value as follows:

$$V_t(T) = F_0(T)(1 + r)^{-(T-t)} - S_t$$

Solve for  $V_t(T)$  where  $F_0(T) = 300.84$ ,  $r = 0.04$ ,  $T = 0.5$ ,  $t = 0.25$ , and  $S_t = 285$ :

$$V_t(T) = 300.84(1.04)^{-(0.25)} - 285$$

$$V_t(T) = \text{INR}12.90 \text{ MTM gain}$$

(2) Solve for  $V_t(T)$  using the same equation and inputs as in Question (1) except that  $r = 0.08$ :

$$V_t(T) = F_0(T)(1 + r)^{-(T-t)} - S_t$$

Solve for  $V_t(T)$  where  $F_0(T) = 300.84$ ,  $r = 0.08$ ,  $T = 0.5$ ,  $t = 0.25$ , and  $S_t = 285$ :

$$V_t(T) = 300.84(1.08)^{-(0.25)} - 285$$

$$V_t(T) = \text{INR}10.11 \text{ MTM gain}$$

VFO's MTM gain on the contract has declined by  $\text{INR}2.79$  ( $12.90 - 10.11$ ). The *higher* risk-free rate increases the opportunity cost of a cash position and *lowers* the present value of the forward price, *reducing* VFO's MTM gain,  $V_t(T)$ , as represented by the present value of  $F_0(T) - S_t$ .

**Pricing and Valuation of Forward Contracts with Additional Costs or Benefits**

The pricing and valuation of forward contracts in this lesson have assumed that there are no cash flows associated with the underlying asset. We showed that the contract MTM value at any time  $t$  equals the difference between the current spot price and the present value of the forward price discounted at the risk-free rate. Now we turn our attention to how the cost of carry, or the net of all costs and benefits related to owning an underlying asset or index, affects the valuation of a forward contract. Recall the relationship between spot and forward prices for underlying assets with ownership benefits or income ( $I$ ) or costs ( $C$ ), which is expressed as a present value at time  $t = 0$  in discrete compounding terms:

$$F_0(T) = (S_0 - \text{PV}_0(I) + \text{PV}_0(C)) (1 + r)^T \quad (6)$$

Equation 6 incorporates the cost of carry and satisfies the no-arbitrage condition at  $t = 0$  (ignoring transaction costs). A forward contract at a price,  $F_0(T)$ , that incorporates these known costs and benefits is neither an asset nor a liability to the buyer or seller at inception:  $V_0(T) = 0$ . Also, since the forward price,  $F_0(T)$ , incorporates the cost of carry, the value at maturity,  $V_T(T)$ , is equal to the difference between the spot price ( $S_T$ ) at maturity and the original forward price,  $F_0(T)$ .

At any time  $t$  over the life of the contract, the MTM value of the forward contract,  $V_t(T)$ , will depend on the difference between the current spot price adjusted for any remaining costs or benefits from time  $t$  through maturity,  $S_t - PV_t(I) + PV_t(C)$ , and the present value of the forward price,  $PV_t F_0(T) = F_0(T)(1 + r)^{-(T-t)}$ . This result is shown in Equation 7 from the long forward counterparty's perspective:

$$V_t(T) = (S_t - PV_t(I) + PV_t(C)) - F_0(T)(1 + r)^{-(T-t)} \quad (7)$$

#### EXAMPLE 4

##### Highest Equity Forward Valuation

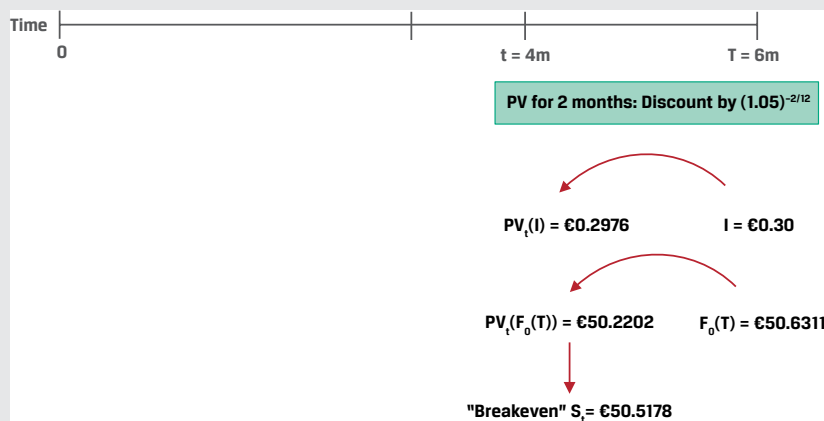
In an earlier example, Highest Capital agreed to deliver 1,000 Unilever (UL) shares to a financial intermediary in six months under a forward contract at a price of EUR 50,631.10, or EUR 50.6311 per share. Unilever pays a quarterly dividend of EUR 0.30 three months after contract inception and at time  $T$ , and the risk-free rate ( $r$ ) is 5%. Calculate the forward contract breakeven price,  $S_t$ , where  $V_t(T) = \text{MTM} = 0$  four months after contract inception if the risk-free rate,  $r$ , remains unchanged at 5%.

Use Equation 7 with  $PV_t(C) = 0$  to solve for  $V_t(T) = 0$  four months after contract inception:

$$V_t(T) = (S_t - PV_t(I)) - F_0(T)(1 + r)^{-(T-t)}$$

First, solve for the present value of the dividend per share,  $PV_t(I)$ , given that the second dividend will be paid in two months:

$$\begin{aligned} PV_t(I) &= \text{EUR}0.30(1.05)^{-0.167} \\ &= \text{EUR}0.2976 \end{aligned}$$



Substitute  $PV_t(I) = \text{EUR}0.2976$  into Equation 5 to solve for  $V_t(T) = 0$ :

$$\begin{aligned} 0 &= (S_t - 0.2976) - 50.6311(1.05)^{-0.167} \\ S_t &= \text{EUR}50.2202 + \text{EUR}0.2976 \\ &= \text{EUR}50.5178 \end{aligned}$$

Highest Capital's breakeven spot rate,  $S_t$  (i.e., where  $V_t(T) = 0$ ), four months after inception of the forward contract is therefore EUR50.5178 per share.

Forward commitments on underlying indexes such as equity indexes or commodity indexes are predominantly exchange-traded index futures contracts. An earlier lesson used the *rate of return* over the life of the contract under continuous compounding to establish the relationship between spot and forward pricing for these instruments. The valuation of index futures and other futures contracts will be addressed in a later lesson.

Recall that for a foreign exchange forward, both spot ( $S_{0,fd}$ ) and forward ( $F_{0,fd}(T)$ ) prices are expressed in terms of units of a *price* currency ( $f$  or foreign currency) per *single* unit of a *base* currency ( $d$  or domestic currency). The spot price versus forward price relationship reflects the *difference* between the foreign risk-free rate,  $r_f$ , and the domestic risk-free rate,  $r_d$ , as shown in Equation 8:

$$F_{0,fd}(T) = S_{0,fd} e^{(r_f - r_d)T} \quad (8)$$

At trade inception at  $t = 0$ , the currency with the lower risk-free rate for the forward period is said to trade at a forward premium—that is, fewer units of currency are required to purchase one unit of the other—while the currency with the higher risk-free rate trades at a forward discount.

At any given time  $t$ , the MTM value of the FX forward is the difference between the *current* spot FX price ( $S_{t,fd}$ ) and the present value of the forward price discounted by the *current* difference in risk-free rates ( $r_f - r_d$ ) for the remaining period through maturity, as shown in Equation 9:

$$V_t(T) = S_{t,fd} - F_{0,fd}(T) e^{-(r_f - r_d)(T-t)} \quad (9)$$

Changes in the interest rate differential ( $r_f - r_d$ ) represent a change in the *relative* opportunity cost between currencies. As described in an earlier lesson, a price change of one currency in terms of another is referred to as *appreciation* or *depreciation*. For example, if fewer USD are required to purchase one EUR, then the USD/EUR exchange rate falls and the USD is said to *appreciate* against the EUR.

A greater interest rate differential—that is, an increase in ( $r_f - r_d$ )—causes the price, or foreign, currency to depreciate on a forward basis and the base, or domestic, currency to appreciate. The following example illustrates the effect of an interest rate change on the FX forward contract MTM value.

#### EXAMPLE 5

##### Rook Point Investors LLC FX Forward MTM

Rook Point Investors LLC has entered into a long one-year USD/EUR forward contract. That is, it has agreed to purchase EUR1,000,000 in exchange for USD1,201,000 in one year. At time  $t = 0$  when the contract is initiated, the USD/EUR spot exchange rate is 1.192 (i.e., USD1.192 = EUR1), the one-year USD risk-free rate is 0.50%, and the one-year EUR risk-free rate is -0.25%.

Describe the MTM impact on the FX forward contract from Rook Point's perspective if the one-year USD risk-free rate instantaneously rises by 0.25% once the contract is initiated, with other details unchanged.

The instantaneous rise in the USD risk-free rate by 0.25% (from 0.5% to 0.75%) increases the difference between the foreign and domestic risk-free rates ( $r_f - r_d$ ), *increasing* the discount rate used to calculate the present value of the forward rate,  $F_{0,fd}(T)$ . Since  $S_{0,fd} > F_{0,fd}(T) e^{-(r_f - r_d)(T-t)}$ ,  $V_0(T) > 0$  from Rook Point's perspective and it realizes an MTM gain.

We can solve for the MTM value at time  $t = 0$  by first rewriting Equation 9:

$$V_t(T) = S_{0,USD/EUR} - F_{0,USD/EUR}(T) e^{-(r_{USD} - r_{EUR})T}$$

Solve for  $V_t(T)$  from Rook Point's perspective, with  $S_{0,fd} = 1.192$  USD/EUR,  $F_{0,fd} = 1.201$  USD/EUR,  $r_f = 0.75\%$ ,  $r_d = -0.25\%$ , and  $T = 1$ .

$$= 1.192 - 1.201 e^{-(0.0075 + 0.0025)}$$

$$= 0.00295 \text{ USD/EUR}$$

Note that this positive exchange rate difference of 0.00295 USD/EUR can be shown in Equation 9 if we instead substitute the USD needed to purchase EUR1,000,000 to arrive at a USD contract value:

$$= \text{USD1,192,000} - \text{USD1,201,000} e^{-(0.0075 + 0.0025)}$$

= USD2,950.15 MTM gain

In this and other examples involving interest rates, we have assumed constant risk-free rates both over time and across maturities. In the next lesson, we will explore how spot and forward prices change for variables, with different prices across maturities.

## QUESTION SET



### Pricing and Valuation of Forward Contracts

1. Identify which MTM situation corresponds to which forward price versus spot price relationship for an underlying asset with no additional costs or benefits.

1. At time $t = 0$ once the forward price is agreed, the spot price of the underlying asset immediately falls and other market parameters remain unchanged.	A. The forward contract seller has an MTM loss.
2. At time $T$ , the forward contract price, $F_0(T)$ , equals the current spot price, $S_T$ .	B. The forward contract seller has an MTM gain.
3. At time $T$ , the forward contract price, $F_0(T)$ , is below the current spot price, $S_T$ .	C. The MTM value of the forward contract is zero.

#### Solution:

1. B is correct. To satisfy the no-arbitrage condition, the original spot price,  $S_0$  at  $t = 0$ , must equal the present value of the forward price discounted at the risk-free rate,  $r$ . Therefore, an immediate fall in the spot price to  $S_0^- < S_0$  results in an MTM gain for the forward contract seller.
2. C is correct. At time  $T$ , the MTM value,  $V_t(T)$ , is equal to contract settlement, or the difference between  $F_0(T)$  and  $S_T$ , which in this case is zero.
3. A is correct. The MTM value to the forward seller upon settlement at time  $T$  equals  $F_0(T) - S_T$ , so the seller has a loss if  $F_0(T) < S_T$ .

2. Identify the following statement as true or false and justify your answer:

At time  $t$ , a forward contract with no additional cash flows has a value equal to the difference between the current spot price and the present value of the forward price. Therefore, the MTM value of the forward contract from the seller's perspective is  $S_t - \text{PV of } F_0(T)$ .

#### Solution:

This statement is false. Although the forward contract MTM equals the difference between the current spot price and the present value of the forward price, the MTM value of the forward contract from the seller's perspective is  $\text{PV of } F_0(T) - S_t$ .

3. Assume that Hightest Capital enters into a six-month forward contract to purchase Unilever (UL) shares at EUR51.23. UL shares pay no dividend, and the risk-free rate across all maturities is 5%. Calculate the forward contract

MTM value from Hightest's perspective in three months' time if the current UL spot rate,  $S_t$ , is EUR50.50 and the risk-free rate does not change.

**Solution:**

Use Equation 5 to solve for the forward contract value in three months from Hightest's perspective:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)}$$

If  $S_t = \text{EUR}50.50$ ,  $F_0(T) = \text{EUR}51.23$ ,  $r = 5\%$ , and  $T - t = 0.25$ :

$$V_t(T) = \text{EUR}50.50 - (\text{EUR}51.23)(1.05)^{-0.25}$$

$$= -\text{EUR}0.1089$$

Hightest therefore has an MTM loss of EUR0.11 on the forward contract in three months' time.

4. Match the following statements with their corresponding MTM situation for an underlying asset with additional costs and benefits.

1. At time $t$ , the present value of the benefits of owning the underlying asset is <i>greater than</i> the present value of the costs.	A. The forward contract has an MTM value <i>equal to</i> the difference between the current spot price and the present value of the forward price.
2. At time $t$ , the present value of the benefits of owning the underlying asset is <i>equal to</i> the present value of the costs.	B. The forward contract has an MTM value <i>greater than</i> the difference between the current spot price and the present value of the forward price.
3. At time $t$ , the present value of the benefits of owning the underlying asset is <i>less than</i> the present value of the costs.	C. The forward contract has an MTM value <i>less than</i> the difference between the current spot price and the present value of the forward price.

**Solution:**

Recall the forward contract MTM value,  $V_t(T)$ , for underlying assets with additional costs and benefits from the long forward counterparty's perspective in Equation 7:

$$V_t(T) = (S_t - \text{PV}_t(I) + \text{PV}_t(C)) - F_0(T)(1 + r)^{-(T-t)}$$

1. C is correct. A higher present value of the benefits of owning the underlying asset versus the present value of the costs will reduce the MTM below the difference between the current spot price and the present value of the forward price. For Equation 7, if  $\text{PV}_t(C) - \text{PV}_t(I) < 0$ , then:

$$V_t(T) < S_t - F_0(T)(1 + r)^{-(T-t)}$$

2. A is correct. If the present values of the remaining benefits and costs of owning the underlying asset offset each other, then the forward contract MTM *equals* the difference between the current spot price and the present value of the forward price. For Equation 7, if  $\text{PV}_t(C) - \text{PV}_t(I) = 0$ , then:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)}$$

3. B is correct. A lower present value of the benefits of owning the underlying asset versus the present value of the costs will increase the MTM beyond the difference between the current spot price and the present value of the forward price. For Equation 7, if  $\text{PV}_t(C) - \text{PV}_t(I) > 0$ , then:

$$V_t(T) > S_t - F_0(T)(1 + r)^{-(T-t)}$$

5. Rook Point Investors LLC has entered into a six-month FX forward contract in which it agrees to sell South African rand (ZAR) and buy EUR at a forward ZAR/EUR price of 17.2506 in six months. The ZAR/EUR spot price is 16.909, the six-month South African risk-free rate is 3.5%, and the six-month EUR risk-free rate is -0.5%. Describe the FX forward MTM impact



from Rook Point's perspective of an immediate appreciation in ZAR/EUR to 16.5 if other parameters are unchanged.

**Solution:**

An immediate appreciation in the ZAR/EUR spot price after contract inception will result in an MTM gain from Rook Point's perspective as the forward seller of ZAR/EUR.

The FX forward MTM from Rook Point's perspective equals the present value of the forward price discounted at the interest rate differential between the foreign currency and the domestic currency minus the spot price:

$$V_0(T) = F_{0,f/d}(T) e^{-(r_f - r_d)T} - S_{0,f/d}$$

Note that ZAR is the price, or foreign, currency and EUR is the base, or domestic, currency, so we can rewrite the equation as:

$$V_0(T) = F_{0,ZAR/EUR}(T) e^{-(r_{ZAR} - r_{EUR})T} - S_{0,ZAR/EUR}$$

If the ZAR price ( $S_{0,ZAR/EUR}$ ) appreciates from 16.909 to 16.5, we can show that Rook Point would have a 0.4090 gain, as follows:

$$\begin{aligned} V_t(T) &= 17.2506e^{-(0.035 - 0.005) \times (0.5)} - 16.5 \\ &= 16.909 - 16.5 \\ &= 0.4090 \end{aligned}$$

## PRICING AND VALUATION OF INTEREST RATE FORWARD CONTRACTS

### 3



explain how forward rates are determined for interest rate forward contracts and describe the uses of these forward rates.

### Interest Rate Forward Contracts

The relationship between spot and forward prices of underlying assets uses a constant risk-free interest rate as the opportunity cost of owning the underlying asset. Unlike equities and commodities, addressed earlier, interest rates are characterized by a term structure—that is, different interest rates exist for different times-to-maturity. While this lesson focuses on interest rates, similar principles apply to other underlying variables with a term structure, including credit spreads and implied volatility as well as foreign exchange, where two interest rate term structures are involved.

#### *Spot Rates and Discount Factors*

The relationships between spot and forward interest rates, established in earlier fixed-income lessons, are key building blocks for interest rate derivatives pricing. These building blocks are usually based on a government benchmark or market reference rate. While most fixed-income instruments have coupon cash flows prior to maturity, interest rate derivatives pricing and valuation are based on the price and yield of single cash flows on a future date. The transformation of these cash flows from one form into another is a first step in the process, as in the following example.

**EXAMPLE 6****Zero Rate**

Assume we observe three most recently issued annual fixed-coupon government bonds, with coupons and prices as follows:

Years to Maturity	Annual Coupon	PV (per 100 FV)
1	1.50%	99.125
2	2.50%	98.275
3	3.25%	98.000

Note that each bond trades at a discount ( $PV < FV$ ). As a first step, we solve for each bond's yield-to-maturity (YTM) using the Excel RATE function ( $=RATE(nper, pmt, pv, [fv], [type])$ ). For example, for the three-year bond, we use  $nper = 3$ ,  $pmt = 3.25$ ,  $pv = -98$ ,  $fv = 100$ , and  $type = 0$  (indicating payment at the end of the period) to solve for the three-year bond YTM of 3.9703%.

Years to Maturity	Annual Coupon	PV (per 100 FV)	YTM
1	1.50%	99.125	2.3960%
2	2.50%	98.275	3.4068%
3	3.25%	98.000	3.9703%

We can use these government yields-to-maturity to solve for a sequence of yields-to-maturity on zero-coupon bonds, or zero rates ( $z_1, \dots, z_N$ ), where  $z_i$  is the zero rate for period  $i$ .

Starting with the one-year bond, which consists of a single cash flow at maturity, we can solve for the one-year zero rate ( $z_1$ ) as follows:

One-year:

$$99.125 = \frac{101.5}{(1+z_1)^1}; z_1 = 2.3960\%$$

The yield-to-maturity rate and the zero rate for a bond with a single cash flow at maturity in one period are identical. Since all cash flows at time  $t = 1$  are discounted at  $z_1$ , we can substitute  $z_1$  into the two-year fixed-coupon bond calculation to solve for the zero-coupon rate at the end of the second period ( $z_2$ ):

Two-year:

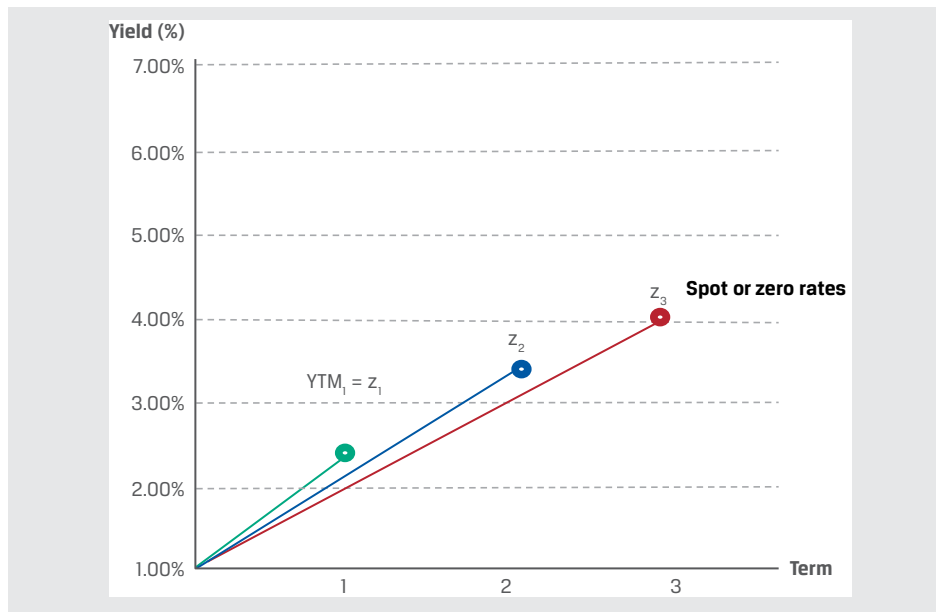
$$98.275 = \frac{2.5}{1.02396} + \frac{102.5}{(1+z_2)^2}; z_2 = 3.4197\%$$

We then substitute both  $z_1$  and  $z_2$  into the three-year bond equation to solve for the zero-coupon cash flow at the end of year three ( $z_3$ ):

Three-year:

$$98.00 = \frac{3.25}{(1.02396)} + \frac{3.25}{(1.034197)^2} + \frac{103.25}{(1+z_3)^3}; z_3 = 4.0005\%$$

The zero rates are summarized in the following diagram:



This process of deriving zero or spot rates from coupon bonds using forward substitution, as shown in Example 6, is sometimes referred to as bootstrapping. The *price* equivalent of a zero rate is the present value of a currency unit on a future date, known as a **discount factor**. The discount factor for period  $i$  ( $DF_i$ ) is:

$$DF_i = \frac{1}{(1 + z_i)^i} \quad (10)$$

For each of the zero rates shown in Example 6, an equivalent discount factor can be derived as follows:

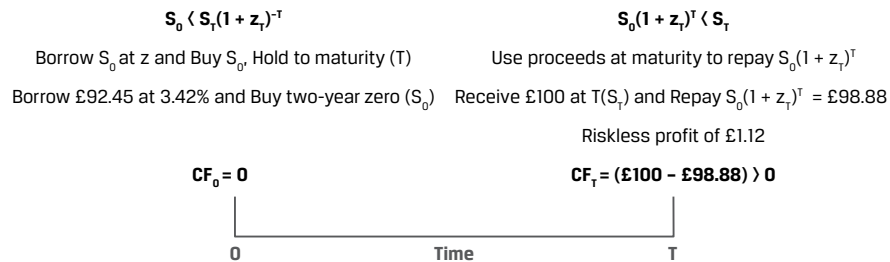
One-year:  $DF_1 = 1/(1 + z_1)$ ;  $DF_1 = 0.976601$

Two-year:  $DF_2 = 1/(1 + z_2)^2$ ;  $DF_2 = 0.934961$

Three-year:  $DF_3 = 1/(1 + z_3)^3$ ;  $DF_3 = 0.888982$

A discount factor may also be interpreted as the price of a zero-coupon cash flow or bond. We can use the discount factor for any period to demonstrate the same no-arbitrage condition from an earlier lesson—namely, that an asset with a known future price must trade at the present value of its future price as determined by using an appropriate discount rate.

For example, assume a two-year GBP risk-free zero rate ( $z_2$ ) of 3.42% and a two-year zero-coupon bond with a face value of GBP100 trading at a price of GBP92.45. The no-arbitrage price of the zero-coupon bond is equal to GBP93.4955:  $GBP100/(1.0342)^2$ . In order to earn a riskless arbitrage profit, we can take the following steps, as shown in Exhibit 6.

**Exhibit 6: Asset with Known Future Price Does Not Trade at Its Present Value**

At time  $t = 0$ :

- Borrow GBP92.45 at the risk-free rate of 3.42%.
- Purchase the two-year zero-coupon note for GBP92.45.

At time  $t = T$  (in two years):

- Receive GBP100 on the zero-coupon bond maturity date.
- Repay loan of GBP98.88— $\text{GBP}92.45 \times (1.0342)^2$ —and earn a riskless arbitrage profit of GBP1.12.

**Forward Rates**

As in the case of other underlying assets, the forward market for interest rates involves a delivery date beyond the usual cash market settlement date. Given the term structures of different interest rates for different maturities, an interest rate forward contract specifies both the *length* of the forward period and the tenor of the underlying rate.

For example, a two-year forward contract that references a three-year underlying interest rate (which starts at the end of year two and matures at the end of year five) is referred to as a “2y3y” forward rate, which we will denote as  $F_{2,3}$ . Short-term market reference rates (MRRs) usually reference a forward rate in months—for instance,  $F_{3m,6m}$  references a six-month MRR that begins in three months and matures nine months from today.

The breakeven reinvestment rate linking a short-dated and a long-dated zero-coupon bond is an **implied forward rate (IFR)**. That is, the implied forward rate is the interest rate for a period in the future at which an investor earns the same return from:

1. investing for a period from today until the forward start date and rolling over the proceeds at the implied forward rate, or
2. investing today through the final maturity of the forward rate.

The fact that these strategies have equal returns establishes the no-arbitrage condition for the implied forward rate. The following example demonstrates how spot rates can be used to derive forward rates.

**EXAMPLE 7****Implied Forward Rate ( $IFR_{1,1}$ )**

We return to the earlier example of three most recently issued annual fixed-coupon government bonds, with coupons and prices as well as yields-to-maturity and zero or spot rates as follows:

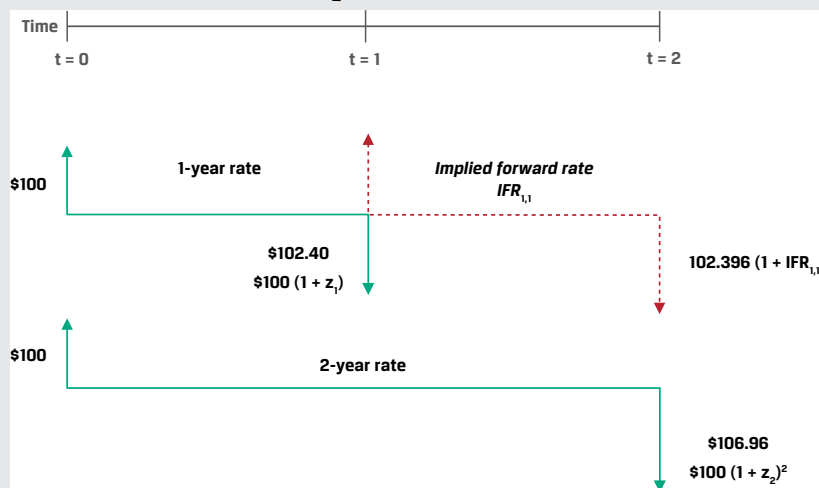
Years to Maturity	Annual Coupon	PV (per 100 FV)	YTM
1	1.50%	99.125	2.3960%
2	2.50%	98.275	3.4068%
3	3.25%	98.000	3.9703%

Years to Maturity	Zero Rate
1	2.3960%
2	3.4197%
3	4.0005%

Using the one-year and two-year zero rates from the prior example, an investor faces the following investment choices over a two-year period:

1. Invest USD100 for one year today at the zero rate ( $z_1$ ) of 2.396% and reinvest the proceeds of USD102.40 at the one-year rate in one year's time, or the "1y1y" implied forward rate ( $IFR_{1,1}$ ).
2. Invest USD100 for two years at the two-year zero rate ( $z_2$ ) of 3.4197% to receive  $USD100(1 + z_2)^2$ , or USD106.96.



In order to arrive at the same return for investment choices 1.) and 2.), we set them equal to each other and solve for  $IFR_{1,1}$  as follows:

$$USD100 \times (1 + z_1) \times (1 + IFR_{1,1}) = USD100 \times (1 + z_2)^2$$

$$USD100 \times (1.02396) \times (1 + IFR_{1,1}) = USD100 \times (1.034197)^2$$

$$IFR_{1,1} = 4.4536\%$$

We can demonstrate that the  $IFR_{1,1}$  of 4.4536% creates identical returns for the first and second strategies by calculating the return on the first strategy as follows:

$$USD100 \times (1.02396) \times (1.044536) = USD106.96$$

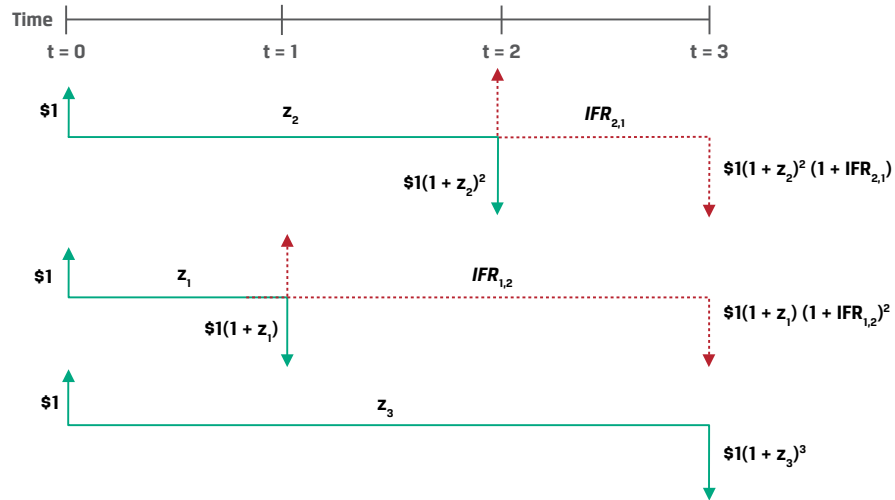
In general, assume a shorter-term bond matures in  $A$  periods and a longer-term bond matures in  $B$  periods. The yields-to-maturity per period on these bonds are denoted as  $z_A$  and  $z_B$ . The first bond is an  $A$ -period zero-coupon bond trading in the cash market. The second is a  $B$ -period zero-coupon cash market bond. The implied forward rate between period  $A$  and period  $B$  is denoted as  $IFR_{A,B-A}$ . It is a forward rate on a bond that starts in period  $A$  and ends in period  $B$ . Its tenor is  $B - A$  periods.

Equation 11 is a general formula for the relationship between the two spot rates ( $z_A, z_B$ ) and the implied forward rate ( $IFR_{A,B-A}$ ):

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B \quad (11)$$

Exhibit 7 shows the possible implied forward rates over three periods.

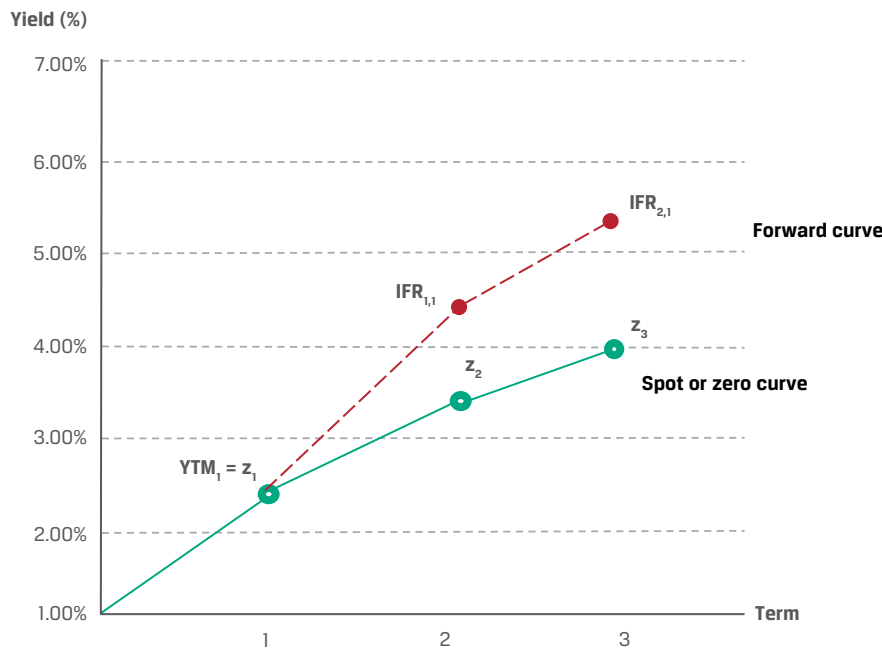
**Exhibit 7: Implied Forward Rate Example**



Using Equation 11, we can solve for the one-period rate in two periods ( $IFR_{2,1}$ ) as follows:

$$\begin{aligned} (1 + z_2)^2 \times (1 + IFR_{2,1})^1 &= (1 + z_3)^3 \\ (1.034197)^2 \times (1 + IFR_{2,1}) &= (1.040005)^3 \\ IFR_{2,1} &= 5.1719\% \end{aligned}$$

A series of forward rates can be used to construct a forward curve of rates with the same time frame that are implied by cash market transactions or may be observed in the interest rate derivatives market. Exhibit 8 summarizes the relationship between spot or zero rates and the forward curve for one-year rates from the earlier government bond example.

**Exhibit 8: Interest Rate Spot or Zero Curve and Forward Curve**

While our examples so far have focused on government benchmark rates, we now turn our attention to short-term market reference rates. Assume, for example, that today we observe a current three-month MRR of 1.25% and a current six-month MRR of 1.75%. How can we solve for the three-month implied forward MRR in three months' time ( $IFR_{3m, 3m}$ )?

In order to apply Equation 11 in this case, we must first ensure that the time frames, or periodicities, of the interest rates are the same. Here, the six-month rate has two periods per year and the three-month rate has four. Recall from an earlier fixed-income lesson that we can convert an annual percentage rate for  $m$  periods per year, denoted as  $APR_m$ , into an annual percentage rate for  $n$  periods per year,  $APR_n$ , as follows:

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n \quad (12)$$

First, we must convert the 1.75% semiannual MRR into a quarterly rate:

$$\begin{aligned} (1 + APR_4/4)^4 &= (1 + 0.0175/2)^2 \\ APR_4 &= 1.74619\% \end{aligned}$$

We can use this to solve Equation 11 for  $IFR_{3m, 3m}$ :

$$\begin{aligned} (1 + z_{3m}/4) \times (1 + IFR_{3m, 3m}/4) &= (1 + z_{6m}/4)^2 \\ (1 + 0.0125/4) \times (1 + IFR_{3m, 3m}/4) &= (1 + 0.0174619/4)^2 \\ IFR_{3m, 3m} &= 2.24299\% \end{aligned}$$

Our result can be confirmed by showing that CNY100,000,000 invested at:

1. 1.25% for three months and reinvested at 2.24299% for the following three months, or
2. 1.75% for six months

will both return CNY100,875,000 in six months:

1. CNY100,875,000 ( $= \text{CNY}100,000,000 \times (1 + 0.0125/4) \times (1 + 0.0224299/4)$ )
2. CNY100,875,000 ( $= \text{CNY}100,000,000 \times (1 + 0.0175/2)$ )

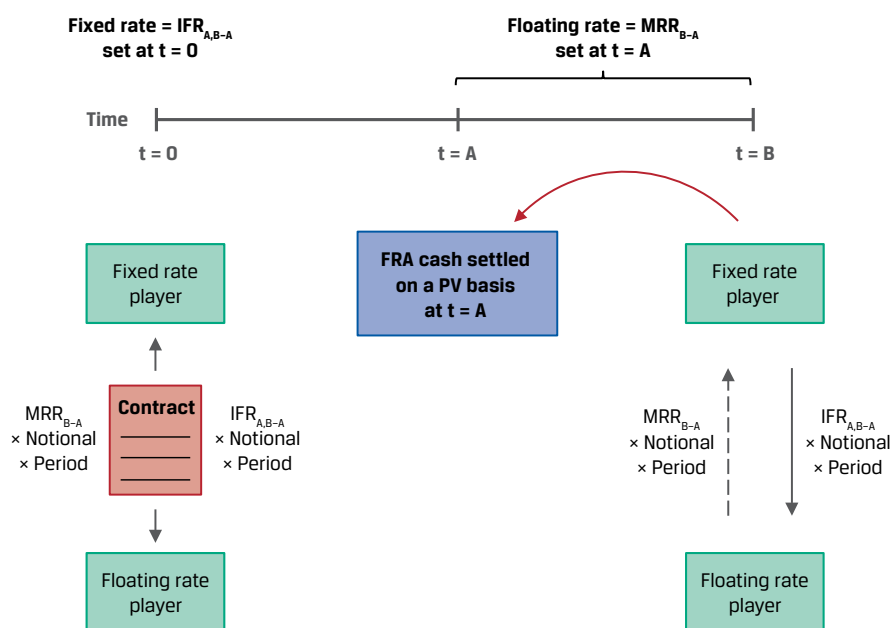
As we will see in the next section, this breakeven reinvestment rate between zero rates of different maturities establishes a no-arbitrage price for a one-period interest rate forward contract.

### Forward Rate Agreements (FRAs)

An OTC derivatives contract in which counterparties agree to apply a specific interest rate to a future period is a **forward rate agreement (FRA)**. The underlying is a hypothetical deposit of a notional amount in the future at a market reference rate that is fixed at contract inception ( $t = 0$ ). The FRA buyer, or long position, agrees to pay the deposit interest based on the agreed upon fixed rate and receives deposit interest based on a market reference rate that begins in  $A$  periods and ends in  $B$  periods (with a tenor of  $B - A$  periods) and is determined on or just before the forward settlement date at time  $t = A$ .

Exhibit 9 shows FRA mechanics at  $t = 0$  and settlement at time  $A$ .

#### Exhibit 9: Forward Rate Agreement (FRA) Mechanics



If Exhibit 9 looks familiar, it is because an FRA is a one-period version of the interest rate swap introduced as a *series* of exchanges in an earlier lesson. Similar to the swap, fixed versus floating payments on an FRA occur on a net basis and the notional is not exchanged but is used solely for interest calculations. The implied forward rates shown in the previous section represent the FRA fixed rate for a given period where no riskless profit opportunities exist. This no-arbitrage interest rate ensures that, similar to other forward contracts, the forward rate agreement has a value of zero ( $V_0(T) = 0$ ) to both parties at inception. FRA settlement and other details are best demonstrated by extending our earlier implied forward rate example.



**EXAMPLE 8****CNY Forward Rate Agreement**

In a prior example, we solved for a three-month implied forward MRR in three months' time ( $IFR_{3m,3m}$ ) of 2.24299%. Yangzi Bank enters into an agreement to pay the three-month rate in three months' time and receive MRR as in Exhibit 9. Yangzi Bank uses the FRA to offset or hedge an underlying liability in three months on which it will owe MRR.

Consider the following FRA term sheet:

**Yangzi Bank CNY Forward Rate Agreement Term Sheet**

<b>Start Date:</b>	[Today]
<b>Maturity Date:</b>	[Three months from Start Date]
<b>Notional Principal:</b>	CNY 100,000,000
<b>Fixed-Rate Payer:</b>	Yangzi Bank
<b>Fixed Rate:</b>	2.24299% on a quarterly actual/360 basis
<b>Floating-Rate Payer:</b>	[Financial Intermediary]
<b>Floating Rate:</b>	Three-month CNY MRR on a quarterly actual/360 basis
<b>Payment Date:</b>	Maturity Date
<b>Business Days:</b>	Shanghai
<b>Documentation:</b>	ISDA Agreement and credit terms acceptable to both parties

Similar to forward agreements for other underlying assets, the FRA settlement amount is a function of the difference between  $F_0(T)$  (here, a forward interest rate,  $IFR_{A,B-A}$ ) and  $S_T$  (the market reference rate  $MRR_{B-A}$ , or the MRR for  $B - A$  periods, which ends at time  $B$ ). In our example,  $MRR_{3m}$  is the three-month market reference rate, which sets in three months' time. FRAs are usually cash settled at the beginning of the period during which the reference rate applies. We calculate the net payment amount from the perspective of Yangzi Bank (the FRA buyer or fixed-rate payer) at the *end* of the interest period as follows:

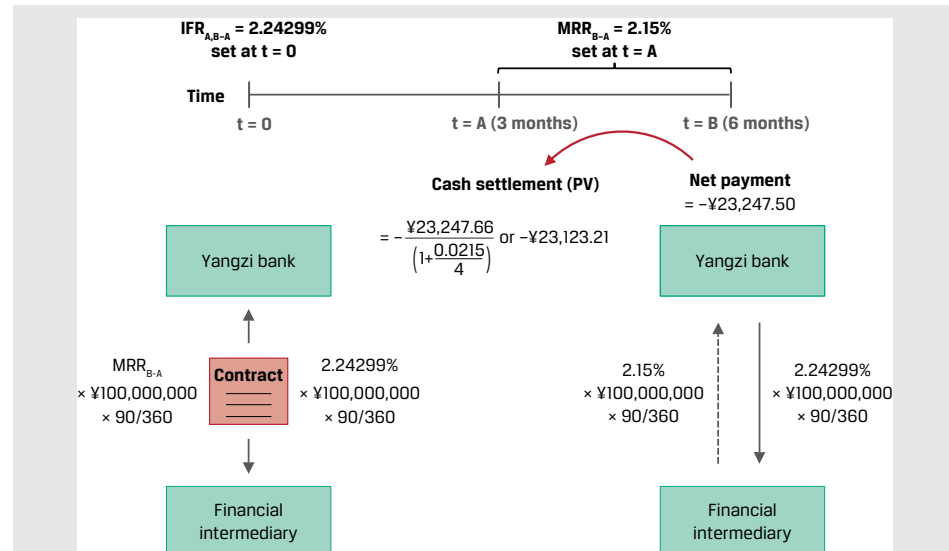
$$\text{Net Payment} = (MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional Principal} \times \text{Period}$$

If  $MRR_{B-A}$  in three months' time sets at 2.15% and we assume a 90-day interest period, we calculate the net payment at the end of the period as follows:

$$\begin{aligned} \text{Net Payment} &= (MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional Principal} \times \text{Period} \\ &= (2.15\% - 2.24299\%) \times \text{CNY}100,000,000 \times 90/360 \\ &= -\text{CNY}23,247.50 \end{aligned}$$

This is the net payment amount at the end of six months. However, the FRA settles at the *beginning* rather than the *end* of the interest period, which is three months in our example. We must therefore calculate the *present value* of the settlement amount to the beginning of the period during which the reference rate applies, using  $MRR_{B-A}$  as the discount rate:

$$\text{Cash Settlement (PV): } -\text{CNY}23,123.21 = -\text{CNY}23,247.50 / (1 + 0.0215/4)$$



In this case, since  $MRR_{B-A}$  sets below the fixed rate, as the fixed-rate payer and floating-rate ( $MRR_{B-A}$ ) receiver, Yangzi Bank must make a net settlement payment to the financial intermediary. Because Yangzi Bank has used the FRA to offset or hedge an underlying liability on which it owes MRR, the net settlement payment it makes on the FRA is offset by a lower MRR payment on its liability.

Forward rate agreements are almost exclusively used by financial intermediaries to manage rate-sensitive assets or liabilities on their balance sheets. The forward rate agreement and its single-period swap equivalent that settles at the end of an interest period form the basic building blocks for interest rate swaps, which are more frequently used by issuers and investors to manage interest rate risk.

### QUESTION SET



### Interest Rate Forward Contracts

1. Determine the correct answers to complete the following sentences: The yield-to-maturity rate and the zero rate for a bond with a single cash flow at maturity in one period are \_\_\_\_\_. The *price* equivalent of a zero rate is the present value of a currency unit on a future date, known as a \_\_\_\_\_.

#### Solution:

The yield-to-maturity rate and the zero rate for a bond with a single cash flow at maturity in one period are *identical*. The *price* equivalent of a zero rate is the present value of a currency unit on a future date, known as a *discount factor*.

2. An analyst observes three- and four-year government benchmark zero-coupon bonds priced at 93 and 90 per 100 face value, respectively. Calculate  $IFR_{3,1}$  (the implied one-year forward rate in three years' time).

#### Solution:

First, use the three-year ( $DF_3$ ) and four-year ( $DF_4$ ) discount factors provided per unit of currency to derive three-year ( $z_3$ ) and four-year ( $z_4$ ) zero rates, which can be calculated as follows:

$$DF_i = \frac{1}{(1+z_i)^i}$$

Therefore:

$$0.93 = 1/(1+z_3)^3; z_3 = 2.4485\%$$

$$0.90 = 1/(1+z_4)^4; z_4 = 2.6690\%$$

Solve for  $IFR_{3,1}$  as follows:

$$(1+z_3)^3 \times (1+IFR_{3,1}) = (1+z_4)^4$$

$$(1.024485)^3 \times (1+IFR_{3,1}) = (1.02669)^4$$

$$IFR_{3,1} = 3.3333\%$$

3. Match the following descriptions with their corresponding interest rate derivative building block.

A. Forward rate agreement	1. The breakeven reinvestment rate linking a short-dated and long-dated zero-coupon bond
B. Implied forward rate	2. The present value of a currency unit on a future date
C. Discount factor	3. A derivative in which counterparties agree to apply a specific interest rate to a future time period

**Solution:**

1. B is correct. The breakeven reinvestment rate linking a short-dated and a long-dated zero-coupon bond is the implied forward rate.
2. C is correct. The price equivalent of a zero rate is the present value of a currency unit on a future date, known as a discount factor.
3. A is correct. A forward rate agreement is a derivative in which counterparties agree to apply a specific interest rate to a future period.

4. A trader observes one-year and two-year zero-coupon bonds that yield 4% and 5%, respectively, and would like to protect herself against a rise in one-year rates a year from now. Explain the position she should take in the FRA contract to achieve this objective and the forward interest rate she should expect on the contract.

**Solution:**

A forward rate agreement involves an underlying hypothetical future deposit at a market reference rate fixed at contract inception ( $t = 0$ ). To protect against higher rates, the trader should enter into a fixed-rate payer FRA in order to realize a gain if one-year spot rates one year from now exceed the current forward rate. A long FRA position, or fixed-rate payer, agrees to pay interest based on the fixed rate and receives interest based on a variable market reference rate determined at settlement. The FRA is priced based on the implied forward rate, or breakeven reinvestment rate between a shorter and a longer zero rate. We can solve for  $IFR_{1,1}$  as follows:

$$(1+z_1) \times (1+IFR_{1,1}) = (1+z_2)^2$$

$$(1.04) \times (1+IFR_{1,1}) = (1.05)^2$$

$$IFR_{1,1} = 6.0096\%$$

5. A counterparty agrees to be the FRA fixed-rate receiver on a one-month AUD MRR in three months' time based on a AUD150,000,000 notional amount. If  $IFR_{3m,1m}$  at contract inception is 0.50% and one-month AUD

MRR sets at 0.35% for settlement of the contract, calculate the settlement amount and interpret the results.

**Solution:**

A short FRA position, or fixed-rate receiver, agrees to pay interest based on a market reference rate determined at settlement and receives interest based on a pre-agreed fixed rate, the implied forward rate ( $IFR_{3m,1m}$ ). Since AUD  $MRR_{B-A}$  at settlement is below the pre-agreed fixed rate, the FRA fixed-rate receiver realizes a gain and receives a net payment based on the following calculation:

$$\begin{aligned}\text{Net Payment} &= (IFR_{A,B-A} - MRR_{B-A}) \times \text{Notional Principal} \times \text{Period} \\ &= (0.50\% - 0.35\%) \times \text{AUD}150,000,000 \times (1/12) \\ &= \text{AUD}18,750 \text{ at the end of the period}\end{aligned}$$

Solve for the present value of settlement given the contract is settled at the beginning of the period, using  $MRR_{B-A}$  as the discount rate:

$$\begin{aligned}\text{Cash Settlement (PV)} &= \text{AUD}18,750.00 / (1 + 0.35\%/12) \\ &= \text{AUD}18,744.53\end{aligned}$$

## PRACTICE PROBLEMS

### The following information relates to questions 1-6

Baywhite Financial is a broker-dealer and wealth management firm that helps its clients manage their portfolios using stand-alone derivative strategies. A new Baywhite analyst is asked to evaluate the following client situations.

1. Match the following definitions with their corresponding forward pricing or valuation component.

1. Equal to the difference between the current spot price (adjusted by remaining costs and benefits through maturity) and the present value of the forward price	A. Forward contract MTM value at inception, $V_0(T)$
2. Future value of the underlying asset spot price ( $S_0$ ) compounded at the risk-free rate incorporating the present value of the costs and benefits of asset ownership	B. Forward contract MTM value between inception and maturity, $V_t(T)$
3. Under no-arbitrage conditions for a given underlying spot price, $S_0$ , adjusted by the costs and benefits, risk-free rate ( $r$ ), and forward price, $F_0(T)$ , this should be equal to zero (ignoring transaction costs).	C. Forward price, $F_0(T)$

2. A Baywhite client currently owns 5,000 common non-dividend-paying shares of Vivivyu Inc. (VIVU), a digital media company, at a spot price of USD173 per share. The client enters into a forward commitment to sell half of its VIVU position in six months at a price of USD175.58. Which of the following market events is most likely to result in the greatest gain in the VIVU forward contract MTM value from the client's perspective?
  - A. An increase in the risk-free rate
  - B. An immediate decline in the VIVU spot price following contract inception
  - C. A steady rise in the spot price of VIVU stock over time
3. A Baywhite client has entered into a long six-month MXN/USD FX forward contract—that is, an agreement to sell MXN and buy USD. The MXN/USD spot exchange rate at inception is 19.8248 (MXN19.8248 = USD1), the six-month MXN risk-free rate is 4.25%, and the six-month USD risk-free rate is 0.5%. Baywhite's market strategist predicts that the Mexican central bank (Banco de Mexico) will surprise the market with a 50 bp short-term rate cut at its upcoming meeting. Which of the following statements best describes how the client's existing FX forward contract will be impacted if this prediction is realized and other parameters remain unchanged?
  - A. The lower interest rate differential between MXN and USD will cause the MXN/USD contract forward rate to be adjusted downward.

- B. The client will realize an MTM gain on the FX forward contract due to the decline in the MXN versus USD interest rate differential.
  - C. The lower interest rate differential between MXN and USD will cause the client to realize an MTM loss on the MXN/USD forward contract.
- 4. A client seeking advice on her fixed-income portfolio observes the price and yield-to-maturity of one-year ( $r_1$ ) and two-year ( $r_2$ ) annual coupon government benchmark bonds currently available in the market. Which of the following statements best describes how the analyst can determine a breakeven reinvestment rate in one year's time to help decide whether to invest now for one or two years?
  - A. As the two-year rate involves intermediate cash flows, divide the square root of  $(1 + r_2)$  by  $(1 + r_1)$  and subtract 1 to arrive at a breakeven reinvestment rate for one year in one year's time.
  - B. Since the first year's returns are compounded in the second year, set  $(1 + r_1)$  multiplied by 1 plus the breakeven reinvestment rate equal to  $(1 + r_2)^2$  and solve for the breakeven reinvestment rate.
  - C. Since the breakeven reinvestment involves a zero-coupon cash flow, first substitute the one-year rate ( $r_1$ ) into the two-year bond price equation to solve for the two-year spot or zero rate ( $z_2$ ), then set  $(1 + r_1) \times (1 + \text{breakeven reinvestment rate}) = (1 + z_2)^2$  and solve for the breakeven reinvestment rate.
- 5. Baywhite Financial seeks to gain a competitive advantage by making margin loans at fixed rates for up to 60 days to its investor clients. Since Baywhite borrows at a variable one-month market reference rate to finance these client loans, the firm enters into one-month FRA contracts on one-month MRR to hedge the interest rate exposure of its margin loan book. Which of the following statements best describes Baywhite's interest rate exposure and the FRA position it should take to hedge that exposure?
  - A. Baywhite faces exposure to a *rise* in one-month MRR over the next 30 days, so it should enter into the FRA as a fixed-rate *payer* in order to benefit from a rise in one-month MRR above the FRA rate and offset its higher borrowing cost.
  - B. Baywhite faces exposure to a *rise* in one-month MRR over the next 30 days, so it should enter into the FRA as a fixed-rate *receiver* in order to benefit from a rise in one-month MRR above the FRA rate and offset its higher borrowing cost.
  - C. Baywhite faces exposure to a *decline* in one-month MRR over the next 30 days, so it should enter into the FRA as a fixed-rate *receiver* in order to benefit from a rise in one-month MRR above the FRA rate and offset its higher borrowing cost.
- 6. Baywhite observes that one-month MRR is 1.2% and two-month MRR is 1.5%. Which of the following rates is closest to the forward rate that Baywhite would expect on 1m1m forward rate agreement?
  - A. 1.80%
  - B. 1.35%

**C.** 3.55%

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## SOLUTIONS

1. B is correct. The forward contract MTM value between inception and maturity,  $V_t(T)$ , is equal to the difference between the current spot price (adjusted by costs and benefits through maturity) and the present value of the forward price.  
2. C is correct. The forward price,  $F_0(T)$ , is the future value of the underlying asset spot price ( $S_0$ ) compounded at the risk-free rate incorporating the present value of the costs and benefits of asset ownership.  
3. A is correct. Under no-arbitrage conditions for a given underlying spot price,  $S_0$ , adjusted by the costs and benefits, risk-free rate ( $r$ ), and forward price,  $F_0(T)$ , the forward contract MTM value at inception,  $V_0(T)$ , should be equal to zero (ignoring transaction costs).
2. B is correct. The original VIVU spot price ( $S_0$ ) at  $t = 0$  must equal the present value of the forward price discounted at the risk-free rate, so an immediate fall in the spot price to  $S_0^- < S_0$  results in an MTM gain for the forward contract seller. A is not correct, since a higher risk-free rate will reduce the contract MTM from the client's perspective by reducing the PV of  $F_0(T)$ , while C will also reduce the forward contract MTM from the seller's perspective.
3. C is correct. A decline in the interest rate differential between MXN and USD will cause the client to realize an MTM loss on the MXN/USD forward contract, while B states that this decline will result in an MTM gain. A is incorrect as the forward price,  $F_0(T)$ , is not adjusted during the contract life.  
Specifically, the original MXN/USD forward exchange rate at inception is equal to 20.20 ( $= 19.8248e^{(.0425 - 0.005) \times 0.5}$ ). If the MXN rate were to decline by 50 bps immediately after the contract is agreed, a new MXN/USD forward contract would be at a forward exchange rate of 20.15 ( $= 19.8248e^{(.0375 - 0.005) \times 0.5}$ ). The MXN would weaken or depreciate against the USD. Since the MXN seller has locked in a forward sale at the original 20.20 versus the new 20.15 rate, the seller's MTM loss is equal to 0.05, or MXN50,000 per MXN1,000,000 ( $= 0.05 \times 1,000,000$ ) notional amount.
4. C is correct. The one-year annual rate equals the one-year zero rate, as it involves a single cash flow at maturity ( $z_1 = r_1$ ). Since the breakeven reinvestment rate involves a single cash flow at maturity, substitute the one-year rate ( $r_1$ ) into the two-year bond price equation to solve for  $z_2$ , then set  $(1 + r_1) \times (1 + \text{breakeven reinvestment rate}) = (1 + z_2)^2$  and solve for the breakeven reinvestment rate ( $IFR_{1,1}$ ).
5. A is correct. As Baywhite faces exposure to a rise in one-month MRR over the next 30 days, it should enter into the FRA as a fixed-rate payer in order to benefit from a rise in one-month MRR above the FRA rate and offset its higher borrowing cost. Both B and C are incorrect, as the fixed-rate *receiver* in an FRA does not benefit but rather must make a higher payment upon settlement if MRR rises.
6. A is correct. The APR of the monthly compounded two-month rate is 1.499%. Dividing  $(1.01499/12)^2$  by  $(1.012/12)$  equals 1.001499. Subtracting 1 and then multiplying by 12 gives 1.7982%. Thus, the approximate forward rate is 1.80%. B is incorrect because this is a simple average of the two spot rates. C is incorrect because this result is derived from simply dividing  $(1.01499/12)$  by  $(1.012/12)$ , then subtracting 1, and then multiplying by 12.



## LEARNING MODULE

## 6

## Pricing and Valuation of Futures Contracts

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	compare the value and price of forward and futures contracts
<input type="checkbox"/>	explain why forward and futures prices differ

## INTRODUCTION

1

Many of the pricing and valuation principles associated with forward commitments are common to both forward and futures contracts. For example, previous lessons demonstrated that forward commitments have a price that prevents market participants from earning riskless profit through arbitrage. It was also shown that long and short forward commitments may be replicated using a combination of long or short cash positions and borrowing or lending at the risk-free rate. Finally, both forward and futures pricing and valuation incorporate the cost of carry, or the benefits and costs of owning an underlying asset over the life of a derivative contract.

We now turn our attention to futures contracts. We discuss what distinguishes them from other forward commitments and how they are used by issuers and investors. We expand upon the daily settlement of futures contract gains and losses introduced earlier and explain the differences between forwards and futures. We also address and distinguish the interest rate futures market and its role in interest rate derivative contracts.

## LEARNING MODULE OVERVIEW



- Futures are standardized, exchange-traded derivatives (ETDs) with zero initial value and a futures price  $f_0(T)$  established at inception. The futures price,  $f_0(T)$ , equals the spot price compounded at the risk-free rate as in the case of a forward contract.
- The primary difference between forward and futures valuation is the daily settlement of futures gains and losses via a margin account. Daily settlement resets the futures contract value to zero at the current futures price  $f_t(T)$ . This process continues until contract maturity and the futures price converge to the spot price,  $S_T$ .

CFA Institute would like to thank Don Chance, PhD, CFA, for his contribution to this section, which includes material derived from material that appeared in *Derivative Markets and Instruments*, featured in the 2022 CFA® Program curriculum.

- The cumulative realized mark-to-market (MTM) gain or loss on a futures contract is approximately the same as for a comparable forward contract.
- Daily settlement and margin requirements give rise to different cash flow patterns between futures and forwards, resulting in a pricing difference between the two contract types. The difference depends on both interest rate volatility and the correlation between interest rates and futures prices.
- The futures price for short-term interest rate futures is given by  $(100 - \text{yield})$ , where yield is expressed in percentage terms. There is a price difference between interest rate futures and forward rate agreements (FRAs) due to convexity bias.
- The emergence of derivatives central clearing has introduced futures-like margining requirements for over-the-counter (OTC) derivatives, such as forwards. This arrangement has reduced the difference in the cash flow impact of ETDs and OTC derivatives and the price difference in futures versus forwards.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following responses is closest to the one-year futures price of a stock with a spot price ( $S_0$ ) of €125 and an annual dividend of €2.50 paid at maturity if the risk-free rate is 1%?

- A. €123.75
- B. €122.50
- C. €126.25

**Solution:**

A is correct. The no arbitrage futures price for an underlying asset with known benefits, such as a dividend, may be determined using the following equation:

$$f_0(T) = [S_0 - PV_0(I)] (1 + r)^T.$$

First, solve for the present value of the dividend  $PV_0(I)$  as follows:

$$€2.48 = (€2.50 / 1.01).$$

Substitute  $PV_0(I)$  into the original equation to solve for  $f_0(T)$ :

$$f_0(T) = €123.75 = (€125 - €2.48)(1.01).$$

2. Which of the following statements regarding the gains or losses of a long forward contract position compared to a long futures contract position is most correct? Assume that the underlying is identical on both contracts and that both contracts have the same time until maturity.
  - A. The daily realized gain or loss of the forward contract position and the futures contract position are equivalent.
  - B. Before the contracts mature, the cumulative realized gains or losses of the forward contract position and the futures contract position are equivalent.

- C. At contract maturity, the cumulative realized gains or losses of the forward contract position and the futures contract position are approximately equivalent.

**Solution:**

C is correct. The two contracts are similar in all respects except for the frequency with which contracts are marked to market. As a result, the cumulative gain or loss is approximately the same when the contracts mature. A is incorrect because the futures contract's daily mark-to-market (MTM) feature creates daily realized gains or losses while the forward contract's gains or losses are realized only at contract maturity. B is incorrect because the response refers to realized gains or losses, and the contracts have not yet matured. Thus, the forward contract has generated no realized gains or losses yet.

3. Identify which of the following situations leads to which relationship between forward and futures prices for forward commitment contracts with otherwise identical terms.

- |   |   |
|---|---|
| 1. Futures prices are <i>positively</i> correlated with interest rates, and interest rates change over the contract period. | A. Forward prices are above futures prices: $F_0(T) > f_0(T)$ . |
| 2. Futures prices are <i>negatively</i> correlated with interest rates, and interest rates change over the contract period. | B. Forward and futures prices are the same: $F_0(T) = f_0(T)$ . |
| 3. Interest rates are constant over the forward commitment contract period.   | C. Futures prices are above forward prices: $f_0(T) > F_0(T)$ . |

**Solution:**

1. C is correct. If futures prices are positively correlated with interest rates, then higher prices lead to futures profits reinvested at rising rates, and lower prices lead to losses that may be financed at lower rates.
2. A is correct. If futures prices are negatively correlated with interest rates, then higher prices lead to futures profits reinvested at lower rates, and lower prices lead to losses that must be financed at higher rates.
3. B is correct. If interest rates are constant over the forward commitment contract period, then forward and futures prices are the same.

4. Which of the following statements most correctly describes a development that has helped reduce the difference in the cash flow impact between forward and futures contracts?

- A. Futures exchanges have moved away from daily mark-to-market recognition of gains and losses on futures contracts.
- B. OTC derivatives have become increasingly subject to central clearing requirements.
- C. Lower volatility in markets has reduced the magnitude of gains and losses in both types of contracts.

**Solution:**

B is correct. Under a central clearing framework for OTC derivatives, financial intermediaries that serve as counterparties are required to post daily margin or eligible collateral to the central counterparty (CCP) in a process very similar to futures margining. Dealers, therefore, often impose similar requirements on derivatives end users. A is incorrect because no such

change has occurred in exchange-traded futures markets. C is incorrect because volatility changes over time are hard to categorize as higher or lower.

## 2

## PRICING OF FUTURES CONTRACTS AT INCEPTION



compare the value and price of forward and futures contracts

When a forward commitment is initiated, no cash is exchanged and the contract is neither an asset nor a liability to the buyer or seller. The value of both a forward contract and a futures contract at initiation is zero:

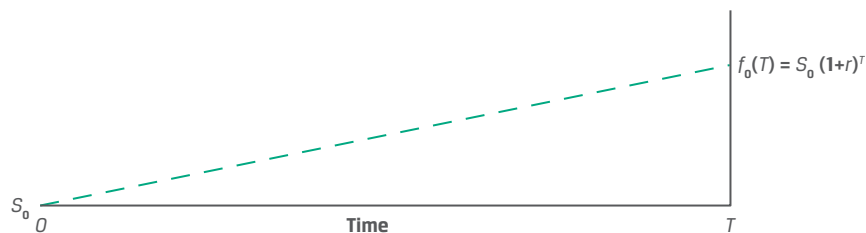
$$V_0(T) = 0.$$

An underlying asset with no cost or benefit has a futures price  $f_0(T)$  at  $t = 0$  of:

$$f_0(T) = S_0(1 + r)^T,$$

where  $r$  is the risk-free rate and  $T$  is the time to maturity. As in the case of a forward contract, the futures price is the spot price compounded at the risk-free rate over the life of the contract. This is shown in Exhibit 1, where the slope of the line is equal to the risk-free rate,  $r$ .

**Exhibit 1: Futures Price at Initiation**



As for forwards, we use discrete compounding as in Equation 2 for futures on individual underlying assets. However, for underlying assets that are comprised of a portfolio—such as an equity, fixed-income, commodity, or credit index—or where the underlying involves foreign exchange with interest rates denominated in two currencies, continuous compounding is the preferred method, as shown in Equation 3:

$$f_0(T) = S_0 e^{rT}.$$

### EXAMPLE 1

#### Procam Investments - Gold Futures Contract

As shown in a previous lesson, Procam Investments purchases a 100-ounce gold futures contract. The current spot price is \$1,770.00 per ounce, the risk-free rate is 2.0%, and we assume gold may be stored at no cost. Calculate the no arbitrage futures price,  $f_0(T)$ , for settlement in 91 days ( $T = 91/365$  or 0.24932).

**Solution**

Using Equation 2:  $f_0(T) = S_0(1 + r)^T$

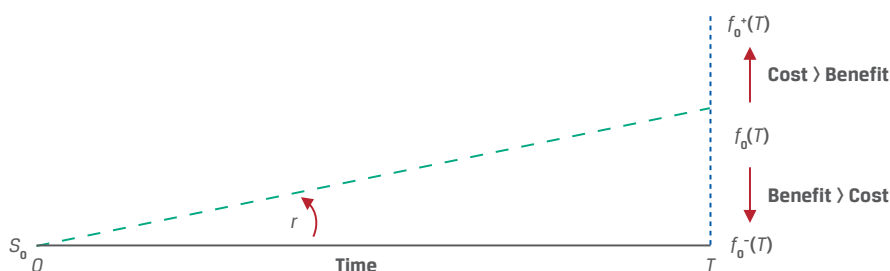
$$\text{\$1,778.76 per ounce} = \text{\$1,770.00} \times (1.02)^{0.24932}$$

$$\text{Contract price} = \text{\$177,876.04} (= 100 \times \text{\$1,778.76})$$

The futures price,  $f_0(T)$ , is identical to the forward price from a previous lesson.

As in the case of a forward, for underlying assets with ownership benefits or income ( $I$ ) or costs ( $C$ ) expressed as a known amount in present value terms at time  $t = 0$ , the spot versus futures price relationship using discrete compounding is shown in Equation 4 and Exhibit 2:

$$f_0(T) = [S_0 - \text{PV}_0(I) + \text{PV}_0(C)] (1 + r)^T.$$

**Exhibit 2: Futures Prices with Underlying Asset Costs and Benefits****EXAMPLE 2****Procam's Gold Futures Contract with Storage Costs**

Procam purchased a gold futures contract in Example 1 at  $f_0(T)$  of \$177,876.04 (or \$1,778.76 per ounce) with  $S_0$  equal to \$1,770 per ounce. How would  $f_0(T)$  change to satisfy the no arbitrage condition if a \$2 per ounce cost of gold storage and insurance were payable at the end of the contract?

The futures price for a commodity with known storage cost amounts may be determined using Equation 4, where  $\text{PV}_0(I) = 0$ :

$$f_0(T) = [S_0 + \text{PV}_0(C)] (1 + r)^T.$$

First, solve for the present value of the storage cost per ounce,  $\text{PV}_0(C)$ , as follows:

$$\text{PV}_0(C) = \$1.99 = [\text{\$2}(1.02)^{-0.24982}].$$

Substitute  $\text{PV}_0(C) = \$1.99$  into Equation 4 to solve for  $f_0(T)$ :

$$f_0(T) = (\text{\$1,770.00} + \$1.99)(1.02)^{-0.24982}$$

$$= \text{\$1,780.78 per ounce}$$

The addition of storage and insurance costs increases the difference between the spot price and futures price. Finally, as in the case of forwards, a futures price,  $f_0(T)$ , significantly *below* the no arbitrage price including cash costs and benefits may indicate the presence of a convenience yield.

## 3

## MTM VALUATION: FORWARDS VERSUS FUTURES

- | compare the value and price of forward and futures contracts

Examples 1 and 2 show the similarities between forward and futures prices at contract inception. Over time, different forward and futures contract features lead to different MTM values for contracts with the same underlying assets and otherwise identical details. Example 3 shows how the daily settlement of gains and losses causes this difference to arise.

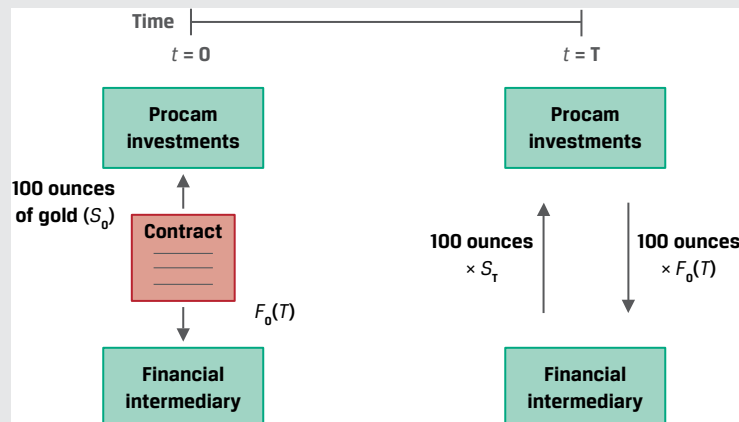
## EXAMPLE 3

## Procam Forward versus Futures Pricing and Valuation

We extend the earlier example to compare forward and futures pricing and valuation. In both cases, Procam Investments enters a cash-settled forward commitment to buy 100 ounces of gold at a price ( $f_0[T] = F_0[T]$ ) of \$1,778.76 per ounce in 91 days, with a risk-free rate of 2% and no gold storage cost.

## Forward Contract

The forward price is  $F_0(T) = \$1,778.76$  per ounce. No cash is exchanged or deposited at inception, and the contract value at inception,  $V_0(T)$ , is zero.



Over time, the forward price,  $F_0(T)$ , does not change, and the MTM at any time,  $[V_t(T)]$ , equals the difference between the current spot price,  $S_t$ , and the present value of the forward price,  $PV_t$  of  $F_0(T)$ , shown from Procams' (the forward buyer's) perspective:

$$V_t(T) = S_t - F_0(T)(1 + r)^{-(T-t)}.$$

For example, say 71 days have elapsed and 20 days remain to maturity,  $T - t = 20/365$  or 0.0548. If the gold spot price, ( $S_t$ ), has fallen by \$50 since inception to \$1,720 per ounce, solve for  $V_t(T)$  as follows:

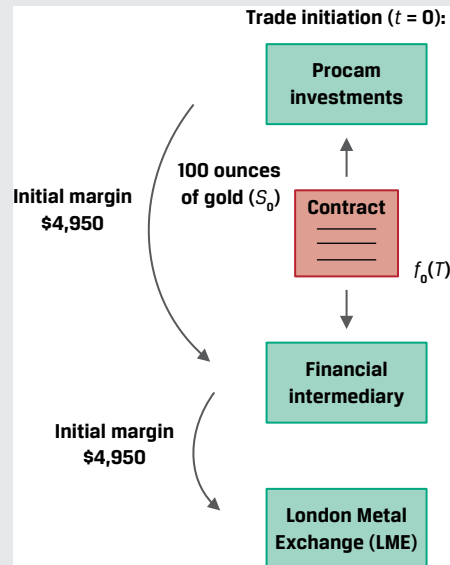
$$V_t(T) = \$1,720 - \$1,778.76(1.02)^{-0.0548}$$

$$= -\$56.83 \text{ per ounce, or a } \$5,683 \text{ MTM loss } (= -\$56.83 \times 100 \text{ ounces}).$$

Under terms of the forward contract, no settlement of the MTM amount occurs until maturity. This process of resetting the contract value to zero each day makes it very unlikely that the futures contract would reach a similar MTM value.

## Futures Contract

The futures price,  $f_0(T)$ , is \$1,778.76 per ounce. As per futures exchange daily settlement rules, the contract buyer and seller must post an initial cash margin of \$4,950 per gold contract (100 ounces) and maintain a maintenance margin of \$4,500 per contract. If a margin balance falls below \$4,500, a counterparty receives a margin call and must immediately replenish its account to the initial \$4,950.



Consider the first day of trading, where the spot gold price, ( $S_0$ ), is \$1,770 per ounce and the opening gold futures price,  $f_0(T)$ , is \$1,778.76 per ounce.

- Assume that the gold futures price,  $f_1(T)$ , falls by \$5 on the first trading day to \$1,773.76 and the spot price,  $S_1$ , ends the day at a no arbitrage equivalent of \$1,765.12 ( $= \$1,773.76(1.02)^{-90/365}$ ).
- Procam realizes a \$500 MTM loss ( $= \$5$  per ounce  $\times$  100 ounces) deducted from its margin account, leaving Procam with \$4,450.
- The MTM value of Procam's futures contract resets to zero at the futures closing price of \$1,773.76 per ounce.
- Since Procam's margin account balance has fallen below the \$4,500 maintenance level, it must deposit \$500 to return the balance to the \$4,950 initial margin.

## Futures versus Forward Price and Value over Time

Using the same details, we compare the futures and forward price and value over two trading days. Assume that day two trading opens at day one's closing spot and futures prices. The following table shows the comparison:

Beginning of Day 2 Trading				
Contract Type	Contract Price	Contract MTM	Realized MTM	Margin Deposit
Forward	$F_0(T) = \$177,876$	-\$498	\$0	\$0
Futures	$f_1(T) = \$177,376$	\$0	-\$500	\$4,950

The forward MTM contract value,  $V_t(T)$ , equals the difference between the current spot price,  $S_1 = \$1,765.12$ , and the present value of the original forward price,  $PV_t[F_0(T)]$ , here with 90 days remaining to maturity,  $T - t = 90/365$  or 0.24657:

$$V_t(T) = \$1,765.12 - \$1,778.76(1.02)^{-0.24657}$$

= \$4.98 per ounce.

Extending our example to the beginning of day three at the prior day's close, assume a \$4 per ounce fall in the gold futures price on day two,  $f_2(T)$ , to \$1,769.76 and a no arbitrage equivalent spot price decline,  $S_2$ , to \$1,761.24 (= \$1,769.76[1.02]<sup>-89/365</sup>). The following table shows the summary on day three:

#### Beginning of Day 3 Trading

Contract Type	Contract Price	Contract MTM	Realized MTM	Margin Deposit
Forward	$F_0(T) = \$177,876$	-\$895	\$0	\$0
Futures	$f_2(T) = \$176,976$	\$0	-\$400	\$4,550

The forward MTM contract value,  $V_t(T)$ , equals the difference between the current spot price,  $S_2 = \$1,761.24$ , and the present value of the original forward price,  $PV_t[F_0(T)]$ , here with 89 days remaining to maturity,  $T - t = 89/365$  or 0.24384:

$$V_t(T) = \$1,761.24 - \$1,778.76(1.02)^{-0.24384}$$

= \$8.95 per ounce.

Example 3 demonstrates the key differences in the price and value of forward and futures contracts over time. The forward contract price,  $F_0(T)$ , remains fixed until the contract matures. Forward contract MTM value changes are captured by the difference between the current spot price,  $S_t$ , and the present value of the forward price,  $PV_t[F_0(T)]$ . This forward contract MTM is not settled until maturity, giving rise to counterparty credit risk over time since no cash is exchanged from inception of the contract to its maturity or expiration. Futures contract prices fluctuate daily based upon market changes. The daily settlement mechanism resets the futures MTM to zero, and variation margin is exchanged to settle the difference, reducing counterparty credit risk. The *cumulative* realized MTM gain or loss on a futures contract is approximately the same as for a comparable forward contract. We will explore these differences in the next lesson after first turning our attention to forward and futures contracts on market reference rates.

## 4

### INTEREST RATE FUTURES VERSUS FORWARD CONTRACTS



compare the value and price of forward and futures contracts

In an earlier lesson on interest rate forward contracts, zero rates derived from coupon bonds were used to derive a future investment breakeven rate (or implied forward rate). The implied forward rate was shown to equal the no arbitrage fixed rate on a forward rate agreement (FRA) under which counterparties exchange a fixed-for-floating cash flow at a time in the future.

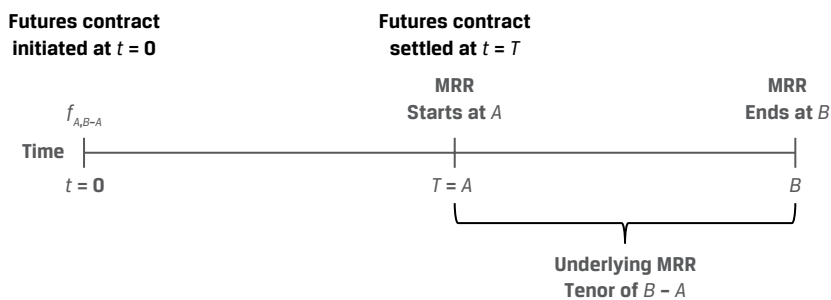


Futures markets on short-term interest rates offer market participants a highly liquid, standardized alternative to FRAs. Interest rate futures contracts are available for monthly or quarterly market reference rates for successive periods out to final contract maturities of up to ten years in the future. Although the underlying variable is the market reference rate (MRR) on a hypothetical deposit on a future date as for forward rate agreements, interest rate futures trade on a *price* basis as per the following general formula:

$$f_{A,B-A} = 100 - (100 \times \text{MRR}_{A,B-A}),$$

where  $f_{A,B-A}$  represents the futures price for a market reference rate for  $B-A$  periods that begins in  $A$  periods ( $\text{MRR}_{A,B-A}$ ), as shown in Exhibit 3.

### Exhibit 3: Interest Rate Futures Contract Mechanics



For example, we may solve for the implied three-month MRR rate in three months' time (where  $A = 3\text{m}$ ,  $B = 6\text{m}$ ,  $B - A = 3\text{m}$ ) if an interest rate futures contract is trading at a price of 98.25 using Equation 5:

$$f_{3\text{m},3\text{m}}: 98.25 = 100 - (100 \times \text{MRR}_{A,B-A})$$

$$\text{MRR}_{3\text{m},3\text{m}} = 1.75\%.$$

This  $(100 - \text{yield})$  price convention results in an inverse price/yield relationship but is *not* the same as the price of a zero-coupon bond at the contract rate. A long futures position involves earning or receiving MRR in  $A$  periods, whereas a short position involves paying MRR in  $A$  periods. The interest rate exposure profile for long and short futures contracts are as follows:

- Long futures contract (lender): Gains as prices rise, future MRR falls
- Short futures contract (borrower): Gains as prices fall, future MRR rises

In an earlier lesson, Yangzi Bank enters into an FRA as a fixed-rate payer to hedge a liability on which it owes MRR in the future, realizing a gain on the FRA contract as rates rise. Note that this would be equivalent to taking a *short* position on a CNY MRR futures contract if one were available. Exhibit 4 summarizes the relationship between futures and FRAs.

**Exhibit 4: Interest Rate Futures versus FRAs**

Contract Type	Gains from Rising MRR	Gains from Falling MRR
Interest rate futures	Short futures contract	Long futures contract
Forward rate agreement	Long FRA: FRA fixed-rate payer (FRA floating-rate receiver)	Short FRA: FRA floating-rate payer (FRA fixed-rate receiver)

Interest rate futures daily settlement occurs based on price changes, which translate into **futures contract basis point value (BPV)** as follows:

$$\text{Futures Contract BPV} = \text{Notional Principal} \times 0.01\% \times \text{Period.}$$

For example, assuming a \$1,000,000 notional for three-month MRR of 2.21% for one quarter (or 90/360 days), the underlying deposit contract value would be:

$$\$1,005,525 = \$1,000,000 \times [1 + (2.21\% / 4)].$$

Consider how a one basis point (0.01%) change in MRR affects contract value:

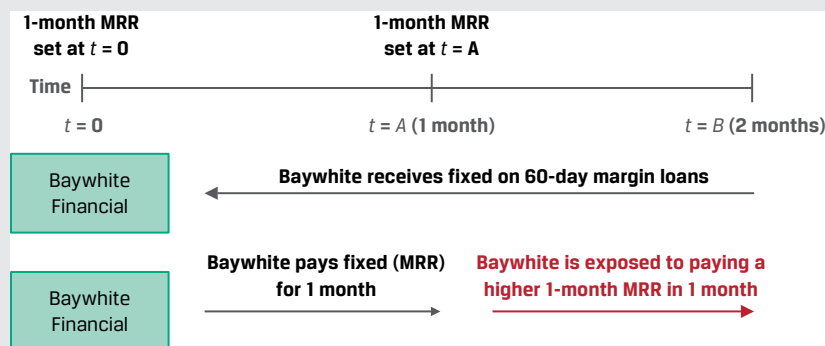
$$1 \text{ bp increase (2.22\%): } \$1,005,550 = \$1,000,000 \times [1 + (2.22\% / 4)].$$

$$1 \text{ bp decrease (2.20\%): } \$1,005,500 = \$1,000,000 \times [1 + (2.20\% / 4)].$$

Both the increase and decrease in MRR by one basis point change the contract BPV by \$25. Short-term interest rate futures are characterized by a fixed linear relationship between price and yield changes. The following example illustrates their use in practice.

**EXAMPLE 4****Interest Rate Futures - Baywhite Margin Loan Book**

In an earlier example, Baywhite Financial offered 60-day margin loans at fixed rates to its clients and borrowed at a variable one-month MRR to finance the loans. Describe Baywhite's residual interest rate exposure and how it may use interest rate futures as a hedge.



The diagram shows that Baywhite faces the risk of higher MRR in one month's time ( $\text{MRR}_{1,1}$ ), which would reduce the return on its fixed margin loans. In the prior example, Baywhite entered an FRA where it agreed to pay fixed one-month MRR and receive floating. If Baywhite were to use an interest rate futures contract instead, it would *sell* a futures contract on one-month MRR. The futures contract BPV for a \$50,000,000 notional amount is:

$$\text{Contract BPV} = \$416.67 (= \$50,000,000 \times 0.01\% \times [1/12]).$$

If Baywhite sells  $f_{1,1}$  for \$98.75 (or  $\text{MRR}_{1,1} = 1.25\%$ ) and settles at maturity at a price of \$97.75 ( $\text{MRR}_{1,1} = 2.25\%$ ), it would expect to have a *cumulative* gain on the contract through maturity equal to \$41,667 (= Contract BPV  $\times$  100 bps).

## QUESTION SET



## Valuation and Pricing of Futures Contracts

1. Identify which of the following features corresponds to which type of forward commitment contract.

- |  |  |
|--|--|
| 1. The daily change in contract price is used to determine and settle the MTM.   | A. Forward contract                      |
| 2. Inclusion of storage and insurance costs increases the difference between the spot and forward commitment contract price. | B. Futures contract                      |
| 3. The contract price established at inception remains unchanged over time.  | C. Both a forward and a futures contract |

**Solution:**

1. B is correct. The futures contract price change at the close of each trading day is used to determine the daily MTM settlement via the margin account.
2. C is correct. Storage and insurance costs increase the forward commitment price for both forward and futures contracts.
3. A is correct. The forward contract price,  $F_0(T)$ , established at  $t = 0$  remains unchanged and is used to calculate the MTM settlement at maturity.

2. Calculate the correct answer to fill in the blank and justify your response: An investor entered a short oil futures contract position three months ago on 1,000 barrels at an initial price of \$69.00 per barrel. The constant risk-free rate is 0.50%. Daily oil spot and futures prices for the final 10 days of trading are shown in the following table. The change in the investor's futures contract value on day  $T - 5$  is closest to \_\_\_\_\_.

Day	Crude Oil Spot Price (\$)	Crude Oil Futures Price (\$)
T-10	69.62	68.69
T-9	69.01	68.11
T-8	66.88	66.15
T-7	65.18	64.77
T-6	66.72	66.02
T-5	68.59	68.01
T-4	68.80	68.08
T-3	68.93	68.32
T-2	69.43	69.15
T-1	69.36	69.18
T	70.03	70.03

**Solution:**

The answer is \$(1,990). The MTM of the investor's futures position is the daily futures price change on Day  $T - 5$  per barrel multiplied by 1,000 barrels:

$$V_{T-5}(T) = - [ -f_{T-5}(T) - f_{T-6}(T) ] \times 1,000 = - (68.01 - 66.02) \times 1,000 \\ = - \$1,990.$$

Note the negative sign refers to the investor's short futures position. The investor realizes a loss as the futures price rises due to the short position.

3. Determine the correct answers to complete the following sentences: The daily settlement mechanism resets the futures MTM to \_\_\_\_\_, and margin is exchanged to settle the difference. The \_\_\_\_\_ realized MTM gain or loss on a futures contract is approximately the same as for a comparable forward contract.

**Solution:**

The daily settlement mechanism resets the futures MTM to *zero*, and margin is exchanged to settle the difference. The *cumulative* realized MTM gain or loss on a futures contract is approximately the same as for a comparable forward contract.

4. Identify the following statement as true or false and justify your answer: An FRA fixed-rate receiver (floating-rate payer) position is equivalent to a long interest rate futures contract on MRR, as both positions realize a gain as MRR falls below the initial fixed rate.

**Solution:**

The statement is true. An FRA fixed-rate receiver (floating-rate payer) position realizes a gain as MRR falls as the counterparty receives the fixed MRR and owes the floating MRR in the future. A long futures contract price is based on  $(100 - \text{yield})$ , which rises as yield-to-maturity (MRR) falls.

5. From May 2020 to January 2021, the three-month SONIA (Sterling Overnight Index Average) futures contract expiring in June 2021 traded at a price above 100. Describe the interest rate scenario implied by this futures price and justify your response.

**Solution:**

The future interest rate scenario implied by the futures price above 100 is a negative SONIA interest rate in June 2021. For example, if we consider the futures price for three-month SONIA one year forward as of June 2020 from Equation 5:

$$f_{1y,3m} = 100 - (100 \times \text{MRR}_{1y,3m}).$$

If  $f_{1y,3m} > 100$ , this implies that  $\text{MRR}_{1y,3m} < 0$ .

## 5

## FORWARD AND FUTURES PRICE DIFFERENCES



explain why forward and futures prices differ

Despite their similar symmetric payoff profile at maturity, differences exist between forward and futures valuation and pricing because of different cash flow profiles over the life of a futures versus a forward contract with otherwise similar characteristics. The distinguishing features of a futures contract are the posting of initial margin, daily mark-to-market, and settlement of gains and losses.

These features limit the MTM value of a futures contract to the daily gain or loss since the previous day's settlement. When that value is paid out in the daily settlement via the margin account, the futures price resets to the current settlement price and the MTM value goes to zero. Forward contracts, on the other hand, involve privately negotiated credit terms (which sometimes involve cash or securities collateral) and do not require daily MTM cash settlement. Forward contract settlement occurs at maturity in a one-time cash settlement of the cumulative change in contract value.

The different patterns of cash flows for forwards and futures can lead to a difference in the pricing of forwards versus futures. Forward and futures prices are identical under certain conditions, namely:

- if interest rates are constant, or
- if futures prices and interest rates are uncorrelated.

On the other hand, violations of these assumptions can give rise to differences in pricing between these two contracts. For example, if futures prices are positively correlated with interest rates, long futures contracts are more attractive than long forward positions for the same underlying and maturity. The reason is because rising prices lead to futures profits that are reinvested in periods of rising interest rates, and falling prices lead to losses that occur in periods of falling interest rates. The price differential will also vary with the volatility of interest rates.

A negative correlation between futures prices and interest rates leads to the opposite interpretation, with long forward positions being more desirable than long futures positions. In general, the more desirable contract will tend to have the higher price.

## INTEREST RATE FORWARD AND FUTURES PRICE DIFFERENCES

# 6

- ☐ | explain why forward and futures prices differ

The short maturity of most futures contracts and the ability of most market participants to borrow near risk-free rates for these maturities typically results in little to no distinction between futures and forward prices. An exception to this is the so-called **convexity bias**, which arises given the difference in price changes for interest rate futures versus forward contracts, as illustrated in the following example.

### EXAMPLE 5

#### Interest Rate Forwards versus Futures

Let us return to an example from the prior lesson with an interest rate futures contract of \$1,000,000 notional for three-month MRR of 2.21% for one quarter (or 90/360 days). Recall that the underlying deposit contract value was:

$$\$1,005,525 = \$1,000,000 \times [1 + (2.21\% / 4)].$$

The contract BPV was shown to be \$25 ( $= \$1,000,000 \times 0.01\% \times [1/4]$ ).

Consider in contrast a \$1,000,000 notional FRA on three-month MRR in three months' time with an identical 2.21% rate. The net payment on the FRA is based upon the difference between MRR and the implied forward rate (IFR):

$$\text{Net Payment} = (\text{MRR}_{B-A} - \text{IFR}_{A,B-A}) \times \text{Notional Principal} \times \text{Period}.$$

For example, if the observed MRR in three months is 2.22% (+0.01%), the net payment *at maturity* would be \$25 ( $= \$1,000,000 \times 0.01\% \times [1/4]$ ). However, the settlement of an FRA is based upon the present value of the final cash flow discounted at MRR, so:

$$\text{Cash Settlement (PV): } \$24.86 = \$25 / (1 + 0.0222 / 4).$$

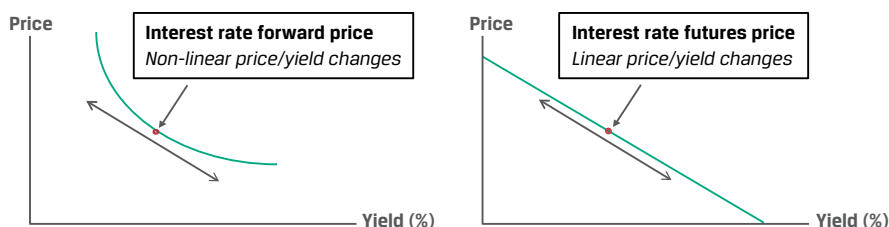
If we increase the magnitude of the MRR change at settlement and compare these changes between a long interest rate futures position and a short receive-fixed (pay floating) FRA contract, we arrive at the following result:

$\text{MRR}_{3m,3m}$	Short FRA Cash Settlement (PV)	Long Futures Settlement
2.01%	\$497.50	\$500
2.11%	\$248.69	\$250
2.21%	\$0	\$0
2.31%	(\$248.56)	(\$250)
2.41%	(\$497.01)	(\$500)

Although the settlement values differ due to different conventions across these instruments, note that while the futures contract has a fixed linear payoff profile for a given basis point change, the FRA settlement does not.

In the FRA contract in Example 5, we see that the percentage price change is greater (in absolute value) when MRR falls than when it rises. Although the difference here is very small due to the short forward period, note that this non-linear relationship is the convexity property, which characterizes fixed-income instruments from earlier lessons, as shown in Exhibit 5.

#### Exhibit 5: Convexity Bias



The discounting feature of the FRA, which is not present in the futures contract, leads to a convexity bias that is greater for longer discounting periods. You will recall from an earlier lesson that a discount factor is the *price* equivalent of a zero rate and is the present value of a currency unit on a future date, which may also be interpreted as the price of a zero-coupon cash flow or bond. We will show later how this discount factor is used to price interest rate swaps and other derivatives.

## EFFECT OF CENTRAL CLEARING OF OTC DERIVATIVES

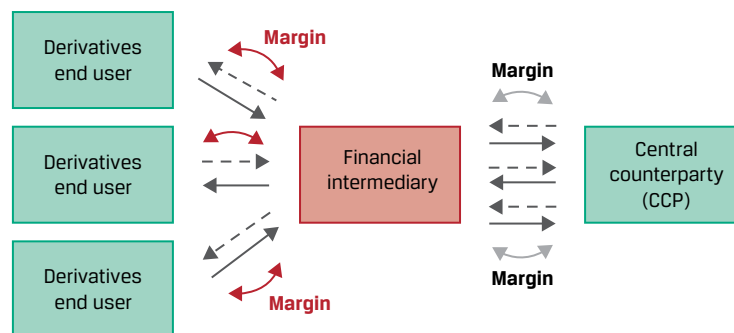
7

- ☐ explain why forward and futures prices differ

In periods of market and/or counterparty financial stress, large price movements combined with a derivative counterparty's inability to meet a margin call may force the closeout of a futures transaction prior to maturity. An OTC forward contract with more flexible credit terms, however, may remain outstanding.

The advent of derivatives central clearing, introduced in an earlier lesson, has created futures-like margining requirements for OTC derivative dealers who buy and sell forwards to derivatives end users. Dealers who are required to post cash or highly liquid securities to a central counterparty often impose similar requirements on derivatives end users. These dealer margin requirements reduce the difference in the cash flow impact of exchange-traded and OTC derivatives. This arrangement between dealers and their counterparties, shown in Exhibit 6, has been added to the original central clearing diagram from an earlier lesson.

**Exhibit 6: Margin Requirements for Centrally Cleared OTC Derivatives**



Investors who actively use both exchange-traded futures or OTC forwards must therefore maintain sufficient cash or eligible collateral to fulfill margin or collateral requirements. Market participants must also consider the financing, transaction, and administrative costs of maintaining these positions when using derivatives in a portfolio.

### QUESTION SET



#### Forward and Futures Prices

1. Identify the following statement as true or false and justify your answer: If futures prices are positively correlated with interest rates, long futures contracts are more attractive than long forward positions for the same underlying and maturity.

**Solution:**

The statement is true. If futures contract prices rise as interest rates rise, a long futures contract holder can reinvest futures contract profits at higher interest rates.

2. An investor seeks to hedge its three-month MRR exposure on a £25,000,000 liability in two months and observes an implied forward rate today ( $IFR_{2m,3m}$ ) of 2.95%. Calculate the settlement amounts if the investor enters a long pay-fixed (receive floating) FRA and a short futures contract, and compare and interpret the results if  $MRR_{2m,3m}$  settles at 3.25%.

**Solution:**

Solve for the pay-fixed FRA Cash Settlement (PV) value as follows:

$$\text{Net Payment} = (MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional Principal} \times \text{Period}$$

$$= £18,750 (= [3.25\% - 2.95\%] \times £25,000,000 \times [1/4]).$$

The present value based upon  $MRR_{2m,5m}$  of 3.25% is:

$$= £18,598.88 (= £18,750 / [1 + 0.0325/4]).$$

For the futures contract, contract BPV is equal to:

$$\text{Contract BPV} = £625 (= £25,000,000 \times 0.01\% \times [1/4]).$$

For a 30-basis point increase in MRR ( $= 3.25\% - 2.95\%$ ), the short futures contract will realize a price appreciation of £18,750 ( $= £625 \times 30$ ). Both contracts result in a gain from the investor's perspective as MRR rises. However, the futures settlement is larger due to the discounting of the FRA final payment to the settlement date.

3. Explain why short futures contracts are more attractive than short forward positions if futures prices are negatively correlated with interest rates for positions with the same underlying and maturity.

**Solution:**

The reason that short futures contracts are more attractive than short forward positions if futures prices are negatively correlated with interest rates is because falling prices lead to futures profits that are reinvested in periods of rising interest rates, and rising prices lead to losses that occur in periods of falling interest rates.

4. Identify the following statement as true or false and justify your answer:

The convexity bias between interest rate futures and interest rate forwards causes the percentage price change to be greater (in absolute value) when MRR rises than when it falls for a forward than for a futures contract.

**Solution:**

The statement is false. The convexity bias between interest rate futures and interest rate forwards causes the percentage price change to be greater (in absolute value) when MRR *falls* than when it *rises* for a forward contract, as opposed to a futures contract.

5. Explain how central clearing of derivatives reduces the difference in futures and forward prices for the same underlying and maturity periods.

**Solution:**

The central clearing of derivatives has created futures-like margining requirements for OTC derivative dealers who buy and sell forwards to derivatives end users. Dealers who are required to post cash or highly liquid securities to a central counterparty often impose similar requirements on derivatives end users. This arrangement between derivative dealers and their



counterparties will reduce the difference in the cash flow impact of ETD and OTC derivatives. Hence, any price difference between ETD futures and OTC forwards will be reduced.

## PRACTICE PROBLEMS

### The following information relates to questions 1-4

Ace Limited is a financial intermediary active in both futures and forward markets. You have been hired as an investment consultant and asked to review Ace's activities and answer the following questions.

1. Ace serves as a futures commission merchant to assist several of its commodity trading adviser (CTA) clients to clear and settle their futures margin positions with the futures exchange. Ace is reviewing the copper futures market for a CTA client considering a long copper futures position for the first time. Details of the copper futures market are as follows:

#### CME Copper Futures Contract Specifications

<b>Contract Maturities:</b>	Monthly [from 1 month to 15 months]
<b>Contract Size:</b>	25,000 pounds
<b>Delivery Type:</b>	Cash settled
<b>Price Quotation:</b>	\$ per pound
<b>Initial Margin:</b>	\$10,000 per contract
<b>Maintenance Margin:</b>	\$6,000 per contract
<b>Final Maturity:</b>	Last CME business day of contract month
<b>Daily Settlement:</b>	CME Trading Operations calculates daily settlement values based on its published procedures

Today's copper spot price is \$4.25 per pound, and the constant risk-free rate is 1.875%. Each contract has a \$10 storage cost payable at the end of the month. Which of the following statements best characterizes the margin exposure profile of Ace's CTA client if it enters a one-month copper futures contract?

- A. The CTA will be expected to post \$10,000 initial margin and would receive a margin call if the copper futures price were to immediately fall below \$4.10 per pound or below a price of \$102,425 per contract.
  - B. The CTA would be expected to post \$10,000 in initial margin and would receive a margin call at any time over the life of the contract if the copper futures price were to immediately fall below \$3.86 per pound or below a price of \$96,425 per contract.
  - C. The CTA will be expected to post \$10,000 initial margin, but we cannot determine the exact futures price at which a margin call will occur as the futures MTM is settled each day and the contract value resets to zero.
2. One of Ace's investor clients has entered a long six-month forward transaction with Ace on 100 shares of Xenaliya (XLYA), a non-dividend-paying technology stock. The stock's spot price per share,  $S_0$ , is €85, and the risk-free rate is a constant 1% for all maturities. Ace has hedged the client transaction with a long

six-month XLYA futures contract at a price  $f_0(T)$  of €85.42 and posted initial margin of €1,000. Three months after the forward and futures contracts are initiated, XLYA announces a strategic partnership with a major global technology firm, and its spot share price jumps €15 on the day's trading to close at €123. Which of the following statements best characterizes the impact of the day's trading on the MTM value of the forward versus the futures contract?

- A. Ace's client realizes an MTM gain of approximately €1,500 ( $= €15 \times 100$ ) on its margin account, which Ace must deposit at the end of the day to cover its margin call.
- B. Ace's client benefits from an MTM unrealized gain on its forward contract with Ace, and Ace has a corresponding MTM gain of approximately €1,500 ( $= €15 \times 100$ ) deposited in its margin account by the exchange.
- C. Because Ace has entered a hedge of its client's long forward position on XLYA by executing a futures contract with otherwise identical terms, the two contract MTM values exactly offset one another and no cash is exchanged on either transaction.

3. Identify which of the following corresponds to which description.

- |  |   |
|--|---|
| 1. Long interest rate futures position       | A. Results in a gain when MRR settles above the initial forward commitment rate at maturity |
| 2. Pay fixed (receive floating) FRA contract | B. Results in a loss when MRR settles above the initial forward commitment rate at maturity |
| 3. Receive fixed (pay floating) FRA contract | C. Has a forward commitment price that will increase as short-term interest rates fall      |

4. Ace's investor clients usually use OTC forward transactions that Ace must clear with a central counterparty. Which of the following statements related to the impact on Ace from clearing these positions is most accurate?

- A. If Ace's counterparties enter long forward contracts whose prices are positively correlated with interest rates, Ace will have to post more collateral to central counterparties than for otherwise similar futures contracts, since rising prices will lead to counterparty MTM gains reinvested at higher rates.
  - B. If Ace's counterparties enter short forward contracts whose prices are negatively correlated with interest rates, Ace will have to post less collateral to central counterparties than for otherwise similar futures contracts, since falling prices will lead to counterparty MTM gains reinvested at higher rates.
  - C. Since Ace is required to post collateral (cash or highly liquid securities) to the central counterparty to clear its client forward transactions, Ace will face similar margining requirements to those of standardized exchange-traded futures markets.
-

## SOLUTIONS

1. A is correct. The CTA will face a margin call if the copper contract price falls by more than \$4,000, or \$0.16 ( $= \$4,000/25,000$ ) per pound. We may solve for the price at which the CTA receives a margin call by first solving for the initial futures contract price,  $f_0(T)$ , at contract inception as follows:
 
$$f_0(T) = [S_0 + PV_0(C)] (1 + r)^T.$$
 Solve for  $PV_0(C)$  per pound as follows:
 
$$PV_0(C) = \$9.98 (= \$10[1.01875^{-(1/12)}]).$$
 Substitute  $PV_0(C) = \$9.98$  into Equation 4 to solve for  $f_0(T)$ :
 
$$f_0(T) = [(\$4.25 \times 25,000) + \$9.98](1.01875^{-(1/12)})$$

$$f_0(T) = \$106,425 \text{ per contract } (\approx \$4.257 \text{ per pound}).$$
 So,  $\$106,425 - \$4,000 = \$102,425$  per contract, and  $\$4.257 - \$0.16 = \$4.10$  per pound.  
 B is incorrect as it assumes there is no maintenance margin, and while C may be true under some circumstances, the change in A is immediate (occurs at trade inception).
2. B is correct. The long investor client forward position with Ace benefits from an MTM gain on its forward contract with Ace, but no cash is exchanged until maturity. Ace receives a deposit in its futures margin account equal to the daily MTM futures contract gain, which if spot and futures prices change by approximately the same amount will be equal to €1,500 ( $\text{€}15 \times 100$ ).
3. 1. C is correct. The futures contract price changes daily based upon a (yield – 100) quoting convention, so its price will increase as yields fall and vice versa. The fixed rate on an FRA does not change for the life of the contract.  
 2. A is correct. An FRA fixed-rate payer (floating-rate receiver) will realize a gain on the contract upon settlement (equal to the present value of the difference between the fixed rate and MRR multiplied by the contract notional over the specified interest period) if MRR settles above the initial fixed rate on the contract.  
 3. B is correct. If the MRR settles above the initial forward commitment rate at maturity, the FRA fixed-rate payer has an MTM loss on the contract.
4. C is correct. Mandatory central clearing requirements impose margin requirements on financial intermediaries similar to those of standardized exchange-traded futures markets, who often pass these costs and/or requirements on to their clients. Answers A and B are incorrect, as the MTM gains on the forward contracts are not realized until maturity.

## LEARNING MODULE

## 7

## Pricing and Valuation of Interest Rates and Other Swaps

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	describe how swap contracts are similar to but different from a series of forward contracts
<input type="checkbox"/>	contrast the value and price of swaps

### INTRODUCTION

1

Swap contracts were introduced earlier as a firm commitment to exchange a series of cash flows in the future, with interest rate swaps where fixed cash flows are exchanged for floating payments being the most common type. Subsequent lessons addressed the pricing and valuation of forward and futures contracts across the term structure, which form the building blocks for swap contracts.

In this lesson, we will explore how swap contracts are related to these other forward commitment types. While financial intermediaries often use forward rate agreements or short-term interest rate futures contracts to manage interest rate exposure, issuers and investors usually prefer swap contracts, since they better match rate-sensitive assets and liabilities with periodic cash flows, such as fixed-coupon bonds, variable-rate loans, or known future commitments. It is important for these market participants not only to be able to match expected future cash flows using swaps but also to ensure that their change in value is consistent with existing or desired underlying exposures. The following lessons compare swap contracts with forward contracts and contrast the value and price of swaps.

#### LEARNING MODULE OVERVIEW



- A swap contract is an agreement between two counterparties to exchange a *series* of future cash flows, whereas a forward contract is a *single* exchange of value at a later date.

- Interest rate swaps are similar to forwards in that both contracts are firm commitments with symmetric payoff profiles and no cash is exchanged at inception, but they differ in that the fixed swap rate is constant, whereas a series of forward contracts has different forward rates at each maturity.
- A swap is priced by solving for the par swap rate, a fixed rate that sets the present value of all future expected floating cash flows equal to the present value of all future fixed cash flows.
- The value of a swap at inception is zero (ignoring transaction and counterparty credit costs). On any settlement date, the value of a swap equals the current settlement value plus the present value of all remaining future swap settlements.
- A swap contract's value changes as time passes and interest rates change. For example, a rise in expected forward rates increases the present value of floating payments, causing a mark-to-market (MTM) gain for the fixed-rate payer (floating-rate receiver) and an MTM loss for the fixed-rate receiver (floating-rate payer).

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Identify which of the following characteristics matches which forward commitment contract.

- |   |   |
|---|---|
| 1. Involves periodic settlements based on the difference between a fixed rate established for each period and market reference rate (MRR) | A. Both an interest rate swap and a series of forward rate agreements |
| 2. Has a symmetric payoff profile and a value of zero to both counterparties at inception   | B. A series of forward rate agreements (FRAs)                         |
| 3. Involves periodic settlements based on the difference between a constant fixed rate and the MRR  | C. Interest rate swap   |

#### Solution:

1. B is correct. A series of FRAs involves periodic settlements based on the difference between a fixed rate established for each period and the MRR.
2. A is correct. Both an interest rate swap and a series of forward rate agreements have a symmetric payoff profile and a value of zero to both counterparties at inception.
3. C is correct. An interest rate swap involves periodic settlements based on the difference between a constant fixed rate and the MRR.

2. Which of the following transactions would allow a fixed-income portfolio manager to gain from falling interest rates?

- A. Buy a floating-rate bond
- B. Enter into a receive-fixed, pay-floating interest rate swap

**C.** Enter into a pay-fixed, receive-floating interest rate swap

**Solution:**

B is correct. A fixed-income portfolio manager seeking to gain from falling interest rates may consider entering a *receive-fixed, pay-floating* interest rate swap rather than purchasing bonds. The fixed-rate payments become more valuable as interest rates decline. A is incorrect as the floating-rate bond interest rate payments decline as interest rates decline, thus the bond does not increase in value. C is incorrect as the fixed interest rate payments become more costly as interest rates decline.

3. Which of the following statements provides a correct description of a pay-fixed, receive-floating interest rate swap position?

- A.** Long a floating-rate note priced at the MRR and short a fixed-rate bond with a coupon equal to the fixed swap rate
- B.** Long a fixed-rate bond with a coupon equal to the fixed swap rate and short a floating-rate note priced at the MRR
- C.** Long a floating-rate note priced at the MRR

**Solution:**

A is correct. An interest rate swap is economically equivalent to a long and short position in underlying debt securities. In the case of a pay-fixed, receive-floating swap, the cash flow received reflects the long position. In this case, the party receives floating payments, so this is like buying a floating-rate note. The pay-fixed portion of the swap is like selling a fixed-rate bond and paying fixed coupons to the bond buyer. B is incorrect as this response is economically equivalent to entering into a receive-fixed, pay-floating swap. C is incorrect because the response does not reflect the short position.

4. An investor enters into a 10-year, pay-fixed EUR100 million swap at a rate of 1.12% versus six-month EUR MRR. Assume six-month EUR MRR sets today at 0.25%. Which of the following is closest to the correct calculation of the periodic settlement value of the swap from the investor's perspective in six months' time?

- A.** EUR870,000
- B.** -EUR870,000
- C.** -EUR435,000

**Solution:**

C is correct. From the investor's (fixed-rate payer's) perspective, the periodic settlement value of the swap is equal to

$$\text{Periodic settlement value} = (\text{MRR} - s_N) \times \text{Notional amount} \times \text{Period} \\ = -\text{EUR}435,000 = (0.25\% - 1.12\%) \times \text{EUR}100 \text{ million} \times 0.5 \text{ years.}$$

Since EUR MRR has set below the fixed swap rate, the fixed-rate payer must make a net payment to the fixed-rate receiver at the end of the interest period. Both A and B responses omit the period of the swap (0.5 years) so are incorrect. A is also incorrect because the reversed sign on the answer would properly reflect the counterparty's periodic settlement value, not the investor's.

## 2

## SWAPS VS. FORWARDS

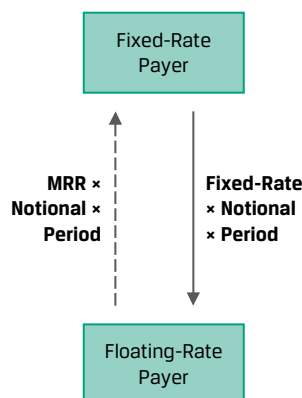


describe how swap contracts are similar to but different from a series of forward contracts

A swap contract is an agreement between two parties to exchange a *series* of future cash flows, while a forward contract is an agreement for a *single* exchange of value at a later date. Although this lesson focuses on interest rate swaps, similar principles apply to other underlying variables where a series of cash flows are exchanged on a future date.

An earlier lesson showed how implied forward rates may be derived from spot rates. An implied forward rate for a given period in the future is equivalent to the forward rate agreement (FRA) fixed rate for that same period for which no riskless profit opportunities exist. The single cash flow of an FRA is similar to a single-period swap, as shown in Exhibit 1.

**Exhibit 1: Swap and FRA Payoff Profile**

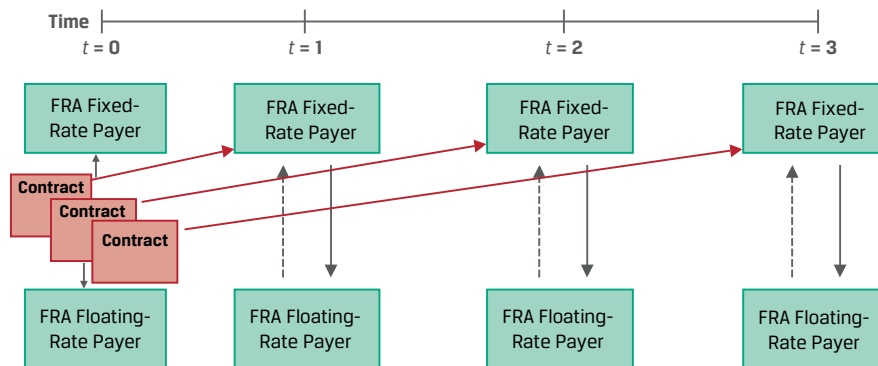
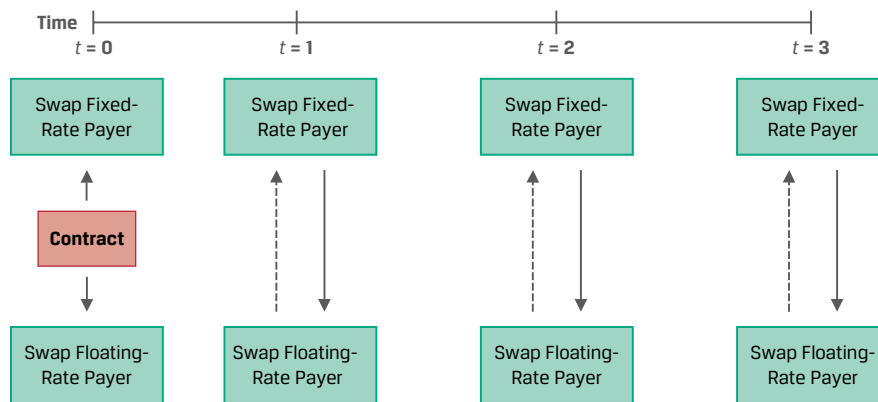


In each case, the net difference between a fixed rate agreed on at inception and an MRR set in the future is used as the basis for determining cash settlement on a given notional principal over a specific time period. For example, a fixed-rate payer on a swap or FRA will realize a gain if the MRR sets at a rate higher than the agreed-on fixed rate and will receive a net payment from the floating-rate payer. However, as we saw in an earlier lesson, the FRA has a single settlement, which occurs at the *beginning* of an interest period, while a standard swap has periodic settlements, which occur at the *end* of each respective period.

Other similarities between interest rate forwards and swaps include the symmetric payoff profile and the fact that no cash flow is exchanged upfront. Both interest rate forward and swap contracts involve counterparty credit exposure.

Since interest rates are characterized by a term structure, different FRA fixed rates usually exist for different times to maturity. In contrast, a standard interest rate swap has a *constant* fixed rate over its life, which includes multiple periods. This relationship is shown visually in Exhibit 2 and numerically in Example 1, which extends an earlier spot and forward rate example.



**Exhibit 2: Series of FRAs vs. Standard Interest Rate Swap****Series of Forward Rate Agreements (at Different Fixed Rates)****Standard Interest Rate Swap (At A Constant Fixed Rate)****EXAMPLE 1****Swaps as a Combination of Forwards**

Recall from an earlier lesson that three recently issued annual fixed-coupon government bonds had the following coupons, prices, yields-to-maturity, and zero (or spot) rates:

Years to Maturity	Annual Coupon	PV (per 100 FV)	YTM	Zero Rates
1	1.50%	99.125	2.3960%	2.3960%
2	2.50%	98.275	3.4068%	3.4197%
3	3.25%	98.000	3.9703%	4.0005%

We solve for the implied forward rate ( $IFR_{A,B-A}$ ), the break-even reinvestment rate for a period starting in the future (at  $t = A$ ) between short-dated ( $z_A$ ) and longer-dated ( $z_B$ ) zero rate using the following formula:

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B. \quad (1)$$

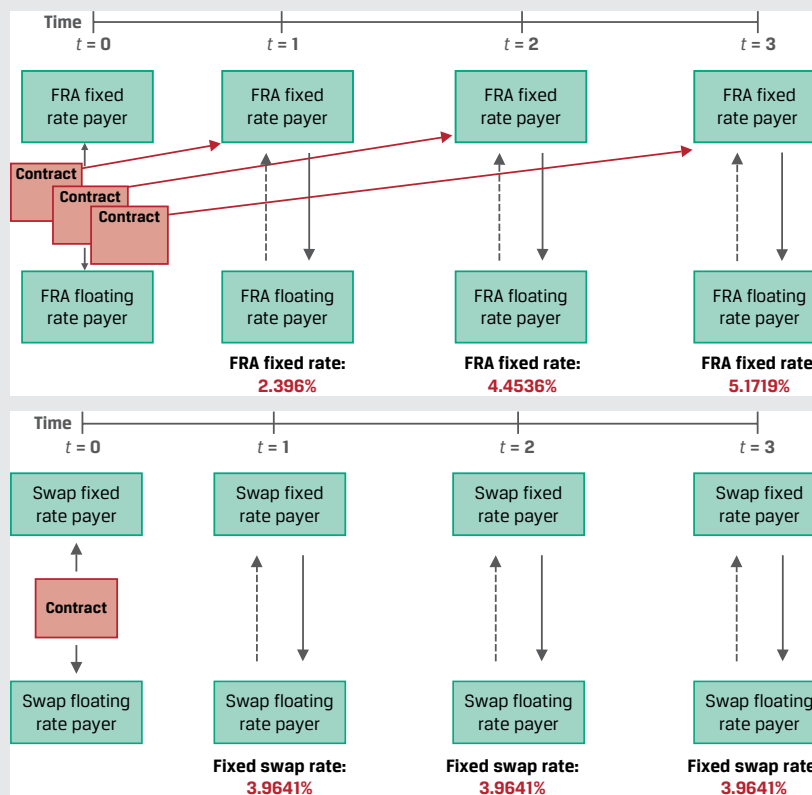
The respective spot rate at time  $t = 0$  ( $IFR_{0,1}$ ) and the implied forward rates in one year ( $IFR_{1,1}$ ) and in two years ( $IFR_{2,1}$ ) are as follows:

$$\text{IFR}_{0,1} = 2.3960\% = (1.023960) - 1.$$

$$\text{IFR}_{1,1} = 4.4536\% = (1.034197)^2 / (1.02396) - 1.$$

$$\text{IFR}_{2,1} = 5.1719\% = (1.040005)^3 / (1.034197)^2 - 1.$$

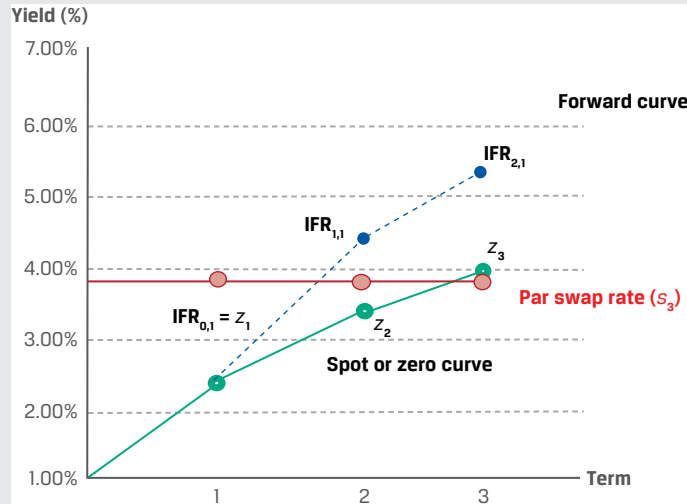
One way to create a forward commitment for multiple periods with an initial value of zero would be to use a series of forward rate agreements, exchanging cash flows based on the respective implied forward rate (i.e., the FRA fixed rate) for each period. However, the fixed rate would *vary* for each period based on the term structure of interest rates. Instead, a standard interest rate swap is characterized by a *constant* fixed rate over multiple periods. These rates are shown for both alternatives below.



The method used to solve for a swap rate was introduced in an earlier fixed-income lesson. The *par rate* (PMT) was shown to be the fixed rate at which a fixed-coupon bond has a price equal to par (or 100) using a sequence of zero rates ( $z_i$  for period  $i$ ) as market discount factors, as follows:

$$100 = \frac{\text{PMT}}{(1+z_1)^1} + \frac{\text{PMT}}{(1+z_2)^2} + \dots + \frac{\text{PMT} + 100}{(1+z_N)^N}. \quad (2)$$

A standard interest rate swap represents an exchange of fixed payments (with *no* final principal payment) at a constant rate for a series of floating-rate cash flows expected to equal the respective implied forward rates at time  $t=0$ . We must therefore modify the par bond rate calculation in Equation 2 to solve for PMT as a **par swap rate**.

**Exhibit 3: Par Swap Rate, Spot, and Forward Curve**

The par swap rate is the fixed rate that equates the present value of all future expected floating cash flows to the present value of fixed cash flows.

$\Sigma \text{PV}(\text{Floating payments}) = \Sigma \text{PV}(\text{Fixed payments})$ , or

$$\sum_{i=1}^N \frac{\text{IFR}}{(1+z_i)^i} = \sum_{i=1}^N \frac{s_i}{(1+z_i)^i} \quad (3)$$

In our three-period example, we use the implied forward rates, IFR, to solve for  $s_3$ , or the three-year swap rate:

$$\frac{\text{IFR}_{0,1}}{(1+z_1)} + \frac{\text{IFR}_{1,1}}{(1+z_2)^2} + \frac{\text{IFR}_{2,1}}{(1+z_3)^3} = \frac{s_3}{(1+z_1)} + \frac{s_3}{(1+z_2)^2} + \frac{s_3}{(1+z_3)^3}$$

Substitute each of these building blocks into Equation 3 to solve for  $s_3$ :

$$\begin{aligned} 0.111017 &= \frac{2.396\%}{1.02396} + \frac{4.4537\%}{(1.034197)^2} + \frac{5.1719\%}{(1.040005)^3} \\ &= \frac{s_3}{1.02396} + \frac{s_3}{(1.034197)^2} + \frac{s_3}{(1.040005)^3} \\ 0.111017 &= 2.800545 \times s_3 \\ s_3 &= 3.9641\% \end{aligned}$$

The three-year swap rate of 3.9641% may be interpreted as a multiperiod breakeven rate at which an investor would be indifferent to

- *paying* the fixed swap rate and *receiving* the respective forward rates or
- *receiving* the fixed swap rate and *paying* the respective forward rates.

For this reason, we may think of the fixed swap rate as an internal rate of return on the implied forward rates through the maturity of the swap as of  $t = 0$ . Exhibit 3 also demonstrates that while the combined value of the equivalent forward contracts is zero at time  $t = 0$ , some individual forward exchanges may have a positive present value and some will have a negative present value. For example, a fixed-rate receiver (floating-rate payer) on the swap in Example 1 would expect the following cash flow in one year's time:

- *Receive* the fixed swap rate ( $s_3$ ) of 3.9641%.
- *Pay* the initial floating rate ( $\text{IFR}_{0,1}$ ) of 2.396%.

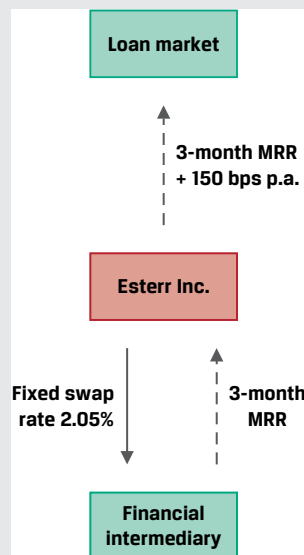
- Receive a net payment of 1.5681% ( $= 3.9641\% - 2.396\%$ ) on the notional for the period.

Derivative end users, such as issuers and investors, tend to use swaps more often than individual interest rate forward contracts for a number of reasons. For example, the ability to match cash flows of underlying assets or liabilities allows issuers to transform their exposure profile as in Example 2.

## EXAMPLE 2

### Esterr Inc. Swap to Fixed

Recall from an earlier lesson that Esterr Inc. has a CAD250 million floating-rate term loan at three-month Canadian MRR plus 150 basis points with three and a half years to maturity paid quarterly. Esterr enters into a CAD250 million interest rate swap contract to pay a fixed quarterly swap rate of 2.05% and receive a three-month floating MRR on a notional principal of CAD250 million based on payment dates that match the term loan. The combined loan and swap exposure may be shown as follows:



Under the swap arrangement, Esterr can fix its interest expense for the term loan and avoids the administrative burden of multiple forward contracts at different forward rates. Consider the following cash flow scenarios on the interest rate swap and term loan (assuming each interest period is 0.25 year):

#### Scenario 1:

CAD MRR sets at 3.75%. As the fixed-rate payer, Esterr

- pays the fixed swap rate of 2.05% and receives CAD MRR of 3.75%,
- receives a net swap payment of 1.70% for the quarter, and
- makes a term loan payment of 5.25% ( $= 3.75\% + 1.50\%$ ).

Esterr's interest expense is 3.55% ( $= 5.25\%$  paid  $- 1.70\%$  received).

#### Scenario 2:

CAD MRR sets at 1.25%. As the fixed-rate payer, Esterr

- pays the fixed swap rate of 2.05% and receives CAD MRR of 1.25%,
- pays a net swap payment of 0.80% for the quarter, and

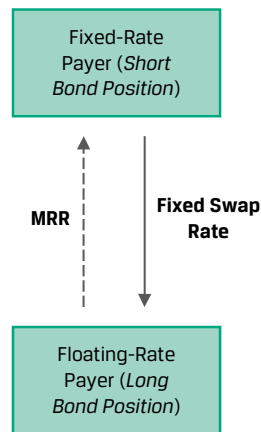
- makes a term loan payment of 2.75% (= 1.25% + 1.50%).

Esterr's interest expense is 3.55% (= 2.75% paid + 0.80% paid).

Regardless of where CAD MRR sets each period, Esterr's interest expense for each quarterly interest period is approximately CAD2,218,750 (= 3.55%, [equal to the 2.05% swap rate + 1.50% loan spread] × CAD250 million × 0.25 year).

As a fixed-income instrument with periodic fixed cash flows through maturity, an interest rate swap should be expected to have risk and return features similar to those of a fixed-coupon bond of a similar maturity. This feature makes interest rate swaps a more attractive alternative to forward rate agreements for investors in managing fixed-income exposures as well. Consider the following comparison of a cash bond position to paying or receiving a fixed swap rate:

#### Exhibit 4: Using Swaps to Manage Fixed-Income Exposure



Instrument	Position	Higher interest rates		Lower interest rates	
Cash bond	Long fixed bond	Loss		Gain	
	Short floating-rate note				
Interest rate swap	Receive fixed	Loss		Gain	
	Pay floating				
Cash bond	Short fixed bond	Gain		Loss	
	Long floating-rate note				
Interest rate swap	Pay fixed	Gain		Loss	
	Receive floating				

While the next lesson will explore the value and price of interest rate swaps in greater detail, Exhibit 4 demonstrates the similarity between a long (short) bond position and a receive (pay) fixed interest rate swap. Given their greater liquidity than and similar benchmark exposure profile to individual bond positions, active fixed-income portfolio managers often use swaps rather than underlying securities to adjust their interest rate exposure. For example, if interest rates are expected to fall, a portfolio manager may choose to receive fixed on a swap rather than purchase underlying bonds to realize a gain in a lower rate environment.

As mentioned earlier, FRAs are primarily used by financial intermediaries to manage their rate-sensitive positions on a period-by-period basis. Issuers and investors typically opt for the greater efficiency of interest rate swaps to manage their interest rate exposures. As we will see in later lessons, the greater liquidity of interest rate swaps has also led to their use both as a bond pricing benchmark and an underlying variable for other derivative instruments.

### QUESTION SET



### Interest Rate Swaps vs. Forward Contracts

1. Determine the correct answers to fill in the blanks: A fixed-rate payer on a swap or FRA will realize a(n) \_\_\_\_\_ if the MRR sets at a rate higher than the agreed-on fixed rate and will \_\_\_\_\_ a net payment \_\_\_\_\_ the floating-rate payer.

**Solution:**

A fixed-rate payer on a swap or FRA will realize a *gain* if the MRR sets at a rate higher than the agreed-on fixed rate and will *receive* a net payment *from* the floating-rate payer.

2. Identify which of the following characteristics matches which forward commitment contract.

- |   |   |
|---|---|
| 1. The price of this contract is the implied forward rate, or the breakeven reinvestment rate, for a period starting in the future. | A. Interest rate swap   |
| 2. Involves counterparty credit risk  | B. Forward rate agreement   |
| 3. Has a constant fixed rate for which the present value of future fixed versus floating cash flow exchanges is equal to zero       | C. Both an interest rate swap and an interest rate forward contract |

**Solution:**

1. B is correct. The breakeven reinvestment rate, or implied forward rate, is the no-arbitrage FRA fixed rate.
2. C is correct. Both an interest rate swap and a forward rate agreement involve counterparty credit risk.
3. A is correct. An interest rate swap has a constant fixed rate for which the present value of fixed versus floating cash flow exchanges equals zero.

3. Identify which of the following benefits of using swaps over forwards are most applicable to which derivative end users.

- |   |                               |
|---|-------------------------------|
| 1. Swaps allow these end users to match the periodic cash flows of a specific balance sheet liability to transform their interest rate exposure profile.      | A. Both issuers and investors |
| 2. Swaps enable these end users to actively adjust their interest rate exposure profile without buying or selling underlying securities                       | B. Issuers                    |
| 3. Swaps involving a series of cash flows enable these end users to avoid the administrative burden of entering into and managing multiple forward contracts. | C. Investors                  |

**Solution:**

1. B is correct. Swaps allow issuers to match the periodic cash flows of a specific balance sheet liability to transform their interest rate exposure profile, as shown in Example 2.
2. C is correct. Swaps enable investors to actively adjust their interest rate exposure profile without buying or selling underlying securities.
3. A is correct. Both issuers and investors benefit from a reduced administrative burden of entering one interest rate swap for a series of cash flows rather than multiple individual forward contracts.

4. Identify the following statement as true or false, and justify your response:  
The market reference rate (MRR) is the internal rate of return on the implied forward rates over the life of an interest rate swap.

**Solution:**

False. The fixed swap rate is the internal rate of return on the implied forward rates over the life of an interest rate swap.

5. Explain how an active fixed-income portfolio manager might use an interest rate swap rather than underlying bonds to realize a gain in a lower-interest rate environment, and justify your response.

**Solution:**

A manager may choose to receive fixed, pay floating on an interest rate swap, with the fixed cash flow stream being similar to owning a fixed-coupon bond. If interest rates are expected to fall, the manager will realize an MTM gain in a lower-rate environment.

## SWAP VALUES AND PRICES

# 3



contrast the value and price of swaps

In the prior lesson, we showed a swap price (or par swap rate) to be a periodic fixed rate that equates the present value of all future expected floating cash flows to the present value of fixed cash flows. The swap rate ( $s_N$  for  $N$  periods) is equivalent to a forward rate,  $F_0(T)$ , that satisfies the no-arbitrage condition. Similar to other forwards, the initial contract value (ignoring transaction and counterparty credit costs) is zero ( $V_0(T) = 0$ ).

In contrast to other forward commitments, which involve a *single* settlement at maturity, a swap contract involves a *series* of periodic settlements with a final settlement at contract maturity. Recall from an earlier lesson that the value of a forward contract at maturity from the long forward (or forward buyer's) perspective is  $V_T(T) = S_T - F_0(T)$ , where  $S_T$  is the spot price at maturity and  $F_0(T)$  is the forward price. For a swap with periodic exchanges, the current MRR is the "spot" price and the fixed swap rate,  $s_N$ , is the forward price. Restating this result for the fixed-rate payer on a swap, the periodic settlement value is

$$\text{Periodic settlement value} = (\text{MRR} - s_N) \times \text{Notional amount} \times \text{Period}. \quad (4)$$

The value of a swap on any settlement date equals the current settlement value in Equation 4 *plus* the present value of all remaining future swap settlements. Although swap market conventions vary, for purposes of this lesson we will assume the MRR sets at the beginning of each interest period, with the same periodicity and day count convention as the swap rate. The fixed versus floating difference is exchanged at the end of each period.

### EXAMPLE 3

#### Esterr Inc. Swap Value and Price

Esterr entered a 3.5-year CAD250 million interest rate swap contract under which it pays a fixed quarterly swap rate of 2.05% and receives three-month CAD MRR. The fixed swap *price* of 2.05% paid by Esterr remains constant over the life of the contract. While the first three-month CAD MRR is *known* at  $t = 0$ , the remaining 13 MRR settings are *unknown* but are *expected* to equal the respective implied forward rates (IFRs) for each period through maturity. A prior lesson showed how IFRs are derived from zero rates, which were then used to solve for the fixed swap rate by setting the present value of fixed payments equal to the present value of floating payments.

The *value* of Esterr's swap contract will change as time passes and interest rates change. We first consider the passage of time with no change to expected interest rates. That is, the MRR sets each period based on the implied forward rates at trade inception. Assume the following implied forward rates apply to Esterr's swap at  $t = 0$ :

Period	IFR	Rate
1	$\text{IFR}_{0,3m}$	0.50%
2	$\text{IFR}_{3m,3m}$	0.74%
3	$\text{IFR}_{6m,3m}$	0.98%
4	$\text{IFR}_{9m,3m}$	1.22%
5	$\text{IFR}_{12m,3m}$	1.46%
6	$\text{IFR}_{15m,3m}$	1.70%
7	$\text{IFR}_{18m,3m}$	1.94%
8	$\text{IFR}_{21m,3m}$	2.18%
9	$\text{IFR}_{24m,3m}$	2.43%

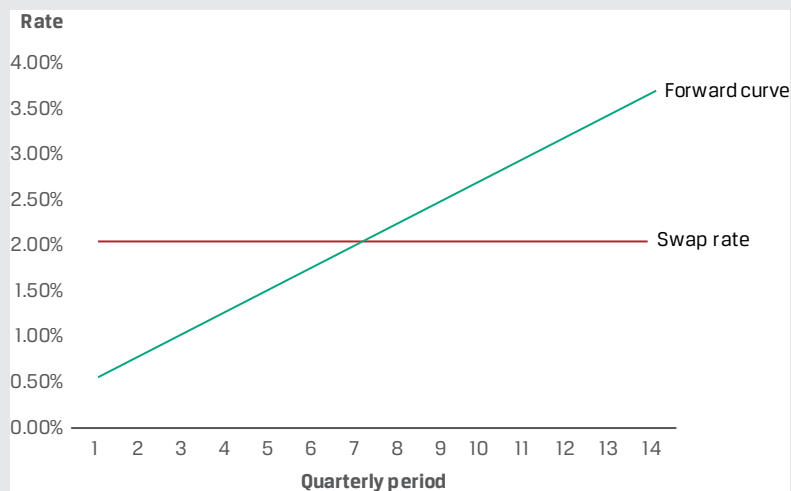


Period	IFR	Rate
10	$IFR_{27m,3m}$	2.67%
11	$IFR_{30m,3m}$	2.91%
12	$IFR_{33m,3m}$	3.15%
13	$IFR_{36m,3m}$	3.39%
14	$IFR_{39m,3m}$	3.78%

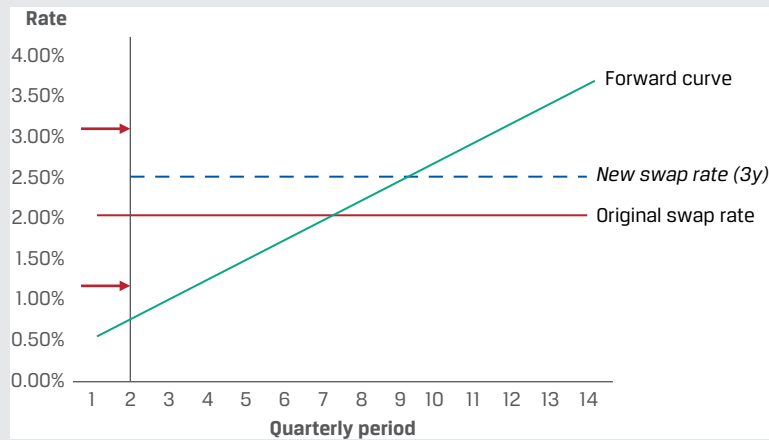
Based on these forward rates, Esterr expects to *make* a net swap payment each quarter through the seventh period—since  $(MRR - s_N) = (1.94\% - 2.05\%) < 0$ —and *receive* a net quarterly swap payment starting in the eighth period, where  $(MRR - s_N) = (2.18\% - 2.05\%) > 0$ . Using Equation 1, consider the periodic settlement values for the first two periods from Esterr's perspective as the fixed-rate payer:

- **Period 1:**  $-\text{CAD}968,750 = (0.5\% - 2.05\%) \times \text{CAD}250 \text{ million} \times 0.25$ .
- **Period 2:**  $-\text{CAD}818,750 = (0.74\% - 2.05\%) \times \text{CAD}250 \text{ million} \times 0.25$ .

What is the swap MTM *value* from Esterr's perspective immediately after the second periodic settlement if forward rates remain unchanged? Note that the swap *price* (the fixed swap rate of 2.05%) was set at inception to equate the present value of fixed versus floating payments.



As we move forward in time with *no* change to interest rate expectations, the present value of remaining floating payments rises above the present value of fixed payments at 2.05%, as Esterr has made a net settlement payment in the first two periods.



As the diagram shows, as time progresses, Esterr realizes an MTM gain on the swap, since

$$\Sigma PV(\text{Floating payments received}) > \Sigma PV(\text{Fixed payments paid}).$$

Note that this result depends on the relative level of IFRs and shape of the forward curve, which in this case is upward sloping.

If we instead consider interest rate changes only, from Esterr's perspective as the fixed-rate payer (and floating-rate receiver), we can show the conditions under which Esterr has a swap MTM gain or loss:

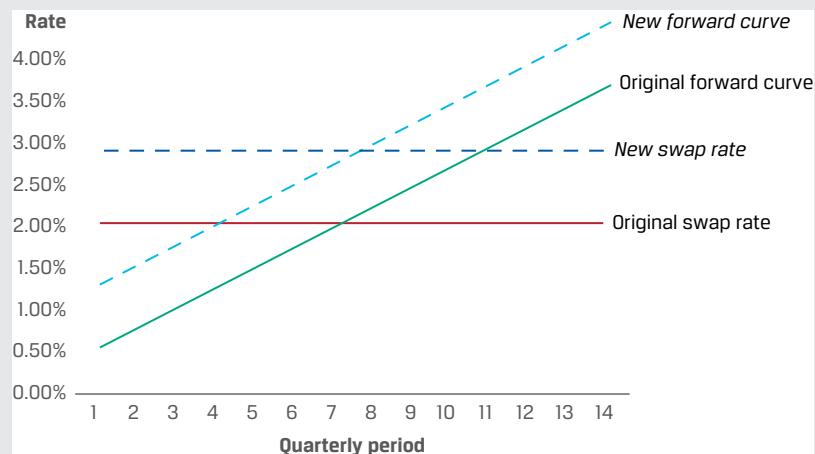
- Esterr realizes an MTM *gain* on the swap as the fixed-rate payer if

$$\Sigma PV(\text{Floating payments received}) > \Sigma PV(\text{Fixed payments paid}).$$

- Esterr realizes an MTM *loss* on the swap as the fixed-rate payer if

$$\Sigma PV(\text{Floating payments received}) < \Sigma PV(\text{Fixed payments paid}).$$

A rise in the expected forward rates after inception will increase the present value of floating payments, while the fixed swap rate will remain the same. We show the effect of an immediate change in interest rates following trade inception in the following diagram:



The new forward curve in this diagram is composed of higher IFRs. If we were to now solve for a fixed swap rate using these higher IFRs by setting floating and fixed cash flows equal to one another, the new swap rate would be above the original swap rate. However, since Esterr has locked in future fixed payments at the *lower* original swap rate while receiving higher expected future MRRs, the swap has a positive MTM value to Esterr.

Another interpretation of an interest rate swap is that the fixed-rate payer (floating-rate receiver) is *long* a floating-rate note (FRN) priced at the MRR and *short* a fixed-rate bond with a coupon equal to the fixed swap rate. Note that the combination of long and short bond positions causes both the purchase and sale prices of the two bonds at inception and the return of principal at maturity cancel one another out, and only the fixed versus variable coupon payments remain. The following example shows how the change in an interest rate swap's value is similar to that of a fixed-income security using an earlier investor swap example.

#### EXAMPLE 4

##### Fyleton Investments

Fyleton Investments entered a five-year, receive-fixed GBP200 million interest rate swap in an example from an earlier lesson to increase the duration of its fixed-income portfolio. Assume in this case that Fyleton receives a fixed swap rate of 2.38% on a semiannual basis versus six-month GBP MRR. Further assume that initial six-month GBP MRR sets at 0.71% and, as in the case of Esterr, the MRR forward curve is upward sloping. How will the value of Fyleton's swap change as time passes and interest rates change?

First, consider the passage of time with no rate changes. The first-period swap settlement value (in six months) from Fyleton's perspective as the fixed-rate receiver is

$$\text{GBP}1,670,000 = (2.38\% - 0.71\%) \times \text{GBP}200 \text{ million} \times 0.5.$$

What is the MTM value from Fyleton's perspective immediately following the first settlement if implied forward rates remain the same as at trade inception?

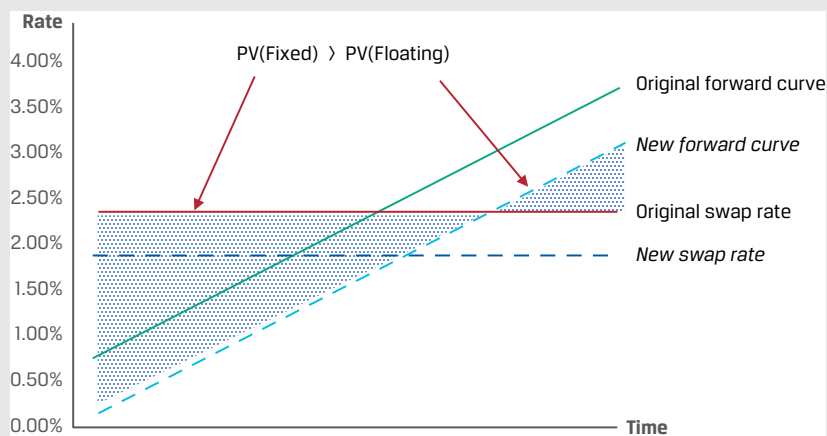
As the fixed-rate *receiver*, Fyleton will realize an MTM *loss* on the swap, since

$$\Sigma \text{PV}(\text{Floating payments paid}) > \Sigma \text{PV}(\text{Fixed payments received}).$$

Second, consider a change in forward rates. A *decline* in expected forward rates immediately following trade inception will *reduce* the present value of floating payments, while the fixed swap rate will remain the same. Fyleton will realize an MTM gain as the fixed-rate receiver, since

$$\Sigma \text{PV}(\text{Fixed payments received}) > \Sigma \text{PV}(\text{Floating payments paid}).$$

This MTM gain is shown by the different size of the shaded areas under the original swap rate using the new forward curve in the following diagram.



Note the similarity between the receive-fixed swap exposure profile and that of a long cash fixed-rate bond position. In the first instance, we would expect a fixed-rate par bond to be priced at a *discount* as time passes if rates rise as per an upward-sloping forward curve, while an FRN priced at the MRR would remain at par. In the second instance, a decline in implied forward rates (or downward shift in the forward curve) would cause a fixed-rate par bond to be priced at a *premium*, while the FRN price would not change. We will explore how term structure and yield curve changes affect swap values and bond prices in greater detail later in the curriculum.

### QUESTION SET



#### Swap Prices and Values

1. Determine the correct answers to fill in the blanks: A rise in the expected forward rates after inception will \_\_\_\_\_ the present value of floating payments, causing a fixed-rate receiver to realize a(n) \_\_\_\_\_ in MTM value on the swap contract.

**Solution:**

A rise in the expected forward rates after inception will *increase* the present value of floating payments, causing a fixed-rate receiver to realize a *decline* in MTM value on the swap contract.

2. Identify the following statement as true or false, and justify your response: The fixed rate on an interest rate swap is priced such that the present value of the floating payments (based on respective implied forward rates for each period) is equal to the present value of the fixed payments at  $t = 0$ .

**Solution:**

This statement is true. The fixed rate on an interest rate swap may be solved by setting the present value of floating payments equal to the present value of fixed payments. We use zero rates to derive implied forward rates, or breakeven reinvestment rates, between a shorter and a longer zero rate. These IFRs represent the respective floating rates that are expected to apply for each future period at time  $t = 0$ .

3. Identify which of the following statements is associated with which position in an interest rate swap contract.

- |   |   |
|---|---|
| 1. Establishes a set of certain net future cash flows on a swap contract at inception             | A. Fixed-rate payer (Floating-rate receiver)            |
| 2. Realizes an MTM gain on a swap contract if the expected future floating-rate payments increase | B. Fixed-rate receiver (Floating-rate payer)            |
| 3. An investor may increase portfolio duration by entering this position in a swap contract       | C. Neither a fixed-rate payer nor a fixed-rate receiver |

**Solution:**

1. C is correct. A swap contract establishes a set of *certain* fixed future cash flows that are exchanged for a set of *expected (uncertain)* floating future cash flows. Therefore, *neither* a fixed-rate payer nor a fixed-rate receiver knows the *net* future cash flows of a swap at inception.

2. A is correct. A fixed-rate payer realizes an MTM gain on a swap contract if the expected future floating-rate payments increase.
3. B is correct. A fixed-rate receiver may increase portfolio duration by entering this position on a swap, because receiving fixed is similar to a long bond position.

4. Determine the correct answers to fill in the blanks: The value of a swap on any settlement date equals the \_\_\_\_\_ settlement value plus the present value of all remaining \_\_\_\_\_ swap settlements.

**Solution:**

The value of a swap on any settlement date equals the *current* settlement value plus the present value of all remaining *future* swap settlements.

5. Describe how an investor may use an interest rate swap to reduce the duration of a fixed-income portfolio, and justify your response.

**Solution:**

A pay-fixed swap is similar to a short bond position that reduces duration, because the fixed-rate payer (floating-rate receiver) is effectively long a floating-rate note priced at the MRR and short a fixed-rate bond with a coupon equal to the fixed swap rate.

## PRACTICE PROBLEMS

### The following information relates to questions 1-6

Ace Limited is a financial intermediary that is active in forward and swap markets with its issuer and investor clients. You have been asked to consult on a number of client situations to determine the best course of action.

1. Identify which of the following statements is associated with which Ace counterparty swap position.
 

<p>1. Ace's counterparty with this swap position will realize an MTM gain if implied forward rates rise.</p> <p>2. Ace's counterparty with this swap position will make a net payment if the initial market reference rate sets above the fixed swap rate.</p> <p>3. Ace's counterparty with this position will face an initial swap contract value (ignoring transaction and counterparty credit costs) of zero.</p>	<p>A. A receive-fixed interest rate swap</p> <p>B. A pay-fixed interest rate swap</p> <p>C. Both a receive-fixed and a pay-fixed interest rate swap</p>
---	---
  
2. Ace's client is an asset manager with a significant portion of its fixed-rate bond investment portfolio maturing soon. Ace intends to reinvest the proceeds in five-year bond maturities. Which of the following describes the *best* course of action in the derivatives market for Ace's client to address its bond reinvestment risk?
  - A. Ace's client should consider *receiving* fixed on a cash-settled five-year forward-starting swap that starts and settles in three months in order to best address its bond reinvestment risk.
  - B. Ace's client should consider *paying* fixed on a cash-settled five-year forward-starting swap starting in three months in order to best address its bond reinvestment risk.
  - C. Ace's client should consider entering a *series* of forward rate agreements (FRAs) from today until five years from now under which it pays a fixed rate and receives a floating rate each period ending in five years to address its bond reinvestment risk.
  
3. Ace enters a 10-year GBP interest rate swap with a client in which Ace receives an initial six-month GBP MRR of 1.75% and pays a fixed GBP swap rate of 3.10% for the first semiannual period. Which of the following statements best describes the value of the swap from Ace's perspective three months after the inception of the trade?
  - A. Ace has an MTM *loss* on the swap, because it owes a net settlement payment to its counterparty equal to 1.35% multiplied by the notional and period.

- B. Ace has an MTM *gain* on the swap, because once it makes the first known net payment to its counterparty, the remainder of the future net fixed versus floating cash flows must have a positive present value from Ace's perspective.
- C. While the present value of fixed and future cash flows was set to zero by solving for the swap rate at inception, we do not have enough information to determine whether the swap currently has a positive or negative value from Ace's perspective following inception.
4. Ace enters a 10-year GBP interest rate swap with a client in which Ace receives an initial six-month GBP MRR of 1.75% and pays a fixed GBP swap rate of 3.10% for the first semiannual period. Six months later, Ace and its counterparty settle the first swap payment, and no change has occurred in terms of future interest rate expectations. Which of the following statements best describes the value of the swap from Ace's perspective?
- A. Ace has an MTM gain on the swap, because once it makes the first known net payment to its counterparty, the remainder of the future net fixed versus floating cash flows must have a positive present value from Ace's perspective.
- B. Ace has an MTM loss on the swap, because once it receives the first known payment from its counterparty, the remainder of the future net fixed versus floating cash flows must have a negative present value from Ace's perspective.
- C. While the present value of fixed and future cash flows was set to zero by solving for the swap rate at inception, we do not have enough information to determine whether the swap currently has a positive or negative value from Ace's perspective following inception.
5. At time  $t = 0$ , Ace observes the following zero rates over three periods:

Periods	Zero Rates
1	2.2727%
2	3.0323%
3	3.6355%

Which of the following best describes how Ace arrives at a three-period par swap rate ( $s_3$ )?

- A. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we discount each zero rate back to the present using zero rates and solve for  $s_3$  to get 2.961%.
- B. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we first solve for the implied forward rate per period using zero rates, then discount each implied forward rate back to the present using zero rates, and solve for  $s_3$  to get 3.605%.

- C. Since the par swap rate represents the fixed rate at which the present value of fixed and future cash flows equal one another, we first solve for the implied forward rate per period using zero rates, then discount each zero rate back to the present using implied forward rates, and solve for  $s_3$  to get 3.009%.
6. Ace's issuer client has swapped its outstanding fixed-rate debt to floating to match asset portfolio cash flows that generate an MRR-based return. Which of the following statements *best* describes how Ace's MTM credit exposure to the issuer changes if interest rates rise immediately following trade inception?
- A. Since the client receives fixed and pays floating swap, it faces an MTM loss on the transaction as rates rise, *increasing* Ace's MTM exposure to the client.
- B. Since the client receives fixed and pays floating swap, it faces an MTM gain on the transaction as rates rise, *decreasing* Ace's MTM exposure to the client.
- C. Since the swap's value is equal to the current settlement plus future expected settlement amounts, we do not have enough information to determine whether Ace's MTM exposure to the client increases or decreases.
-



## SOLUTIONS

1. B is correct. A pay-fixed swap counterparty will realize an MTM gain if implied forward rates rise.  
2. A is correct. A receive-fixed swap counterparty will make a net payment if the initial market reference rate sets above the fixed swap rate.  
3. C is correct. Both a receive-fixed and a pay-fixed swap counterparty will face an initial swap contract value (ignoring transaction and counterparty credit costs) of zero.
2. A is correct. Ace's client should consider *receiving* fixed on a five-year swap. A receive-fixed swap has a risk and return profile similar to that of a *long* fixed-rate bond position. Ace's client would therefore expect to have a similar MTM gain or loss on the swap position as if it had purchased a five-year bond at inception.
3. C is correct. At time  $t = 0$ , the present value of fixed and future cash flows was set to zero by solving for the swap rate at inception. Although the current settlement value is known, we cannot determine whether the swap has a positive or negative value from Ace's perspective three months later without further information—specifically, the current level of future forward rates.
4. A is correct. Ace makes the first net payment because the fixed-rate payment is greater than the floating rate received. Given no change in forward interest rates, this implies that the remaining net cash flows must have positive present value to Ace. B is incorrect as this response states the opposite compared to the prior response. C is incorrect because we have information about forward rate expectations.
5. B is correct. Since the expected floating cash flows on the swap are the implied forward rates, we first use zero rates to solve for IFRs using Equation 1:

$$(1 + z_A)^4 \times (1 + \text{IFR}_{A,B-A})^{B-A} = (1 + z_B)^B.$$

We may solve for these rates as  $\text{IFR}_{0,1} = 2.2727\%$ ,  $\text{IFR}_{1,1} = 3.7975\%$ , and  $\text{IFR}_{2,1} = 4.8525\%$ . We then substitute the respective IFRs discounted by zero rates into the following equation to solve for  $s_3$ :

$$\frac{\text{IFR}_{0,1}}{(1 + z_1)} + \frac{\text{IFR}_{1,1}}{(1 + z_2)^2} + \frac{\text{IFR}_{2,1}}{(1 + z_3)^3} = \frac{s_3}{(1 + z_1)} + \frac{s_3}{(1 + z_2)^2} + \frac{s_3}{(1 + z_3)^3}.$$

Solving for the left-hand side of the equation, we get

$$\begin{aligned} 0.10159 &= \frac{2.2727\%}{1.022727} + \frac{3.7975\%}{(1.030323)^2} + \frac{4.8525\%}{(1.036355)^3} \\ &= \frac{2.2727\%}{1.022727} + \frac{3.7975\%}{(1.030323)^2} + \frac{4.8525\%}{(1.036355)^3} \end{aligned}$$

Solving for the right-hand side, we get

$$2.81819 s_3 = \left[ \frac{1}{1.022727} + \frac{1}{(1.030323)^2} + \frac{1}{(1.036355)^3} \right] \times s_3.$$

$$s_3 = 3.605\% = 0.10159 \div 2.81819.$$

A is incorrect, because it discounts zero rates, not IFRs, back to the present using zero rates, while C incorrectly discounts zero rates by the respective IFRs.

6. A is correct. Since the client receives fixed and pays floating swap, in a rising-rate environment,  $\Sigma \text{PV}(\text{Floating payments}) > \Sigma \text{PV}(\text{Fixed payments})$ , and it will

therefore owe more in future floating-rate settlements than it will receive in fixed-rate settlements, resulting in an MTM loss for the client and an *increase* in Ace's MTM exposure.

## LEARNING MODULE

## 8

## Pricing and Valuation of Options

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain the exercise value, moneyness, and time value of an option
<input type="checkbox"/>	contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims
<input type="checkbox"/>	identify the factors that determine the value of an option and describe how each factor affects the value of an option

## INTRODUCTION

1

Option contracts are contingent claims in which one of the counterparties determines whether and when a trade will settle. Unlike a forward commitment with a value of zero to both counterparties at inception, an option buyer pays a premium to the seller for the right to transact the underlying in the future at a pre-agreed price. The contingent nature of options affects their price as well as their value over time.

In the first lesson, we explore three features unique to contingent claims related to an option's value versus the spot price of the underlying: the exercise, or intrinsic, value; the relationship between an option's spot price and its exercise price, referred to as "moneyness"; and the time value. We then turn to how the arbitrage and replication concepts introduced earlier for forward commitments differ when applied to contingent claims with an asymmetric payoff profile. Finally, we identify and describe factors that determine the value of an option. These lessons focus on European options, which can be exercised only at expiration.

## LEARNING MODULE OVERVIEW



- An option's value comprises its exercise value and its time value. The exercise value is the option's value if it were *immediately* exercisable, while the time value captures the *possibility* that the passage of time and the variability of the underlying price will increase the profitability of exercise at maturity.
- Option moneyness expresses the relationship between the underlying price and the exercise price. A put or call option is "at the money" when the underlying price equals the exercise price. An option is more

likely to be exercised if it is “in the money”—with an underlying price above (for a call) or below (for a put) the exercise price—and less likely to be exercised if it is “out of the money.”

- Due to their asymmetric payoff profile, options are characterized by no-arbitrage price bounds. The lower bound is a function of the present value of the exercise price and the underlying price, while the upper bound is the underlying price for a call and the exercise price for a put.
- As in the case of forward commitments, the replication of option contracts uses a combination of long (for a call) or short (for a put) positions in an underlying asset and borrowing or lending cash. The replicating transaction for an option is based on a proportion of the underlying, which is closely associated with the moneyness of the option.
- The underlying price, the exercise price, the time to maturity, the risk-free rate, the volatility of the underlying price, and any income or cost associated with owning the underlying asset are key factors in determining the value of an option.
- Changes in the volatility of the underlying price and the time to expiration will usually have the same directional effect on put and call option values. Changes to the exercise price, the risk-free rate, and any income or cost associated with owning the underlying asset have the opposite effect on call options versus put options.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements correctly describes the lower bound of a call option's value?

- A. The underlying's price minus the present value of the option's exercise price
- B. The underlying's price minus the option's exercise price or zero, whichever is greater
- C. The underlying's price minus the present value of the option's exercise price or zero, whichever is greater

**Solution:**

C is correct. The lower bound of a call price is the underlying's price minus the present value of its exercise price or zero, whichever is greater. A is incorrect as the response omits the fact that the lower bound is zero if the underlying's price is less than the present value of the exercise price. B is incorrect as it omits the present value term.

2. Match the following statements about replication strategies with their associated derivative instrument(s):

- |   |  |
|---|--|
| 1. At time $t = 0$ , lend at the risk-free rate and sell the underlying at $S_0$ .                            | A. Neither a call option nor a put option replication strategy |
| 2. The replication strategy is executed at inception and is settled at maturity with no adjustment over time. | B. A put option replication strategy                           |
| 3. At time $t = T$ , sell the underlying at $S_T$ and repay the loan of $X$ .                                 | C. A call option replication strategy if exercised             |

**Solution:**

1. B is correct. At time  $t = 0$ , a put option replication strategy involves lending at the risk-free rate and selling the underlying at  $S_0$ .
2. A is correct. As both call and put options involve a non-linear payoff profile, their replication strategy requires adjustment over time as the likelihood of exercise changes.
3. C is correct. A call option replication strategy if exercised involves repaying the loan of  $X$  and selling the underlying at  $S_T$  at time  $t = T$ .

3. A European call option with three months remaining to maturity on an underlying stock with no additional cash flows has an exercise price ( $X$ ) of GBP 50, a risk-free rate of 2%, and a current underlying price ( $S_t$ ) of GBP 57.50. If the current call option price is GBP 10, which response below most closely shows the correct exercise value and the time value of the option?

- A. Exercise value = GBP 7.50; Time value = GBP 2.50
- B. Exercise value = GBP 7.75; Time value = GBP 2.25
- C. Exercise value = GBP 0; Time value = GBP 10

**Solution:**

B is correct. An option's value comprises its exercise value plus its time value. The exercise value of a call option is  $\text{Max}(0, S_t - \text{PV}(X))$  and is calculated as follows:

$$\text{Call Option Exercise Value} = \text{Max}(0, S_t - X(1 + r)^{-(T-t)})$$

$$\text{Max}(0, \text{GBP } 57.50 - \text{GBP } 50(1.02)^{-0.25})$$

$$= \text{GBP } 7.75$$

The exercise value is positive, as the current underlying price exceeds the present value of the exercise price. The time value is the difference between the option price and the exercise value, representing the possibility that the option payoff at maturity will exceed the current exercise value due to a favorable price change:

$$\text{Call Option Time Value} = c_t - \text{Max}(0, S_t - X(1 + r)^{-(T-t)})$$

$$= \text{GBP } 2.25 (= \text{GBP } 10 - \text{GBP } 7.75)$$

The time value is always positive and declines to zero at maturity ( $t = T$ ). A is incorrect as the present value term is omitted from the exercise value calculation. C is incorrect as it implies the option has zero exercise value.

4. Match the following underlying price and exercise price relationships with their associated put option:

- |                         |                                   |
|-------------------------|-----------------------------------|
| 1. $S_T = 100, X = 100$ | A. An at-the-money put option     |
| 2. $S_T = 110, X = 100$ | B. An in-the-money put option     |
| 3. $S_T = 90, X = 100$  | C. An out-of-the-money put option |

**Solution:**

Put options are in the money when  $S_T < X$ , at the money when  $S_T = X$ , and out of the money when  $S_T > X$ . Therefore:

1. A is correct. Since  $S_T = X = 100$ , this is an at-the-money put option.
2. C is correct. Since  $S_T > X$ , this is an out-of-the-money put option.
3. B is correct. Since  $S_T < X$ , this is an in-the-money put option.

5. Match the following changes in a factor affecting option value (holding other factors constant) with their corresponding option value change:

- |  |   |
|--|---|
| 1. A higher exercise price ( $X$ )                     | A. Decreases the value of both a call option and a put option |
| 2. A higher underlying price ( $S_T$ )                 | B. Decreases the value of a call option                       |
| 3. A decline in the volatility of the underlying price | C. Decreases the value of a put option                        |

**Solution:**

1. B is correct. A higher exercise price decreases the value of a call option; for a given underlying price at maturity ( $S_T$ ), the call option settlement value of  $\text{Max}(0, S_T - X)$  will decrease for a higher  $X$ .
2. C is correct. A higher underlying price ( $S_T$ ) will decrease the value of a put option. Since a put option is the right to sell an underlying, the put option settlement value of  $\text{Max}(0, X - S_T)$  will fall as  $S_T$  rises.
3. A is correct. A decline in the volatility of the underlying price will decrease the value of both a call option and a put option. Lower price variability of the underlying will reduce the probability of a higher positive exercise value for a call or a put option without affecting the downside case where the option expires unexercised.

6. Which of the following statements provides the correct description as to how a call option's value changes if the income on the underlying declines unexpectedly, holding all else equal.

- A. The call option value increases.
- B. The call option value decreases.
- C. The call option value does not change.

**Solution:**

A is correct. Income or other, non-cash benefits (such as convenience yield) accrue to the owner of an underlying asset but not to the owner of a derivative, whose value is based on the underlying. A call option on an underlying with income has lower value than an identical call option on the same underlying without income. Thus, a decline in income on an underlying asset increases the value of a call option.

## OPTION VALUE RELATIVE TO THE UNDERLYING SPOT PRICE

2

- ☐ | explain the exercise value, moneyness, and time value of an option

As shown in earlier lessons, the non-linear or asymmetric payoff profile of an option causes us to approach these derivative instruments differently than for a forward commitment. When evaluating these derivatives, whose value depends critically on whether the spot price crosses an exercise threshold at maturity, buyers and sellers frequently rely on three measures—exercise value, moneyness, and time value—to gauge an option's value over the life of the contract. Recall from an earlier lesson that American options can be exercised at any time, while European options can be exercised only at maturity. This lesson focuses solely on European options with no additional cost or benefit of owning the underlying asset.

## OPTION EXERCISE VALUE

3

- ☐ | explain the exercise value, moneyness, and time value of an option

An option buyer will exercise a call or put option at maturity only if it returns a positive payoff—that is:

- $(S_T - X) > 0$  for a call
- $(X - S_T) > 0$  for a put

If not exercised, the option expires worthless and the option buyer's loss equals the premium paid.

At any time *before* maturity ( $t < T$ ), buyers and sellers often gauge an option's value by comparing the underlying spot price ( $S_t$ ) with the exercise price ( $X$ ) to determine the option's exercise value at time  $t$ . This is the option contract's value *if* the option were exercisable at time  $t$ . The exercise value for a call and a put option at time  $t$  incorporating the time value of money is the difference between the spot price ( $S_t$ ) and the present value of the exercise price ( $PV(X)$ ), as follows:

$$\text{Call Option Exercise Value: } \text{Max}(0, S_t - X(1 + r)^{-(T-t)}) \quad (1)$$

$$\text{Put Option Exercise Value: } \text{Max}(0, X(1 + r)^{-(T-t)} - S_t) \quad (2)$$

If we assume an exercise price,  $X$ , equal to the forward price,  $F_0(T)$ , the exercise value of a call option is the *same* as the value of a long forward commitment at time  $t$  ( $V_t(T)$ ). This forward commitment was shown earlier to equal  $S_t - PV(F_0(T))$ , *provided that*  $S_t > PV(X)$ . That is, for a call option where  $F_0(T) = X$ :

$$\text{If } S_t > PV(X): \quad S_t - PV(F_0(T)) = \text{Max}(0, S_t - PV(X))$$

Note that this comparison *ignores* the upfront call option premium paid by the option buyer ( $c_0$  at time  $t = 0$ ).

**EXAMPLE 1****Put Option Exercise Value**

Consider the case of a one-year put option with an exercise price ( $X$ ) of EUR 1,000 and a risk-free rate of 1%. What is the exercise value of the option in six months if the spot price ( $S_t$ ) equals EUR 950?

Use Equation 2 to solve for  $\text{Max}(0, \text{PV}(X) - S_t)$ :

$$X(1 + r)^{-(T-t)} - S_t$$

$$= \text{EUR } 45.04 (= \text{EUR } 1,000(1.01)^{-0.5} - \text{EUR } 950)$$

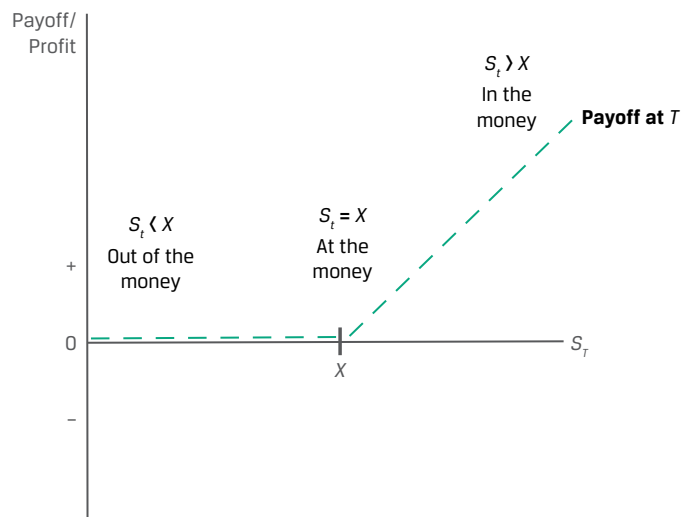
**4****OPTION MONEYNES**

explain the exercise value, moneyness, and time value of an option

An option's exercise value at any time  $t$  was shown to be its current payoff. The relationship between the option's total value and its exercise price expresses the option's **moneyness**. Examples of in-the-money (ITM) options include a call option whose underlying spot price is *above* the exercise price ( $X$ ) and a put option with an underlying spot price *below* the exercise price. When the underlying price is *equal to* the exercise price, the put or call option is said to be at the money (ATM). When the underlying price is *below (above)* the exercise price for a *call (put)* option, the option is less likely to be exercised and is said to be out of the money (OTM).

Also, the *degree* to which an option is in or out of the money affects the sensitivity of an option's price to underlying price changes. For example, a so-called **deep-in-the-money option**, or one that is highly likely to be exercised, usually demonstrates a nearly one-to-one correspondence between option price and underlying price changes. A **deep-out-of-the-money option**, which is very *unlikely* to be exercised, demonstrates far less option price sensitivity for a given underlying price change. In contrast, relatively small price changes in the underlying for an at-the-money option often determine whether the option will be exercised. Moneyness is often used to compare options on the same underlying but with different exercise prices and/or times to maturity. Exhibit 1 shows the moneyness of a call option at maturity and summarizes these relationships.



**Exhibit 1: Call Option Moneyness at Maturity**

Moneyiness	Call option ( $c_t$ )	Put option ( $p_t$ )
In the money (ITM)	$S_t > X$	$S_t < X$
At the money (ATM)	$S_t = X$	$S_t = X$
Out of the money (OTM)	$S_t < X$	$S_t > X$

**EXAMPLE 2****Put Option Moneyness**

Recall in Example 1 that at time  $t$ , a put option with six months remaining to maturity had an exercise price ( $X$ ) of EUR 1,000 and an underlying spot price ( $S_t$ ) of EUR 950. Describe the moneyness of the put option at time  $t$ .

Given that the underlying spot price is below the exercise price, the put option is in the money by EUR 50.

**OPTION TIME VALUE****5**

- ☐ explain the exercise value, moneyness, and time value of an option

While the exercise value of an option reflects its *current* payoff, an additional component of an option's value is derived from its remaining *time* to maturity. Although European options can be exercised only at maturity, they can be purchased or sold prior to maturity at a price ( $c_t$  or  $p_t$  for a call or put, respectively) that reflects the option's *future expected* payoff. A longer time until expiration usually means a higher potential dispersion of the future underlying price for a given level of volatility.

Similarly, an increase in volatility at a specific underlying price for a given time to expiration increases option value for the same reason. We will explore these factors further in a later lesson.

The time value of an option is equal to the difference between the current option price and the option's current payoff (or exercise value):

$$\text{Call Option Time Value: } \text{Max}(0, S_t - X(1+r)^{-(T-t)}) \quad (3)$$

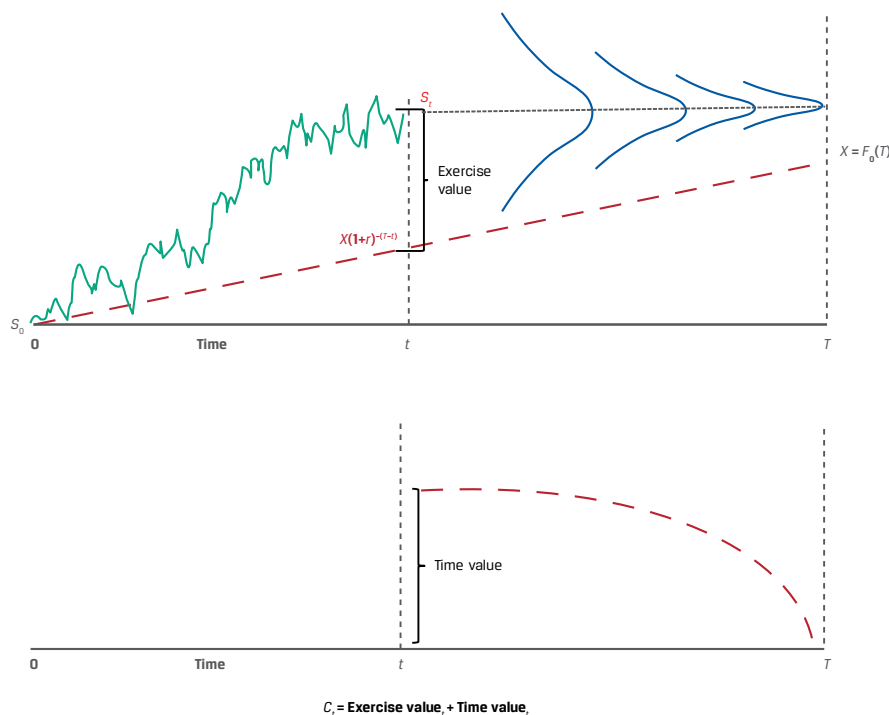
$$\text{or: } c_t = \text{Max}(0, S_t - X(1+r)^{-(T-t)}) + \text{Time Value}$$

$$\text{Put Option Time Value: } p_t - \text{Max}(0, X(1+r)^{-(T-t)} - S_t) \quad (4)$$

$$\text{or: } p_t = \text{Max}(0, X(1+r)^{-(T-t)} - S_t) + \text{Time Value}$$

That is, the current option price is equal to the sum of its exercise value and time value. As Exhibit 2 shows, the time value of an option is always positive but declines to zero at maturity, a process referred to as **time value decay**.

**Exhibit 2: Exercise Value and Time Value of a Call Option**



### EXAMPLE 3

#### Put Option Time Value

Example 1 showed that a one-year put option with an exercise price ( $X$ ) of EUR 1,000 had an exercise value of EUR 45.04 with six months remaining to maturity when the spot price ( $S_t$ ) was EUR 950. If we observe a current put option price ( $p_t$ ) of EUR 50, what is the time value of the put option?

Use Equation 4 to solve for  $p_t - \text{Max}(0, \text{PV}(X) - S_t)$ :

$$p_t - \max(0, X(1+r)^{-(T-t)} - S_t)$$

$$= \text{EUR } 4.96 (= \text{EUR } 50 - \text{EUR } 45.04)$$

## QUESTION SET



## Option Value relative to Underlying Spot Price

1. Describe the similarities and differences between the exercise value of a long put option position with an exercise price ( $X$ ) equal to the forward price,  $F_0(T)$ , and a short forward position payoff at maturity with the same underlying details.

**Solution:**

The exercise value of a put option with an exercise price of  $X = F_0(T)$  at maturity ( $\text{Max}(0, X - S_T)$ ) is the *same* as the payoff of a short forward commitment at maturity ( $F_0(T) - S_T$ ) *as long as* the exercise price is greater than the underlying price. That is, for  $F_0(T) = X$ :

$$\text{If } X > S_T: F_0(T) - S_T = \text{Max}(0, X - S_T)$$

These payoff profiles will *differ* if  $S_T > X$ , since the short forward payoff at maturity will require a payment from the seller to the buyer, while the put option owner will allow the option to expire unexercised. Note that this payoff comparison ignores the upfront option premium paid by the put option buyer ( $p_0$ ) at time  $t = 0$ .

2. A European put option with three months remaining to maturity on an underlying stock with no additional cash flows has an exercise price ( $X$ ) of GBP 50, a risk-free rate of 2%, and a current underlying price ( $S_t$ ) of GBP 55. If the current put option price is GBP 5, calculate the exercise value and the time value and interpret the results.

**Solution:**

An option's value comprises its exercise value plus its time value. The exercise value of a put option is  $\text{Max}(0, \text{PV}(X) - S_t)$  and can be calculated as follows:

$$\text{Put Option Exercise Value} = \text{Max}(0, X(1+r)^{-(T-t)} - S_t)$$

$$\text{Max}(0, \text{GBP } 50(1.02)^{-0.25} - \text{GBP } 55)$$

$$= 0$$

The exercise value is zero, as the current underlying price exceeds the present value of the exercise price. The time value is the difference between the option price and the exercise value and represents the possibility that the likelihood and profitability of exercise at maturity may increase due to a favorable price change:

$$\text{Put Option Time Value} = p_t - \text{Max}(0, X(1+r)^{-(T-t)} - S_t)$$

$$= \text{GBP } 5 (= \text{GBP } 5 - \text{GBP } 0)$$

The time value is always positive and declines to zero at maturity ( $t = T$ ). Since the put option is out of the money, its value consists *solely* of time value.

3. Match the following underlying price and exercise price relationships with their associated call option:

1. $S_T = 60, X = 50$	A. An at-the-money call option
2. $S_T = 50, X = 50$	B. An in-the-money call option
3. $S_T = 40, X = 50$	C. An out-of-the-money call option

**Solution:**

Call options are in the money when  $S_T > X$ , at the money when  $S_T = X$ , and out of the money when  $S_T < X$ . Therefore:

1. B is correct. Since  $S_T > X$ , this is an in-the-money call option.
2. A is correct. Since  $S_T = X = 50$ , this is an at-the-money call option.
3. C is correct. Since  $S_T < X$ , this is an out-of-the-money call option.

4. Describe how the moneyness of an option affects how the option's value will change for a given change in the price of the underlying.

**Solution:**

An increase in the moneyness of an option will increase the sensitivity of its value to changes in the underlying price. For example, an option that is very likely to be exercised will have a nearly one-to-one change in option value for a given change in the underlying price, while an option that is unlikely to be exercised will have a relatively small change in value for a given change in the underlying price.

## 6

## ARBITRAGE



contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims

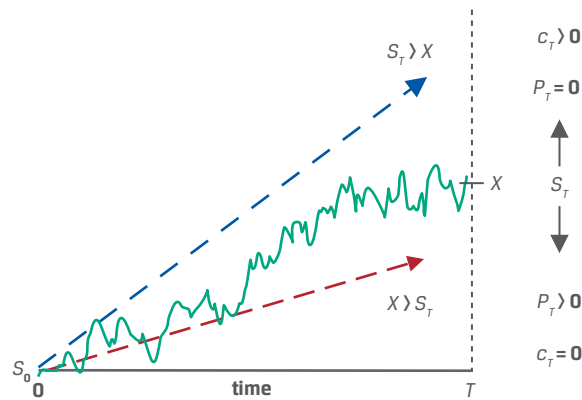
An earlier lesson showed that riskless arbitrage opportunities arise if the “law of one price” does not hold—that is, an identical asset trades at different prices in different places at the same time. In the case of a derivative, we establish no-arbitrage conditions based on the payoff profile at maturity. As shown earlier, forward commitments with a *symmetric* payoff profile settle based on the difference between the forward price,  $F_0(T)$ , at contract inception ( $t = 0$ ) and the underlying price,  $S_T$ , at contract maturity ( $t = T$ ), or  $(S_T - F_0(T))$  for a long forward position. For an underlying with no additional cash flows, it was shown that assets with a known future price must have a spot price equal to the future price discounted at the risk-free rate,  $r$ :  $S_0 = S_T(1 + r)^{-T}$ . For purposes of this lesson, we ignore any costs or benefits of owning the underlying other than the opportunity cost (or the risk-free rate,  $r$ ).

Contingent claims are characterized by *asymmetric* payoff profiles, introduced earlier. In the case of European options representing the right to purchase an underlying (or call option,  $c$ ) or the right to sell an underlying (or put option,  $p$ ) at a given exercise price ( $X$ ) at maturity, the payoff profiles at maturity for an option buyer were shown to be:

$$c_T = \text{Max}(0, S_T - X) \quad (5)$$

$$p_T = \text{Max}(0, X - S_T) \quad (6)$$

### Exhibit 3: European Option Exercise at Expiration



Recall that, unlike forward commitments with an initial price of zero, the option buyer pays the seller a premium ( $c_0$  for a call and  $p_0$  for a put at time  $t = 0$ ), so the option buyer's *profit* at maturity was introduced earlier as follows (ignoring the time value of money):

$$\Pi = \text{Max}(0, S_T - X) - c_0 \quad (7)$$

$$\Pi = \text{Max}(0, X - S_T) - p_0 \quad (8)$$

Equations 7 and 8 show the key distinction between forward commitments and contingent claims for purposes of arbitrage. The forward buyer enters into the contract with no cash paid upfront and has an unlimited gain or loss (bounded by zero for an underlying such as a stock that cannot have a negative price) at maturity as the underlying price rises or falls. The option buyer, in contrast, will exercise an option at maturity only if it is in the money. This *conditional* nature of option payoff profiles leads us to establish upper and lower no-arbitrage price bounds at any time  $t$ .

A call option buyer will exercise only if the spot price ( $S_T$ ) exceeds the exercise price ( $X$ ) at maturity. The *lower* bound of a call price is therefore the underlying's price minus the present value of its exercise price or zero, whichever is greater. In other words, an option which trades below its exercise value violates the no-arbitrage condition. The call buyer will not pay more for the right to purchase an underlying than the price of that underlying, which is the *upper* bound:

$$\text{Max}(0, S_t - X(1+r)^{-(T-t)}) < c_t \leq S_t \quad (9)$$

$$c_{t, \text{Lower bound}} = \text{Max}(0, S_t - X(1+r)^{-(T-t)}) \quad (10)$$

$$c_{t, \text{Upper bound}} = S_t \quad (11)$$

A put option buyer will exercise only if the spot price,  $S_T$ , is below  $X$  at maturity. The exercise price,  $X$ , therefore represents the *upper* bound on the put value. The *lower* bound is the present value of the exercise price minus the spot price or zero, whichever is greater:

$$\text{Max}(0, X(1+r)^{-(T-t)} - S_t) < p_t \leq X \quad (12)$$

$$p_{t, \text{Lower bound}} = \text{Max}(0, X(1+r)^{-(T-t)} - S_t) \quad (13)$$

$$p_{t, \text{Upper bound}} = X \quad (14)$$

#### EXAMPLE 4

##### Call Option Upper and Lower Bounds

Consider a one-year call option with an exercise price,  $X$ , of EUR 1,000. The underlying asset,  $S_0$ , trades at EUR 990 at time  $t = 0$  and the risk-free rate,  $r$ , is 1%. What are the no-arbitrage upper and lower bounds in six months' time if the underlying asset price,  $S_t$ , equals EUR 1,050?

As the option buyer will exercise only if  $S_T > X$  at  $t = T$ , the *lower* bound is equal to  $S_t - \text{PV}(X)$  or zero, whichever is greater:

$$c_{t, \text{Lower bound}} = \text{Max}(0, S_t - X(1+r)^{-(T-t)})$$

$$c_{t, \text{Lower bound}} = \text{Max}(0, \text{EUR } 1,050 - \text{EUR } 1,000(1.01)^{-0.5})$$

$$c_{t, \text{Lower bound}} = \text{Max}(0, \text{EUR } 54.96)$$

The call buyer will not pay more than  $S_t$  for the right to purchase the underlying:

$$c_{t, \text{Upper bound}} = S_t$$

$$c_{t, \text{Upper bound}} = \text{EUR } 1,050$$

## 7

### REPLICATION

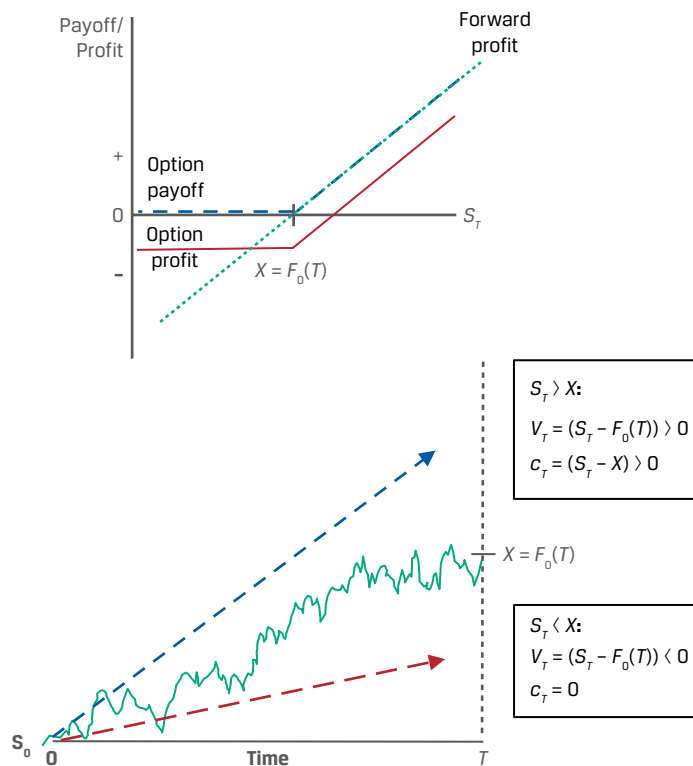


contrast the use of arbitrage and replication concepts in pricing forward commitments and contingent claims

In an earlier lesson, we used replication to create forward commitment cash flows, utilizing a combination of long or short positions in an underlying asset and borrowing or lending cash. The ability of market participants to use replication strategies ensures that the law of one price holds and no riskless arbitrage profit opportunities exist.

Recall from an earlier lesson that a call option is *similar* to a long forward position in that it increases in value as the underlying price rises but *differs* in that the call option settles only if there is a gain upon exercise. This apparent symmetry for positive outcomes is shown in Exhibit 4, where the exercise price,  $X$ , is equal to the forward price,  $F_0(T)$ .

## Exhibit 4: Call Option versus Long Forward Position



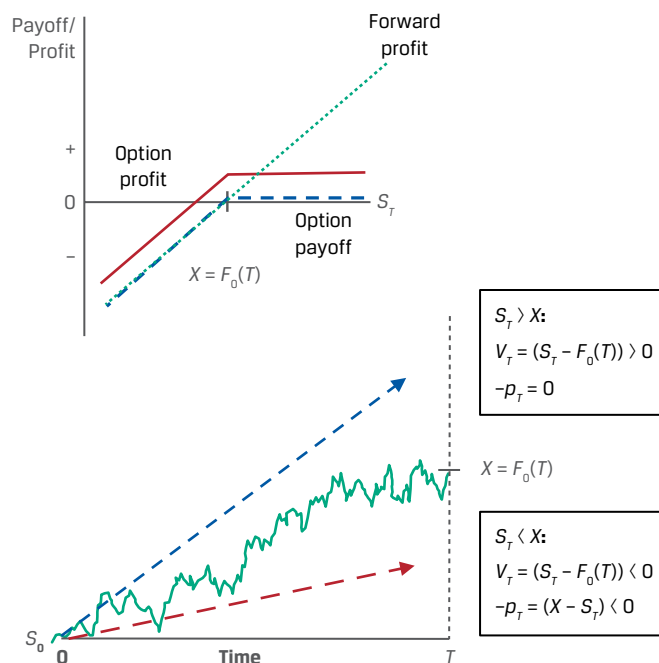
In order to replicate the call option at contract inception ( $t = 0$ ), we also must borrow at the risk-free rate,  $r$ , and use the proceeds to purchase the underlying at a price of  $S_0$ . At option expiration ( $t = T$ ), unlike in the case of the forward commitment, there are two possible replication outcomes:

- **Exercise** ( $S_T > X$ ): Sell the underlying for  $S_T$  and use the proceeds to repay  $X$ .
- **No exercise** ( $S_T < X$ ): No settlement is required.

If exercise were certain, we would borrow  $X(1 + r)^{-T}$  at inception, as in the case of the forward. However, since it is uncertain, we instead borrow a *proportion* of  $X(1 + r)^{-T}$  based on the *likelihood* of exercise at time  $T$ , which is closely associated with the moneyness of an option. The non-linear payoff profile of an option requires that the replicating transaction be adjusted as this likelihood changes, while the replicating trades for a forward commitment remain constant. Option replication will be addressed in greater detail in later lessons.

We now turn our attention to put option replication. It was shown earlier that a *short* put position with an exercise price equal to the forward price ( $X = F_0(T)$ ) mirrors the outcomes for a *long* forward position when the underlying price at maturity is *below* the forward price ( $S_T < X = F_0(T)$ ). The sold put *decreases* in value as the underlying price falls, but it settles only if there is a gain upon exercise. This relationship is shown in Exhibit 5, where the exercise price ( $X$ ) is equal to the forward price,  $F_0(T)$ .

## Exhibit 5: Put Option versus Short Forward Position



In order to replicate the put option at contract inception ( $t = 0$ ), we must sell the underlying short at a price of  $S_0$  and lend the proceeds at the risk-free rate,  $r$ . At option expiration ( $t = T$ ), unlike in the case of the forward commitment, there are two possible replication outcomes:

- **Exercise** ( $S_T < X$ ): Purchase the underlying for  $S_T$  from the proceeds of the risk-free loan.
- **No exercise** ( $S_T > X$ ): No settlement is required.

If exercise were certain, we would lend  $X(1 + r)^{-T}$  at inception, as in the case of the forward. However, as in the case of the call option, the asymmetric payoff profile requires adjustment over time based on the *likelihood* of exercise.

In the next lesson, we will turn our attention to the factors that drive the likelihood of option exercise prior to maturity.

## QUESTION SET



## Arbitrage and Replication

1. Determine the correct answers to complete the following sentence: The lower bound of a call price is the underlying's price \_\_\_\_\_ the present value of its \_\_\_\_\_ price or zero, whichever is greater.

**Solution:**

The lower bound of a call price is the underlying's price *minus* the present value of its *exercise* price or zero, whichever is greater.

2. A six-month European call option on an underlying stock with no additional cash flows has an exercise price ( $X$ ) of GBP 50, an initial underlying price



( $S_0$ ) of GBP 49.75, and a risk-free rate of 2%. Calculate the lower bound of the call price in three months' time if  $S_t = \text{GBP } 65$ .

**Solution:**

As the call option buyer will exercise only if  $S_T > X$  at maturity, the lower bound is equal to  $S_t - \text{PV}(X)$  or zero in three months' time, whichever is greater:

$$c_{t, \text{Lower bound}} = \text{Max}(0, S_t - X(1+r)^{-(T-t)})$$

Given  $S_t = \text{GBP } 65$ ,  $X = \text{GBP } 50$ ,  $r = 2\%$ , and  $(T - t) = 0.25$ , we can solve for the call option's lower bound as follows:

$$\begin{aligned} c_{t, \text{Lower bound}} &= \text{Max}(0, \text{GBP}65 - \text{GBP}50(1.02)^{-(0.25)}) \\ &= \text{GBP } 15.25 \end{aligned}$$

3. Match the following statements about replication strategies with their associated derivative instrument(s):

- |  |   |
|--|---|
| 1. At time $t = 0$ , borrow at the risk-free rate and purchase the underlying at $S_0$ . | A. Both a call option and a put option replication strategy |
| 2. The strategy requires adjustment over time as the likelihood of exercise changes.     | B. A call option replication strategy                       |
| 3. At time $t = T$ , receive the loan repayment and purchase the underlying at $S_T$ .   | C. A put option replication strategy if exercised           |

**Solution:**

1. B is correct. At time  $t = 0$ , a call option replication strategy involves borrowing at the risk-free rate and purchasing the underlying.
2. A is correct. As both call and put options have a non-linear payoff profile, the replication strategy requires adjustment over time as the likelihood of exercise changes.
3. C is correct. A put option replication strategy if exercised involves receiving the loan repayment and purchasing the underlying at  $S_T$  at time  $t = T$ .

4. Explain the key difference between the changes in the replication of a contingent claim versus a forward commitment over the life of the contract.

**Solution:**

An option has an *asymmetric* payoff profile, since the option buyer will exercise only if the exercise value of the option is positive. Since the likelihood of option exercise changes over time, transactions replicating an option contract must be adjusted over time. A forward commitment has a *symmetric* payoff profile that will settle with certainty at a future date, and therefore the replicating transactions do not require adjustment over the contract life.

## 8

## FACTORS AFFECTING OPTION VALUE



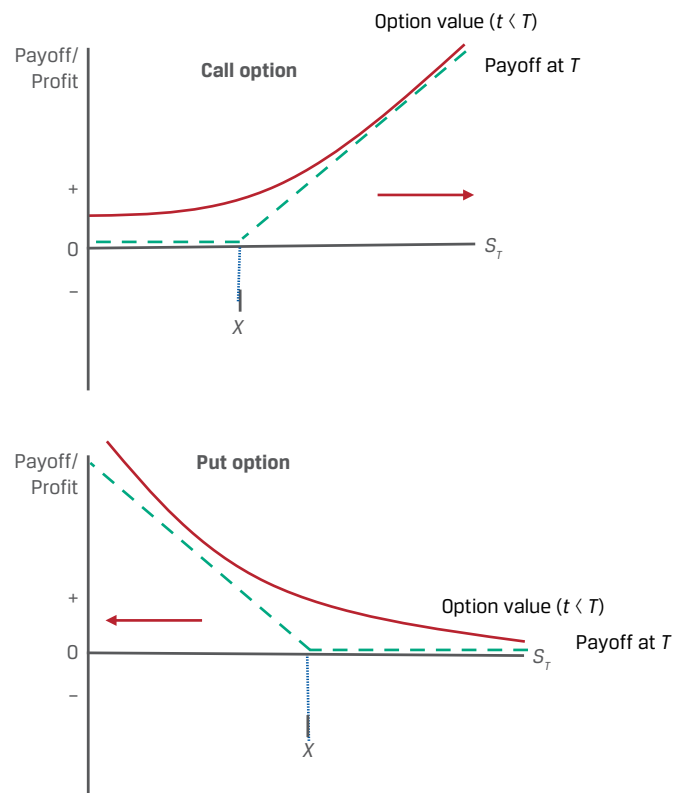
identify the factors that determine the value of an option and describe how each factor affects the value of an option

We now turn our attention to identifying and describing factors that affect an option's value. Several of these factors are common to both forward commitments and options, and a number are unique to contingent claims. Factors that determine the value of an option include the value of the underlying, the exercise price, the time to maturity, the risk-free rate, the volatility of the underlying price, and any income or cost associated with owning the underlying asset.

## Value of the Underlying

Changes in the value of the underlying will have the same directional effect on the *right* to transact the underlying under an option contract as on the *obligation* to transact under a forward commitment. For example, a call option and a long forward position will both *appreciate* if the spot price of the underlying *rises*, while a put option and a short forward position will both *appreciate* if the spot price of the underlying *falls*, as shown in Exhibit 6.

**Exhibit 6: Call and Put Option Value versus Underlying Value**



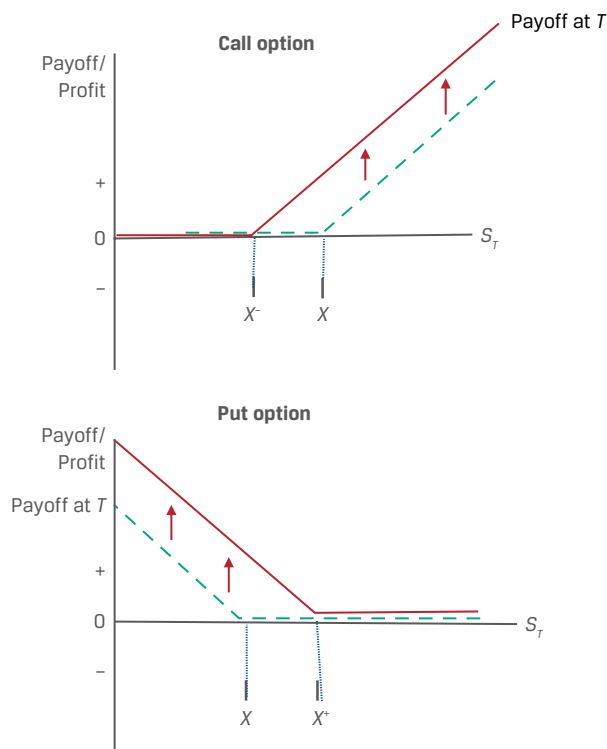
A key distinction between forward commitments and contingent claims is the *magnitude* of a derivative's price change for a given change in the underlying value. For a forward commitment, the derivative's value is a *linear* (one-for-one) function of the underlying price, while for an option it is a *non-linear* relationship that depends on the likelihood that an option buyer will exercise in the future. The more likely it is that the option buyer will exercise (e.g., when the option is in the money), the more sensitive the option's value to the underlying price. This likelihood depends on the relationship between the value of the underlying and the exercise price, a feature unique to option contracts that we turn to next.

## Exercise Price

The exercise price is the threshold that determines whether an option buyer chooses to transact at contract maturity. For a call option representing the right to buy the underlying, the exercise price represents a *lower* bound on the option's exercise value at maturity, leading to a *higher* option value for a *lower* exercise price. As call options settle based on  $\text{Max}(0, S_T - X)$  at expiry, a lower  $X$  will increase both the likelihood of exercise and the settlement value if exercised.

The exercise price of a put option, in contrast, is an *upper* bound on its exercise value at maturity. A *higher* exercise price at which a put option buyer has the right to sell the underlying will therefore *increase* the value of the option. Exhibit 7 shows these differing effects of exercise price changes for both call and put options.

**Exhibit 7: Call and Put Option Value versus Exercise Price**



## Time to Expiration

Recall from a prior lesson that the time value, or difference between an option's price and its exercise value, represents the likelihood that favorable changes to the underlying price will increase the profitability of exercise. Unlike a long forward position where the buyer is equally likely to experience an increase or a decrease in the underlying price over a longer period, the one-sided (or asymmetric) payoff profile of an option allows the buyer to ignore any outcomes where the option will expire unexercised.

For a call option, a longer time to expiration will increase the option's value in *all* cases. The price appreciation potential of an underlying is essentially unlimited and increases over longer periods, while the downside is limited to the loss of the premium.

A put option representing the right to sell the underlying also usually benefits from the passage of time. In the case of a put option, a longer time to expiration offers greater potential for price *depreciation* below the exercise price, while the loss is limited to the premium if the underlying price rises. However, as put option buyers await the *sale* of the underlying in order to receive  $(X - S_T)$  upon exercise, a longer time to expiry *reduces* the present value of the payoff. While less common, in some cases a longer time to expiration will *lower* a put's value, especially for deep-in-the-money puts with a longer time to expiration and a higher risk-free rate of interest.

## Risk-Free Interest Rate

An earlier lesson showed the risk-free rate to be the opportunity cost of holding an asset, which extends to the no-arbitrage valuation of derivatives. For example, an option's exercise value was shown to be equal to the difference between the spot price and the *present value* of the exercise price:

$$\text{Call Option Exercise Value: } \text{Max } (0, (S_t - \text{PV}(X)))$$

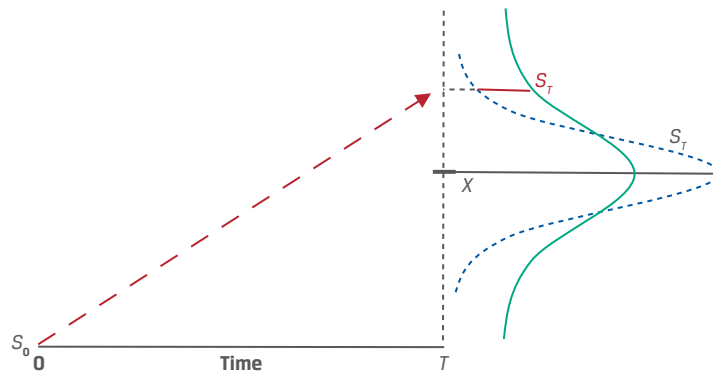
$$\text{Put Option Exercise Value: } \text{Max } (0, (\text{PV}(X) - S_t))$$

A *higher* risk-free rate therefore *lowers* the present value of the exercise price *provided* an option is in the money. A higher risk-free rate will *increase* the exercise value of a call option and *decrease* the exercise value of a put option. Note that the risk-free rate does not directly affect the time value of an option.

## Volatility of the Underlying

Volatility is a measure of the expected dispersion of an underlying asset's future price movements. Higher price volatility of the underlying increases the likelihood of a higher positive exercise value without affecting the downside case in which the option expires unexercised, as shown in Exhibit 8. Note that this effect will be the same for both call options and put options.

For example, as volatility *rises*, a wider range of possible underlying prices *increases* an option's time value and the likelihood that it will end up in the money. *Lower* volatility will *reduce* the time value of both put and call options.

**Exhibit 8: Volatility and Option Value****Income or Cost Related to Owning Underlying Asset**

In a prior lesson, the benefits and costs of owning an underlying asset over time were shown to affect the relationship of spot versus forward prices. For example, income or other, non-cash benefits (such as convenience yield) accrue to the owner of an underlying asset but not to the owner of a derivative. This effect holds for both forward commitments and contingent claims. Income or other benefits of ownership *decrease* the value of a call and *increase* the value of a put. Carry costs (such as storage and insurance for commodities) have the opposite effect, *increasing* the value of a call option and *decreasing* the value of a put. A summary of the factors that affect an option's value is provided in Exhibit 9, with the sign on the option value referring to the impact (positive or negative) of an *increase* in the factor.

**Exhibit 9: Factors That Affect the Value of an Option**

Factor	Call Value	Put Value
Value of the underlying	+	-
Exercise price	-	+
Time to expiration	+	+/-
Risk-free interest rate	+	-
Volatility of the underlying	+	+
Income/cost related to owning the underlying	-/+	+/-

## QUESTION SET



## Factors That Affect Option Value

1. Determine the correct answers to complete the following sentence: For a call option representing the right to buy the underlying, the exercise price represents a \_\_\_\_\_ bound on an option's exercise value at maturity, leading to a \_\_\_\_\_ option value for a lower exercise price.

**Solution:**

For a call option representing the right to buy the underlying, the exercise price represents a *lower* bound on an option's exercise value at maturity, leading to a *higher* option value for a lower exercise price.

2. Match the following changes in factors affecting option value (holding other factors constant) with their corresponding option value change:

- |   |   |
|---|---|
| 1. A lower exercise price ( $X$ )                   | A. Increases the value of both a call option and a put option |
| 2. A lower underlying price ( $S_T$ )               | B. Increases the value of a call option                       |
| 3. A rise in the volatility of the underlying price | C. Increases the value of a put option                        |

**Solution:**

- B is correct. A lower exercise price increases the value of a call option, since for a given underlying price at maturity of  $S_T$ , the call option settlement value of  $\text{Max}(0, S_T - X)$  will increase for a lower  $X$ .
- C is correct. A lower underlying price ( $S_T$ ) will increase the value of a put option. Since a put option is the right to sell an underlying at the exercise price ( $X$ ), the put option settlement value of  $\text{Max}(0, X - S_T)$  will increase as  $S_T$  declines.
- A is correct. A rise in the volatility of the underlying price will increase the value of both a call option and a put option. Greater price variability of the underlying will increase the probability of a higher positive exercise value for a call or a put option without affecting the downside case where the option expires unexercised.

3. Determine the correct answers to complete the following sentences: A key distinction between forward commitments and contingent claims is the \_\_\_\_\_ of a derivative's price change for a given change in the underlying value. For a forward commitment, the derivative value is a \_\_\_\_\_ function of the underlying price, while for an option it is a \_\_\_\_\_ relationship.

**Solution:**

A key distinction between forward commitments and contingent claims is the *magnitude* of a derivative's price change for a given change in the underlying value. For a forward commitment, the derivative value is a *linear* function of the underlying price, while for an option it is a *non-linear* relationship.

4. Explain the effect of a *decrease* in the risk-free rate of interest on the value of put and call options and justify your answer.

**Solution:**

A *decrease* in the risk-free rate *increases* the present value of an option's exercise price ( $PV(X)$ ). The exercise value for an *in-the-money call* option will fall, since  $(S_t - PV(X))$  is lower for a higher  $PV(X)$ , while the exercise value for an *in-the-money put* option will rise, since  $(PV(X) - S_t)$  rises for a higher  $PV(X)$ . Note that the time value of an option is not directly affected by the risk-free rate.

## PRACTICE PROBLEMS

### The following information relates to questions 1-9

The Viswan Family Office (VFO) owns non-dividend-paying shares of Biomian Limited that are currently priced ( $S_0$ ) at INR 295 per share. VFO's CIO is considering an offer to sell shares at a forward price ( $F_0(T)$ ) of INR 300.84 per share in six months based on a risk-free rate of 4%. You have been asked to advise on the purchase of a put option or the sale of a call option with an exercise price ( $X$ ) equal to the forward price ( $F_0(T)$ ) as alternatives to a forward share sale.

1. Which of the following statements correctly describes the put option and call option premiums given the relationship between Biomian's current price of INR 295 and the exercise price equal to the forward price of INR 300.84?
  - A. The call option's premium consists solely of time value, and the put option's premium consists solely of exercise value.
  - B. The call option's premium consists solely of exercise value, and the put option's premium consists solely of time value.
  - C. The call option's premium consists solely of time value, and the put option's premium consists solely of time value.
2. Which of the following statements correctly describes an important distinction between the two strategies of buying a put option and selling a call option with respect to option time value decay?
  - A. Time value decay is a benefit to both option strategies.
  - B. Time value decay is a cost to both option strategies.
  - C. Time value decay is a benefit to selling a call option and a cost to buying a put option.
3. VFO is considering the purchase of the put option to hedge against a decline in Biomian's share price. Which of the following statements best characterizes the trade-off between the put and the forward based on no-arbitrage pricing?
  - A. The gain on the forward sale will *equal* the purchased put option's profit at maturity *provided* the put option ends up in the money at maturity.
  - B. The loss on the forward sale will *exceed* the loss on the purchased put at maturity if Biomian's share price exceeds the forward price by more than the initial put premium paid.
  - C. We do not have enough information to answer this question, since we do not know the time value of the option at maturity.
4. In evaluating the purchased put strategy (with  $X = F_0(T)$ ), the CIO has asked you to consider selling the put in three months' time if its price appreciates over that period. Which of the following best characterizes the no-arbitrage put price at



that time?

- A. As VFO will exercise only if the spot price is below the exercise price, the lower bound of the put price is the greater of zero and the present value of the spot price minus the exercise price.
  - B. As VFO will exercise only if the spot price is below the exercise price, the upper bound of the put price equals the present value of the exercise price minus the spot price.
  - C. The put price can be no greater than the forward price and no less than the greater of zero and the present value of the exercise price minus the spot price.
5. VFO is considering a sold call strategy to generate income from the sale of a call. In your scenario analysis of the sold call option alternative, VFO has asked you to value the call option in three months' time if Biomian's spot price is INR 325 per share. Given an estimated call price of INR 46.41 at that time, which of the following correctly reflects the relationship between the call's exercise value and its time value?
- A. The call's exercise value is INR 24.16, and its time value is INR 22.25.
  - B. The call's exercise value is INR 27.10, and its time value is INR 19.31.
  - C. The call's exercise value is INR 20.99, and its time value is INR 25.42.
6. Which of the following statements correctly describes how VFO could replicate selling a call option on Biomian if exercise is certain?
- A. Borrow  $X(1 + r)^{-T}$  at the risk-free rate and use the proceeds to buy Biomian stock at the current spot price,  $S_0$ . At expiration, sell the Biomian stock and use the proceeds to pay off the loan.
  - B. Sell short Biomian stock at the current spot price,  $S_0$ , and use the proceeds to lend  $X(1 + r)^{-T}$  at the risk-free rate. At expiration, receive  $X$  as repayment of the risk-free loan and buy back the Biomian stock.
  - C. Sell short Biomian stock at the current spot price,  $S_0$ , and borrow  $X(1 + r)^{-T}$  at the risk-free rate. At expiration, pay off the risk-free loan of  $X$  and buy back the Biomian stock.
7. In comparing the sold call and purchased put strategies at the forward price, VFO's CIO is concerned about how an increase in the volatility of the underlying Biomian shares affects option premiums. Which of the following statements about volatility change and its effect on strategy is most accurate?
- A. An increase in the volatility of the underlying shares has the *same* effect on call and put option values, so this change should not affect VFO's strategy decision.
  - B. Since changes in the volatility of the underlying shares have the *opposite* effect on put versus call options, this change will increase the attractiveness of the put strategy versus the call strategy.

- C. An increase in the volatility of the underlying shares will increase both the *cost* of the purchased put strategy and the premium received on the sold call strategy, so this change will increase the attractiveness of the call strategy versus the put strategy.
  - 8. In comparing the Biomian purchased put and sold call strategies, which of the following statements is most correct about how the call and put values are affected by changes in factors other than volatility?
    - A. Changes in the time to expiration and the risk-free rate have a similar directional effect on the put and call strategies, while changes in the exercise price tend to have the opposite effect.
    - B. Changes in the time to expiration tend to have a similar directional effect on the put and call strategies, while changes in the exercise price and the risk-free rate tend to have the opposite effect.
    - C. Changes in the risk-free rate have a similar directional effect on the put and call strategies, while changes in the exercise price and the time to expiration tend to have the opposite effect.
  - 9. VFO is concerned about the potential for increasing interest rates in the future. Which of the following statements provides the most correct description as to how rising rates after entering into the two option strategies would affect the option valuations?
    - A. Rising risk-free rates would make the selling a call option strategy more advantageous to VFO because call options increase in value with higher risk-free rates.
    - B. Rising risk-free rates would make the buying a put option strategy more advantageous to VFO because the company locks in the put option premium at lower interest rates.
    - C. Rising risk-free rates are a negative for both option strategies.
-

## SOLUTIONS

1. C is correct. The exercise value of both the call and put options are zero because the present value of the exercise price is INR 295, which is equivalent to the stock price of INR 295. Thus, both option premiums reflect time value only. A is incorrect as this statement does not properly account for the discounting of the exercise price. B is incorrect as the call option's premium does not reflect exercise value.
2. C is correct. The time value component of an option premium declines toward zero as time passes toward the expiration date. A purchased option declines in value, all else equal, because of the time value decline. A sold option increases in value, all else equal, because of the time value decline. A is incorrect as time value decay is a cost to the purchased put option strategy, and B is incorrect as time value decay is a benefit to the sold option strategy.
3. The correct answer is B. The loss on the forward sale will be greater than the loss on the purchased put at maturity if Biomian's share price exceeds the forward price by more than the initial put premium. VFO's downside return is limited to the put premium paid, while the forward sale has unlimited downside as Biomian shares appreciate. A is incorrect as it does not take the put premium paid into account, while C is incorrect as the time value of an option is equal to zero at maturity.
4. The correct answer is C. The put exercise price,  $X$  (equal to  $F_0(T)$  in this case), represents the *upper* bound on the put value, while the *lower* bound is the greater of the present value of the exercise price minus the spot price and zero:

$$\text{Max}(0, X(1+r)^{-(T-t)} - S_t) < p_t \leq X$$

A is incorrect, as the *lower* bound of the put price is the greater of zero and the present value of the exercise price minus the spot price, not the present value of the spot price minus the exercise price. B is incorrect, as the *lower*, not the upper, bound of the put price equals the present value of the exercise price minus the spot price.

5. The correct answer is B. The option price is equal to the sum of the exercise value and the time value. A call option's exercise value is equal to the greater of zero and the spot price minus the present value of the exercise price:

$$\begin{aligned} &\text{Max}(0, S_t - X(1+r)^{-(T-t)}) \\ &= \text{Max}(0, \text{INR}325 - \text{INR}300.84(1.04)^{-0.25}) \\ &= \text{INR}27.10 \end{aligned}$$

The time value is equal to the call price minus the exercise value, or INR 19.31 (= INR46.41 – INR27.10). A is incorrect as it takes the spot price minus the exercise price as the exercise value, while C calculates the exercise value as the present value of the spot price minus the exercise price.

6. B is correct. To replicate selling a call option, combine shorting the underlying with risk-free lending. This is exactly the opposite strategy to replicate buying a call option in which the underlying is purchased with proceeds from risk-free borrowing. A is incorrect as this statement describes replicating buying a call option. C is incorrect as selling short and borrowing initially creates two cash

inflows at  $t = 0$  followed by two cash outflows at  $t = T$ .

7. The correct answer is C. An increase in the volatility of the underlying share price will increase both the upfront premium received on the sold call option and the premium paid on the purchased put option. Therefore, since the purchased put strategy involves an increased upfront payment made by VFO and the sold call strategy involves an increased premium received, the volatility increase will increase the attractiveness of the call strategy versus the put strategy.
8. The correct answer is B. Changes in the time to expiration tend to have a similar directional effect on the put and call strategies (the only exception being deep-in-the-money put options in some cases), while changes in the exercise price and the risk-free rate tend to have the opposite effect.
9. C is correct. Because both option strategies (buy a put or sell a call) are short strategies, VFO is delaying cash inflows so higher risk-free rates are negative. A is incorrect as this statement describes the value effect from buying a call option, not from selling. B is incorrect as the locked-in premium will decline after rising risk-free rates.

## LEARNING MODULE

## 9

## Option Replication Using Put–Call Parity

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain put–call parity for European options
<input type="checkbox"/>	explain put–call <i>forward</i> parity for European options

## INTRODUCTION

1

Previous lessons examined the payoff and profit profiles of call options and put options, the upper and lower bounds of an option's value, and the factors impacting option values. In doing so, we contrasted the asymmetry of one-sided option payoffs with the linear or symmetric payoff of forwards and underlying assets.

We now extend this analysis further to show that there are ways to *combine* options to have an equivalent payoff to that of the underlying and a risk-free asset as well as a forward commitment. In the first lesson, we demonstrate that the value of a European call may be used to derive the value of a European put option with the same underlying details, and vice versa, under a no-arbitrage condition referred to as put–call parity. In the second lesson, we show how this may be extended to forward commitments and how the put–call parity relationship may be applied to option and other investment strategies. We will focus on European options on underlying assets with no income or benefit.

## LEARNING MODULE OVERVIEW



- Put–call parity establishes a relationship that allows the price of a call option to be derived from the price of a put option with the same underlying details and vice versa.
- Put–call parity holds for European options with the same exercise price and expiration date, representing a no-arbitrage relationship between put option, call option, underlying asset, and risk-free asset prices.
- If put–call parity does not hold, then riskless arbitrage profit opportunities may be available to investors.

- Put–call forward parity extends the put–call parity relationship to forward contracts given the equivalence of an underlying asset position and a long forward contract plus a risk-free bond.
- Under put–call forward parity, we may demonstrate that a purchased put option and a sold call option are equivalent to a long risk-free bond and short forward position, and a sold put and purchased call are equivalent to a long forward and short risk-free bond.
- Put–call parity may be applied beyond option-based strategies in finance—for example, to demonstrate that equity holders have a position equivalent to a purchased call option on the value of the firm with unlimited upside, while debtholders have a sold put option position on firm value with limited upside.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements correctly describes the equivalent to a long position in an underlying according to put–call parity?
  - A. Long a put option on the underlying, short a call option on the underlying, and long a risk-free bond
  - B. Short a put option on the underlying, long a call option on the underlying, and long a risk-free bond
  - C. Short a put option on the underlying, long a call option on the underlying, and short a risk-free bond

**Solution:**

B is correct. Put–call parity demonstrates that a long underlying position is equivalent to a sold put option, a purchased call option, and a long risk-free bond. This is shown in the following equation:

$$\text{Put–call parity: } S_0 + p_0 = c_0 + X(1 + r)^{-T} \text{ implies } S_0 = -p_0 + c_0 + X(1 + r)^{-T}.$$

2. Identify which of the following positions has the same no-arbitrage value as which portfolio under put–call parity:

- |  |   |
|--|---|
| 1. Long call option ( $c_0$ )                | A. Long underlying, short risk-free bond, and short call option |
| 2. Short risk-free bond ( $-X(1 + r)^{-T}$ ) | B. Long underlying, long put option, and short risk-free bond   |
| 3. Short put option ( $-p_0$ )               | C. Short underlying, long call option, and short put option     |

**Solution:**

Recall that the put–call parity relationship may be expressed as

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

1. B is correct. A long call option position is the no-arbitrage equivalent of a long underlying position, a long put option, and a short risk-free bond position.
2. C is correct. A short risk-free bond position is equivalent to a short underlying position, a long call option, and a short put option.

3. A is correct. A short put option is equivalent to a long underlying position, a short risk-free bond, and a short call option.

3. Which of the following statements correctly describes a synthetic protective put position according to put–call forward parity?

- A. A long forward contract on the underlying, a long put option on the underlying, and short a risk-free bond
- B. A short forward contract on the underlying, a long put option on the underlying, and short a risk-free bond
- C. A short forward contract on the underlying, a short put option on the underlying, and short a risk-free bond

**Solution:**

A is correct. The formula for put–call forward parity is as follows:

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}.$$

Rearranging the terms as follows shows the synthetic protective put position on the left-hand side of the equation:

$$F_0(T)(1+r)^{-T} + p_0 - X(1+r)^{-T} = c_0.$$

4. Which of the following statements best describes a shareholder's claim in terms of an option payoff?

- A. Shareholder payoff resembles the payoff of a put option on firm value.
- B. Shareholder payoff resembles the payoff of a covered call option on firm value.
- C. Shareholder payoff resembles the payoff of a call option on firm value.

**Solution:**

C is correct. When considering shareholder claims in option terms, the shareholder payoff resembles a call option on firm value.

## PUT–CALL PARITY

## 2

☐ explain put–call parity for European options

A prior lesson contrasted no-arbitrage pricing conditions and the replication of cash flows for forward commitments and contingent claims. Forwards have zero initial value and their *certain* payoff, which is replicated at inception by borrowing to purchase the underlying or selling the underlying and lending the sale proceeds. Option buyers pay an upfront premium, and their *contingent* payoff profiles lead us to establish upper and lower no-arbitrage price bounds. Option replication is similar to that of a forward but involves borrowing or lending to buy or sell a *proportion* of the underlying, which is adjusted as the moneyness of an option changes. We now extend this analysis using a combination of positions.

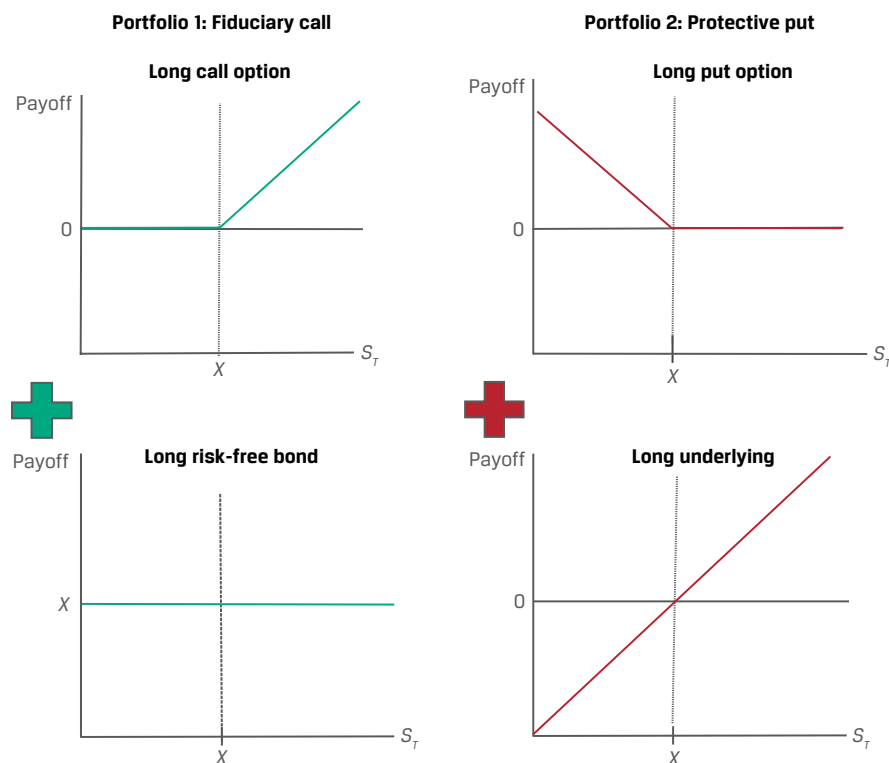
In this section, we show how combining cash and derivative instruments into a portfolio in a certain way enables us to price and value these positions without directly modeling them using no-arbitrage conditions. Consider an investor whose goal is to

benefit from upward movements in the value of an underlying but who wants to protect her investment from downward movements in the underlying's value. Consider the following two portfolios, shown in Exhibit 1:

1. At  $t = 0$ , an investor purchases a call option ( $c_0$ ) on an underlying with an exercise price of  $X$  and a risk-free bond today that pays  $X$  at  $t = T$ . The cost of this strategy is  $c_0 + X(1 + r)^{-T}$ , where we assume the option expires at time  $T$ .
2. At  $t = 0$ , an investor purchases an underlying unit ( $S_0$ ) and a put option on the underlying ( $p_0$ ) with an exercise price of  $X$  at  $t = T$ . The cost of this strategy is  $p_0 + S_0$ .

Exhibit 1 shows the payoff of the individual components of these two portfolios.

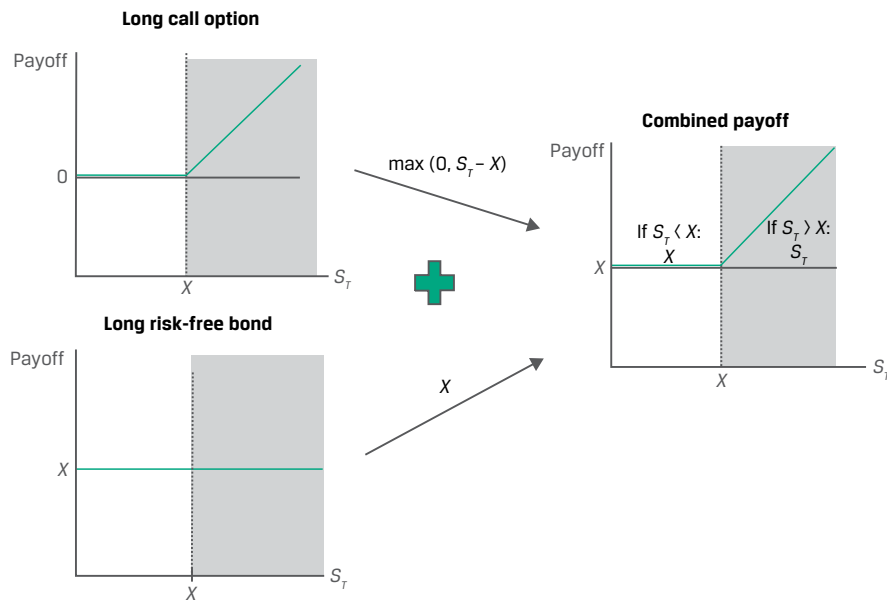
**Exhibit 1: Payoffs at Time  $T$  for Two Portfolios**



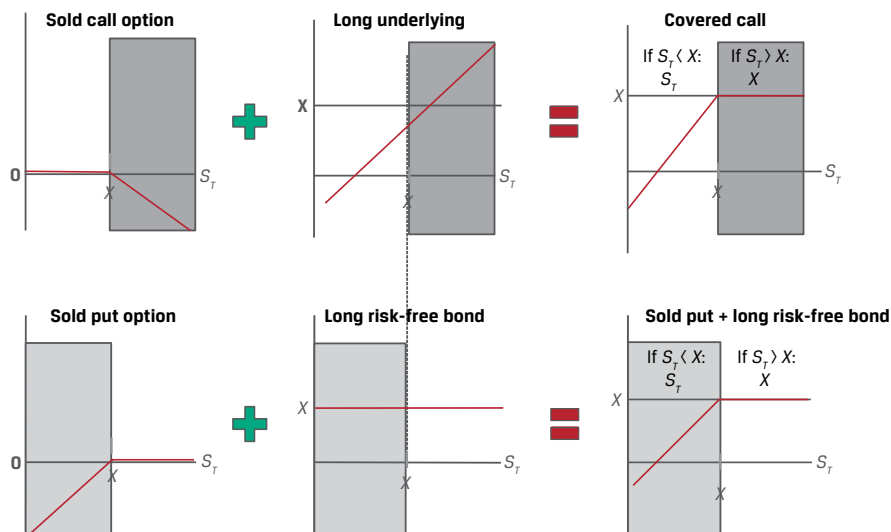
At first glance, these portfolios appear to offer the investor a similar opportunity to benefit from underlying asset appreciation without exposure to an underlying price decline below the exercise price.

In the first case (Portfolio 1), the investor buys a call option with a positive payoff if the underlying asset price rises above the exercise price ( $S_T > X$ ) and invests cash in a risk-free bond. Since the risk-free asset pays  $X$  at time  $T$ , the investor pays  $c_0 + X(1 + r)^{-T}$  at time  $t = 0$ . This combination of a purchased call and a risk-free bond is known as a **fiduciary call** and is shown in Exhibit 2.



**Exhibit 2: Portfolio 1 (Fiduciary Call) Payoff at Time  $T$** 

In the second instance (Portfolio 2) in Exhibit 1, the investor pays  $S_0 + p_0$  at inception and is hedged if the underlying price falls below  $X$ . This strategy of holding an underlying asset and purchasing a put on the same asset is sometimes called a **protective put**. The payoff for Portfolio 2 at time  $T$  is shown in Exhibit 3.

**Exhibit 3: Portfolio 2 (Protective Put) Payoff at Time  $T$** 

Despite their differences, these two portfolios have identical payoff profiles, as evidenced by Exhibit 2 and Exhibit 3. Recall that no-arbitrage conditions require two assets with identical future cash flows to trade at the same price, ignoring transaction costs. In Exhibit 4, we evaluate this in the case of the two portfolios by comparing their cash flows under all possible scenarios at time  $t = T$ .

#### Exhibit 4: Protective Put vs. Fiduciary Call at Expiration

Portfolio Position	Put Exercised ( $S_T < X$ )	No Exercise ( $S_T = X$ )	Call Exercised ( $S_T > X$ )
<b>Protective Put:</b>			
Underlying Asset	$S_T$	$S_T$	$S_T$
Put Option	$X - S_T$	0	0
<b>Total:</b>	$X$	$S_T (= X)$	$S_T$
<b>Fiduciary Call:</b>			
Call Option	0	0	$S_T - X$
Risk-Free Asset	$X$	$X$	$X$
<b>Total:</b>	$X$	$X (= S_T)$	$S_T$

These two portfolios have cash flows that are identical at time  $T$  under each scenario, and the prices of these portfolios must be equal at  $t = 0$  to satisfy the no-arbitrage condition in a relationship commonly referred to as **put–call parity**:

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

In other words, under put–call parity, at  $t = 0$  the price of the long underlying asset plus the long put must equal the price of the long call plus the risk-free asset.

#### EXAMPLE 1

##### Biomian Put–Call Parity

In an earlier example, the Viswan Family Office (VFO) held non-dividend-paying Biomian shares currently priced ( $S_0$ ) at INR295 per share. VFO is considering the purchase of a six-month put on Biomian shares at an exercise price,  $X$ , of INR265. If VFO's chief investment officer observes a traded six-month call option price of INR59 per share for the same INR265 exercise price, what should he expect to pay for the put per share if the relevant risk-free rate is 4%?

From Equation 1, the put–call parity relationship was shown as

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

We can solve for the risk-free bond price as INR259.85 ( $= \text{INR}265(1.04)^{-0.5}$ ) and substitute into Equation 1:

$$\text{INR}295 + p_0 = \text{INR}59 + \text{INR}259.85.$$

$$p_0 = \text{INR}23.85.$$

VFO should expect to pay a six-month put option premium of  $p_0 = \text{INR}23.85$ .

Arbitrage profit opportunities arise if these portfolios trade at different prices. As in the case of individual assets, an investor able to borrow and lend at the risk-free rate can earn a riskless profit if either portfolio is mispriced, as shown in the following example.

**EXAMPLE 2****VFO Put–Call Parity Arbitrage Opportunity**

As in Example 1, the Viswan Family Office holds non-dividend-paying Biomian shares at a price ( $S_0$ ) of INR295 per share. VFO is considering the purchase of a put for which it expects to pay INR23.85 but which is instead currently priced at INR30. Assuming a risk-free rate of 4%, identify the arbitrage opportunity and the steps VFO might take to earn a riskless profit.

From Equation 1, the put–call parity relationship is

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

Substituting the values from Example 1 and the current put price into Equation 1 gives us the following result:

$$\text{INR}295 + \text{INR}30 > \text{INR}59 + \text{INR}259.85,$$

$$\text{so, } S_0 + p_0 > c_0 + X(1 + r)^{-T}.$$

By selling the put and the shares and purchasing the call and the risk-free asset at  $t = 0$ , VFO has a positive cash flow of INR6.15 ( $= \text{INR}295 + \text{INR}30 - \text{INR}59 - \text{INR}259.85$ ), or  $S_0 + p_0 - c_0 - X(1 + r)^{-T}$ , as shown in the following diagram.

Arbitrage Position	Cash Flow at $t = 0$	Put Exercised ( $S_T < X$ )	No Exercise ( $S_T = X$ )	Call Exercised ( $S_T > X$ )
Protective Put:				
Sell Underlying Asset	$S_0$	$-S_T$	$-S_T$	$-S_T$
Sell Put Option	$p_0$	$-(X - S_T)$	0	0
Total:	$S_0 + p_0$	$-X$	$-S_T (= X)$	$-S_T$
Fiduciary Call:				
Buy Call Option	$-c_0$	0	0	$(S_T - X)$
Buy Risk-Free Asset	$-X(1 + r)^{-T}$	$X$	$X$	$X$
Total:	$-c_0 - X(1 + r)^{-T}$	$X$	$X (= S_T)$	$S_T$
Overall Portfolio:	$S_0 + p_0 - c_0 - X(1 + r)^{-T}$	0	0	0

Note that the combined cash flows of the two portfolios are equal to zero under each scenario at time  $t = T$ , leaving VFO with an arbitrage profit of INR6.15.

**OPTION STRATEGIES BASED ON PUT–CALL PARITY****3**

☐ explain put–call parity for European options

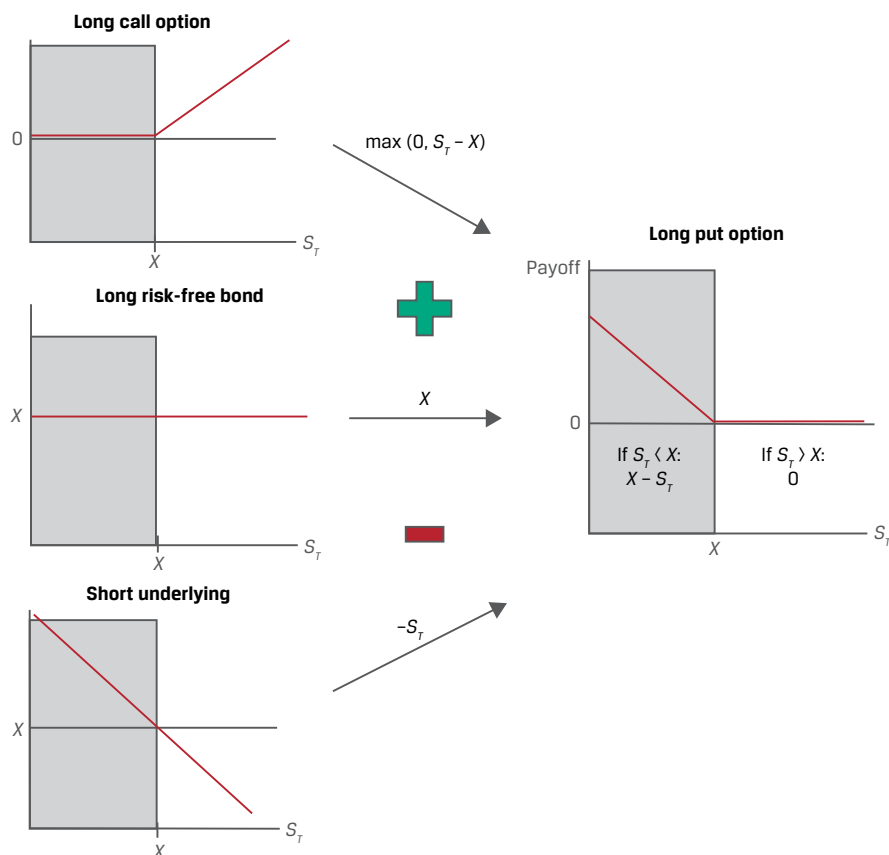
The put–call parity relationship established between call option, put option, underlying asset, and risk-free asset pricing in the previous section provides the foundation for thinking about replication and pricing of individual derivative positions, cash positions, and option-based strategies.

For example, if we rearrange Equation 1 to solve for the put option premium,  $p_0$ , we find that it may be derived from a combination of a long call option ( $c_0$ ), a long risk-free bond ( $X(1 + r)^{-T}$ ), and a short position in the underlying ( $-S_0$ ), as shown in Equation 2 and Exhibit 5.

$$p_0 = c_0 + X(1 + r)^{-T} - S_0. \quad (1)$$

Note that Equation 2 is both a statement of what the price of the put option should be and also sets out a replicating portfolio for that put option using a call option, the underlying, and a risk-free bond, as shown in Exhibit 5.

**Exhibit 5: Put Option as a Long Call, Long Bond, and Short Underlying**



What is more, we may also demonstrate that the asymmetric payoff profiles of put and call options effectively offset one another in a no-arbitrage condition when combined to solve for the price of the underlying, as follows:

$$S_0 = c_0 - p_0 + X(1 + r)^{-T}. \quad (2)$$

Exhibit 6 summarizes the equivalence of these individual positions in terms of replicating positions using other cash and derivative positions.

**Exhibit 6: Replication of Individual Positions under Put–Call Parity**

Position	Underlying ( $S_0$ )	Risk-Free Bond ( $X(1+r)^{-T}$ )	Call Option ( $c_0$ )	Put Option ( $p_0$ )
Underlying ( $S_0$ )	—	Long	Long	Short
Risk-free bond ( $X(1+r)^{-T}$ )	Long	—	Short	Long
Call option ( $c_0$ )	Long	Short	—	Long
Put option ( $p_0$ )	Short	Long	Long	—

These building blocks are used not only to generate riskless profits in the case of mispricing but also to create other option-based strategies, such as in the following example.

**EXAMPLE 3****VFO Covered Call Strategy**

Recall from an earlier example that the Viswan Family Office holds non-dividend-paying Biomian shares currently priced ( $S_0$ ) at INR295 per share. Since VFO's chief investment officer believes Biomian's share price will appreciate over the long term but remain relatively unchanged for the next six months, he would like to *sell* a six-month call option at a INR325 exercise price ( $X$ ) to generate short-term income in what is known as a covered call strategy.

Using put–call parity, how can he replicate this position using a risk-free bond (the risk-free rate is 4%) and a put option, and what is the expected call option premium if the put option has a price of INR56?

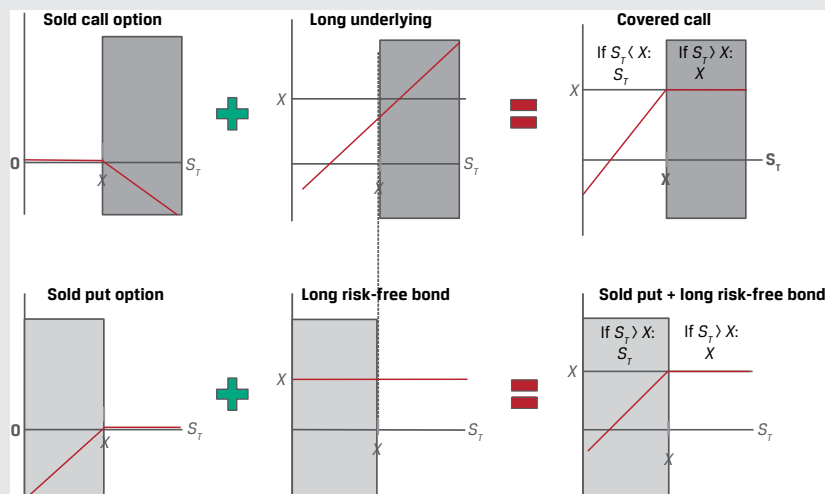
The covered call strategy consists of a *long* position in the underlying and a *short* call option, or ( $S_0 - c_0$ ) at inception. Recall from Equation 1 that put–call parity is shown as

$$S_0 + p_0 = c_0 + X(1+r)^{-T}.$$

We may rearrange these terms to solve for ( $S_0 - c_0$ ):

$$S_0 - c_0 = X(1+r)^{-T} - p_0.$$

The covered call position is therefore equivalent to a *long* risk-free bond and a *short* put option, as shown in the following diagram.



We may solve for the no-arbitrage call price,  $c_0$ , using put–call parity by substituting terms into the following equation:

$$S_0 - c_0 = X(1 + r)^{-T} - p_0.$$

$$\text{INR}295 - c_0 = \text{INR}325(1.04)^{-0.5} - \text{INR}56.$$

$$c_0 = \text{INR}32.31.$$

## QUESTION SET



### Put–Call Parity

1. Determine the correct answers to complete the following sentence: If two portfolios have cash flows that are \_\_\_\_\_ at time  $T$  under each scenario, then the no-arbitrage prices of those portfolios must \_\_\_\_\_ one another at  $t = 0$  in a relationship commonly referred to as put–call parity.

**Solution:**

If two portfolios have cash flows that are *identical* at time  $T$  under each scenario, then the no-arbitrage prices of those portfolios must *equal* one another at  $t = 0$  in a relationship commonly referred to as put–call parity.

2. Identify which of the following positions has the same no-arbitrage value as which portfolio under put–call parity:

- |  |  |
|--|--|
| 1. Short call option ( $-c_0$ )            | A. Short underlying, long risk-free bond, and long call option |
| 2. Long risk-free bond ( $X(1 + r)^{-T}$ ) | B. Short underlying, short put option, and long risk-free bond |
| 3. Long put option ( $p_0$ )               | C. Long underlying, short call option, and long put option     |

**Solution:**

Recall that the put-call parity relationship may be expressed as

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

1. B is correct. A short call option position is the no-arbitrage equivalent of a short underlying position, a short put option, and a long risk-free bond position.
2. C is correct. A long risk-free bond position is equivalent to a long underlying position, a short call option, and a long put option.
3. A is correct. A long put option is equivalent to a short underlying position, a long risk-free bond, and a long call option.

3. Determine the correct answer to complete the following sentence: The combination of a \_\_\_\_\_ call and a risk-free bond position is known as a fiduciary call.

**Solution:**

The combination of a *purchased* call and a risk-free bond position is known as a fiduciary call.

4. Identify the following statement as true or false, and justify your answer:  
The covered call strategy consists of a long position in the underlying and a long call option, or  $(S_0 + c_0)$  at inception.

**Solution:**

The statement is false. The covered call strategy consists of a *long* position in the underlying and a *short* call option, or  $(S_0 - c_0)$  at inception.

## PUT–CALL FORWARD PARITY AND OPTION APPLICATIONS

4

- ☐ | explain put–call *forward* parity for European options

Forward commitment replication was shown in earlier lessons to involve borrowing at the risk-free rate to purchase the underlying in the case of a long position or selling the underlying short and lending the proceeds at the risk-free rate for a short position. We also showed that a long underlying position combined with a short forward resulted in a risk-free return. We now incorporate these forward commitment building blocks into the put–call parity relationship shown in the prior lesson.

## PUT–CALL FORWARD PARITY

5

- ☐ | explain put–call *forward* parity for European options

In an earlier lesson, we learned that a long underlying asset position can be replicated by entering a long forward contract and purchasing the risk-free asset. Combining the synthetic asset with the put–call parity relationship—so, substituting the present value of  $F_0(T)$  for  $S_0$  in Equation 1—we have what is referred to as **put–call forward parity**:

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}. \quad (3)$$

We can demonstrate put–call forward parity by comparing a synthetic protective put position to the protective put and fiduciary call positions. Consider a modification of the portfolios used earlier to demonstrate put–call parity, as follows:

1. At  $t = 0$ , an investor purchases a forward contract and a risk-free bond with a face value equal to the forward price,  $F_0(T)$ , and a put option on the underlying ( $p_0$ ) with an exercise price of  $X$  at  $t = T$ .
2. At  $t = 0$ , an investor purchases a call option ( $c_0$ ) on the same underlying with an exercise price of  $X$  and a risk-free bond that pays  $X$  at  $t = T$ .

The first portfolio has replaced the *cash* underlying position with a *synthetic* underlying position using a forward purchase and a risk-free bond. The combination of the synthetic underlying position and a purchased put on the underlying is known as a **synthetic protective put**. Exhibit 7 demonstrates the equivalence of the protective and synthetic protective puts by comparing their cash flows at both  $t = 0$  and contract maturity,  $T$ .

**Exhibit 7: Protective Put vs. Synthetic Protective Put**

Position	Cash Flow at $t = 0$	Put Exercised ( $S_T < X$ )	No Exercise ( $S_T \geq X$ )
<b>Protective Put:</b>			
Purchased Put ( $p_0$ )	$p_0$	$X - S_T$	0
Cash Underlying ( $S_0$ )	$S_0$	$S_T$	$S_T$
<b>Total:</b>	$p_0 + S_0$	$X$	$S_T$
<b>Synthetic Protective Put:</b>			
Purchased Put ( $p_0$ )	$p_0$	$X - S_T$	0
Forward Purchase	0	$S_T - F_0(T)$	$S_T - F_0(T)$
Risk-Free Bond $F_0(T)(1 + r)^{-T}$	$F_0(T)(1 + r)^{-T}$	$F_0(T)$	$F_0(T)$
<b>Total:</b>	$p_0 + F_0(T)(1 + r)^{-T}$ ( $= p_0 + S_0$ )	$X$	$S_T$

Exhibit 8 compares the future cash flows of the synthetic protective put with the fiduciary call under all possible scenarios at expiration.

**Exhibit 8: Synthetic Protective Put vs. Fiduciary Call**

Portfolio Position	Put Exercised ( $S_T < X$ )	No Exercise ( $S_T = X$ )	Call Exercised ( $S_T > X$ )
<b>Synthetic Protective Put:</b>			
Purchased Put ( $p_0$ )	$X - S_T$	0	0
Forward Purchase	$S_T - F_0(T)$	$S_T - F_0(T)$	$S_T - F_0(T)$
Risk-Free Bond $F_0(T)(1 + r)^{-T}$	$F_0(T)$	$F_0(T)$	$F_0(T)$
<b>Total:</b>	$X$	$S_T (= X)$	$S_T$
<b>Fiduciary Call:</b>			
Purchased Call ( $c_0$ )	0	0	$S_T - X$
Risk-Free Asset	$X$	$X$	$X$
<b>Total:</b>	$X$	$X (= S_T)$	$S_T$

It follows that the cost of the fiduciary call must equal the cost of the synthetic protective put, thereby demonstrating the put–call forward parity relationship:

$$F_0(T)(1 + r)^{-T} + p_0 = c_0 + X(1 + r)^{-T}.$$

If we rearrange these terms, we can demonstrate that a long put and a short call are equivalent to a long risk-free bond and short forward position:

$$p_0 - c_0 = [X - F_0(T)](1 + r)^{-T}. \quad (4)$$

Consider the earlier put–call parity example using a synthetic underlying position, as in the following example.



**EXAMPLE 4****VFO Put–Call Forward Parity**

Consider the Viswan Family Office example using a long forward and a risk-free bond, rather than a cash underlying position as in the prior example. Biomian shares trade at a price ( $S_0$ ) of INR295 per share. VFO is considering the purchase of a six-month put on Biomian shares at an exercise price ( $X$ ) of INR265. If VFO's chief investment officer observes a traded six-month call option price of INR59 per share for the same INR265 exercise price, what should he expect to pay for the put option per share if the relevant risk-free rate is 4%?

From Equation 5, the put–call forward parity relationship is

$$p_0 - c_0 = [X - F_0(T)](1 + r)^{-T}.$$

Substituting terms and solving for  $F_0(T) = \text{INR}300.84 (= \text{INR}295(1.04)^{0.5})$ ,

$$p_0 - \text{INR}59 = (\text{INR}265 - \text{INR}300.84)(1.04)^{-0.5}.$$

$$p_0 = \text{INR}23.86.$$

VFO should expect to pay a six-month put option premium of  $p_0 = \text{INR}23.86$ .

## OPTION PUT–CALL PARITY APPLICATIONS: FIRM VALUE

# 6

- ☐ explain put–call parity for European options
- ☐ explain put–call *forward* parity for European options

The insights established by the put–call parity relationship go well beyond option trading strategies, extending to modeling the value of a firm to describe the interests and financial claims of capital providers—namely, the owners of a firm's equity and the owners of its debt.

Assume that at time  $t = 0$ , a firm with a market value of  $V_0$  has access to borrowed capital in the form of zero-coupon debt with a face value of  $D$ . The market value of the firm's assets,  $V_0$ , is equal to the present value of its outstanding debt obligation,  $PV(D)$ , and equity,  $E_0$ :  $V_0 = E_0 + PV(D)$ .

When the debt matures at  $T$ , the firm's debt and assets are distributed between shareholders and debtholders with two possible outcomes depending on the firm's value at time  $T$  ( $V_T$ ):

1. **Solvency ( $V_T > D$ ):** If the value of the firm ( $V_T$ ) *exceeds* the face value of the debt, or  $V_T > D$ , at time  $T$ , we say the firm is *solvent* and able to return capital to *both* its shareholders and debtholders.
  - Debtholders receive  $D$  and are repaid in full.
  - Shareholders receive the residual:  $E_T = V_T - D$ .

**2. Insolvency ( $V_T < D$ ):** If the value of the firm ( $V_T$ ) is *below* the face value of the debt, or  $V_T < D$ , at time  $T$ , we say the firm is *insolvent*. In the event of insolvency, shareholders receive nothing and debtholders are owed more than the value of the firm's assets. Debtholders therefore receive  $V_T$  to settle their debt claim of  $D$  at time  $T$ .

- Debtholders have a priority claim on assets and receive  $V_T < D$ .
- Shareholders receive the residual,  $E_T = 0$ .

Unlike the *risk-free* bond shown in the prior put–call parity lesson, the firm has *risky* debt, because the bondholders receive  $D$  *only* in the case of solvency (when  $V_T > D$ ). Debtholders therefore demand a premium similar to a put option premium from shareholders in order to assume the risk of insolvency.

Shareholders retain *unlimited* upside potential (if the firm remains solvent and can pay off its debt) and *limited* downside potential (if the firm becomes insolvent). In contrast, debtholder upside is *limited* to receiving debt repayment in the event of solvency, and principal and interest are at risk in the downside event of insolvency.

Next, we examine the respective payoff profiles at maturity more closely. At  $t = T$ ,

- shareholder payoff is  $\max(0, V_T - D)$  and
- debtholder payoff is  $\min(V_T, D)$ .

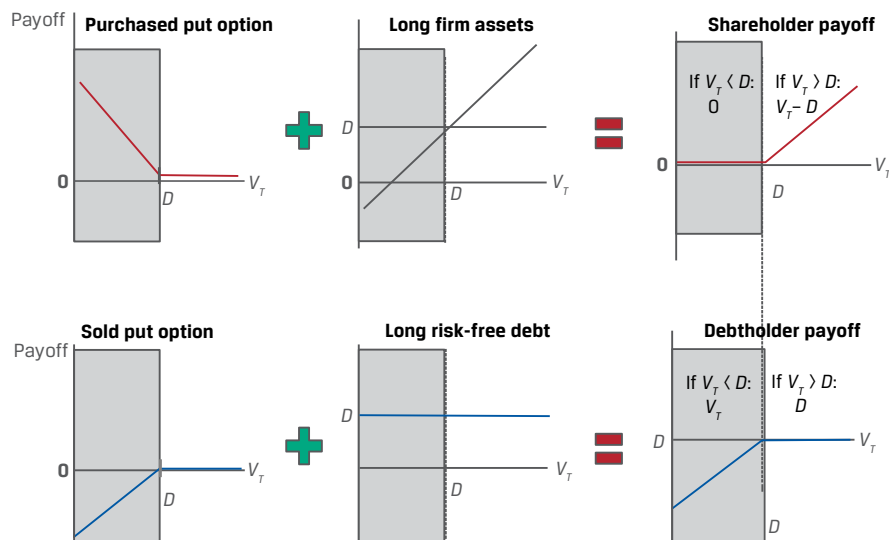
Consider these payoff profiles in terms of options:

- Shareholders hold a long position in the underlying firm's assets ( $V_T$ ) and have purchased a put option on firm value ( $V_T$ ) with an exercise price of  $D$ ; that is,  $\max(0, D - V_T)$ .
- Debtholders hold a long position in a risk-free bond ( $D$ ) and have sold a put option to shareholders on firm value ( $V_T$ ) with an exercise price of  $D$ .

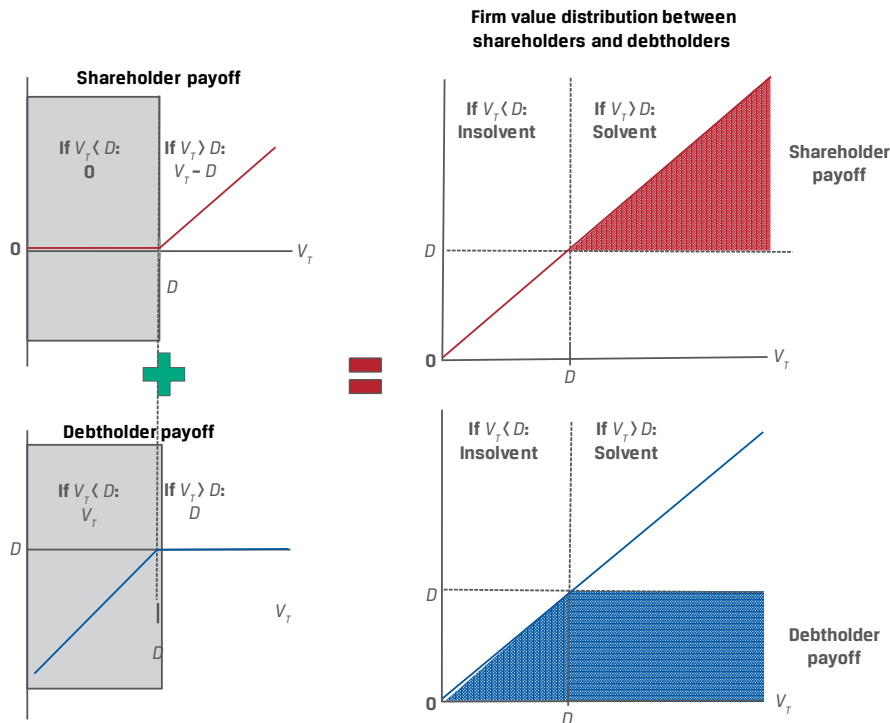
The payoff profiles for shareholders versus debtholders is shown in Exhibit 9.

#### Exhibit 9: Shareholder and Debtholder Payoff at Time $T$

##### Construction of Shareholder and Debtholder Payoffs



## Firm Value Distribution Between Shareholders and Debtholders



Note that the shareholder's combination of a purchased put option and a long position in the firm's assets is equivalent to a *call* option on the firm's assets. The risky debt held by the debtholders is a combination of the risk-free debt,  $D$ , and the put option sold to shareholders.

Revisiting the put–call parity relationship,  $S_0 + p_0 = c_0 + PV(X)$ , from Equation 1, we may substitute the value of the firm at time 0 ( $V_0$ ) for the underlying asset ( $S_0$ ), substitute the debt ( $D$ ) for the risk-free bond ( $X$ ), and solve for  $V_0$  as follows:

$$V_0 + p_0 = c_0 + PV(D).$$

$$V_0 = c_0 + PV(D) - p_0. \quad (5)$$

Equation 6 captures the value of the firm's assets at  $t = 0$  from a shareholder and debtholder perspective. The *shareholder* has a position with a payoff similar to that of a call option on firm value ( $c_0$ ). The *debtholder* has a position of  $PV(D) - p_0$ , or risk-free debt of the firm *plus* a sold put option on firm value. The put option ( $p_0$ ) may be interpreted as the *credit spread* on the firm's debt, or the premium above the risk-free rate the firm must pay to debtholders to assume insolvency risk. This put option increases in value to shareholders as the likelihood of insolvency increases. From a debtholder's perspective, the more valuable the sold put, the more credit risk is present in the firm's debt.

## QUESTION SET



## Put–Call Forward Parity and Option Applications

1. Identify the following statement as true or false, and justify your answer: A key link between put–call parity and put–call forward parity is that the cash underlying position is replaced by a synthetic underlying position using a forward purchase and a risk-free bond.

**Solution:**

The statement is true. The replacement of the cash underlying position by a synthetic underlying position using a forward purchase and a risk-free bond links call option, put option, and forward prices under put–call forward parity.

2. Identify which of the following positions has the same no-arbitrage value as which portfolio under put–call forward parity:

- |  |  |
|--|--|
| 1. Long call option ( $c_0$ )              | A. Forward sale, long risk-free bond, and long call option       |
| 2. Long risk-free bond ( $X(1 + r)^{-T}$ ) | B. Forward purchase, long put option, and short risk-free bond   |
| 3. Long put option ( $p_0$ )               | C. Long forward purchase, short call option, and long put option |

**Solution:**

Recall that the put–call forward parity relationship may be expressed as

$$F_0(T)(1 + r)^{-T} + p_0 = c_0 + X(1 + r)^{-T}.$$

1. B is correct. A long call option position is the no-arbitrage equivalent of a forward purchase, a long put option, and a short risk-free bond position.
2. C is correct. A long risk-free bond position is equivalent to a long forward purchase, a short call option, and a long put option.
3. A is correct. A long put option is equivalent to a forward sale, a long risk-free bond, and a long call option.

3. Describe the cash flows at the time of option expiration for a synthetic protective put.

**Solution:**

A synthetic protective put is the combination of a *synthetic* underlying position (using a forward purchase and a long risk-free bond position equal to the exercise price) and a purchased put.

At the time of option expiration, time  $T$ , there are two possible scenarios:

**Put option exercised ( $X > S_T$ ):** If the put option is exercised, an investor receives  $(X - S_T)$ . The risk-free asset returns  $F_0(T)$ , and the forward purchase returns  $S_T - F_0(T)$ .

The total return equals  $(X - S_T) + [S_T - F_0(T)] + F_0(T) = X$ .

**Put option unexercised ( $X \leq S_T$ ):** If the put option is unexercised, it will expire worthless. The risk-free asset returns  $F_0(T)$ , and the forward purchase returns  $S_T - F_0(T)$ .

The total return equals  $[S_T - F_0(T)] + F_0(T) = S_T$ .

4. Describe how a debtholder's position may be considered similar to the sale of a put option on firm value.

**Solution:**

If the value of the firm ( $V_T$ ) is *below* the face value of its debt outstanding, or  $V_T < D$  at time  $T$ , we say the firm is *insolvent* and debtholders receive less than the face value ( $D$ ) to settle their debt claim. Stated differently, a debtholder's payoff is  $\min(D, V_T) = D - \max(0, D - V_T)$  and equals the debt face value ( $D$ ) minus a put option on firm value ( $V_T$ ) with an exercise price of  $D$ , which represents a *sold* put on firm value.

## PRACTICE PROBLEMS

### The following information relates to questions 1–6

#### South China Sprintwyck Investments (SCSI)

South China Sprintwyck Investments (SCSI) has a Chinese equity portfolio that has outperformed in the first half of the year due to an overweight position in health care industry shares. SCSI is considering option-based alternatives for one of its current overweight positions, ChinaWell Inc. (CWI). CWI has a current price ( $S_0$ ) of CNY127.50 and pays no dividends. The current risk-free rate is 4%. You are a new SCSI analyst hired to evaluate several alternatives for CWI stock.

1. SCSI is considering the sale of a three-month call on CWI shares. If you observe a traded put price with the same underlying details, which of the following statements best describes how to derive the no-arbitrage call price?
  - A. Using put–call parity, the call price may be derived by *subtracting* the traded put price from the underlying price less the present value of the exercise price.
  - B. Using put–call parity, the call price may be derived by *adding* the traded put price to the underlying price less the present value of the exercise price.
  - C. Using put–call parity, the call price may be derived by *adding* the traded put price to the present value of the exercise price minus the underlying price.
2. Due to the recent sharp rise in CWI's share price, SCSI is recommending that clients consider protective put strategies for three- to six-month tenors. If you observe that a six-month CWI call option at an exercise price of CNY120 is trading at a price of CNY22.60, which answer is closest to what you would expect to pay for a six-month put option with the same underlying terms?
  - A. CNY15.10
  - B. CNY9.83
  - C. CNY12.77
3. You realize that the six-month put option on CWI shares is overpriced relative to the no-arbitrage price from Question 2. Which of the following statements best describes the steps you would take to earn a riskless arbitrage profit under this scenario?
  - A. Sell the six-month put option and sell CWI short, investing the proceeds in a call option and a risk-free bond.
  - B. Sell the six-month put option, buy a call option, and borrow at the risk-free rate to buy CWI shares.
  - C. Sell the six-month put option, buy a call option, enter a forward purchase of CWI, and invest in a risk-free bond.

4. In her most recent research note on CWI, SCSI's equity analyst specifically mentions an increase in CWI's leverage ratio as a reason for her bearish outlook on the stock. Applying the put-call parity relationship to the value of the firm, which of the following statements most accurately describes the CWI outlook in terms of option pricing?
- A. The shareholder payoff has improved versus debtholders, because they have sold a put option on the firm value that has appreciated.
  - B. The debtholder payoff has deteriorated versus the shareholders, because they are effectively short a put option on the value of the firm equal to the value of debt, which has appreciated in value.
  - C. The shareholder payoff has improved versus the debtholders, because the debtholders have sold a call option on the firm's assets to the shareholders.
5. Which of the following choices best describes how SCSI could replicate a long risk-free bond return using a forward contract on CWI and call and put options on CWI?
- A. Short a forward contract on CWI, short a put option on CWI, and long a call option on CWI
  - B. Long a forward contract on CWI, long a put option on CWI, and short a call option on CWI
  - C. Long a forward contract on CWI, long a put option on CWI, and long a call option on CWI
6. SCSI observes that a 3-month call option on CWI with an exercise price of CNY 130 trades at a premium of CNY 3. A 3-month forward contract on CWI trades at a forward price of CNY 128.76. Which of the following choices is closest to the correct premium for a 3-month put option with an exercise price of CNY 130 on CWI?
- A. CNY 4.20
  - B. CNY 1.20
  - C. CNY 0.50
-

## SOLUTIONS

1. The correct answer is B. From Equation 1, the put–call parity relationship is

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

We can rearrange these terms to solve for  $c_0$ :  $c_0 = S_0 + p_0 - X(1 + r)^{-T}$ , which shows that the call price may be derived by *adding* the traded put price to the underlying price less the present value of the exercise price.

2. The correct answer is C. If  $c_0 = \text{CNY}22.60$ ,  $S_0 = \text{CNY}127.50$ , and  $PV(X) = \text{CNY}117.67 (= 120(1.04)^{-0.5})$ , we can solve for  $p_0$  using put–call parity:

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

$$p_0 = c_0 + X(1 + r)^{-T} - S_0.$$

$$\text{CNY}12.77 = 22.60 + 117.67 - 127.50.$$

3. The correct answer is A. Since the put option is overpriced, we would sell it to earn the difference between the price at which it is sold and the no-arbitrage price. The put–call parity relationship, from Equation 1, is

$$S_0 + p_0 = c_0 + X(1 + r)^{-T}.$$

We can rearrange this to demonstrate that the put option value is equivalent to a long call option, a long risk-free bond, and a short position in CWI shares:

$$p_0 = c_0 + X(1 + r)^{-T} - S_0.$$

Answer A reflects this long risk-free bond and short CWI combination, which has a payoff of  $X - S_T$  at expiration matching that of the put payoff, whereas Answer B involves a long cash position in CWI and Answer C involves a long synthetic (forward purchase) position in CWI stock.

4. The correct answer is B. The debtholder payoff has deteriorated versus the shareholders, because they are effectively short a put option on the value of the firm equal to the value of debt, which has appreciated in value.

The debtholder payoff is  $\min(D, V_T) = D - \max(0, D - V_T)$  and equals the debt face value ( $D$ ) *minus* a put option on firm value ( $V_T$ ) with an exercise price of  $D$ . Answer A is incorrect because the debtholders, not the shareholders, have sold a put option. C is incorrect, because the shareholders own a call option, but it is not sold by the debtholders.

5. B is correct. Using the put–call forward parity equation as follows:

$$F_0(T)(1 + r)^{-T} + p_0 = c_0 + X(1 + r)^{-T}$$

And rearranging to solve for the risk-free bond gives the following:

$$F_0(T)(1 + r)^{-T} + p_0 - c_0 = X(1 + r)^{-T}.$$

The signs on the forward contract and put option positions are both positive, indicating long positions. The negative sign on the call option indicates a short position.

6. A is correct. The solution can be derived through either put–call parity or put–call forward parity. You can demonstrate the solution using put–call forward parity:

$$F_0(T)(1 + r)^{-T} + p_0 = c_0 + X(1 + r)^{-T}$$

$$128.76(1.04)^{-0.25} + p_0 = 3 + 130(1.04)^{-0.25}$$



$$127.50 + p_0 = 131.73$$

$$p_0 = 4.23.$$



## LEARNING MODULE

## 10

## Valuing a Derivative Using a One-Period Binomial Model

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain how to value a derivative using a one-period binomial model
<input type="checkbox"/>	describe the concept of risk neutrality in derivatives pricing

## INTRODUCTION

## 1

Earlier lessons explained how the principle of no arbitrage and replication can be used to value and price derivatives. The put–call parity relationship linked put option, call option, underlying asset, and risk-free asset prices. This relationship was extended to forward contracts given the equivalence of an underlying asset position and a long forward contract plus a risk-free bond.

Forward commitments can be priced without making assumptions about the underlying asset's price in the future. However, the pricing of options and other contingent claims requires a model for the evolution of the underlying asset's future price. The first lesson introduces the widely-used binomial model to value European put and call options. A simple one-period version is introduced, which may be extended to multiple periods and used to value more complex contingent claims. In the second lesson, we demonstrate the use of risk-neutral probabilities in derivatives pricing.

## LEARNING MODULE OVERVIEW



- The one-period binomial model values contingent claims, such as options, and assumes the underlying asset will either increase by  $R^u$  (up gross return) or decrease by  $R^d$  (down gross return) over a single period that corresponds to the expiration of the derivative contract.
- The binomial model combines an option with the underlying asset to create a risk-free portfolio where the proportion of the option to the underlying security is determined by a hedge ratio.
- The hedged portfolio must earn the prevailing risk-free rate of return; otherwise, riskless arbitrage profit opportunities would be available.

CFA Institute would like to thank Don Chance, PhD, CFA, for his contribution to this section, which includes material derived from material that appeared in *Derivative Markets and Instruments*, featured in the 2022 CFA® Program curriculum.

- Valuing a derivative via risk-free hedging is equivalent to computing the discounted expected payoff of the option using *risk-neutral* probabilities rather than actual probabilities.
- Neither the actual (real-world) probabilities of underlying price increases or decreases nor the expected return of the underlying are required to price an option.
- The one-period binomial model can be extended to multiple periods as well to value more complex contingent claims.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following statements most correctly describes the binomial model for valuing options?
  - A. The model uses the actual probabilities associated with stock price moves up or down.
  - B. The model assumes that a risk-free portfolio can be created by combining the option and the underlying according to a hedge ratio.
  - C. The model is similar to those used for valuing forward and futures contracts.

**Solution:**

B is correct. Unlike forward commitments, contingent claims, such as options, require that we model the future price behavior of the underlying asset because unlike forwards and futures, options have asymmetric payoffs. By modeling the future price behavior, the option and its underlying asset can be combined into a risk-free portfolio. The cost of this portfolio, where the proportion of the option and the underlying asset is set by a hedge ratio, determines the no-arbitrage price of the option. A is incorrect because the actual probabilities of up and down price moves do not factor into the model. C is incorrect because options have asymmetric payoffs, so they must be modeled differently than symmetric-payoff instruments like forward and futures contracts.

2. When using a one-period binomial model to price a call option, an increase in the actual probability of an upward move in the underlying asset will result in the call option price:
  - A. decreasing.
  - B. staying the same.
  - C. increasing.

**Solution:**

The correct answer is B. The call option price will stay the same. The actual (real-world) probabilities of an up or a down price movement in a binomial model do not influence the (no-arbitrage) price of an option.

3. Identify which of the various factor changes has which effect on the no-arbitrage price of a put option based on the one-period binomial model:

- |  |                                      |
|--|--------------------------------------|
| 1. The probability of an upward price movement, $q$ , increases.       | A. Put option price remains the same |
| 2. The spread between the up and down factor, $R^u - R^d$ , increases. | B. Put option price increases        |
| 3. The risk-neutral probability of price, $\pi$ , increases.           | C. Put option price decreases        |

**Solution:**

- The correct answer is A. The probability of an upward price movement,  $q$ , has no impact on value in the one-period binomial option pricing model. Thus, this change would not have any impact on the price of a put option, and the price of the put option would remain the same.
- The correct answer is B. The spread between the up and down factor,  $R^u - R^d$ , increases the range of potential prices, which increases the likelihood that the option ends up in the money. Thus, this change would increase the price of a put option.
- The correct answer is C. The risk-neutral probability of price,  $\pi$ , captures the probability of the price of the underlying increasing. As  $\pi$  increases, the likelihood of the put option ending up in the money decreases.

4. A one-period binomial model assumes that the price of the underlying asset can change from \$16.00 today to either \$20.00 or \$12.00 at the end of the period. If the risk-free rate of return over the period is 5%, which of the following choices is closest to the risk-neutral probability of a price increase?

- A. 0.50  
B. 0.60  
C. 0.625

**Solution:**

B is correct. An increase from \$16.00 to \$20.00 or a decrease from \$16.00 to \$12.00 corresponds to:

$$R^u = \$20.00/\$16.00 = 1.25 \text{ and } R^d = \$12.00/\$16.00 = 0.75.$$

Using the risk-neutral probability ( $\pi$ ) of a price increase:

$$\begin{aligned} \pi &= (1 + r - R^d)/(R^u - R^d) \\ &= (1 + 0.05 - 0.75)/(1.25 - 0.75) = 0.3/0.5 = 0.60. \end{aligned}$$

## BINOMIAL VALUATION

## 2



explain how to value a derivative using a one-period binomial model

The law of one price states that if the payoffs from any two assets (or portfolio of assets) at a given future time are identical in all possible scenarios, then the value of these two assets must also be identical *today*. Forward commitments offer symmetric payoffs at a predetermined price in the future, the value of which are independent of the future price behavior of the underlying asset.

The asymmetric payoff profile of options and other contingent claims makes valuation of these instruments more challenging. Assumptions about future prices are an important component in option valuation given the different payoffs under different scenarios whose likelihood changes over time. Option valuation therefore requires the specification of a model for the future (random) price behavior of the option's underlying asset.

The binomial model is a common tool used to determine the no-arbitrage value of an option. The simplicity of this model makes it attractive, as we only need to make an assumption about the magnitude of the potential upward and downward price changes of the underlying asset in a future time period.

### 3

## THE BINOMIAL MODEL



explain how to value a derivative using a one-period binomial model

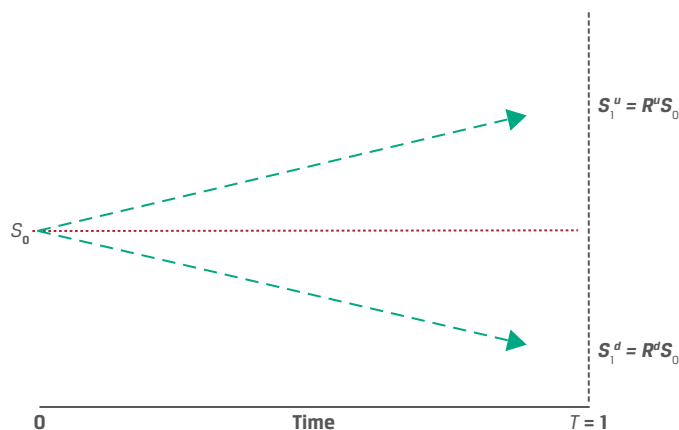
The binomial model builds on a simple idea: Over a given period of time, the asset's price will either go up ( $u$ ) to  $S_1^u > S_0$  or go down ( $d$ ) to  $S_1^d < S_0$ . We do not need to know the future price in advance, because it is determined by the outcome of a random variable. The movement from  $S_0$  to either  $S_1^u$  or  $S_1^d$  can be interpreted as the outcome of a *Bernoulli trial*.

Let us denote  $q$  as the probability of an upward price movement and  $1 - q$  as the probability of a downward price movement. With only two possible outcomes—the price either goes up or down—the sum of probabilities must equal 1. We will also find it useful to define the gross return from an up or a down price move as:

$$R^u = S_1^u / S_0 > 1 \quad (1)$$

$$R^d = S_1^d / S_0 < 1 \quad (2)$$

At first glance, it would appear that knowing  $q$  is crucial in determining the value of an option on any underlying asset. However, knowing  $q$  is not required; only specifying the values of  $S_1^u$  and  $S_1^d$  is needed. The difference between  $S_1^u$  and  $S_1^d$  measures the “spread” of possible future price outcomes. Specifying  $S_1^u$  and  $S_1^d$  (or  $R^u S_0$  and  $R^d S_0$ ) determines the *volatility* of the underlying asset, an important factor in valuing options as shown earlier. Simply stated, the size of the up and down price movements should match the underlying asset volatility, as shown in Exhibit 1.

**Exhibit 1: Price Movement for the Underlying Asset**

Binomial models may be extended to multiple periods where the underlying asset price can move up or down in each period. This extension creates a binomial tree, a powerful way to model more realistic price dynamics. A simple one-period binomial model is sufficient to introduce the pricing methodology and the required steps in the option pricing procedure.

**PRICING A EUROPEAN CALL OPTION****4**

explain how to value a derivative using a one-period binomial model

Consider a one-year European call option with an exercise price ( $X$ ) of €100. The underlying spot price ( $S_0$ ) is €80. The one-period binomial model corresponds to the option's time to expiration of one year.

The binomial model specifies the possible values of the underlying asset in one year, where the option value is a known function of the value of the underlying asset. Further, we assume that  $S_1^d < X < S_1^u$  (i.e., the exercise price of the option [ $X = €100$ ] is between the value of the underlying in the two scenarios)—for example, setting  $S_1^d = €60$  and  $S_1^u = €110$ . Then,  $R^u = 1.375$  and  $R^d = 0.75$ . The value of the call option is as follows:

- At  $t = 0$ , the call option value is  $c_0$ .
- At  $t = 1$ , the call option value is either  $c_1^u$  (if the underlying price rises to  $S_1^u$ ) or  $c_1^d$  (if the underlying price falls to  $S_1^d$ ).

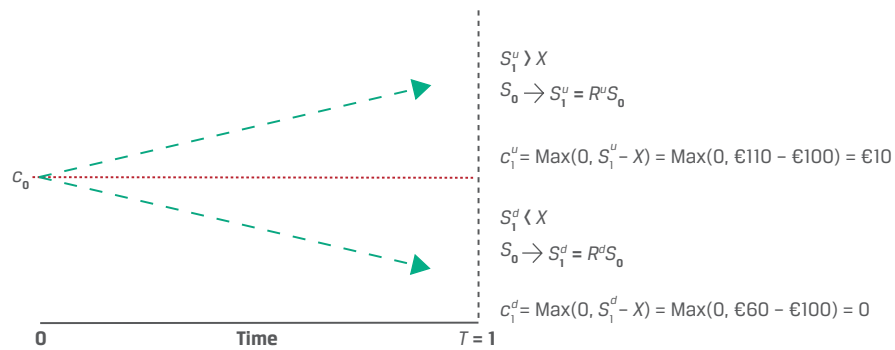
- **Up move to  $S_1^u$ :** Call option ends up *in-the-money*

$$c_1^u = \text{Max}(0, S_1^u - X) = \text{Max}(0, €110 - €100) = €10.$$

- **Down move to  $S_1^d$ :** Call option expires *out-of-the-money*

$$c_1^d = \text{Max}(0, S_1^d - X) = \text{Max}(0, €60 - €100) = 0.$$

Exhibit 2 summarizes the one-period binomial model and the value of the underlying asset and call option.

**Exhibit 2: One-Period Binomial Option Pricing**

The only unknown value is  $c_0$ , which may be determined using replication and no-arbitrage pricing. That is, the value of both the option and its underlying asset in each future scenario may be used to construct a risk-free portfolio. For example, assume that at  $t = 0$  we sell the call option at a price of  $c_0$  and purchase  $h$  units of the underlying asset. Denoting the value of the portfolio as  $V$ , we have the following:

$$V_0 = hS_0 - c_0 \quad (3)$$

$$V_1^u = hS_1^u - c_1^u = h \times R^u \times S_0 - \text{Max}(0, S_1^u - X) \quad (4)$$

$$V_1^d = hS_1^d - c_1^d = h \times R^d \times S_0 - \text{Max}(0, S_1^d - X) \quad (5)$$

$V_0$  represents the initial portfolio investment in the portfolio, and  $V_1^u$  and  $V_1^d$  represent the portfolio value if the underlying price moves up or down, respectively. We must choose  $h$  so that  $V_1^u = V_1^d$  (i.e., the portfolio value is the same in either scenario). The value of the combination,  $V_1$ , will not change if the underlying asset price changes. For portfolio  $V_0$ , the impacts of the changes are as follows:

■ **Up move from  $S_0$  to  $S_1^u$  by  $R^u$ :**

- The asset value changes from €80 to €110 by 1.375, or 37.5%.
- The call option ends up *in-the-money*:

$$c_1^u = \text{Max}(0, S_1^u - X) = \text{Max}(0, €110 - €100) = €10.$$

- Total portfolio value:  $V_1^u = hS_1^u - c_1^u = €110 \times h - €10$ .

■ **Down move from  $S_0$  to  $S_1^d$  by  $R^d$ :**

- The asset value changes from €80 to €60 by 0.75, or -25%.
- The call option expires *out-of-the-money*:

$$c_1^d = \text{Max}(0, S_1^d - X) = \text{Max}(0, €60 - €100) = 0.$$

- Total portfolio value:  $V_1^d = hS_1^d - c_1^d = €60 \times h$ .

Since we create two portfolios at time  $t = 0$  with identical payoffs at option expiry at time  $t = 1$ , we must solve for  $h$ , the ratio between the underlying asset,  $S_0$ , and the call option,  $c_0$ , such that  $V_1^u = V_1^d$ , or  $hS_1^u - c_1^u = hS_1^d - c_1^d$ . Solving for  $h^*$  yields:

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d} \quad (6)$$



where all quantities on the right-hand side are known at  $t = 0$ . Equation 4 gives us the **hedge ratio** of the option, or the *proportion* of the underlying that will offset the risk associated with an option. In our sold call option example,

$$h^* = \frac{c_1^u - c_1^d}{s_1^u - s_1^d} = \frac{€10 - €0}{€110 - €60} = \frac{10}{50} = 0.20.$$

For each call option unit sold, we buy 0.2 units of the underlying asset (or for each underlying asset unit, we must sell 5 call options to equate the portfolio values at  $t = 1$ ). Consider the two scenarios as follows:

▪ **Up move from  $S_0$  to  $S_1^u$  by  $R^u$ :**

- Total portfolio value:  $V_1^u = hS_1^u - c_1^u$

$$= €110 \times 0.20 - €10 = €22 - 10 = €12.$$

▪ **Down move from  $S_0$  to  $S_1^d$  by  $R^d$ :**

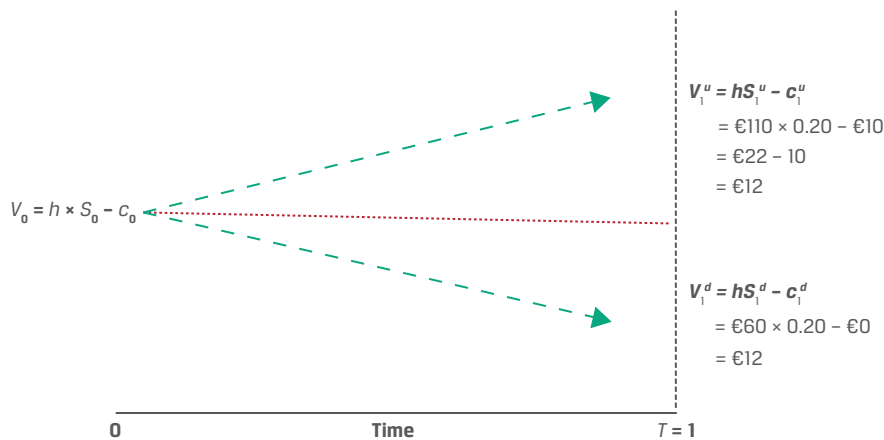
- Total portfolio value  $V_1^d = hS_1^d - c_1^d = €60 \times 0.20 = €12.$

The portfolio values are the same,  $V_1^u = V_1^d$ , which has two implications:

1. We can use either portfolio to value the derivative, and
2. The return  $V_1^u / V_0 = V_1^d / V_0$  must equal one plus the risk-free rate.

Exhibit 3 summarizes these results.

**Exhibit 3: Value of the Hedged Portfolio**



This hedging approach is not specific to call options and can be used for any derivative contract whose value is entirely determined by the underlying asset's value at  $t = 1$ .

To prevent arbitrage, the portfolio value at  $t = 1$ ,

$$V_1 = h \times R^u S_0 - c_1^u = h \times R^d S_0 - c_1^d,$$

should be discounted at the risk-free rate so that:

$$hS_0 - c_0 = \frac{V_1}{1+r}.$$

Based on no-arbitrage pricing and the certain portfolio payoff,  $V_1$ , the value of the call option may be shown as:

$$c_0 = h \times S_0 - V_1 (1 + r)^{-1}. \quad (7)$$

Substituting the information from the previous example, we already established that  $V_1 = €12$ . For an annual risk-free rate of 5%:

$$\begin{aligned} c_0 &= h \times S_0 - V_1 (1 + r)^{-1} \\ &= 0.2 \times €80 - €12 (1.05)^{-1} = €16 - €11.43 = €4.57. \end{aligned}$$

The call option price  $c_0$  is therefore €4.57. We may confirm an investor in the hedge portfolio would earn the risk-free rate ( $r$ ) of 5% by comparing the initial portfolio value  $V_0$  with the portfolio value after one period of  $V_1$  (recalling that  $V_1^u = V_1^d$ ) equal to €12:

$$\begin{aligned} V_0 &= h \times S_0 - c_0 \\ V_0 &= €11.43 = €16 - €4.57 \\ V_1 &= V_0 (1 + r) \\ V_1 &= €12 = €11.43 \end{aligned} \quad (8)$$

### EXAMPLE 1

#### Highest Capital

Highest Capital believes that a particular non-dividend-paying stock is currently trading at \$50 and is considering the sale of a one-year European call option at an exercise price of \$55. Answer the following questions:

1. If the stock price is expected to either go up or down by 20% over the next year, what price should Highest expect to receive for the sold call option? Assume a risk-free rate of 5%.
2. How would the call option price change if the stock price were expected to either go up or down by 40% over the next year?
3. If Highest had a more optimistic outlook on the future stock price (i.e., they estimated a higher probability of the option ending up in-the-money), how would the expected call option price change?
4. What would be the price of a one-year put option at an exercise price of \$55 if the stock price were expected to change by 20%? 40%?

#### Solution to 1:

Denote the initial price of the underlying stock as  $S_0 = \$50$  and the exercise price of the call option as  $X = \$55$ . If the stock price moves up or down by 20%, then:

$$S_1^u = R^u S_0 = 1.2 \times \$50 = \$60.$$

$$S_1^d = R^d S_0 = 0.8 \times \$50 = \$40.$$

The price will either move up to \$60 or down to \$40. Given that the payoff of a call option at expiry is  $\text{Max}(0, S_T - X)$ :

$$c_1^u = \text{Max}(0, S_1^u - X) = \text{Max}(0, \$60 - \$55) = \$5.$$

$$c_1^d = \text{Max}(0, S_1^d - X) = \text{Max}(0, \$40 - \$55) = \$0.$$

The call option value is \$5 if the underlying stock increases in price by 20% and zero if it decreases by 20%. The hedge ratio of the option is:

$$h^* = \frac{(c_1^u - c_1^d)}{(S_1^u - S_1^d)} = \frac{\$5.00 - \$0}{\$60.00 - \$40.00} = \frac{\$5.00}{\$20.00} = 0.25.$$

To create a risk-free portfolio, we can sell a call option and purchase 0.25 units of the underlying asset today. At maturity, the hedged portfolio value is:

$$V_1 = V_1^u = h^* S_1^u - c_1^u = 0.25 \times \$60.00 - \$5.00 = \$10.00.$$

$$V_1 = V_1^d = h^* S_1^d - c_1^d = 0.25 \times \$40.00 - \$0.00 = \$10.00.$$

We can use either  $V_1^u$  or  $V_1^d$  to compute the certain value,  $V_1$ , and the present value of the hedged position today is:

$$V_0 = V_1(1+r)^{-1} = \$10.00 \times 1.05^{-1} = \$9.52.$$

The call option value is:

$$c_0 = h^* S_0 - V_0 = 0.25 \times \$50.00 - \$9.52 = \$12.50 - \$9.52 = \$2.98.$$

The no-arbitrage price of the call option should be \$2.98. If not, then investors would be able to construct a synthetic risk-free asset using the option and its underlying asset with a higher return than the risk-free rate.

Note that the hedge ratio (0.25 units in this example) is positive in this case since the derivative is sold and the underlying is purchased. A negative hedge ratio implies that *both* the derivative and the underlying are purchased or sold to create the hedge. Also, while the hedge ratio is usually expressed as a fraction of an underlying unit, option contracts are usually traded in larger size, allowing a round number of underlying assets to be purchased or sold as a hedge. It is important that the *ratio* (4:1 in this case) of options to underlying units is maintained in the portfolio.

### Solution to 2:

If the stock price changes by 40%, then the call option payoff at expiry is:

$$c_1^u = \text{Max}(0, S_1^u - X) = \text{Max}(0, \$70 - \$55) = \$15.$$

$$c_1^d = \text{Max}(0, S_1^d - X) = \text{Max}(0, \$30 - \$55) = 0.$$

The hedge ratio of the option is:

$$h^* = \frac{(c_1^u - c_1^d)}{(S_1^u - S_1^d)} = \frac{\$15 - \$0}{\$70 - \$30} = \frac{\$15}{\$40} = 0.375.$$

At maturity, the perfectly hedged portfolio is worth:

$$V_1 = V_1^u = h^* S_1^u - c_1^u = 0.375 \times \$70 - \$15 = \$11.25.$$

$$V_1 = V_1^d = h^* S_1^d - c_1^d = 0.375 \times \$30 - \$0 = \$11.25.$$

The present value of the hedged position at  $t = 0$  is:

$$V_0 = V_1(1+r)^{-1} = \$11.25 \times 1.05^{-1} = \$10.71.$$

Then, the call option price at  $t = 0$  is:

$$c_0 = h^* S_0 - V_0 = 0.375 \times \$50 - \$10.71 = \$18.75 - \$10.71 = \$8.04.$$

A higher expected price change indicates higher volatility. An increase in the range of future price changes increases the value of the call option.

**Solution to 3:**

Since the actual probabilities of an up or a down move in the underlying asset do not affect the no-arbitrage value of the option, the option price that Hightest may charge should not change. Hightest can offset the risk of selling the call by purchasing  $h^*$  units of the underlying asset, so any directional views on the stock price do not affect the hedge position.

**Solution to 4:**

Using put–call parity,  $c_0 - p_0 = S_0 - X(1 + r)^{-T}$ , and rearranging terms to solve for the put price, when the stock price changes by 20% and the call option price is \$2.98, the result can be calculated as:

$$p_0 = c_0 - S_0 + X(1 + r)^{-T} = \$2.98 - \$50 + \$55(1 + 0.05)^{-1} = \$5.36.$$

When the underlying stock price changes by 40% and the call option price is \$8.04, the put option price is:

$$p_0 = c_0 - S_0 + X(1 + r)^{-T} = \$8.04 - \$50 + \$55(1 + 0.05)^{-1} = \$10.42.$$

**QUESTION SET****Binomial Valuation of Options**

1. Determine the correct answers to fill in the blanks: To price a contingent claim, such as an option, a model for the \_\_\_\_\_ of the underlying asset is needed due to the \_\_\_\_\_ nature of the contract's payoff.

**Solution:**

To price a contingent claim, such as an option, a model for the *future price behavior* of the underlying asset is needed due to the *asymmetric* nature of the contract's payoff.

2. Describe the main difference between pricing a contingent claim and pricing a forward commitment.

**Solution:**

The symmetric nature of a forward commitment's payoff (i.e., the obligation to transact at maturity) allows the commitment to be perfectly replicated without the need to model the future price behavior of the underlying asset. However, the asymmetric nature of a contingent claim's payoff (i.e., the right but *not* the obligation to transact at maturity) *does* require the future price behavior to be modeled.

3. If a one-period binomial model is used to price an at-the-money put option, which of the following statements is *most* accurate? The option will be:

- A. in-the-money if the price moves up.
- B. out-of-the-money if the price moves up.
- C. at-the-money if the price moves up or down.

**Solution:**

The correct answer is B; the option will be out-of-the-money if the price moves up. An at-the-money put option has an exercise price equal to the underlying asset price. Therefore, a price decrease will result in the put op-

tion moving *in the money*, and a price increase will result in the put option moving *out of the money*.

4. Explain how increasing the up gross return,  $R^u$ , and/or decreasing the down gross return,  $R^d$ , in a one-period binomial model would influence the price of a call and a put option.

**Solution:**

In a one-period binomial model, the volatility of the underlying asset is represented by the spread between the up gross return,  $R^u$ , and the down gross return,  $R^d$ . Therefore, if either the up gross return increases or the down gross return decreases (or both), the price of the underlying asset at maturity will be more volatile. If all else remains equal, then the price of both call and put options will increase when the underlying asset is expected to have a higher volatility over the life of the option.

5. A put option on a non-dividend-paying stock has an exercise price,  $X$ , of £21 and six months left to maturity. The current stock price,  $S_0$ , is £20, and an investor believes that the stock's price in six months' time will be either 10% higher or 10% lower.

- Describe how the investor can construct a perfectly hedged portfolio using the put option and its underlying stock.
- What will the value of the hedged portfolio be in the scenario that the stock price rises and the scenario that the stock price falls (assume a risk-free rate is 4%)?
- What is the no-arbitrage price of the put option?

**Solution:**

- a. Denote the initial price of the underlying stock as  $S_0 = £20.00$  and the exercise price of the put option as  $X = £21.00$ . If the stock price moves up by 10%, then:

$$S_1^u = R^u S_0 = 1.1 \times £20.00 = £22.00.$$

If the stock price moves down by 10%, then

$$S_1^d = R^d S_0 = 0.9 \times £20.00 = £18.00.$$

Given that the payoff of a put option at expiry is  $\text{Max}(0, X - S_T)$ :

$$p_1^u = \text{Max}(0, X - S_1^u) = \text{Max}(0, £21.00 - £22.00) = £0.$$

$$p_1^d = \text{Max}(0, X - S_1^d) = \text{Max}(0, £21.00 - £18.00) = £3.00.$$

The put option will be worth £3.00 if the underlying stock decreases in price by 10% and worthless if it increases by 10%. The hedge ratio of the option is:

$$h^* = \frac{(p_1^u - p_1^d)}{(S_1^u - S_1^d)} = \frac{£0 - £3}{£22 - £18} = \frac{£3}{£4} = -0.75.$$

So, to create a risk-free portfolio, we can buy the put option and buy 0.75 units of the underlying asset.

- b. At maturity, the value of the perfectly hedged portfolio is:

$$V_1 = V_1^u = h^* S_1^u + p_1^u = 0.75 \times £22 + £0 = £16.50.$$

$$V_1 = V_1^d = h^* S_1^d + p_1^d = 0.75 \times £18 + £3.00 = £16.50.$$

We can use either  $V_1^u$  or  $V_1^d$  to compute the certain value,  $V_1$ , and the present value of the hedged position today is:

$$V_0 = V_1(1+r)^{-1} = £16.50 \times 1.04^{-0.5} = £16.18.$$

c. The price of the put option is:

$$p_0 = V_0 - h^* S_0 = £16.18 - 0.75 \times £20 = £16.18 - £15 = £1.18.$$

The no-arbitrage price of the put option should be £1.18.

6. Determine the correct answers to fill in the blanks: When the \_\_\_\_\_ applies, the rate of return on all (real or synthetic) risk-free assets should equal the \_\_\_\_\_.

**Solution:**

When the *law of one price* applies, the rate of return on all (real or synthetic) risk-free assets should equal the *risk-free rate*.

## 5

### RISK NEUTRALITY

☐ describe the concept of risk neutrality in derivatives pricing

As shown in the prior lesson, an option's value is not affected by actual (real-world) probabilities of underlying price increases or decreases. This realization contributed to the growing use of options as it became easier to agree on a given option's value. As we will see in this lesson, only the expected volatility—that is, gross returns  $R^u$  and  $R^d$  introduced earlier—and not the expected return are required to price an option.

We may generalize the relationship between an option's value and that of the hedge portfolios from Equation 5 of the prior lesson using the example of a call option  $c_0$  as follows:

$$c_0 = \frac{(\pi c_1^u + (1-\pi) c_1^d)}{(1+r)^T}. \quad (9)$$

The value of the call option today,  $c_0$ , is computed as the expected value of the option at expiration,  $c_1^u$  and  $c_1^d$ , discounted at the risk-free rate,  $r$ . This shows the derivative's value as similar to any other asset in that it equals the present value of expected future cash flows. In this case, these cash flows are weighted by assumed probabilities that are consistent with risk-neutral returns on the underlying.

The **risk-neutral probability** ( $\pi$ ) is the computed probability used in binomial option pricing by which the discounted weighted sum of expected values of the underlying,  $S_1^u = R^u S_0$  and  $S_1^d = R^d S_0$ , equal the current option price. Specifically, this probability is computed using the risk-free rate and assumed up gross return and down gross return of the underlying as in Equation 7.

$$\pi = \frac{1+r-R^d}{R^u-R^d}. \quad (10)$$

More specifically,  $\pi$ , is the risk-neutral probability of an increase in the underlying price to  $S_1^u = R^u S_0$ , and  $(1-\pi)$  is that of a decrease,  $S_1^d = R^d S_0$ .

Substituting the details from our earlier example, where  $R^u S_0 = €110$ ,  $R^d S_0 = €60$ ,  $S_0 = €80$ , and  $X = €100$ ,  $R^u = 1.375$  and  $R^d = 0.75$  and assuming an annual risk-free rate of 5%:

$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1 + 0.05 - 0.75}{1.375 - 0.75} = 0.48.$$

$$c_0 = \frac{(\pi c_1^u + (1 - \pi) c_1^d)}{(1 + r)^T}$$

$$= [0.48 \times \text{Max}(0, €110 - €100) + 0.52 \times \text{Max}(0, €60 - €100)] / 1.05$$

$$= €4.80 / 1.05 = €4.57, \text{ which matches our earlier result from the prior lesson.}$$

In Equation 7, the risk-neutral probabilities are determined solely by the up and down gross returns,  $R^u$  and  $R^d$ , representing underlying asset volatility and the risk-free rate used to calculate the present value of future cash flows. This no-arbitrage derivative value established separately from investor views on risk is referred to as **risk-neutral pricing**.

The use of risk-neutral pricing goes well beyond the simple one-period binomial tree and may be applied to any model that uses future underlying asset price movements, as we will see later in the curriculum.

## EXAMPLE 2

### Highest Capital (revisited)

Revisiting Example 1 and the European call option sold by Highest Capital, we can now explore the option price using risk-neutral pricing. Answer the following questions:

1. What is the risk-neutral probability of an up move and a down move in the one-period binomial model described in Example 1?
2. Demonstrate how this risk-neutral probability can be used to arrive at the no-arbitrage price of the call option.
3. What would be the value of a European *put* option on the same stock with the same exercise price and expiration date?
4. Confirm that the call option price computed in question 2 and the put option price computed in question 3 both satisfy the put–call parity relationship.

#### Solution to 1:

The risk-neutral probability of an up move, denoted  $\pi$ , is:

$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1 + 0.05 - 0.8}{1.2 - 0.8} = 0.625.$$

The risk-neutral probability of a down move is therefore:

$$1 - \pi = 1 - 0.625 = 0.375.$$

#### Solution to 2:

The call option price today is given by the (risk-neutral) expected value of the option payoff at maturity, discounted at the risk-free rate,  $r$ . In Equation 6:

$$c_0 = \frac{(\pi c_1^u + (1 - \pi) c_1^d)}{(1 + r)^T}$$

$$= [0.625 \times \text{Max}(0, \$60 - \$55) + 0.375 \times \text{Max}(0, \$40 - \$55)] / 1.05$$

$$= \frac{\$3.125}{1.05} = \$2.98$$

which matches the value in Example 1.

### Solution to 3:

Since Equation 6 is valid for any European option, the put option value with an exercise price of \$55 is:

$$\begin{aligned}
 p_0 &= \frac{(\pi p_1^u + (1 - \pi)p_1^d)}{(1 + r)^T} \\
 &= [0.625 \times \text{Max}(0, \$55 - \$60) + 0.375 \times \text{Max}(0, \$55 - \$40)]/1.05 \\
 &= \$5.625/1.05 = \$5.36.
 \end{aligned}$$

Note that we have used the same risk-neutral probability  $\pi$  as in Questions 1 and 2, since these values are a function of the binomial model for the underlying asset for a given asset volatility and risk-free rate, not the specific option being priced.

### Solution to 4:

European call and put options with the same exercise price and maturity date must satisfy put-call parity as defined by:

$$S_0 + p_0 = c_0 + X(1 + r)^{-T} = \$50 + \$5.36 = \$2.98 + \$55 \times 1.05^{-1} = \$55.36,$$

which confirms the relationship.

## QUESTION SET



### Hedging and Risk Neutrality

- Which of the following factors influences the value of an option price when using a binomial model?
  - The risk-free rate of return
  - The level of investors' risk aversion
  - The probability of an upward price move

#### Solution:

The correct answer is A, the risk-free rate of return. The value of an option is determined by its risk-neutral expectation discounted at the risk-free rate. In a one-period binomial model, the risk-neutral probabilities are determined only by the risk-free rate over the life of the option and the underlying asset's volatility (as measured by the up and down gross returns,  $R^u$  and  $R^d$ ). Because of the ability to construct a perfect hedge of the option using the underlying asset, an option's price is independent of investors' risk aversion and the probability of the underlying price moving up (or down).

- If the underlying asset price in a one-period binomial model can increase by 15% or decrease by 10% over the period and the prevailing risk-free rate



over the period is 4%, what is the risk-neutral probability of an asset pricing decrease?

**Solution:**

If the underlying asset price in a one-period binomial model can increase by 15%, then the up gross return is  $R^u = 1.15$ . Similarly, if the price can decrease by 10%, then the down gross return is  $R^d = 0.9$ .

The risk-neutral probability of an upward price move is:

$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1 + 0.04 - 0.9}{1.15 - 0.9} = 0.56.$$

The risk-neutral probability of a downward price move is  $1 - 0.56 = 0.44$ .

3. Which of the following *best* describes the risk-neutral pricing interpretation of the one-period binomial option pricing formula?

- A. The real-world expected payoff discounted at the risk-free rate
- B. The risk-neutral expected payoff discounted at the risk-free rate
- C. The risk-neutral expected payoff discounted at a risk-adjusted rate

**Solution:**

The correct answer is B, the risk-neutral expected payoff discounted at the risk-free rate. The risk-neutral pricing interpretation of the option pricing formula states that the value of an option today is its risk-neutral expected value at maturity, discounted at the risk-free rate.

4. Determine the correct answers to fill in the blanks: If a call option is trading at a higher price than that implied from the binomial model, investors can earn a return in excess of the risk-free rate by \_\_\_\_\_ at the risk-free rate, \_\_\_\_\_ the call, and \_\_\_\_\_ the underlying.

**Solution:**

If a call option is trading at a higher price than that implied from the binomial model, investors can earn a return in excess of the risk-free rate by *borrowing* at the risk-free rate, *selling* the call, and *buying* the underlying. A synthetic risk-free asset can be created by this strategy that earns a return higher than the risk-free rate. Selling the over-priced call will provide a higher cash inflow than is required to generate the risk-free rate of return.

5. A stock's price is currently ¥8,000. At the end of one month when its options expire, the stock price is either up by 5% or down by 15%. If the risk-free rate is -0.20% for the period, what is the value of a put option with a strike price of ¥7,950?

- A. ¥333.67
- B. ¥299.60
- C. ¥236.93

**Solution:**

The correct answer is B, ¥299.60. Using risk-neutral pricing, we can determine the risk-neutral probability as:

$$\pi = \frac{1 + r - R^d}{R^u - R^d} = \frac{1 - 0.002 - 0.85}{1.05 - 0.85} = 0.74.$$

The risk-neutral probability of a down move is therefore  $1 - \pi = 1 - 0.74 = 0.26$ . The value of a put option with an exercise price of ¥7,950 is:

$$p_0 = \frac{(\pi p_1^u + (1 - \pi)p_1^d)}{(1 + r)^T}$$

$$= [0.74 \times \text{Max}(0, ¥7,950 - ¥8,400) + 0.26 \times \text{Max}(0, ¥7,950 - ¥6,800)] / (1 - 0.002)$$

$$= [0.26 \times ¥1,150] / 0.998 = ¥299.60.$$

## PRACTICE PROBLEMS

### The following information relates to questions 1-4

Privatbank Kleinert KGaA, a private wealth manager in Munich, has a number of clients with large holdings in the German fintech firm SparCoin AG. Kleinert's analyst is concerned about a drop in SparCoin's share price in the next year and is recommending to clients that they consider purchasing a one-year put with an exercise price of €100. SparCoin's spot price ( $S_0$ ) is €105.25, and it pays no dividends. The risk-free rate is 0.37%.

1. Kleinert's analyst estimates a 50-50 chance that the price of SparCoin will either increase by 12% or decline by 10% at the put option's expiration date. Which of the following statements best describes the no-arbitrage option price implied by this assumption?
  - A. Since there is a 50% chance that the stock will fall to €94.73, there is a 50-50 chance of a €5.27 payout upon exercise and the no-arbitrage put is therefore worth €2.64 ( $= €5.27 / 2$ ).
  - B. Since there is a 50% chance that the stock will fall to €94.73, there is a 50-50 chance of a €5.27 payout upon exercise and given the risk-neutral probability of 0.47, the no-arbitrage put price is €2.48 ( $= €5.27 \times 0.47$ ).
  - C. Since there is a 50% chance that the stock will fall to €94.73 and the risk-neutral probability is 0.47, the no-arbitrage put price is €2.78 ( $= €5.27 \times \{[1 - 0.47]/1.0037\}$ ).
2. If Kleinert's clients observe that the one-year put option with a €100 exercise price is trading at €2.50, which of the following statements best describes how Kleinert's clients could take advantage of this to earn a risk-free return greater than 0.37% over the year.
  - A. Kleinert should purchase the put option and also purchase approximately 0.23 shares per option to match the hedge ratio.
  - B. Kleinert should purchase the put option and purchase 50% of the underlying shares given the 50-50 chance the stock will fall and the put option exercised.
  - C. Kleinert should purchase the put option and purchase 47% of the underlying shares to match the risk-neutral probability of put exercise.
3. If risk-free investments yielded a higher return over the next year, which of the following statements best describes how this would affect the no-arbitrage value of the put option on SparCoin shares?
  - A. An increase in the risk-free rate will have *no effect* on SparCoin's put option price, as it is solely a function of the probability and degree of share price increase or decrease upon option expiration.

- B.** An increase in the risk-free rate will *increase* the value of the put option, as it will increase the risk-neutral probability of a price decline.
    - C.** An increase in the risk-free rate will *decrease* the value of the put option, as it will both increase the risk-neutral probability of a price increase  $\pi$  and decrease the present value of the expected option payoff.
  - 4. If the expected percentage increase and decrease in SparCoin's share price were to *double*, which of the following is the closest estimate of the one-year put option price with an exercise price of €100?
    - A.** The one-year put option price will rise to €7.90.
    - B.** The one-year put option price will rise to €8.50.
    - C.** The one-year put option price will rise to €7.40.
-

## SOLUTIONS

1. The correct answer is C. A 12% increase in the stock price gives:

$$S_1^u = R^u S_0 = 1.12 \times €105.25 = €117.88.$$

The put option will expire unexercised:

$$p_1^u = \text{Max}(0, X - S_1^u) = \text{Max}(0, €100 - €117.88) = €0.$$

Alternatively, a 10% price decrease gives:

$$S_1^d = R^d S_0 = 0.9 \times €105.25 = €94.73.$$

The put option will pay off:

$$p_1^d = \text{Max}(1, X - S_1^d) = \text{Max}(0, €100 - €94.73) = €5.27.$$

To price this option, the risk-neutral pricing formula gives the risk-neutral probability  $\pi$  as:

$$\pi = (1 + 0.0037 - 0.9)/(1.12 - 0.9) = 0.47.$$

The no-arbitrage price is:

$$p_0 = \frac{(\pi \times p_1^u + (1 - \pi) p_1^d)}{(1 + r)}$$

$$p_0 = (0.47 \times €0 + 0.53 \times €5.27)/(1 + 0.0037) = €2.79/1.0037 = €2.78.$$

2. The correct answer is A. If the put option can be purchased for less than the no-arbitrage price, then a potential arbitrage opportunity is available. In this case, Kleinert's clients should purchase the underpriced put option and buy  $h^*$  units of SparCoin's stock. The hedge ratio,  $h^*$ , is calculated as:

$$h^* = \frac{(p_1^u - p_1^d)}{(S_1^u - S_1^d)} = \frac{€0 - €5.27}{€117.88 - €94.73} = \frac{-€5.27}{€23.15} = -0.2276.$$

Note that the negative hedge ratio implies that both the put option and underlying are purchased or sold to create a hedge. This initial purchase of the put option and stock will cost:

$$€2.50 + 0.2276 \times €105.25 = €26.45.$$

Should the stock price decrease, the value of this portfolio will be:

$$V_1 = V_1^d = h^* S_1^d + p_1^d = 0.2276 \times €94.73 + €5.27 = €26.83.$$

The strategy generates a risk-free return of  $(€26.83 - €26.45)/€26.45 = 1.44\%$ , which is greater than the 0.37% return on other available risk-free investments.

3. The correct answer is C. Rising interest rates reduce the value of a put option. Increasing the risk-free rate will increase the risk-neutral probability of a price increase  $\pi$  and decrease the present value of the expected option payoff. Since the value of a put option is inversely related to the price of the underlying asset, an increased probability of an upward price move will reduce the expected payoff from the put. Consequently, both of these effects will reduce the put option value as the return on risk-free investments increases.
4. The correct answer is B. A 24% (previously 12%) increase in the stock price gives:

$$S_1^u = R^u S_0 = 1.24 \times \text{€}105.25 = \text{€}130.51.$$

The put option will expire unexercised:

$$p_1^u = \text{Max}(0, X - S_1^u) = \text{Max}(0, \text{€}100.00 - \text{€}130.51) = \text{€}0.$$

Alternatively, a 20% (previously 10%) decrease gives:

$$S_1^d = R^d S_0 = 0.8 \times \text{€}105.25 = \text{€}84.20.$$

The put option will pay off:

$$p_1^d = \text{Max}(0, X - S_1^d) = \text{Max}(0, \text{€}100 - \text{€}84.20) = \text{€}15.80.$$

To price this option, the risk-neutral pricing formula gives the risk-neutral probability as:

$$\pi = (1 + 0.0037 - 0.8)/(1.24 - 0.8) = 0.46.$$

The no-arbitrage price is:

$$p_0 = \frac{(\pi \times p_1^u + (1 - \pi) p_1^d)}{(1 + r)} = \frac{(0.46 \times \text{€}0 + 0.54 \times \text{€}15.80)}{(1 + 0.0037)} = \frac{\text{€}8.53}{1.0037} = \text{€}8.50.$$

# **Alternative Investments**





## LEARNING MODULE

## 1

## Alternative Investment Features, Methods, and Structures

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	describe features and categories of alternative investments
<input type="checkbox"/>	compare direct investment, co-investment, and fund investment methods for alternative investments
<input type="checkbox"/>	describe investment ownership and compensation structures commonly used in alternative investments

## INTRODUCTION

1

Alternative Investments are grouped together not because they have similar features but instead because they have characteristics distinct from traditional investments. Investing in alternatives can be done through fund investing, co-investing, or direct investing. Alternative investments typically offer investors greater diversification and higher expected returns than traditional investments but often involve longer-term, illiquid investments in less efficient markets. Investing in alternatives requires specialized knowledge. Alternative investments typically rely on more complex and richer compensation structures than traditional investments in order to better align manager and investor incentives over longer periods.

### LEARNING MODULE OVERVIEW



- Alternative investments are investments other than ownership of traditional asset classes (public equity and fixed-income instruments and cash) and include private capital, real assets, and hedge funds.
- Private capital includes private equity and private debt. Real assets include real estate, infrastructure, and natural resources. Hedge funds may invest across both traditional and alternative asset classes and are distinguished by their investment approach, which often includes leverage, derivatives, or other strategies.

- Investors often consider alternative investments in pursuit of greater portfolio diversification and/or increased expected returns. In doing so, they usually face longer investment periods, reduced liquidity, and less efficient markets than for more traditional assets.
- Alternative investment fund investors fully outsource the control and management of investments in exchange for relatively high fees, while co-investment and direct investment methods involve greater investor effort and control over the selection and management of assets in exchange for relatively lower fees.
- Another common type of alternative investment structure is a limited partnership in which responsibilities are flexibly allocated between investors and managers—with managers as general partners and investors as limited partners. Limited partnerships usually have more complex compensation structures, which include both management and performance fees.
- Additional alternative investment structures include trusts and limited liability companies.

#### LEARNING MODULE SELF-ASSESSMENT



1. Identify which of the following choices is *most likely* an alternative investment:
  - A. An investment in a hedge fund focused on traditional assets
  - B. Shares in a manufacturing firm traded on the Bursa Malaysia exchange
  - C. A euro foreign exchange future purchased on the Chicago Mercantile exchange

**Solution:**

The correct answer is A. An investment in a hedge fund, even one that purchases traditional exchange-traded assets, is considered an alternative investment. B is incorrect because shares traded on a public exchange, such as the Bursa Malaysia exchange, are considered traditional, not alternative, investments. C is incorrect because a euro foreign exchange future purchased on a public exchange, such as the Chicago Mercantile exchange, is considered a traditional, not an alternative, investment.

2. An advantage of investing in alternative investments *most likely* is:
  - A. high liquidity.
  - B. low investment fees.
  - C. higher expected returns.

**Solution:**

The correct answer is C. Investors are often attracted to alternative investments seeking greater diversification and/or higher expected returns. A is incorrect because investors usually face longer investment periods, reduced liquidity, and less efficient markets with alternative investments than with more traditional assets. B is incorrect because alternative investments often carry higher fees, including performance and/or incentive fees.

3. Investors with limited experience *most likely* enter into alternative investments through:

- A. co-investing.
- B. fund investing.
- C. direct investing.

**Solution:**

The correct answer is B. Investors with limited resources and/or experience generally enter into alternative investments through fund investing, where the investor contributes capital to a fund and the fund identifies, selects, and makes investments on the investor's behalf. A is incorrect because co-investing is more appropriate for investors who already have some experience investing in funds; in co-investing, the investor invests in assets indirectly through the fund but also possesses rights (known as co-investment rights) to invest directly in the same assets. C is incorrect because direct investing, which occurs when an investor makes a direct investment in an asset without the use of an intermediary, is typically reserved for larger and more sophisticated investors.

4. When an investor invests in an asset without the use of an intermediary, it is called:

- A. co-investing.
- B. fund investing.
- C. direct investing.

**Solution:**

The correct answer is C. In direct investing, an investor makes a direct investment in an asset without the use of an intermediary. A is incorrect because in co-investing, an investor invests in assets indirectly through a fund but also possesses rights (known as co-investment rights) to invest directly in the same assets. B is incorrect because for fund investing, an investor contributes capital to a fund and the fund, not the investor, identifies, selects, and makes investments on the investor's behalf.

5. Which statement regarding alternative investment partnership structures is *most* accurate?

- A. The fund manager has limited liability for anything that goes wrong.
- B. The fund manager is a limited partner, and investors are general partners.
- C. Investors' upfront cash outflow can be a small portion of their total commitment to the partnership.

**Solution:**

The correct answer is C. Limited partners (LPs) are outside investors who own a fractional interest in the partnership based on the amount of their initial investment and the terms set out in the partnership documentation. LPs commit to future investments, and their upfront cash outflow can be a small portion of their total commitment to the fund. A is incorrect because the fund manager is the fund's general partner (GP) who runs the business and theoretically bears unlimited liability for anything that goes wrong. B is incorrect because the fund manager is the fund's general partner, not a limited partner, and the investors are the limited partners.

6. After failing to meet the hurdle rate, which of the following would a general partner still *most likely* receive as compensation?

- A. Carried interest
- B. Management fee
- C. Committed capital

**Solution:**

The correct answer is B. Alternative investment funds are usually structured with a management fee typically ranging from 1% to 2% of assets under management (e.g., for hedge funds) or 1% to 2% of committed capital (e.g., for private equity funds). (Committed capital is the total amount of money that the limited partners have committed to the fund's future investments.) A performance fee (also referred to as an incentive fee, carried interest, or carry) is applied based on excess returns. The partnership agreement usually specifies that the performance fee is earned only after the fund achieves a return known as a "hurdle rate." The hurdle rate is a minimum rate of return that the general partner must exceed in order to earn the performance fee. A is incorrect because "carried interest" is another name for a performance fee, which is earned only after the fund achieves its hurdle rate. C is incorrect because committed capital is the total amount of money that the limited partners have committed to the fund's future investments, not a fee to the general partner.

## 2

## ALTERNATIVE INVESTMENT FEATURES



describe features and categories of alternative investments

Alternative investments are investments other than ownership of public equity securities, fixed-income instruments, or cash that represent the more traditional asset classes. These investments are referred to as alternatives to traditional asset classes because of their characteristics and the way they are structured. Investors are often attracted to alternative investments when seeking greater diversification and/or higher expected returns in exchange for what are often longer-term, illiquid investments in less efficient markets. The features of these investments necessitate specific skills and information to evaluate their performance and include unique factors investors must consider if adding them to a portfolio.

### Alternative Investments: Features and Categories

Some alternative investment features are shared with traditional public debt and equity securities, while others are significantly different. Features that may distinguish alternative investments include the following:

- The need for specialized knowledge to value cash flows and risks
- Typically low correlation of returns with more traditional asset classes
- Illiquidity, long investment time horizons, and large capital outlays

These features lead to the following alternative investment characteristics:

- Different investment structures due to the challenges of direct investment

- Incentive-based fees to address/minimize information asymmetry between managers and investors
- Performance appraisal challenges

For example, while many alternative investments have equity or debt characteristics, they often require a larger or longer financial commitment due to an underlying investment's extended life cycle or different investment methods and vehicles used to align the capabilities and incentives of managers and investors over time. Unlike individual securities, the size and type of some alternative investments may also be prohibitively large for certain investors. For these reasons, most investors limit alternative investments to that portion of their portfolio designated to fund obligations several years in the future. Sophisticated investors with the longest investment time horizons, such as large pension funds, sovereign wealth funds, and not-for-profit endowments, tend to allocate a larger share of their portfolio to these assets.

Alternative investment categories include private capital, real assets, and hedge funds.

## Private Capital

**Private Capital** is a broad term for funding provided to companies that is sourced from neither the public equity nor the public debt markets. Capital that is provided in the form of equity investments is called private equity, whereas capital that is provided as a loan or other form of debt is called private debt.

Private equity and private debt are alternative investments with features similar to public equity and public debt. For example, both private and public equity investors are company owners with residual claims to future cash flows and dividends. However, while investors in private equity may have full access to company information and latitude to influence day-to-day management and strategy decisions, investors in publicly traded equity receive only publicly available information, such as annual reports and periodic financial statements, with voting rights limited to decisions requiring shareholder approval.

**Private equity** refers to investment in privately owned companies or in public companies with the intent to take them private. In general, private equity is used in the *mature* life cycle stage or for firms in *decline*, with leveraged buyouts being a key approach. Private equity managers often use the greater control and flexibility of private versus public ownership to make management and strategy changes, including closing, selling, or reorganizing lines of business to increase profitability over a several-year period. **Venture capital** is a specialized form of private equity whereby ownership capital is used for non-public companies in the *early* life cycle or *startup* phase, where often an idea or business plan exists with a limited operation or customer base.

### EXAMPLE 1

#### Venture Capital vs. Private Equity

Heartfield Digital is an early-stage digital media venture established 18 months ago. Heartfield plans to convert conventional music and art collection rights to digital form for sale and distribution. Its founders are seeking early-stage investors in order to conduct market research, build partnerships, and initiate operations.

In contrast, Arguston Inc. is a mid-sized manufacturing firm in a mature industry that is experiencing a decline in profitability. Arguston's share price has stagnated, and given its high-cost structure and dwindling operating cash flow, Arguston lacks the scale to make necessary technological upgrades to maintain competitiveness. A prospective private equity investor might consider an

investment plan to restructure Arguston's operations, acquire a smaller competitor, and/or create efficiencies, perhaps by updating the plant and equipment. In several years, Arguston may emerge as a more profitable independent company or as an attractive acquisition target for a competitor.

Technically, venture capital (VC) is a form of private equity. The main difference is that while private equity investors prefer stable companies, VC investors usually come in during the startup phase. Venture capital is usually given to small companies with huge growth potential, such as Heartfield Digital, while broader types of private equity financing would be more appropriate for a mature firm, such as Arguston.

For **private debt**, in addition to private loans or bonds, venture debt is extended to early-stage firms with little or no cash flow, while distressed debt (introduced in a separate fixed-income lesson) involves public or private debt of corporate issuers believed to be close to or in bankruptcy that could benefit from investors with capital restructuring skills.

## Real Assets

In contrast to financial assets, **real assets** generally are tangible physical assets, such as real estate (for example, land or buildings) and natural resources, but also include such intangibles as patents, intellectual property, and goodwill. Real assets either generate current or expected future cash flows and/or are considered a store of value. **Real estate** includes borrowed or ownership capital in buildings or land. Developed land includes commercial and industrial real estate, residential real estate, and infrastructure. Commercial real estate includes land and buildings where private business activity is the primary cash flow source, whereas residential real estate's cash flows stem from rents or mortgage payments by households. Publicly traded forms of real estate include real estate investment trusts (REITs), which are issuers of equity securities, and mortgage-backed debt securities, which are introduced and discussed in a fixed-income lesson.

**Infrastructure** is a special type of real asset that typically involves land, buildings and other long-lived fixed assets that are intended for public use and provide essential services. Bridges and toll roads are common examples of tangible infrastructure assets. Infrastructure may be developed either solely by governments or through a **public-private partnership (PPP)** in which private investors also have a stake. For example, a public-private partnership might be used in order to attract long-term private investment for a broadband internet investment. Infrastructure assets create cash flows either *directly* in the form of fees, leases, or other compensation for access rights or *indirectly* by promoting economic growth and supporting a government's ability to generate increased tax revenue on future economic activity. When private investors are involved, a contract known as a **concession agreement** usually governs the investor's obligations to construct and maintain infrastructure as well as the exclusive right to operate and earn fees for a pre-determined period.

### EXAMPLE 2

#### Public-Private Partnership for Infrastructure Projects in Indonesia

PT Indonesia Infrastructure Finance (IIF) is a private national company established in 2010 by the government of Indonesia to accelerate and improve private participation in infrastructure development in Indonesia. Together with the World Bank, the Asian Development Bank, and other institutions, the IIF

provides infrastructure financing and advisory services for commercially viable infrastructure projects. This approach has facilitated development of PPP projects, increased equity investment in Indonesian infrastructure projects, and increased institutional awareness and capacity to implement environmental, health, and safety and social issues. Looking ahead, the IIF anticipates the need to spend USD150 billion over a five-year period to construct power plants and toll roads. While the government will be able to fund 30% of this cost, the remainder will be financed by the private sector through PPPs.

**Natural resources** involve either less developed land, which itself is the source of economic value, or naturally occurring standardized products that are harvested, extracted, and/or refined. Less developed land includes farmland, timberland, or land for exploration for natural resource deposits, such as minerals or energy. Sources of return for these types of less developed land include expected price appreciation over time and cash flows. For example, farmland generates crop yields or agricultural lease payments, future timber harvests generate timberland income, and mineral or drilling rights to extract and refine natural resources can provide income. In some cases, these investments may be considered for environmental, social, and governance (ESG) purposes—for example, when promoting sustainable farming practices for agriculture or creating carbon offsets for timberland, as in the following example.

### EXAMPLE 3

#### Using Timberland to Create Carbon Offsets

Companies eager to offset emissions are paying timberland owners not to cut acres of trees. Growing trees absorb carbon in the atmosphere.

- Companies that pay timber owners not to harvest trees receive credits in the form of carbon offsets. Large companies, which inevitably generate emissions in the course of doing business, can fund carbon offsets to demonstrate to their investors, customers, and others (including regulators) that they are serious about reducing pollution and helping the environment.
- For large and small owners of timberland, choosing not to harvest makes sense, too. Their trees continue to grow, leading to higher future volumes of timber, and they earn non-timber income in the meantime.

In other cases, undeveloped land may also have future potential for commercial, residential, or infrastructure development.

Standardized, traded goods known as **commodities** include plant, animal, energy, and mineral products used in goods and services production. Commodities do not themselves generate cash flows but, rather, are ultimately sold by commodity producers to commodity consumers for economic use. Investors seek to benefit from commodity price changes based on their future economic use as well as a lower correlation of returns versus other asset classes over the economic cycle. With their lower correlation of returns with other asset classes, commodities also can serve as a countercyclical holding and as an inflation hedge.



**EXAMPLE 4****Rising Demand for Lithium, an Increasingly Important Commodity**

Demand for lithium is growing rapidly as demand for electric vehicles (EVs) climbs. Lithium is popular with battery manufacturers because, as the least dense metal, it stores a large proportion of energy relative to its weight.

The sales of battery electric and plug-in hybrid electric cars exceeded 2 million for the first time in 2019. China dominates this market, accounting for more than half of all current EV sales worldwide. An industry source has forecast that EV sales will grow from 2.5 million in 2020 to approximately 11 million in 2025 and will exceed 30 million by 2030, garnering approximately 32% of the total market share for new car sales at that time.

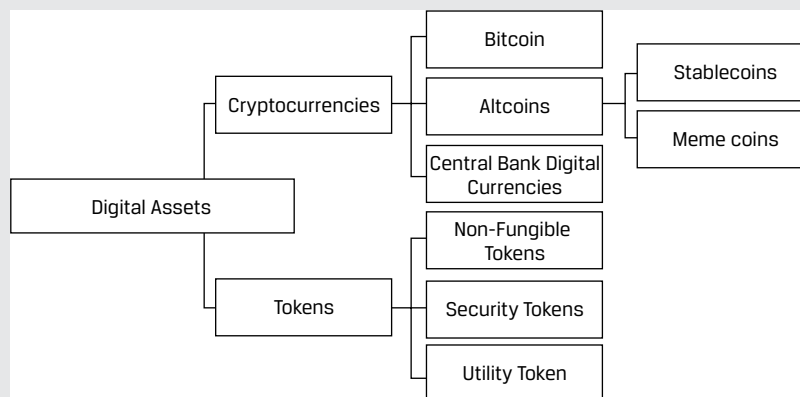
Demand for lithium will rise accordingly. The growth in lithium demand is expected to foster new lithium mining and more production in more countries worldwide.

Other real alternative assets include tangible collectible assets, such as fine art, wine, rare coins, watches, and other rare assets, as well as intangible assets, such as patents, and litigation, and so-called **digital assets**. “Digital assets” is the umbrella term covering assets that can be created, stored, and transmitted electronically and have associated ownership or use rights.

**DIGITAL ASSETS**

The term “digital assets” covers a wide variety of assets, such as cryptocurrencies, tokens (security and utility), and digital collectables (such as digital art). The following diagram provides a simple breakdown of digital assets.

**Digital Assets diagram**



Cryptocurrency and tokens utilize cryptography, an advanced encryption technique that assures the authenticity of crypto assets. The key difference between these two classes of digital asset is that cryptocurrencies have their own blockchains, whereas crypto tokens are built on an existing blockchain.

For example, cryptocurrencies are the native asset of a blockchain—such as Bitcoin (BTC) or Ethereum (ETH). However, tokens are created as part of a platform that is built on an existing blockchain, such as the many ERC-20 tokens that make up the Ethereum ecosystem. The Ethereum ecosystem is used as a network for decentralized applications, such as non-fungible token (NFT) marketplaces and decentralized finance (DeFi) projects.



Digital assets are covered in more detail in a subsequent alternative investments learning module.

## Hedge Funds

**Hedge funds** are private investment vehicles that may invest in public equities or publicly traded fixed-income assets, private capital, and/or real assets, but they are distinguished by their investment *approach* rather than by the investments themselves. Hedge funds make frequent use of leverage, derivatives, short selling, and other investment strategies, which often results in a substantially different risk and return profile from that of merely buying and holding the underlying assets in an investment portfolio. Investors may also invest in a portfolio of hedge funds, often referred to as a **fund of funds**.

### QUESTION SET



1. Tangible physical assets that generate current or expected future cash flows and/or are considered a store of value are *best* labeled as:

- A. real assets.
- B. private equity.
- C. venture capital.

**Solution:**

The correct answer is A. In contrast to financial assets, real assets are generally tangible physical assets that generate current or expected future cash flows and/or are considered a store of value. Major categories of real assets include real estate and natural resources, as well as intangibles such as patents. B is incorrect because private equity, considered an alternative investment, is non-publicly traded capital that is invested directly in private companies (or in public companies that are being taken private). It is typically used to invest in firms in the mature life-cycle stage or in decline. C is incorrect because venture capital is ownership capital used for non-public companies in their early life cycle or startup phase.

2. Contrast private equity and venture capital.

**Solution:**

Private capital is used at different times in a company's life cycle and in different forms. Most private equity is used in the mature life cycle stage or for firms in decline. Private equity managers often use the greater control and flexibility of private versus public ownership to make management and strategy changes including closing, selling, or reorganizing lines of business to increase profitability over a several-year period. In contrast, venture capital is used for non-public companies with high growth potential in their early life cycle or startup phase. Venture capital is essentially a specialized form of private equity; it represents a small portion of the entire private equity market by value.

3. Identify which statement about a digital asset is *most* accurate. A digital asset:

- A. includes digital art but not cryptocurrencies.

- B.** is anything that can be stored and transmitted electronically and has associated ownership or use rights.
- C.** must adhere to very specific designs or requirements in order to work within the limited types of technology that support it.

**Solution:**

The correct answer is B. Digital assets continue to evolve and vary in terms of design and application. Digital assets can be thought of as anything that can be stored and transmitted electronically and has associated ownership or use rights. A is incorrect because digital assets include cryptocurrencies, tokens (security and utility), and digital collectables (such as digital art). C is incorrect because digital assets may take many forms (such as digital tokens and virtual currencies) and may use various underlying technologies. They are not limited to specific designs or technology.

4. Determine the correct answers to fill in the blanks: Alternative investment categories include \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

**Solution:**

Alternative investment categories include *private capital*, *real assets*, and *hedge funds*.

## 3

## ALTERNATIVE INVESTMENT METHODS



compare direct investment, co-investment, and fund investment methods for alternative investments

Investors seeking greater diversification and higher expected returns from alternative investments must consider how best to enter into such an investment. The long-term, illiquid nature of many alternative investments along with the specialized knowledge involved in evaluating and overseeing these investments make investors more dependent on manager decisions over longer time periods. Large, sophisticated investors can address this issue by taking greater control over the investment process, while other, less sophisticated investors must outsource this process and seek to align manager incentives with investor objectives.

## Alternative Investment Methods

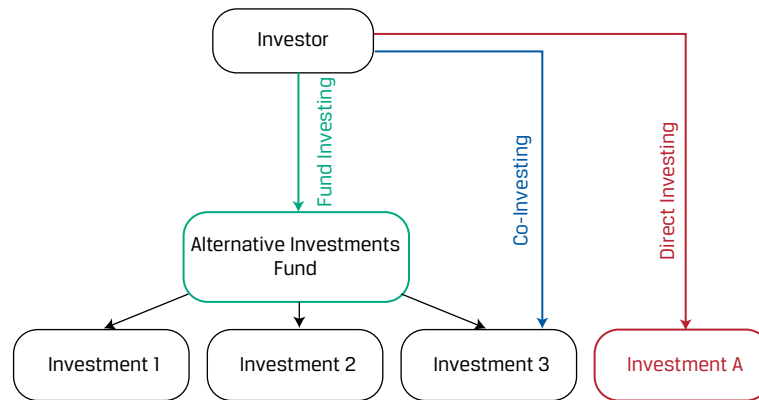
Investors can access alternative investments in three ways:

- Fund investment (such as a in a PE fund)
- Co-investment into a portfolio company of a fund
- Direct investment into a company or project (such as infrastructure or real estate)

Institutional investors typically begin investing in alternative investments via funds. Then, as they gain experience, they may begin to invest via co-investing and direct investing. The largest and most sophisticated direct investors (such as some

sovereign wealth funds) compete with fund managers for access to the best investment opportunities. Exhibit 1 shows an illustration of the three methods of investing in alternative investments. We will refer to this exhibit several times during this lesson.

**Exhibit 1: Three Methods of Investing in Alternative Assets**

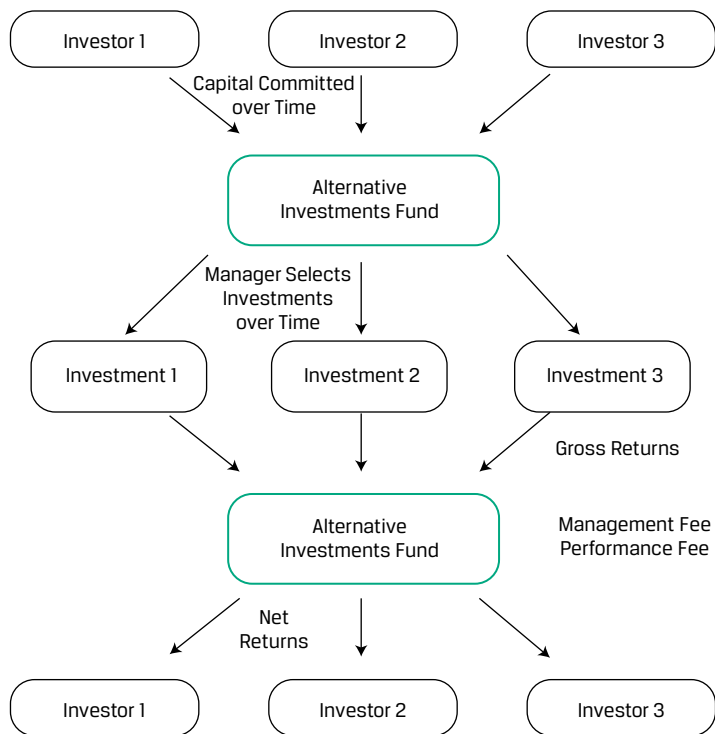
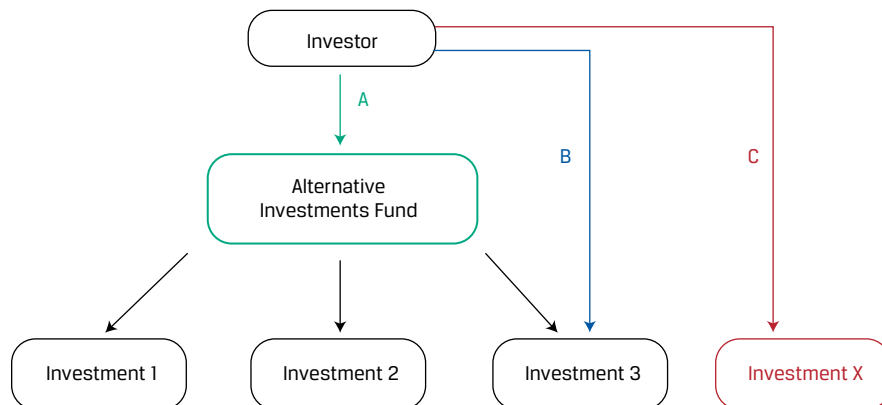


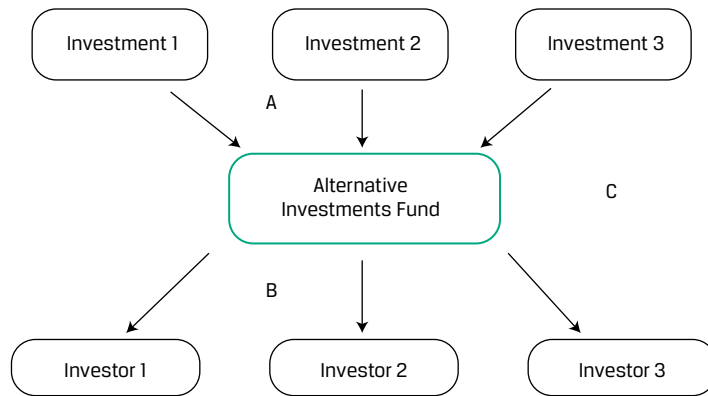
## Fund Investment

Investors with limited resources and/or experience generally enter into alternative investments through **fund investing**, where the investor contributes capital to a fund and the fund identifies, selects, and makes investments on the investor's behalf. For the fund's services, the investor is charged a management fee, plus a performance fee if the fund manager delivers superior results versus a hurdle rate or benchmark. In Exhibit 1, the investor invests in the "alternative investments fund." The fund itself invests in three investments: Investments 1, 2, and 3. Fund investing can be viewed as an indirect method of investing in alternative assets.

Fund investors have little or no leeway in the sense that their investment decisions are limited to either investing in the fund or not. Fund investors typically have neither the sophistication nor the experience to invest directly on their own. Furthermore, fund investors are typically unable to affect the fund's underlying investments. Note that fund investing is available for all major alternative investment types, including hedge funds, private capital, real estate, infrastructure, and natural resources.

Allocation to alternative assets requires distinct specialized skills that many investors likely do not possess. These investors can achieve this exposure through fund investing, where one or more investors contribute capital to an investment management company that identifies, selects, manages, and monitors investments on the investors' behalf, as shown in Exhibit 2.

**Exhibit 2: Alternative Investment Fund Structure****A. Investments**

**B. Returns**

Fund investment structures for alternative investments differ substantially from traditional public equity and fixed-income fund or ETF investments. For example, alternative funds usually involve (1) the pre-commitment of funds prior to investment selection and an extended period during which the fund may not be sold, (2) higher management fees with more complex fee structures, and (3) less frequent transparency on periodic returns and fund positions versus equity or fixed-income funds. Investors in alternative funds therefore usually compensate managers using a performance-based, as opposed to flat, fee structure to better align manager and investor incentives over longer periods. Important terms are provided in a term sheet; a sample term sheet for fund investment with Tenderledge Investments LLC is provided in Exhibit 3. Further details of these terms are discussed in subsequent lessons.

**Exhibit 3: Tenderledge Investment Fund VIII Limited Partner Agreement  
Term Sheet**
**Tenderledge Investment Fund VIII, L.P.  
Term Sheet**

<b>Fund</b>	Tenderledge Investment Fund VIII, L.P
<b>General Partner</b>	Tenderledge Investment LLC
<b>Fund Manager</b>	Tenderledge Investments
<b>Maximum Size</b>	Not to exceed USD750 million
<b>General Partner's Commitment</b>	Equal to at least 2% of the aggregate Commitments of the Limited Partners
<b>Initial Closing Date</b>	The date the General Partner determines that aggregate Commitments equal or exceed USD500 million
<b>Final Closing Date</b>	Twelve (12) months from the Initial Closing Date
<b>Term of the Fund</b>	Ten (10) years from the Initial Closing Date
<b>Investment Policy</b>	To provide attractive, long-term investment return from a diversified portfolio of alternative investments
<b>ESG</b>	In accordance with the General Partner's environmental, social, and governance policy
<b>Management Fee</b>	1.5% per annum of the commitment of each limited partner
<b>Hurdle Rate</b>	A hard hurdle rate of 10%

**Tenderledge Investment Fund VIII, L.P.****Term Sheet**

<b>Performance Fee</b>	20% of fund returns in excess of the specified hard hurdle rate
<b>Side Letters</b>	If any Side Letter grants more favorable rights to any Partner than those provided to other Partners, the more favorable rights will be granted to all other Partners

**Co-Investment**

Once investors have some experience investing in funds, prior to investing directly themselves, many investors gain direct investing experience via **co-investing**, where the investor invests in assets *indirectly* through the fund but also possesses rights (known as co-investment rights) to invest *directly* in the same assets. Through co-investing, an investor is able to make an investment *alongside* a fund when the fund identifies deals; the investor is not limited to participating in the deal solely by investing in the fund. Exhibit 1 illustrates the co-investing method: The investor invests in one deal (labeled “Investment 3”) indirectly via fund investing while investing an additional amount directly via a co-investment. Co-investing allows investors to expand their investment knowledge, skills, and experience beyond what they would gain from taking a fund-only investment approach. Co-investing also provides investors access to an investment opportunity at a lower fee than they would owe as fund-only investors in the same asset. Co-investors can learn from the fund’s process to eventually pursue direct investments themselves. Co-investors weigh the benefits of greater control and lower fees versus higher oversight costs.

Managers benefit from choosing one or more co-investors to

- accelerate investment timing when available funds and expected inflows are insufficient for a specific deal,
- expand the scope of available new investments, and
- increase diversification of an existing pool of fund investments.

**EXAMPLE 5****Co-Investment Opportunity**

Moreton Bay Pension Plan is an investor in Tenderledge LLC Alternatives Fund. Tenderledge has identified a take-private transaction in Fancy Roofing Co. that requires a USD1.5 billion capital investment. However, the fund concentration limit allows Tenderledge to invest only up to USD1 billion in any one investment. Tenderledge offers the additional USD0.5 billion to Moreton Bay Pension Plan and other investors in the fund as a co-investment on a reduced fee and no carry basis. In this case, the co-investment allows Tenderledge the ability to secure the investment without needing to bring an additional fund manager into the transaction, and the co-investors gain additional exposure to Fancy Roofing Company at a reduced management fee and zero performance fees.

**Direct Investment**

The largest, most sophisticated investors with sufficient skills and knowledge to manage individual alternative investments often do so via **direct investing** *without* the use of an intermediary, as previously shown in Exhibit 1 (labeled “Investment A”).

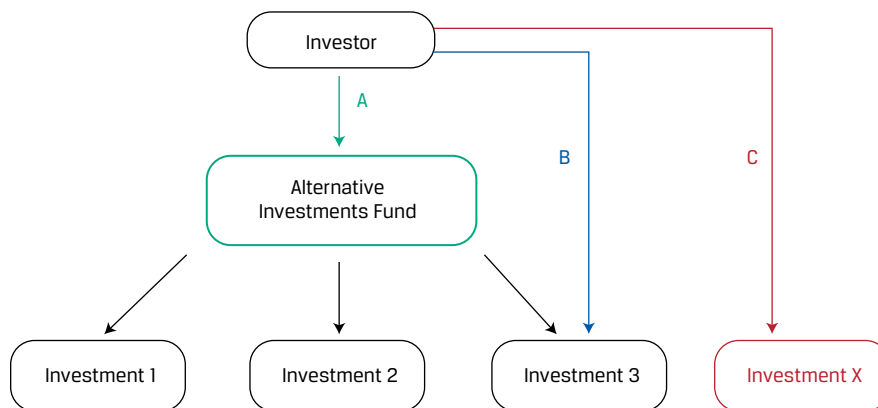
Direct investors retain maximum flexibility and control when it comes to investment choice, methods of financing, and timing. In the case of private equity, this involves the purchase of a direct stake in a private company without the use of a fund managed by an external asset manager or general partner. It also requires the direct investor to have the resources to provide the specialized knowledge, skills, and oversight capabilities that direct investment requires. Although the direct investment approach usually applies to private capital and real estate, some very large investors, such as pensions and sovereign wealth funds, also invest directly in infrastructure and natural resources.

**EXAMPLE 6****Direct Investment in Renewable Energy**

Singapore sovereign wealth fund GIC announced in 2021 that it will directly invest USD240 million in a Singapore-based energy firm, Arctic Green Energy, becoming an equity partner in the firm. The investment will help Arctic Green Energy expand its global operations and increase its capability in geothermal energy. Geothermal energy is derived from hot underground springs and is a renewable, economically competitive, and sustainable alternative to using fossil fuels for heating and cooling. Arctic Green Energy uses geothermal resources to generate power and produce clean heat. GIC has noted that investing sustainably is one of its core long-term investment mandates.

**QUESTION SET**

1. Referring to the diagram below, identify the label that best corresponds to the three methods of investing in alternative assets:



- |    |                     |
|----|---------------------|
| A. | 1. Co-investing     |
| B. | 2. Fund investing   |
| C. | 3. Direct investing |

**Solution (as seen in Exhibit 1 of this lesson):**

A. 2 is correct. In fund investing, the investor contributes capital to a fund and the fund identifies, selects, and makes investments on the investor's behalf.

B. 1 is correct. In co-investing, the investor invests in assets indirectly through the fund but also possesses rights (known as co-investment rights) to invest directly in the same assets.

C. 3 is correct. In direct investing, an investor makes a direct investment in an asset (labeled “Investment X” in the diagram) without the use of an intermediary.

2. Determine the correct answers to fill in the blanks: The \_\_\_\_\_ and \_\_\_\_\_ nature of many alternative investments and the \_\_\_\_\_ required to evaluate and oversee these investments make investors more dependent on manager decisions over longer time periods.

**Solution:**

The *long-term* and *illiquid* nature of many alternative investments and the *specialized knowledge* required to evaluate and oversee these investments make investors more dependent on manager decisions over longer time periods.

3. Identify two reasons investment managers offer co-investment opportunities to investors. Managers might choose to add co-investors for such reasons as:

- 1.
- 2.

**Solution:**

Managers might choose to add co-investors for such reasons as:

- accelerating investment timing when available funds and expected inflows are insufficient for a specific deal.
- expanding the scope of available new investments.
- increasing diversification of an existing pool of fund investments.

4. Identify one reason an investor would elect to participate in each of the following alternative investment methods:

1. Fund investing
2. Co-investing
3. Direct investing

**Solution:**



1. Fund investing	Fund investing gives investors with limited resources or experience an entrance into alternative investing.
2. Co-investing	Investors who have better specific skills and greater ability to allocate investments to individual assets may select a more active investment approach while retaining manager involvement. This results in lower fees for the investor than for a purely fund-based approach. Also, co-investors select and manage an investment jointly with a general fund manager, which potentially gives them greater control and higher returns than they could earn in a fund-only structure.
3. Direct investing	Direct investing offers investors flexibility and control when it comes to choosing their investments, selecting their preferred methods of financing, and planning their approach.

## ALTERNATIVE INVESTMENT STRUCTURES

# 4



describe investment ownership and compensation structures commonly used in alternative investments

Beyond the direct or indirect method of investing in alternatives, the illiquidity, complexity, and long-term nature of these investments require more complex structures to bridge potential gaps between manager and investor interests. Alternative investment structures may explicitly address both the roles and responsibilities of investors and managers to address these gaps. In addition, alternative investment structures tailor the distribution of returns between these two parties to better align the incentives (or interests) between manager and investor.

For example, managers may require investors to be responsible for future capital contributions, while investors may place restrictions on manager investment selection to avoid conflicts of interest or hostile takeovers among other investment criteria. Performance-based compensation structures, which can include minimum return requirements for investors, delayed payouts, and/or the ability to reclaim incentive compensation in the event of poor fund performance, encourage managers to maximize returns in the best interest of investors.

### Alternative Investment Ownership and Compensation Structures

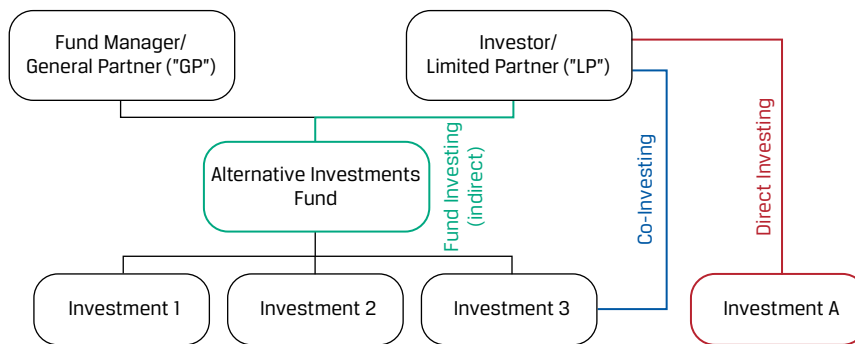
In this section, we discuss the ownership and compensation structures of alternative investments.

#### Ownership Structures

Alternative investment vehicles often take the form of partnerships in order to maximize flexibility in the investment structure to allocate business risk and return and to distribute special responsibilities between investors and managers as required. Limited partnerships, introduced in a corporate issuer lesson, involve at least one general

partner (GP) with theoretically unlimited liability who is responsible for managing the fund. Limited partners (LPs) are outside investors who own a fractional interest in the partnership based on the amount of their initial investment and the terms set out in the partnership documentation. Exhibit 4 shows the basic GP/LP structure together with the various investment approaches (fund, co-investment, direct) introduced earlier.

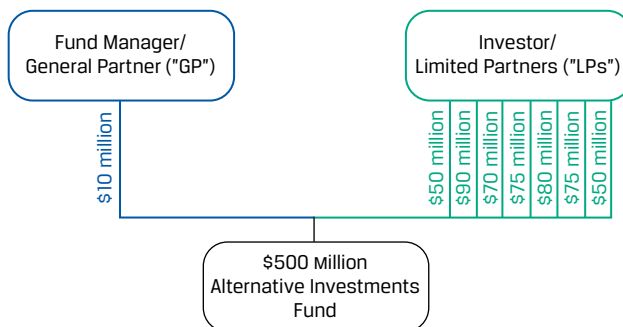
**Exhibit 4: Basic GP/LP Structure and Various Investment Approaches**



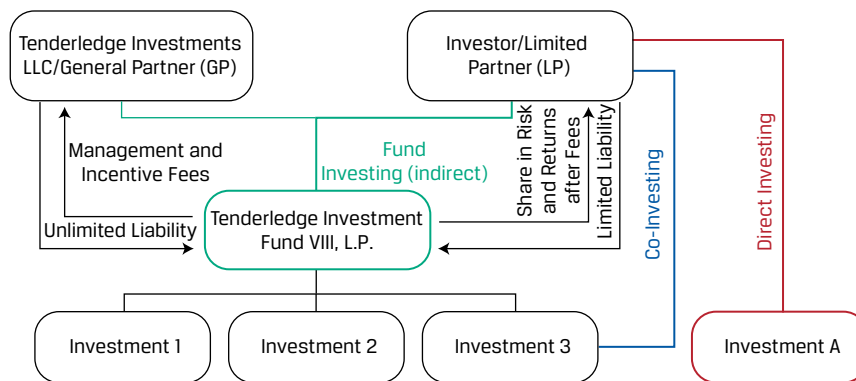
LPs commit to future investments, and the upfront cash outflow can be a small portion of their total commitment to the fund. Funds set up as limited partnerships typically have a limit on the number of LPs allowed to invest in the fund. LPs play passive roles and are not involved with the management of the fund (although co-investment rights allow for the LPs to make additional direct investments in the portfolio companies); the operations and decisions of the fund are controlled solely by the GP.

Limited partners (LPs) have their liability capped at the amount of their investment in the partnership, as shown in Exhibit 5.

**Exhibit 5: Example Limited Partnership Structure**



A restricted number of limited partners hold a fractional interest in the fund. LP investors must generally meet certain minimum regulatory net worth, institutional, or other requirements, as so-called **accredited investors**, to access these investments, which are less regulated than general public offerings. The GP (Tenderledge Investments LLC in Exhibit 6) agrees to manage the fund's operations under an agreed standard of care and perform such activities as buying or selling assets, borrowing funds, establishing reserves, or entering into contracts on behalf of the fund.

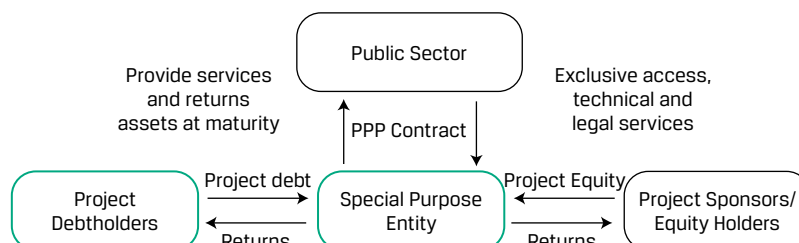
**Exhibit 6: Limited Partnership (Tenderledge Investment Fund)**

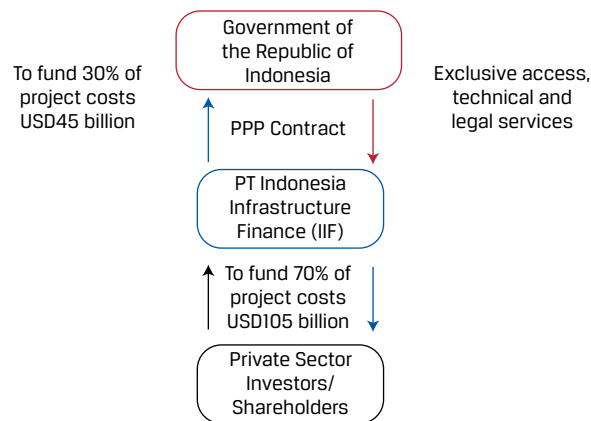
A GP may manage multiple funds at a time. A **limited partnership agreement (LPA)** establishes terms of an LP as governed by a limited partnership agreement.

Key features of an LPA include the distribution of profits and losses (covered in detail below); manager roles and responsibilities, such as investment criteria and restrictions; and terms governing transfers, withdrawals, and dissolution of the agreement.

Adjustments to LP terms are sometimes made to address the unique legal, regulatory, or reporting requirements of a specific investor. In this case, a supplemental document known as a **side letter** is issued between a GP and one or more LPs with terms that override or modify the original LPA terms. These terms might include such features as increased investor ability to transfer investments to a related or successor fund; first right of refusal and other similar clauses to outline potential treatment (regarding fees, co-investment rights, secondary sales, and potentially other matters) in comparison to other LPs; ability to forgo a contractual capital contribution (known as an excusal right); or ability to receive additional investment reporting. One feature of a side letter might include a “most favored nation” clause ensuring any more favorable or additional terms negotiated outside of the LPA with other investors will also apply to a particular LP. The customized features of the LPA and any side letters stand in contrast to a standardized indenture applicable to all bond investors in the case of public fixed-income securities.

Different specialized structures are commonly adopted for other alternative investments. For example, infrastructure investors frequently enter into public–private partnerships, which are agreements between the public sector and the private sector to finance, build, and operate public infrastructure, as shown in Exhibit 7. Exhibit 8 shows the PPP structure for PT Indonesia Infrastructure Finance (IIF), as discussed in Example 2.

**Exhibit 7: Public–Private Partnership**

**Exhibit 8: Public–Private Partnership Structure of IIF**

Infrastructure projects often involve a special purpose entity that raises borrowed and ownership capital for the construction and operation of a specific road, bridge, or other long-lived asset under the terms of a concession agreement, after which the asset is sold or returned to a public sector entity.

Real estate or natural resource fund investors are often classified as unitholders in what is referred to as a **master limited partnership (MLP)**, which has similar features to the limited partnership described earlier but is usually a more liquid investment that is often publicly traded. Other forms of more liquid investments in alternative assets are real estate investment trusts (REITs), commodity funds, and various exchange-traded funds (ETFs). In the case of direct real estate investments, joint ventures are also a common partnership structure. Infrastructure and real estate will be discussed further in a subsequent alternative investment learning module.

## Compensation Structures

The asymmetry in information between the general partner, with specialized knowledge and control, and the limited partners in alternative investments means that more complex compensation structures are used to better align general and limited partner incentives. Most funds that own public equity or debt securities charge management fees as a fixed percentage of assets under management. Alternative investment funds, in contrast, usually combine a higher management fee (often 1%–2% of assets under management) with a **performance fee** (also referred to as an incentive fee or **carried interest**) based on a percentage of periodic fund returns.

While hedge funds and REITs typically charge a management fee on assets under management, private equity funds often levy this fee on **committed capital**, which consists of the total amount that LPs have promised to fund future investments. Private equity funds raise committed capital and draw down on those commitments, generally over three to five years, when they have a specific investment to make. The life of a typical private equity fund is 10 years. Note that the management fee is typically based on committed capital, *not* invested capital; the committed-capital basis for management fees is an important distinction from hedge funds, whose management fees are based on assets under management (AUM). Using committed capital as the basis for management fee calculations reduces the incentive for GPs to deploy the committed capital as quickly as possible (in order to increase near-term management fees). This allows the GPs to be selective about deploying capital into investment

opportunities. In addition, since the GP has so much influence on the value of the assets, it would be inappropriate to pay management fees on the basis of the value of assets under management.

Performance fees are often subject to a minimum fund return or **hurdle rate** (also known as a “preferred return”), as well as other modifications, to align manager incentives as closely as possible with those of investors over long investment periods. Hurdle rate agreements sometimes distinguish between a **hard hurdle rate**, where the manager earns fees on annual returns in excess of the hurdle rate, or a **soft hurdle rate**, where the fee is calculated on the entire return when the hurdle is exceeded. With a soft hurdle, GPs are able to catch up performance fees once the hurdle threshold is exceeded.

If we ignore management fees and assume a single-period fund rate of return of  $r$ , a hard hurdle rate of  $r_h$ , and a GP performance fee ( $p$ ) as a percentage of total return, then the GP's rate of return ( $r_{GP}$ ) is as follows:

$$r_{GP} = \max[0, p(r - r_h)]. \quad (1)$$

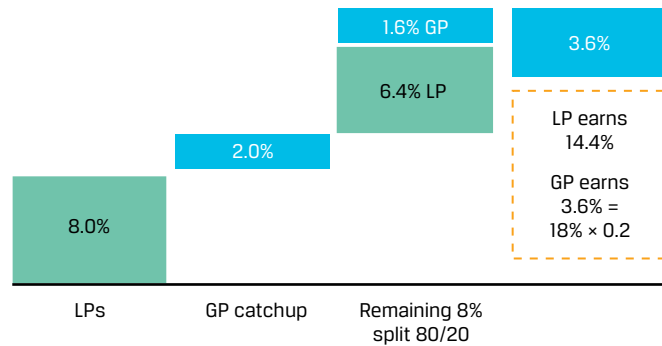
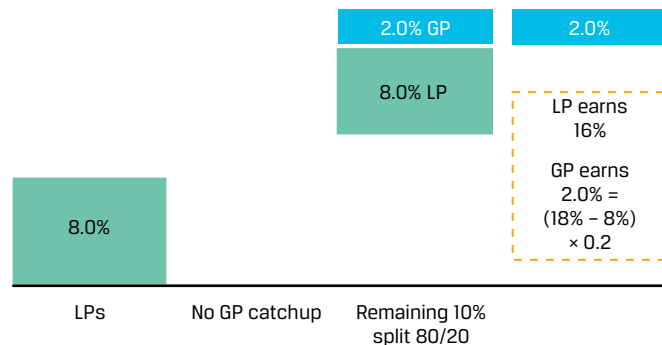
For example, if a fund earns  $r$  of 18% over a period and receives a performance fee of 20% ( $p$ ) of returns in excess of the hurdle rate,  $r_h$ , of 8%, then  $r_{GP}$  is equal to 2%:

$$2\% = \max[0, 20\%(18\% - 8\%)].$$

Performance fee modifications include rewarding managers for exceeding the hurdle rate or generating returns on specific deals or penalizing them by limiting or requiring the return of performance fees for declining performance over time. The acceleration of performance fees once a fund exceeds the soft hurdle rate is known as a **catch-up clause**, as illustrated in Exhibit 9. Under the catch-up clause using the prior example, the GP earns an *immediate* 2% (catch-up return of  $r_{cu}$ ) once the hurdle is exceeded, resulting in a 1.6% higher return than without the catch-up feature. In other words, the LP will receive 100% of all cash distributions until it earns the hurdle rate of 8%. Then, the GP will receive 100% of distributions until the split of profit is 80%/20%. The remaining distributions will be split 80%/20%. The GP's rate of return ( $r_{GP}$ ) from Equation 1 with a catch-up clause becomes

$$r_{GP} = \max[0, r_{cu} + p(r - r_h - r_{cu})]. \quad (2)$$

$$3.6\% = \max[0, 2.0\% + 20\%(18\% - 8\% - 2\%)].$$

**Exhibit 9: Catch-Up Clause Illustration****A. GP Returns with a Catch-Up Clause****B. GP Returns without a Catch-Up Clause**

In other instances, managers are penalized with fee reductions in cases of poor or declining performance. One example of this in the case of hedge funds is the use of a so-called **high-water mark**, which reflects the fund's peak value as of a performance calculation date net of fees. If the fund's value subsequently declines below the high-water mark, the hedge fund manager may not charge performance fees until the fund value exceeds the previous high-water mark. The use of high-water marks seeks to reward managers for sustained performance and protect LPs from paying twice for the same returns.

A **clawback** provision, in contrast, actually grants LPs the right to reclaim a portion of the GP's performance fee. Clawback provisions are usually activated when a GP exits successful deals early on but incurs losses on deals later in the fund's life. For most alternative investments, investor high-water marks carry over into new calendar years, but in the case of hedge funds, an investor may no longer claw back incentive fees paid for a prior calendar year if portfolio losses are incurred later. Given the generally more illiquid and longer-term nature of their holdings, private equity and real estate investments are more likely to contain clawback clauses for the entire life of the portfolio.

Finally, as in the case of asset-backed securities described in fixed-income lessons, alternative investments often use a waterfall structure to determine the distribution of cash flows to GPs and LPs. In alternative investments, GPs usually receive a disproportionately larger share of the total profits relative to their initial investment, which incentivizes them to maximize profitability.

There are two types of waterfalls: *deal-by-deal* (or *American*) waterfalls and *whole-of-fund* (or *European*) waterfalls.

- Deal-by-deal waterfalls are more advantageous to the GP because performance fees are collected on a per-deal basis, allowing the GP to get paid before LPs receive both their initial investment *and* their preferred rate of return (i.e., the hurdle rate) on the entire fund.
- In whole-of-fund waterfalls, all distributions go to the LPs as deals are exited and the GP does not participate in any profits until the LPs receive their initial investment and the hurdle rate has been met. In contrast to deal-by-deal waterfalls, whole-of-fund waterfalls occur at the aggregate fund level and are more advantageous to the LPs.

Exhibit 10 and Exhibit 11 illustrate how the cash flow to the GP differs in a deal-by-deal waterfall with a clawback provision versus a whole-of-fund waterfall, respectively. In the first instance, the fund initially grants the GP payouts of \$2 million, \$3 million, and \$8 million because Investments 1, 2, and 3 generate a profit when sold. However, all subsequent investments either break even or return a loss, meaning that over the course of the fund's life, it breaks even at an aggregate level. Thus, the GP must return all early payouts, to compensate for subsequent losses.

#### Exhibit 10: Deal-by-Deal (American) Waterfall Example—with Clawback Provision

Investment no.	Year		Amount (\$mm)		Profit		
	Invested	Sold	Invested	Sold	\$mm	%	GP at 20%
1	1	4	\$10	\$20	\$10	26.0%	\$2
2	2	5	\$20	\$35	\$15	20.5%	\$3
3	2	7	\$40	\$80	\$40	14.9%	\$8
4	3	7	\$20	\$20	—	—	—
5	3	8	\$35	\$25	(\$10)	neg	(\$2)
6	4	9	\$25	\$20	(\$5)	neg	(\$1)
7	5	9	\$30	\$—	(\$30)	neg	(\$6)
8	5	10	\$20	\$—	(\$20)	neg	(\$4)
Total	1	10	\$200	\$200	—	—	—

In the second instance, the GP would receive no payouts until the LPs have received their initial investment and the hurdle rate has been met at the aggregate level. Since the fund only breaks even at the aggregate level, the GP would receive no performance payouts.

#### Exhibit 11: Whole-of-Fund (European) Waterfall Example

Investment no.	Year		Amount (\$mm)		Profit		
	Invested	Sold	Invested	Sold	\$mm	%	GP at 20%
1	1	4	\$10	\$20	\$10	26.0%	—
2	2	5	\$20	\$35	\$15	20.5%	—
3	2	7	\$40	\$80	\$40	14.9%	—
4	3	7	\$20	\$20	—	—	—

Investment no.	Year		Amount (\$mm)		Profit		
	Invested	Sold	Invested	Sold	\$mm	%	GP at 20%
5	3	8	\$35	\$25	(\$10)	neg	—
6	4	9	\$25	\$20	(\$5)	neg	—
7	5	9	\$30	\$—	(\$30)	neg	—
8	5	10	\$20	\$—	(\$20)	neg	—
Total	1	10	\$200	\$200	—	—	—

### QUESTION SET



1. Compensation structures for alternative investments *most likely* are:

- A. less complex than those of traditional investments.
- B. equally complex as those of traditional investments.
- C. more complex than those of traditional investments.

**Solution:**

C is correct. The illiquidity, complexity, and long-term nature of alternative investments require more complex structures to bridge potential gaps between manager and investor interests. Alternative investment structures may explicitly address both the roles and responsibilities of investors and managers to address these gaps. In addition, alternative investment structures tailor the distribution of returns between these two parties to minimize the divergence of incentives between manager and investor. A is incorrect because the compensation structures for alternative investments are more complex, not less complex, than those for traditional investments. B is incorrect because the compensation structures for alternative investments are more complex than, not equally complex as, those for traditional investments.

2. Identify the following statement as true or false: Limited partners (LPs) are involved in the management of the alternative investment fund in which they invest; they assist the general partner (GP) in the operations and decisions of the fund.

- A. True
- B. False

**Solution:**

False. LPs play passive roles and are not involved in the management of the fund (although co-investment rights allow LPs to make additional direct investments in the portfolio companies); the operations and decisions of the fund are controlled solely by the GP.

3. Calculate the general partner's performance fee earned based on the following terms:

Single-period fund rate of return	20%
Hard hurdle rate	10%



GP performance fee	18%
Catch-up clause	none

A. 1.6%

B. 1.8%

C. 2.0%

**Solution:**

The correct answer is B, 1.8%, determined as follows:

A GP's rate of return when there is no catch-up clause is calculated as

$$r_{GP} = \max[0, p(r - r_h)],$$

where

Symbol	Stands for
$r_{GP}$	GP's rate of return
$p$	GP's performance fee
$r$	Single-period fund rate of return
$r_h$	Hard hurdle rate

Using the terms noted results in a rate of return for the GP of 1.8%:

$$r_{GP} = \max[0, 18\%(20\% - 10\%)].$$

$$r_{GP} = 1.8\%.$$

A is incorrect because it is calculated using a single-period fund rate of return of 18%, a hard hurdle rate of 10%, and a GP performance fee of 20%.

C is incorrect because it is calculated using a single-period fund rate of return of 18%, a hard hurdle rate of 8%, and a GP performance fee of 20%.

4. Determine the correct answers to fill in the blanks: A \_\_\_\_\_ agrees to manage a fund's operations under an agreed standard of care and to perform such activities as buying or selling assets, borrowing funds, establishing reserves, or entering into contracts on behalf of the fund. A \_\_\_\_\_ establishes the terms of a limited partnership and governs the actions and decisions of the limited partnership.

**Solution:**

A general partner agrees to manage a fund's operations under an agreed standard of care and to perform such activities as buying or selling assets, borrowing funds, establishing reserves, or entering into contracts on behalf of the fund. A limited partnership agreement (LPA) establishes the terms of a limited partnership and governs the actions and decisions of the limited partnership.

5. Identify the investment type in which each of the following investments *best* fits.

Investment	Traditional Investment	Alternative Investment
A. Alibaba shares traded on the Frankfurt Stock Exchange		
B. Real estate holdings		
C. Hedge fund shares invested in common stock		
D. An investment in an infrastructure project via a public–private partnership		
E. The most junior tranche in an asset-backed security		

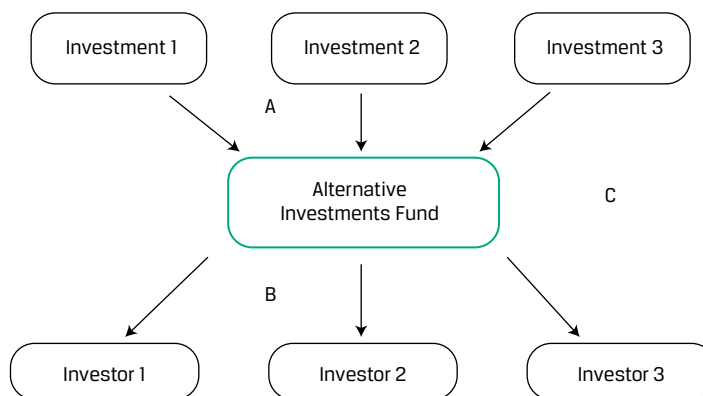
**Solution:**

Investment	Traditional Investment	Alternative Investment
A.	Publicly traded equity shares are a traditional, not an alternative, investment.	
B.		Real estate is a real, tangible physical asset that will generate current and/or future cash flows.
C.		Hedge funds are private investment vehicles, not traditional ones, even though they may invest in public equities or fixed income, private capital, and/or real assets. They are distinguished by their investment approach rather than the types of investments they make via the fund.
D.		Infrastructure is a special type of real asset typically involving land, buildings, and other long-lived fixed assets that are intended for public use and provide essential services. When private investors get involved in such projects, they participate through a public–private partnership.
E.	The most junior tranche in an asset-backed security is still a fixed-income instrument. Fixed-income instruments are traditional investments.	

## PRACTICE PROBLEMS

1. Which of the following documents provides tailored terms for a specific investor?
  - A. A side letter
  - B. An excusal right
  - C. A limited partnership agreement
2. Identify the fee approach that *most directly* encourages private equity fund managers to invest selectively, not just quickly. Management fees:
  - A. based on committed capital
  - B. combined with an incentive fee
  - C. based on a fixed percentage of assets under management
3. Which performance fee modification grants limited partners the right to reclaim a portion of the general partner's performance fee?
  - A. Catch-up clause
  - B. High-water mark
  - C. Clawback provision
4. Which of the following features is *most likely* associated with an alternative investment rather than a traditional one?
  - A. Illiquidity
  - B. Smaller capital outlays
  - C. Shorter investment time horizons
5. Assets developed through a public–private partnership typically involve:
  - A. long-lived fixed assets intended for public use.
  - B. ownership capital used in the early life cycle of a venture.
  - C. commercial real estate where private business activity is the primary cash flow source.

6.



Referring to the diagram above, identify the label (A, B, or C) that *best* corresponds to each of the following financial measures

A.	1. Management fee
B.	2. Net return
C.	3. Gross return

7. Which of the following options is the *most likely* ranking of the alternative investment methods—from most desirable to least—for the following pension fund? A well-endowed university pension fund with significant in-house knowledge, skills, and oversight capabilities is seeking alternative investments to diversify its existing asset portfolio and produce higher returns than more traditional investments while retaining as much control as possible.
- A. fund investment, co-investment, direct investment
  - B. direct investment, co-investment, fund investment
  - C. co-investment, direct investment, fund investment
8. From the following options, identify the *most important* reason that investors in alternative funds compensate managers using a performance-based fee structure. To:
- A. penalize managers for poor or declining performance over time
  - B. better align manager and investor incentives over longer periods
  - C. protect themselves from paying managers twice for the same returns

## SOLUTIONS

1. The correct answer is A. In addition to limited partnership agreements (LPAs), a supplemental document known as a side letter also may be negotiated. Side letters are agreements between the general partner and one or more limited partners that exist outside the LPA and whose terms override or modify the original limited partnership agreement terms. A side letter is negotiated to address the unique legal, regulatory, or reporting requirements of a specific investor. B is incorrect because an excusal right gives an investor the right to forgo a capital contribution or to not participate in a particular type of investment. An excusal right is granted when it has been stipulated in a side letter, not independently. C is incorrect because the limited partnership agreement establishes and governs the terms of a limited partnership; its terms apply to all limited partners, not just to a specific investor who requires tailored terms.
2. The correct answer is A. Private equity funds typically calculate their management fee based on committed capital, which is the total amount that the limited partners have promised to fund future investments, rather than based on assets under management. The committed-capital basis for management fees is an important distinction from hedge funds, whose management fees are typically based on assets under management. Having committed capital as the basis for management fee calculations reduces the incentive for GPs to deploy the committed capital as quickly as possible to grow their fee base and thus allows the GPs to be selective about deploying capital into investment opportunities. B is incorrect because alternative investment funds usually combine a management fee with a performance fee paid when fund returns exceed a specified hurdle rate. Although this combined fee approach is typical for alternative investment funds, it is not what encourages managers to invest selectively, rather than quickly. C is incorrect because typically hedge funds and REITs charge a management fee on assets under management, while private equity funds instead levy management fees on committed capital.
3. C is correct. A clawback provision grants limited partners the right to reclaim a portion of the general partner's performance fee. Clawback provisions are usually activated when a GP exits successful deals early on but incurs losses on deals later in the fund's life. A is incorrect because a catch-up clause allows a general partner to accelerate performance fees once a fund exceeds a specified soft hurdle rate. B is incorrect because the high-water mark identifies the fund's peak value as of a performance calculation date net of fees; it is not a performance fee modification.
4. A is correct. Investors are often attracted to alternative investments if they are seeking greater diversification and higher expected returns than traditional investments can deliver; alternative investments are often longer-term, illiquid investments in less efficient markets. B is incorrect because alternative investments often require larger capital outlays, not smaller ones than traditional investments. C is incorrect because alternative investments typically require long investment time horizons compared to those of traditional investments.
5. A is correct. In a public-private partnership (PPP), both governments and private investors are involved in funding and completing long-lived fixed assets intended for public use and/or to provide essential services. B is incorrect because ownership capital used for non-public companies in the early life cycle of a venture pertains to venture capital, not to public-private partnerships. C is incorrect because typically a public-private partnership is used to fund and develop infrastructure (i.e., long-lived fixed assets intended for public use and to provide

essential services), not commercial real estate for private business activity.

6. A. 2 is correct. In alternative fund investing, the fund manager pays the net return (gross return less management fees) to investors.  
B. 3 is correct. The returns generated by fund investments are gross returns. From these, management deducts its fees, paying the remainder (net fees) to fund investors.  
C. 1 is correct. Management fees and performance fees are how alternative fund managers are compensated for managing the fund and its investments.
7. B is correct. It shows the three alternative investment methods from most control to least. Direct investing offers investors maximum flexibility and control when it comes to investment choice, methods of financing, and timing. Co-investors gain more control than fund-only investors but not as much as direct investors. Fund investors have the least control; their decisions are limited to either investing in the fund or not. A is incorrect because it ranks the three investment methods from least control to most. C is incorrect because it puts co-investment, the alternative investment method with some control but not the most, in front of direct investment, which offers maximum control and flexibility.
8. B is correct. Investors in alternative funds usually compensate managers using a performance-based, versus flat, fee structure to better align manager and investor incentives over longer periods. A is incorrect because while performance fees/fee structures can penalize managers for declining performance, that is not the most important reason for a performance-based fee structure. C is incorrect because although calculation of a manager's performance may include reference to a fund's peak value (high-water mark) in order to avoid paying twice for the same returns, this is not the most important reason that investors in alternative funds compensate using a performance-based fee structure.

## LEARNING MODULE

## 2

## Alternative Investment Performance and Returns

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	describe the performance appraisal of alternative investments
<input type="checkbox"/>	calculate and interpret alternative investment returns both before and after fees

## INTRODUCTION

1

Investment performance measurement on common asset classes, such as public equity and debt, is relatively straightforward. In alternative investments, there are unique features that make this asset class somewhat complicated when it comes to measurement of investment risk and return. In this learning module, we first discuss these unique characteristics and the challenges they pose to performance appraisal. Then we explain the various features in the complex fee arrangement in alternative investments and the nuances when it comes to calculating investor returns.

### LEARNING MODULE OVERVIEW



- Alternative investments differ from traditional asset classes in that they involve longer time horizons, unique patterns of cash flows, the use of leverage, illiquid positions, more complex fee structures, different tax and accounting treatment, and so on. In addition, returns are usually less normally distributed for alternative investments than for traditional investments.
- The investment life cycle is usually longer and involves three phases: capital commitment, capital deployment, and capital distribution.
- Internal rate of return (IRR) is often the preferred measure for alternative investment returns. The multiple of invested capital (MOIC) is often used as a shortcut measure, but it ignores the timing of cash flows.
- Customized and complex compensation arrangements seek to align manager and investor incentives. Special provisions also exist for the lockup and redemption of capital from investors.

- In addition to a base management fee, alternative investments often charge additional performance fees based on a percentage of periodic fund returns. When calculating fees and investors' net returns, different features have to be considered, such as founder share class, either/or fee structure, hurdle rate, and high-water mark and clawback clauses.
- It is difficult to generalize performance appraisal for these investments because returns may vary depending on how and when a particular investor invested in a particular vehicle.

### LEARNING MODULE SELF-ASSESSMENT



1. In which part of the investment life cycle of a private equity investment should investors generally expect a positive cash flow?

- A. Capital commitment
- B. Capital deployment
- C. Capital distribution

**Solution:**

C is correct. In the initial capital commitment phase, fees and expenses are immediately incurred prior to capital deployment, and assets may generate little or no income during this first phase. In the capital deployment phase, cash outflows typically exceed inflows as funds are deployed. Only in the capital distribution phase can excess income be generated from the invested properties and substantial capital gains be realized upon the sale of assets.

2. Why is IRR preferred for performance measurement for alternative investments?

- A. IRR is commonly used for other asset classes.
- B. IRR is easy and intuitive to calculate.
- C. IRR takes into account the timing of cash flows in long-lived alternative investments.

**Solution:**

C is correct. IRR is seldom used to measure investment performance of other asset classes with publicly quoted market prices. Although IRR is complicated to calculate and involves assumptions on opportunity costs and reinvestment rates, it is the best metric to evaluate long-lived alternative investments because it takes into account the unique timing of cash flows in the investment life cycle of alternative investments.

3. Which of the following statements regarding hedge fund fee structure is correct?

- A. The periodic returns of all investors in the same fund must be identical.
- B. Hedge funds usually charge a performance fee based on a percentage of periodic return above a certain threshold.



- C. The management and performance fee rates are always the same for all investors in the same fund.

**Solution:**

B is correct. A hedge fund usually charges both a flat management fee and an additional performance fee based on a percentage of periodic fund returns. Periodic performance results may vary based on which investor has invested and when the investor invested into the fund. Besides, a particular investor may face significantly lower incentive fees if she invests more capital in a fund at an earlier phase or is willing to accept greater restrictions on redemptions.

4. A \$100 million hedge fund charges all its investors a 2% management fee and a 20% performance fee if the periodic return, net of management fee, exceeds a 5% hard hurdle rate. All fees are deducted based on the end-of-year value. If the fund makes a gross return (before fees) of 8% for the year, what is the investor's return, net of fees, *closest* to (ignoring any high-water mark provisions)?

- A. 4.67%  
B. 5.67%  
C. 5.84%

**Solution:**

B is correct. If the hedge fund makes 8% gross return for the year, its net asset value has grown to \$108 million before any fees are deducted.

Management fee =  $\$108 \times 2\% = \$2.16$  million.

Performance fee =  $[(\$108 - \$2.16) - (\$100 \times \$1.05\%)] \times 20\% = \$0.168$  million.

Net asset value after fee deduction =  $\$108 - \$2.16 - \$0.168 = \$105.672$  million.

Net investor return =  $(\$105.672 - \$100)/\$100 \approx 5.67\%$ .

5. A €100 million private equity fund has a preferred return of 5% per annum, 20% carried interest with full catch-up, and standard clawback clauses. The fund realizes a gross gain of 50% in two years before it distributes all its capital back to its LPs. Ignoring management fees, the total carried interest to the GP for the two years is *closest* to:

- A. €8 million.  
B. €10 million.  
C. €16 million.

**Solution:**

B is correct. After two years, the net asset value of the fund has grown to  $\text{€}100 \text{ million} \times 150\% = \text{€}150 \text{ million}$  shortly before distribution. The preferred return to the LPs for the two years totaled  $\text{€}100 \text{ million} \times 5\% \times 2 = \text{€}10 \text{ million}$ . (Note that annual preferred return is typically not compounded.) Next, the GP is allowed full catch-up until the GP's carried interest has caught up to the 20% of the total profit accounted for so far, or €2.5 million ( $\text{€}10 \text{ million} / 0.8 \times 0.2$ ). The remaining profit of  $50 - 10 - 2.5 = \text{€}37.5 \text{ million}$  is then split 80/20 between the LPs and GP; that is, the GP will get another €7.5 million ( $\text{€}37.5 \text{ million} \times 20\%$ ). Therefore, the total carried interest to the GP is  $2.5 + 7.5 = \text{€}10 \text{ million}$ , which is exactly 20% of the €50 million gain.

# 2

## ALTERNATIVE INVESTMENT PERFORMANCE



describe the performance appraisal of alternative investments

The unique features, form, and structure of alternative investments must be considered when evaluating the relative performance *between* alternative investments and when comparing their performance to that of more common asset classes over time. In particular, such features as staggered capital commitments over time, longer required investment horizons, reduced liquidity, and less efficient markets highlighted in prior lessons must be factored into the performance appraisal for alternative investments. Alternative investment returns are usually less normally distributed and therefore require different measures of risk and return than those used for more traditional asset classes.

### Alternative Investment Performance Appraisal

Appraising the performance of alternative investments requires more scrutiny in certain areas than traditional asset classes do.

### Comparability with Traditional Asset Classes

Public equity and debt securities share several characteristics that facilitate the comparison of their performance over a particular period. These standardized claims involve no further required capital commitments and provide identical claims to periodic cash flows, such as dividends in the case of shareholders or contractual bond coupons and principal for debtholders. Prices of publicly traded securities are often continuously quoted, with large peer groups of similar investments available and common indexes used to benchmark returns. Performance appraisal of publicly traded securities is thus straightforward to implement and evaluate.

In contrast, alternative investments are customized investments whose distinctive features complicate performance appraisal between investments and across asset classes. These features include

- the *timing* of cash inflows and outflows for specific investments,
- the use of borrowed funds,
- the *valuation* of individual portfolio positions over specific phases of the investment life cycle, and
- more complex fee structures and tax and accounting treatment.

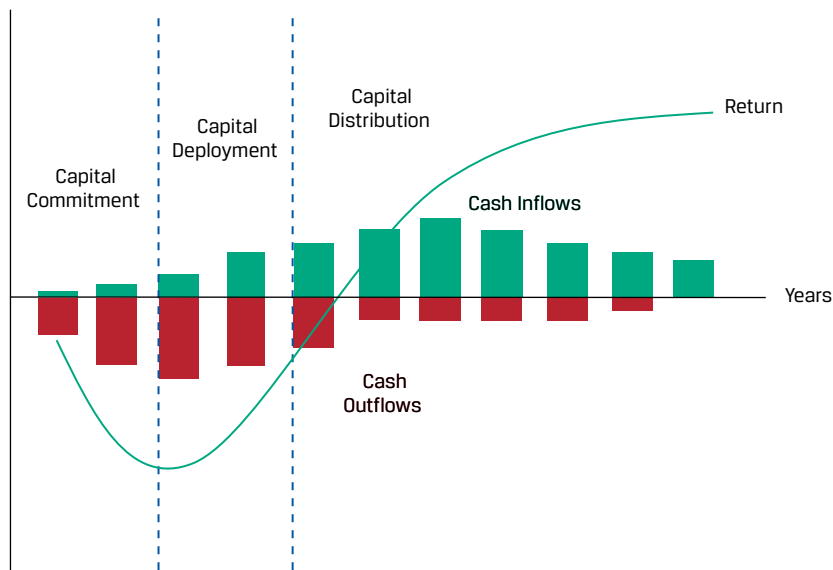
### Performance Appraisal and Alternative Investment Features

When appraising alternative investments, four areas to focus on include the life cycle phase of the investment, the amount of borrowed funds used to maintain the market position, the valuation of the assets, and the fee structure of the fund.

### Investment Life Cycle

Unlike public debt or equity securities purchased in the primary or secondary market, alternative investments usually involve a longer investment life cycle with distinct phases characterized by net cash outflows and inflows that complicate periodic return comparisons. Life cycle phases and timing vary across alternative investment types but generally fall into three distinct periods, as shown in Exhibit 1.

**Exhibit 1: Investment Life Cycle**



- **Capital commitment:** Alternative managers identify and select appropriate investments with either an immediate or a delayed commitment of capital (known as a capital call) that may be in an early-stage company in the case of venture capital, a more mature firm for private equity, or one or more properties in the case of real estate. Returns are usually negative over this phase because fees and expenses are immediately incurred prior to capital deployment and assets may generate little or no income during this first phase.
- **Capital deployment:** Over this second phase, alternative managers deploy funds to engage in construction or make property improvements in the case of real estate or infrastructure, incur expenses in the turnaround phase of a mature company in the case of private equity, or initiate operations for a startup using venture capital. Cash outflows typically exceed inflows, with management fees further reducing returns.
- **Capital distribution:** When the turnaround strategy, startup phase, or property improvements are completed and if the investment is successful, the underlying assets appreciate in price and/or generate income in excess of costs, causing fund returns to accelerate. The fund may realize substantial capital gains from liquidating or exiting its investments, which may involve an initial public offering (IPO) for venture capital or the sale of properties in the case of real estate.

The so-called **J-curve effect** (because it resembles the letter J) shown in Exhibit 1 represents the initial negative return in the capital commitment phase followed by an acceleration of returns through the capital deployment phase. Returns often level off as capital is distributed to investors, investments are sold, and the fund is closed.

As a result of the cash inflows and outflows that occur over the investment life cycle, an internal rate of return is often used as an initial approach to calculate investment returns for these investments, which include private equity and real estate investments.

The performance assessment in both private equity and real estate investments depends far more on the timing and magnitude of cash flows in and out of the investments, and these are often hard to standardize and anticipate. Given the long time horizon, the application of different tax treatments can have a non-trivial impact on after-tax investment returns.

As a general rule, the best way to start evaluating such investments is with the IRR, taking into account the respective cash flows into an investment and the timing thereof, versus the magnitude and the timing of the cash flows returned by the investment (inclusive of tax benefits).

In an independent, fixed-life private equity fund, the decisions to raise money, take money in the form of capital calls, and distribute proceeds are all at the discretion of the private equity manager. Timing of cash flows is an important part of the investment decision process. The private equity manager should thus be rewarded or penalized for the results of those timing decisions, and the calculation of an IRR is key for doing so.

Although the determination of an IRR involves certain assumptions about a financing rate to use for outgoing cash flows (typically a weighted average cost of capital) and a reinvestment rate to use for incoming cash flows (which must be assumed and may or may not actually be earned), the IRR is the key metric used to assess longer-term alternative investments in the private equity and real estate worlds. IRR calculations will be covered in more depth in the Corporate Issuers topic area.

### EXAMPLE 1

#### Peterburgh Capital, LLC

Peterburgh Capital, LLC, a private equity vehicle, is considering investment in various companies and is expecting the following cash flow pattern from these investments:

Year	Cash Inflow (USD)	Cash Outflow (USD)
0	—	2,500,000
1	—	4,000,000
2	—	1,700,000
3	500,000	1,000,000
4	1,000,000	—
5	3,000,000	—
6	12,500,000	—

IRR can be calculated by the standard formula  $\sum_{t=0}^n \frac{CF_t}{(1+r)^t} = 0$ , or using a financial calculator,  $r \approx 14.596\%$ .

As is true for any IRR calculation, assumptions regarding the opportunity cost of outgoing cash flows and the reinvestment rate for incoming cash flows will affect the results.

Because of this complexity, a shortcut methodology often used by both private equity and real estate managers involves simply citing a **multiple of invested capital (MOIC)**, or money multiple, on total invested capital (which is paid-in capital less management fees and fund expenses). Here, one simply measures the total value of all realized investments and residual asset values (assets that may still be awaiting their ultimate sale) relative to an initial total investment. MOIC is calculated as follows:

$$\text{MOIC} = (\text{Realized value of investment} + \text{Unrealized value of investment}) / \text{Total amount of invested capital.}$$

(1)

Although the MOIC ignores the timing of cash flows, it is easier to calculate, and it is intuitively easier to understand when someone says he received two or three times his initial investment. But how long it takes to realize this value does matter. A 2× return on one's initial investment would be great if the return were collected over two years but far less compelling if it took 15 years to realize.

In general, because private equity and real estate investments involve longer holding periods, there is less emphasis on evaluating them in terms of shorter-term portfolio correlation benefits. After a private equity fund has fully drawn in its monetary commitments, interim accounting values for a private equity partnership become less critical for a period of time because no incoming or outgoing cash flows may immediately hinge on such valuations. During this “middle period” in the life of a private equity fund, accounting values may not always be particularly reflective of the future potential realizations (and hence the expected returns) of the fund. It is not that the value of the investments is not actually rising and falling in the face of economic influences; rather, accounting conventions simply leave longer-lived investments marked at their initial cost for some time or make only modest adjustments to carrying value until clearer impairments or realization events take place.

#### KNOWLEDGE CHECK: MOIC CALCULATION



Himitsu, a private equity firm, makes an initial investment of JPY3.8 billion into ZZZ company in Year 0. Eight years later, it sells its stake in ZZZ for JPY8.5 billion. Additional capital investments were made in Year 2 and in Year 3 for JPY1.2 billion and JPY200 million, respectively.

1. Calculate the MOIC.

**Solution:**

$$\text{MOIC} = 8.5 / (3.8 + 1.2 + 0.2) = 1.63\times.$$

	Amount	Year
Invested Capital	(3,800,000)	0
	—	1
Additional Capital	(1,200,000)	2
Additional Capital	(200,000)	3
Liquidity Event	8,500,000	8
<b>MOIC</b>	<b>1.63×</b>	
<b>IRR</b>	<b>20%</b>	

### Use of Borrowed Funds

Alternative investments may use borrowed funds to increase investment returns. This form of financial leverage has the effect of magnifying both gains and losses by allowing investors to take a market position that is larger than the capital committed. Consider a cash investment  $V_c$  with a periodic rate of return  $r$ . If we assume an investor is able to borrow at a periodic rate of  $r_b$  to increase the size of its investment by borrowed funds of  $V_b$ , we can calculate a simple *leveraged* rate of return  $r_L$  for the period as follows:

$$r_L = \text{Leveraged portfolio return/Cash position} = [r \times (V_c + V_b) - (V_b \times r_b)] / V_c. \quad (2)$$

We can rearrange Equation 2 to show the relationship between the cash portfolio return,  $r$ , and the leveraged rate of return,  $r_L$ , as follows:

$$r_L = r + V_b/V_c(r - r_b). \quad (3)$$

Hedge funds leverage their portfolios by using derivatives or borrowing capital from **prime brokers**, negotiating with them to establish margin requirements, interest, and fees in advance of trading. In a typical **margin financing** arrangement, the prime broker essentially lends the hedge fund the shares, bonds, or derivatives, and the hedge fund deposits cash or other collateral into a margin account with the prime broker based on certain fractions of the investment positions. The margin account represents the hedge fund's net equity in its positions. The minimum margin required depends on the riskiness of the investment portfolio and the creditworthiness of the hedge fund.

Leverage is a large part of the reason that some hedge funds either earn larger-than-normal returns or suffer significant losses. If the margin account or the hedge fund's equity in a position declines below a certain level, the lender initiates a margin call and requests that the hedge fund put up more collateral. An inability to meet margin calls can have the effect of magnifying or locking in losses because the hedge fund may have to liquidate (close out) the losing position. This liquidation can lead to further losses if the order size is sufficiently large to move the security's market price before the fund can sufficiently eliminate the position. Under normal conditions, the application of leverage may be necessary for yielding meaningful returns from given quantitative, arbitrage, or relative value strategies. But with added leverage comes increased risk.

### EXAMPLE 2

#### Leverage by Lupulus Opportunity Fund LLC

Lupulus Opportunity Fund LLC, a hedge fund that has USD100 million of capital, ordinarily uses leverage to invest in a variety of equity-linked notes.

*Scenario 1:* Suppose Lupulus's underlying positions return 8%. If it could add leverage of USD50 million to the portfolio at a funding cost of 4%, what would have been the leveraged return?

Using Equation 3, the leveraged return can be calculated as follows:

$$V_c = 100; V_b = 50.$$

$$r_L = 0.08 + (50/100)(0.08 - 0.04) = 10\%.$$

*Scenario 2:* Suppose Lupulus's underlying positions incur a loss of 2% instead of earning a gain. What would have been the leveraged return if Lupulus had borrowed USD50 million at 4%?

Again, using Equation 3,  $V_c = 100$  and  $V_b = 50$

$$r_L = -0.02 + (50/100)(-0.02 - 0.04) = -5\%.$$

In other words, Lupulus would have magnified its portfolio loss from  $-2\%$  to  $-5\%$  by adding leverage of USD50 million.

*Scenario 3:* If Lupulus's underlying positions make a gain of  $6\%$ , what is the breakeven borrowing rate at which Lupulus is indifferent to adding leverage to the portfolio?

From Equation 3, Lupulus would be indifferent about adding leverage when the magnification effect of leverage equals 0; that is,

$$\text{when } r_L = r, \text{ or } (V_b/V_c)(r - r_b) = 0,$$

$$r_b = r = 6\%.$$

As the previous example shows, the leveraged rate of return will exceed that of an equivalent cash portfolio if the market return on assets exceeds the borrowing rate. This example assumes the investor has unlimited access to borrow freely on an unsecured basis. The access to borrowed funds or lack thereof is an important factor in evaluating the risk of leveraged investments.

### Valuation

Alternative assets are often characterized by illiquidity, which makes performance appraisal over time challenging and periodic comparison with common asset classes difficult. Accounting rules dictate that investments must be recorded at their **fair value** for financial reporting purposes. The fair value of an investment is a market-based measure based on observable or derived assumptions to determine a price that market participants would use to exchange an asset or liability (often referred to as the *exit price* for a seller) in an orderly transaction at a specific time. Assumptions used to measure fair value follow a three-level hierarchy, as shown in Exhibit 2:

**Exhibit 2: Fair Value Hierarchy**

Level	Description	Sample Application and Method
<b>Level 1</b>	Quoted prices in active markets for identical asset/liability that may be accessed as of measurement date	Exchange-traded public equity securities (observed closing market price)
<b>Level 2</b>	Inputs other than quoted market prices in Level 1 that are directly or indirectly observable for an asset/liability	Over-the-counter interest rate derivatives (pricing model using quoted market prices)
<b>Level 3</b>	Unobservable inputs are used to measure fair value for asset/liability in which there is little, if any, market activity as of the measurement date	Private equity or real estate investments (cash flow projection models with reasonably available market participant assumptions)

While common asset classes often rely on Level 1 inputs, the interim valuation of private equity, real estate, and other less frequently traded assets based on Level 3 inputs is more challenging. Interim accounting values may be less critical for the partnership itself over a period where no incoming or outgoing cash flows are expected. The lack of new market information over time may thus anchor the value of these long-lived investments at or near initial cost, with adjustments to carrying value when impairments or realization events occur. The relatively stable accounting valuations of

these strategies may give investors a false sense that they are less correlated with and less volatile than other investments, although a more realistic picture may emerge if managers are forced to liquidate a portfolio prematurely.

For Level 3 asset pricing, regardless of the model used by a manager in such circumstances, that model should be independently tested, benchmarked, and calibrated to industry-accepted standards to ensure a consistency of approach. Because of the potential for conflicts of interest when applying estimates of value, hedge funds must develop procedures for in-house valuation, communicate these procedures to clients, and adhere to them consistently.

Notwithstanding best practice, the very nature of assets that can be valued only on a “mark-to-model” basis can and should be a focus for the alternative asset investor. A model may reflect an imperfect theoretical valuation and not a true liquidation value. The illiquid nature of these assets means that estimates, rather than observable transaction prices, may well have factored into any valuation. As a result, returns may be smoothed or overstated and the volatility of returns, understated. As a generalized statement, any investment vehicle that is heavily involved with Level 3 priced assets deserves increased scrutiny and due diligence.

### Fees

Alternative investment fees also vary from those for common asset classes, which typically involve a flat management fee. Alternative investments often levy additional performance fees based on a percentage of periodic fund returns. Performance appraisal for these investments can be difficult to generalize, because results may vary significantly based on *which* investor has invested *when* in a particular vehicle.

For example, an investor may face significantly lower incentive fees if she invests *more* capital in a fund at an *earlier* phase or is willing to accept *greater* restrictions on redemptions. Also, an investor entering an alternative fund following a sharp drop in value may incur performance fees if the fund rises, while an earlier investor who experienced the sharp decline in value from its peak may be exempt from such fees for the same period. These and other details of these complex alternative investment fee arrangements and their effect on investment return are the subject of the following lesson.

### QUESTION SET



1. Which of the following is *not* a factor that makes comparison of performance between alternative investments and public securities difficult?
  - A. Alternative investments charge higher fees.
  - B. The use of leverage in alternative investments magnifies their risk and return measures.
  - C. The fair value of portfolio positions in alternative investments may not be readily available.

#### Solution:

A is correct. Although an alternative investment may charge a higher fee, it is not the absolute fee level but the complexity of the fee arrangement that makes alternative investment appraisal unique compared to other common asset classes.

Alternative investments often involve the use of explicit leverage, which has the effect of magnified gains and losses. Alternative assets are often characterized by illiquidity with unobservable market prices, making performance appraisal over time and periodic comparison with common asset classes challenging.



2. Describe the J-curve effect in alternative investments.

**Solution:**

The J-curve effect in alternative investments describes the initial negative return in the capital commitment phase followed by an acceleration of returns through the capital deployment phase. Returns often level off as capital is distributed to investors, investments are sold, and the fund is closed.

3. A private equity closes a fund with a capital commitment of €750 million. It has a capital call of €500 million initially and another €250 million at the end of Year 1. The management fee is 2% per annum. At the end of Year 5, a total of €1.0 billion is distributed to its investors, and the fund is left with €500 million in asset value. The multiple of invested capital (MOIC) after five years is *closest* to:

- A. 1.3×
- B. 2.0×
- C. 2.2×

**Solution:**

C is correct. Using Equation 1,  $MOIC = (\text{Realized value of investment} + \text{Unrealized value of investment}) / \text{Total amount of invested capital}$ , where invested capital equals total paid-in capital less management fees and fund expenses. MOIC is different from the IRR measure because it ignores the timing of cash flows.

Total paid-in capital =  $500 + 250 = 750$ .

Total management fee for 5 years =  $750 \times 0.02 \times 5 = 75$ .

Total invested capital =  $750 - 75 = 675$ .

$MOIC = (1,000 + 500) / 675 \approx 2.2\times$ .

4. If the market return of underlying portfolio positions is expected to be 10% and the fund manager adds 100% explicit leverage to its capital at a borrowing rate of 3%, what is the expected leveraged return?

**Solution:**

Using Equation 3,

$$r_L = r + (V_b/V_c)(r - r_b) = 0.10 + (100/100)(0.10 - 0.03) = 17\%.$$

5. Match the fair value classification of assets to their description for financial reporting purpose:

Fair Value Classification Description

1. Level 1 assets	(a) Inputs other than quoted market prices are directly or indirectly observable for valuation on measurement date
2. Level 2 assets	(b) Unobservable inputs are used to evaluate fair value for the assets
3. Level 3 assets	(c) Quoted prices in active markets for identical asset available on measurement date

**Solution:**

1. Level 1 assets: (c) Quoted prices in active markets for identical asset available on measurement date

2. Level 2 assets: (a) Inputs other than quoted market prices are directly or indirectly observable for valuation on measurement date
3. Level 3 assets: (b) Unobservable inputs are used to evaluate fair value for the assets

# 3

## ALTERNATIVE INVESTMENT RETURNS



calculate and interpret alternative investment returns both before and after fees

Hedge funds often use complex strategies to achieve outsized returns with low correlation with the broader market. These more complex strategies require more sophisticated portfolio management tools and a larger range of skills, making them more expensive to run. Rather than paying a very high flat management fee, investors demand that some of the compensation is aligned to the performance delivered by the strategy in the form of a performance fee.

Apart from the performance fee, there are also other customized, complex compensation arrangements that seek to align manager and investor incentives. These structures are designed to reward investors for early involvement, larger investments, and/or longer lockup periods. Managers benefit from applying specialized knowledge and skills to achieve steadily rising returns in excess of a designated hurdle rate. These complex fee structures impact returns for different investors in the same fund, as well as returns before and after fees across various alternative investments.

Another factor that can lock in or magnify losses for hedge funds is investor redemptions. Redemptions frequently occur when a hedge fund is performing poorly. Redemptions may require the hedge fund manager to liquidate some positions and potentially receive particularly disadvantageous prices when forced to do so by redemption pressures, while also incurring transaction costs.

Funds sometimes charge a **redemption fee** to discourage redemption and to offset the transaction costs for remaining investors in the fund. A **notice period** is a period (typically 30–90 days) in advance that investors may be required to notify a fund of their intent to redeem some or all their investment. This notice period allows the fund manager an opportunity to liquidate a position in an orderly fashion without magnifying the losses. A **lockup period** is the minimum holding period before investors are allowed to make withdrawals or redeem shares from a fund. This provision is intended to allow the hedge fund manager the required time to implement and potentially realize a strategy's expected results. If the fund receives a drawdown request shortly after a new investment, the lockup period forces the investors who made the request to stay in the fund for a period of time rather than be allowed to immediately withdraw. In addition, funds sometimes impose a **gate**, a provision that when implemented limits or restricts redemptions for a period of time, usually at the discretion of the fund manager. Investors should be aware of their liquidity needs before investing in a fund with restrictive provisions.

A hedge fund's ability to demand a long lockup period while raising a significant amount of investment capital depends a great deal on the reputation of either the firm or the hedge fund manager. Funds of hedge funds may offer more redemption flexibility than is afforded to direct investors in hedge funds because of special redemption arrangements with the underlying hedge fund managers, the maintenance of added cash reserves, access to temporary bridge-loan financing, or the simple avoidance of less liquid hedge fund strategies.

Ideally, redemption terms should be designed to match the expected liquidity of the assets being invested in, but even with careful planning, an initial drawdown can turn into something far more serious when it involves illiquid and obscure assets. These left-tailed loss events are not easily modeled for hedge funds.

## Alternative Investment Returns

### *Custom Fee Arrangements*

Alternative investments often involve customized fee arrangements combining management and performance-based fees that vary based on the size, timing, and/or terms of investor participation in the investment over time. Examples include the following:

- **Fees based on liquidity terms and asset size:** Limited partnerships may charge different rates depending on the liquidity terms that an investor is willing to accept (longer lockups resulting in lower fees), and managers may discount their fees for larger investors or for placement agents who introduced these investors. Different investors in the same fund may face different fee structures. For example, management fees for large LPs might range from 0.5% to 1.5%, with incentive fees reduced to 10%–15%, depending on the mandate. Such reductions can be meaningful in terms of net realized returns. However, smaller funds with strong performance (and capacity constraints) are able to maintain higher fees and may even decline business from larger investors rather than agree to a lower fee.
- **Founders shares:** As a way to entice early participation in startup funds, managers sometimes offer incentives known as **founders class shares**. Founders shares entitle investors to a lower fee structure, such as a 1.5% management fee and a 10% performance fee rather than a more standard 2% management fee and a 20% performance fee, and may apply only to the first \$100 million in assets invested, although cutoff thresholds vary. Another alternative is to reduce the fees for early founders share investors once the fund achieves a critical mass or performance targets.
- **“Either/or” fees:** Major institutional investors, such as the Teacher Retirement System of Texas, have demanded that alternative investment funds accept an **either/or fee** agreement by choosing *between* fixed management and variable performance fees. Managers agree *either* to charge a lower, 1% management fee (to cover expenses during down years) *or* to accept a higher, 30% incentive fee above a mutually agreed-upon annual hurdle (to incentivize and reward managers during up years), whichever is greater. Major investors offering larger commitments may negotiate such novel fee structures designed to reward returns in excess of a benchmark, while smaller investors, such as high-net-worth individuals with smaller commitments, usually face more traditional fees.

## Alternative Investment Return Calculations

Return calculations vary among alternative investments based on the form of the investments, as described in an earlier lesson. While more liquid alternative investments, such as REITs, commodity index exchange-traded funds, or other frequently traded investments, have a simple management fee structure similar to common assets, those with longer life cycles, illiquidity, and less transparency, such as private equity, hedge funds, and real estate, often use performance fees with modifications to create incentives for managers to act in the best interest of investors.

The impact of different fee arrangements and their effect on the resulting returns to investors is best illustrated using a series of examples. If we assume fixed GP management fees as a percentage of assets under management (AUM) of  $r_m$ , beginning-of-period assets of  $P_0$ , end-of-period assets of  $P_1$ , and a GP performance fee ( $p$ ) that is a percentage of total return, the GP's return in currency terms ( $R_{GP}$ ) is as follows:

$$R_{GP} = (P_1 \times r_m) + \max[0, (P_1 - P_0) \times p]. \quad (4)$$

And we may solve for an investor's periodic rate of return,  $r_i$ , as follows:

$$r_i = (P_1 - P_0 - R_{GP})/P_0. \quad (5)$$

### EXAMPLE 3

#### Kettleside Timberland LP

Kettleside is a timberland investment management organization with \$100 million of initial investment capital. It charges a 1% management fee based on year-end AUM ( $r_m$ ) and a 20% performance fee ( $p$ ). In its first year, Kettleside generates a 30% return. Assume management fees are calculated using an end-of-period valuation.

1. What are Kettleside's fees if the performance and management fees are calculated independently? What is an investor's effective return given this fee structure?

**Solution:**

Using Equation 4 with  $P_0$  equal to \$100 million,  $P_1$  of \$130 million,  $r_m = 1\%$ , and  $p = 20\%$ ,

$$\begin{aligned} R_{GP} &= \$130 \text{ million} \times 1\% + \max[0, (\$130 \text{ million} - \$100 \text{ million}) \times 20\%] \\ &= \$7.3 \text{ million.} \end{aligned}$$

$$\begin{aligned} r_i &= (\$130 \text{ million} - \$100 \text{ million} - \$7.3 \text{ million})/\$100 \text{ million} \\ &= 22.7\%. \end{aligned}$$

2. What are Kettleside's fees if the performance fee is calculated from the return *net* of the management fee? What is an investor's net return given this fee structure?

**Solution:**

In order to solve for GP return where performance fee is calculated *net* of management fees, we must modify Equation 4 as follows:

$$R_{GP(Net)} = (P_1 \times r_m) + \max\{0, [P_1(1 - r_m) - P_0] \times p\}. \quad (6)$$

Using Equation 6 with  $P_0$  of \$100 million,  $P_1$  of \$130 million,  $r_m = 1\%$ , and  $p = 20\%$ ,

$$R_{GP(Net)} = \$130 \text{ million} \times 1\% + \max\{0, [\$130 \text{ million}(0.99) - \$100 \text{ million}] \times 20\%\}$$

$$= \$7.04 \text{ million.}$$

$$r_i = (\$130 \text{ million} - \$100 \text{ million} - \$7.04 \text{ million})/\$100 \text{ million}$$

$$= 22.96\%.$$

The previous example demonstrates how fee calculations affect investor returns, with a higher return when fees are calculated on a net basis. Alternative investment databases and indexes usually report fund performance net of aggregated fees. If fee structures vary, the actual net-of-fee returns earned by various investors may vary from the quoted return.

One fee structure variation involves the modification of performance fees using hurdle rates and high-water marks. In the next example, we consider the effect of both on investor returns using the same details from the prior example.

#### EXAMPLE 4

##### Kettleside Timberland LP Performance Fee Modifications

As in the prior case, Kettleside Timberland LP has initial capital of \$100 million, charging a 1% management fee based on year-end AUM ( $r_m$ ) and a 20% performance fee ( $p$ ).

1. If Kettleside's fee agreement specifies a 6% hurdle rate with performance fees based on returns *in excess of* the hurdle rate, what are Kettleside's fees assuming the performance fee is calculated *net* of the management fee? What is an investor's net return given this fee structure?

##### Solution:

To solve for Kettleside's fees with a hard hurdle rate calculated net of management fees, we make a further adjustment to Equation 4 by incorporating the hurdle rate,  $r_h$ , as follows:

$$R_{GP(Net \text{ with Hurdle})} = (P_1 \times r_m) + \max\{0, [P_1(1 - r_m) - P_0 \times (1.06)] \times p\} \quad (7)$$

$$R_{GP(Net \text{ with Hurdle})}$$

$$= \$130 \text{ million} \times 1\% + \max\{0, [\$130 \text{ million}(0.99) - \$100 \text{ million}(1.06)] \times 20\%\}$$

$$= \$5.84 \text{ million.}$$

$$r_i = (\$130 \text{ million} - \$100 \text{ million} - \$5.84 \text{ million})/\$100 \text{ million}$$

$$= 24.16\%.$$

2. In the second year, Kettleside fund value declines to \$110 million. The fee structure is as specified in Question 1 but also includes the use of a high-water mark ( $P_{HWM}$ ) computed net of fees. What are Kettleside's fees in the

second year? What is an investor’s net return for the second year given this fee structure?

Solution:

We must again alter Equation 4 to include the high-water mark ( $P_{HWM}$ ) provision, as follows:

$$R_{GP(High-Water\ Mark)} = (P_2 \times r_m) + \max[0, (P_2 - P_{HWM}) \times p], \tag{8}$$

where  $P_{HWM}$  is defined as the maximum fund value at the end of any *previous* period net of fees. We may solve for investor return  $r_i$  in Period 2 as follows:

$$r_i = (P_2 - P_1 - R_{GP})/P_1. \tag{9}$$

$$\begin{aligned} R_{GP(High-Water\ Mark)} &= \$110\text{ million} \times 1\% + \max[0, (\$110\text{ million} - \$122.7\text{ million}) \times 20\%] \\ &= \$1.1\text{ million.} \end{aligned}$$

$$\begin{aligned} r_i &= (\$110\text{ million} - \$122.7\text{ million} - \$1.1\text{ million})/\$122.7\text{ million} \\ &= -11.247\%. \end{aligned}$$

The beginning capital position in the second year for the investors is \$130 million – \$7.3 million = \$122.7 million. The ending capital position at the end of the second year is \$110 million – \$1.1 million = \$108.9 million.

3. In the third year, Kettleside’s fund value increases to \$128 million. The fee structure is as specified in Questions 1 and 2 of Example 4. What are Kettleside’s fees in the third year? What is an investor’s net return for the third year given this fee structure?

Solution:

We amend Equations 8 and 9 to reflect returns for the third period and calculate as follows:


$$R_{GP(High-Water\ Mark)} = (P_3 \times r_m) + \max[0, (P_3 - P_{HWM}) \times p].$$

$$r_i = (P_3 - P_2 - R_{GP})/P_2.$$

Note that the high-water mark,  $P_{HWM}$ , is the highest value of the fund after fees in all previous years. In Kettleside’s case, it was \$122.7 million, the ending value in the first year,  $P_1$ .

Kettleside Timberland LP Performance Fee Modifications

Year	Fund Value (\$m), after Fees
0	100.00
1	122.70
2	108.90



High-Water Mark

$$\begin{aligned} R_{GP(High-Water\ Mark)} &= \$128\text{ million} \times 1\% + \max[0, (\$128\text{ million} - \$122.7\text{ million}) \times 20\%] \\ &= \$2.34\text{ million.} \end{aligned}$$

$$r_i = (\$128\text{ million} - \$108.9\text{ million} - \$2.34\text{ million})/\$108.9\text{ million}$$

= 15.39%.

The beginning capital position in the third year for the investors is \$110 million – \$1.1 million = \$108.9 million. The ending capital position for the third year is \$128 million – \$2.34 million = \$125.66 million, which represents a new high-water mark to be applied the following year for this investor.

Performance fee modifications may have similar or different effects on the periodic investor returns depending on the timing of an investment. For example, if two Kettleside investors were to purchase the fund at different times and had otherwise similar fee structures, they would both realize a fee reduction in the case of a hard hurdle equal to  $P_t \times r_h \times p$ , or the product of the end-of-period fund value for year  $t$ , the hurdle rate, and the performance fee. However, in the case of a high-water mark, the time-dependent nature of this fee modification gives different results for an investor who enters the fund at a later date, as in the following example.

#### EXAMPLE 5

##### Kettleside Timberland LP High-Water Mark for New Investor

At the end of Year 2, Kettleside Timberland LP has capital of \$108.9 million. Consider the Year 3 returns of a *new* investor assuming the same fund performance and fee structure—namely, a 1% management fee based on year-end AUM ( $r_m$ ), a 20% performance fee ( $p$ ), and a high-water mark provision.

$$R_{GP(High-Water\ Mark)} = (P_3 \times r_m) + \max[0, (P_3 - P_{HWM}) \times p].$$

$$r_i = (P_3 - P_2 - R_{GP})/P_2.$$

$$\begin{aligned} R_{GP(High-Water\ Mark)} &= \$128 \text{ million} \times 1\% + \max[0, (\$128 \text{ million} - \$108.9 \text{ million}) \times 20\%] \\ &= \$5.1 \text{ million.} \end{aligned}$$

$$\begin{aligned} r_i &= (\$128 \text{ million} - \$108.9 \text{ million} - \$5.1 \text{ million})/\$108.9 \text{ million} \\ &= 12.856\%. \end{aligned}$$

Note that the new investor in Example 5 realizes *no* high-water mark reduction in fees as in the prior case and therefore faces a *lower* periodic return than the investor participating since the fund's inception.

In other instances, the timing of returns can have a meaningful impact on manager fees and investor returns, as shown in the following example of a clawback provision.

#### EXAMPLE 6

##### Tenderledge Opportunity Fund LP—Clawback Provision

Tenderledge Opportunity Fund makes \$20 million in new investments, evenly divided with \$10 million into Arguston Inc. (a leveraged buyout) and \$10 million to Heartfield Digital (an early-stage venture). One year later, Arguston is sold to a strategic buyer for \$22 million after costs. Two years later, Heartfield Digital fails and Tenderledge is unable to recoup any of its original investment.

1. If Tenderledge's fee agreement as general partner (GP) specifies a 20% performance fee of aggregate profits ( $p$ ) with a clawback provision, which performance fees will Tenderledge accrue and what will it ultimately receive?

**Solution:**

Gain in the Arguston investment: \$22 million – \$10 million = \$12 million.

Loss in the Heartfield Digital investment: \$0 – \$10 million = –\$10 million.

Aggregate gain of Tenderledge after two years = \$12 million – \$10 million = \$2 million.

Tenderledge would initially accrue 20% of the \$12 million aggregate profit from the sale of Arguston at the end of the first year, or \$12 million  $\times$  20% = \$2.4 million. This amount is typically held in escrow for the benefit of the GP but not paid.

The failure of Heartfield Digital in Year 2 reduces the original \$12 million gain by \$10 million, so the aggregate fund gain at the end of Year 2 is only \$2 million. This net profit results in a performance fee of \$400,000 (= \$2 million  $\times$  20%). Tenderledge would then have to return \$2 million of the previously accrued performance fees to LP investor capital accounts due to the clawback provision.

**KNOWLEDGE CHECK FEE AND RETURN CALCULATIONS**



AWJ Capital is a hedge fund with \$100 million of initial investment capital. It charges a 2% management fee based on year-end AUM and a 20% incentive fee. In its first year, AWJ Capital has a 30% return. Assume management fees are calculated using end-of-period valuation.

1. What are the fees earned by AWJ if the incentive and management fees are calculated independently? What is an investor's effective return given this fee structure?

**Solution:**

AWJ fees:

\$130 million  $\times$  2% = \$2.6 million management fee.

(\$130 million – \$100 million)  $\times$  20% = \$6 million incentive fee.

Total fees to AWJ Capital = \$8.6 million.

2. What are the fees earned by AWJ assuming that the incentive fee is calculated from the return net of the management fee? What is an investor's net return given this fee structure?

**Solution:**

\$130 million  $\times$  2% = \$2.6 million management fee.

(\$130 million – \$100 million – \$2.6 million)  $\times$  20%  
= \$5.48 million incentive fee.

Total fees to AWJ Capital = \$8.08 million.

Investor return = (\$130 million – \$100 million – \$8.08 million)/\$100 million



= 21.92%.

3. If the fee structure specifies a hurdle rate of 5% and the incentive fee is based on returns in excess of the hurdle rate, what are the fees earned by AWJ assuming the performance fee is calculated net of the management fee? What is an investor's net return given this fee structure?

**Solution:**

$\$130 \text{ million} \times 2\% = \$2.6 \text{ million management fee.}$

$(\$130 \text{ million} - \$100 \text{ million} - \$5 \text{ million} - \$2.6 \text{ million}) \times 20\%$   
 $= \$4.48 \text{ million incentive fee.}$

Total fees to AWJ Capital = \$7.08 million.

Investor return =  $(\$130 \text{ million} - \$100 \text{ million} - \$7.08 \text{ million})/\$100 \text{ million}$   
 $= 22.92\%.$

4. In the second year, the fund value declines to \$110 million. The fee structure is as specified for Question 1 but also includes the use of a high-water mark (computed net of fees). What are the fees earned by AWJ in the second year? What is an investor's net return for the second year given this fee structure?

**Solution:**

$\$110 \text{ million} \times 2\% = \$2.2 \text{ million management fee.}$

No incentive fee because the fund has declined in value.

Total fees to AWJ Capital = \$2.2 million.

Investor return =  $(\$110 \text{ million} - \$2.2 \text{ million} - \$121.4 \text{ million})/\$121.4 \text{ million}$   
 $= -11.20\%.$

The beginning capital position in the second year for the investors is \$130 million – \$8.6 million = \$121.4 million. The ending capital position at the end of the second year is \$110 million – \$2.2 million = \$107.8 million.

5. In the third year, the fund value increases to \$128 million. The fee structure is as specified in Questions 1 and 4. What are the fees earned by AWJ in the third year? What is an investor's net return for the third year given this fee structure?

**Solution:**

$\$128 \text{ million} \times 2\% = \$2.56 \text{ million management fee.}$

$(\$128 \text{ million} - \$121.4 \text{ million}) \times 20\% = \$1.32 \text{ million incentive fee.}$

The \$121.4 million represents the high-water mark established at the end of Year 1.

Total fees to AWJ Capital = \$3.88 million.

Investor return =  $(\$128 \text{ million} - \$3.88 \text{ million} - \$107.8 \text{ million})/\$107.8 \text{ million}$   
 $= 15.14\%.$  The ending capital position at the end of Year 3 is \$124.12 million. This amount is the new high-water mark.

## Relative Alternative Investment Returns and Survivorship Bias

Investors seeking higher risk-adjusted returns with low correlation with common asset classes in alternative investments often track their performance based on *relative* returns. As is the case for more common asset classes, returns on individual alternative investments are usually compared to a benchmark of investments with similar features. These benchmarks may be interpreted differently or take on different characteristics in the case of alternative investments. For example, the use of a composite benchmark for private equity or real estate investments may be misleading if a specific investment is in a different life cycle phase than most of its peers. However, return comparisons between such investments of the same vintage year on an annual or “since inception” basis lead to more accurate results. That said, lockups and illiquidity may prevent an investor from reacting to underperformance by selling an investment.

Hedge fund indexes deserve greater scrutiny given changes to the universe of funds included in a benchmark over time. For example, studies suggest that over a quarter of all hedge funds fail within the first three years due to performance problems that lead to investor defections and fund closure. The exclusion of failed funds from a given benchmark is a form of selection bias that can lead investors to overly optimistic return expectations known as **survivorship bias**. Survivorship bias is a major problem among hedge fund indexes that include only current investment funds and exclude those funds that are no longer available. A second form of bias relates to how and when hedge fund returns are initially included in a benchmark index. For example, a fund manager may launch several hedge fund investments at once and include only the most successful funds in an index a couple of years after inception. The subsequent inclusion or “backfilling” of prior performance data on a selective basis serves to increase average reported returns in what is known as **backfill bias**. Because of survivorship and backfill biases, hedge fund indexes may not reflect actual average hedge fund performance but, rather, only the returns of those hedge funds that initially performed best and/or have not failed.

### EXAMPLE 7

#### Comparison of Returns: Investment Directly into a Hedge Fund or through a Fund of Hedge Funds

An investor is contemplating investing €100 million in either the ABC Hedge Fund (ABC HF) or the XYZ Fund of Funds (XYZ FOF). XYZ FOF has a “1 and 10” fee structure and invests 10% of its AUM in ABC HF. ABC HF has a standard “2 and 20” fee structure with no hurdle rate. Management fees are calculated on an annual basis on AUM at the beginning of the year. For simplicity, assume that management fees and incentive fees are calculated independently. ABC HF has a 20% return for the year before management and incentive fees.

1. Calculate the return to the investor from investing directly in ABC HF.

#### Solution:

ABC HF has a profit before fees on a €100 million investment of €20 million ( $= €100 \text{ million} \times 20\%$ ). The management fee is €2 million ( $= €100 \text{ million} \times 2\%$ ), and the incentive fee is €4 million ( $= €20 \text{ million} \times 20\%$ ). The return to the investor is 14% [ $= (20 - 2 - 4)/100$ ].

2. Calculate the return to the investor from investing in XYZ FOF. Assume that the other investments in the XYZ FOF portfolio generate the same

return before management fees as those of ABC HF and that XYZ FOF has the same fee structure as ABC HF.

**Solution:**

XYZ FOF earns a 14% return, or €14 million profit after fees on €100 million invested with hedge funds. XYZ FOF charges the investor a management fee of €1 million ( $= €100 \text{ million} \times 1\%$ ) and an incentive fee of €1.4 million ( $= €14 \text{ million} \times 10\%$ ). The return to the investor is 11.6% [ $= (14 - 1 - 1.4)/100$ ].

3. Why would the investor choose to invest in a fund of funds instead of a hedge fund given the effect of the “double fee” demonstrated in the answers to Questions 1 and 2?

**Solution:**

This scenario assumes that returns are the same for all underlying hedge funds. In practice, this result will not likely be the case, and XYZ FOF may provide due diligence expertise and potentially valuable diversification. In addition, the underlying hedge fund might be closed to new investors and investing in the FOF may be the only way to access the hedge fund.

## QUESTION SET



### 1. Soft Hurdle

A real estate investment fund has deployed \$100 million initial capital to purchase a property. The fund has a soft hurdle preferred return to investors of 8% per annum and an 80%/20% carried interest incentive split thereafter (with a standard catch-up clause). At the end of Year 2, the property is sold for a total of \$160 million.

Ignoring management fees, what are the correct distributions to the LPs and the GP?

**Solution:**

With a soft hurdle arrangement, the carried interest is calculated on the entire annual gross return as long as the set hurdle is exceeded. To calculate the distributions of gain, one needs to construct a waterfall of cash flows. First, the LPs would be due their \$100 million initial investment. Then, they would be due \$16 million (8% preferred return on initial capital for two years).

The soft hurdle has been met, and the GP is due the carried interest until 20% of the profits generated is received, or \$4 million (2% for two years), which would be paid to the GP next as a catch-up to the achieved hurdle return.

The residual amount would be \$160 million – \$100 million – \$16 million – \$4 million = \$40 million. This amount would then be split 80% to the LPs and 20% to the GP, or \$32 million and \$8 million, respectively.

So, the total payout with a soft annual hurdle of 8% of the \$160 million would end up with the following waterfall:

	LP	GP
Return of Capital	\$100 m	
8% Preferred per Annum	\$16 m	
GP Catch-Up 20%		\$4 m
80%/20% Split	\$32 m	\$8 m
<b>Total Payout</b>	<b>\$148 m</b>	<b>\$12 m</b>

## 2. Hard Hurdle

Following Question 1, how would the distributions to the LPs and GP have been different if the real estate investment fund had a hard hurdle of 8% per annum and no catch-up clause?

### Solution:

If the fund had a hard hurdle rate instead (i.e., no catch-up clause), only the amount above the \$100 return of capital and \$16 million preferred return would be subject to the 20% carried interest incentive to the GP:  $20\% \times \$44 \text{ million} = \$8.8 \text{ million}$ —quite a bit less than the carried interest payment with the soft hurdle. The LPs would be due the balance of \$35.2 million ( $= \$44 \text{ million} - \$8.8 \text{ million incentive}$ ). This would result in the following total payout:

	LP	GP
Return of Capital	\$100 m	
8% Preferred per Annum	\$16 m	
80%/20% Split above Hurdle	\$35.2 m	\$8.8 m
<b>Total Payout</b>	<b>\$151.2 m</b>	<b>8.8 m</b>

## 3. Calculating Net Return

Capricorn Fund of Funds invests GBP100 million in each of Alpha Hedge Fund and ABC Hedge Fund. Capricorn Fund of Funds has a “1 and 10” fee structure. Management fees and incentive fees are calculated independently at the end of each year. After one year, net of their respective management and incentive fees, Capricorn’s investment in Alpha is valued at GBP80 million and Capricorn’s investment in ABC is valued at GBP140 million. The annual return to an investor in Capricorn Fund of Funds, net of fees assessed at the fund-of-funds level, is closest to:

- A. 7.9%.
- B. 8.0%.
- C. 8.1%.

### Solution:

A is correct, because the net investor return is 7.9%, calculated as follows: First, note that “1 and 10” refers to a 1% management fee and a 10% incentive fee.

End-of-year capital = GBP140 million + GBP80 million = GBP220 million.

Management fee = GBP220 million  $\times$  1% = GBP2.2 million.

Incentive fee = (GBP220 million – GBP200 million) × 10% = GBP2 million.

Total fees to Capricorn = GBP2.2 million + GBP2 million = GBP4.2 million.

Investor net return = (GBP220 – GBP200 – GBP4.2)/GBP200 = 7.9%.

If, however, the incentive fee is calculated after deduction of management fees (instead of being calculated independently), then the incentive fee would become (GBP220 million – GBP200 million – GBP2.2 million) × 10% = GBP1.78 million

Investor net return would have become (GBP220 – GBP200 – GBP2.2 – GBP1.78)/GBP200 ≈ 8.0%.

#### 4. IRR vs. MOIC

Match the advantages and disadvantages of IRR and MOIC as performance measures for long-lived alternative investments:

Return Metrics	Advantages/Disadvantages
1. IRR	A. ignores timing of cash flows. B. considers timing of cash flows. C. is more complicated to calculate.
2. MOIC	D. is easy to calculate and understand. E. requires assumptions on opportunity costs and reinvestment rates. F. is the preferred measure for long-lived alternative investments. G. is commonly used by private equity and real estate investors as a shortcut.

#### Solution:

1. IRR (F) is the preferred measure for long-lived alternative investments, because it (B) considers timing of cash flows, but it (C) is more complicated to calculate and (E) requires assumptions on opportunity costs and reinvestment rates.
2. MOIC (G) is commonly used by private equity and real estate investors as a shortcut, because it (D) is easy to calculate and understand, but it (A) ignores timing of cash flows.

#### 5. Either/or Fee Structure

A closed-end infrastructure fund with initial capital of €100 million has an either/or fee structure under which the GP can either charge a 1% management fee or accept a higher 25% incentive fee, whichever is higher. All fees are calculated based on end-of-period net asset value. Standard high-water mark provisions apply.

The fund returns for the first five years are as follows:

Year	NAV (€ millions)	Gross Return
1	98.00	-2%
2	93.10	-5%

Year	NAV (€ millions)	Gross Return
3	108.00	16%
4	129.60	20%
5	176.26	36%

Please calculate the fees received by the GP for each of the five years.

**Solution:**

The management and incentive fees that can be received by the GP in each year are tabulated as follows:

Year	NAV (€ millions)	Gross Return	Management Fee (%)	Incentive Fee (%)	Total Fee (€ millions)
1	98.00	−2%	1%	—	0.98
2	93.10	−5%	1%	—	0.93
3	108.00	16%	—	25% (108 − 100) × 25%	2.00
4	129.60	20%	—	25% (129.6 − 108.0) × 25%	5.40
5	176.26	36%	—	25% (176.26 − 129.60) × 25%	11.67
				Cumulative fees	<b>20.98</b>

Note that because the fund made a loss in Year 1 and in Year 2, there is no incentive fee and the GP can receive only the 1% management fee. In Year 3, when the fund made a profit, the incentive fee can be charged on the gain above the last high-water mark, which was the initial €100 million in Year 0. Since the management fee for Year 3 would have been just €1.08 million (= €108 million × 1%), the GP will be better off receiving the incentive fee of €2 million. Likewise, for Years 4 and 5, the GP can choose to receive the incentive fee at 25% of the profits in those two years without earning any management fee.

#### 6. Hedge Fund Indexes

A common problem with hedge fund indexes is the upward bias due to:

- A. backfill bias only.
- B. survivorship bias only.
- C. both backfill and survivorship bias.

**Solution:**

C is correct. Both backfill bias and survivorship bias are common in hedge fund indexes.

Survivorship bias refers to the selection bias in the index due to the exclusion of failed funds from a given benchmark. It leads to overly optimistic return expectations. Backfill bias refers to hedge funds including only the most successful funds in an index a few years after inception. The subsequent inclusion or “backfilling” of prior performance data on a selective basis serves to increase average reported returns. Both of these biases result in an upward bias in hedge fund indexes because they may reflect not actual average hedge fund performance but, rather, only the returns of those funds that initially performed best and/or have not failed.

## PRACTICE PROBLEMS

1. The following information applies to Rotunda Advisers, a hedge fund:

- \$288 million in AUM as of prior year end
- 2% management fee (based on year-end AUM)
- 20% incentive fee calculated:
  - Net of management fee
  - Using a 5% soft hurdle rate
  - Using a high-water mark (high-water mark is \$357 million)
- Current-year fund gross return is 25%.

The total fee earned by Rotunda in the current year is closest to:

- A. \$7.20 million.
  - B. \$20.16 million.
  - C. \$21.60 million.
2. A hedge fund with net capital of GBP500 million has borrowed an additional GBP200 million at 4.5% per annum. The current-year return of the fund is 15%. What would have been the return if the fund had not added any leverage?
- A. 10.7%
  - B. 12.0%
  - C. 19.2%
3. A common problem for the “mark-to-model” valuation of private equity funds is *most likely*:
- A. a violation of accounting rules.
  - B. an understatement of portfolio risk.
  - C. an understatement of interim portfolio return.
4. A commodity hedge fund has three investors:
- €100 million from Investor A invested at Year 0,
  - €100 million from Investor B invested at the beginning of Year 2, and
  - €100 million from Investor C invested at the beginning of Year 3.

The gross returns before fees of the fund are as follows:

Year	Annual Gross Return	Investor A's Investment (€ millions)	Investor B's Investment (€ millions)	Investor C's Investment (€ millions)
0	—	100		—
1	20%	—		—
2	–15%	—	100	—

Year	Annual Gross Return	Investor A's Investment (€ millions)	Investor B's Investment (€ millions)	Investor C's Investment (€ millions)
3	15%	—		100
4	10%	—		—

The management fee is 2% based on end-of-year value. The incentive fee is 20% above the high-water mark and is calculated based on end-of-year value net of management fee.

Which investor has earned the highest net return after the end of Year 4?

- A. Investor A
- B. Investor B
- C. Investor C

## The following information relates to questions 5-6

Buyout Capital, LLC, is a private equity fund that has the following characteristics:

- Capital committed: \$200 million
- Preferred return: 8% soft hurdle, with full catch-up
- Fund distribution: after five years
- Management fee: none
- Carried interest: 20% above preferred return
- Waterfall structure: American (deal by deal) with clawback

The fund made five investments, tabulated as follows:

Investment No.	Year		Amount (\$ m)		Profit	
	Invested	Sold	Invested	Sold	\$ m	%
1	0	4	40	60	20	10.67%
2	0	4	40	100	60	25.74%
3	1	5	40	50	10	5.74%
4	1	5	40	120	80	31.61%
5	2	5	40	30	(10)	neg.
<b>Total</b>			<b>200</b>	<b>360</b>	<b>160</b>	<b>12.47%</b>

5. What is the total carried interest to the GP?
  - A. \$30 million
  - B. \$32 million
  - C. \$34 million
6. Buyout Capital, LLC, is a private equity fund that has the following



characteristics:

- Capital committed: \$200 million
- Preferred return: 8% soft hurdle, with full catch-up
- Fund distribution: after five years
- Management fee: none
- Carried interest: 20% above preferred return
- Waterfall structure: European (whole of fund)

The fund made five investments that are tabulated as follows:

Investment No.	Year		Amount (\$ m)		Profit	
	Invested	Sold	Invested	Sold	\$ m	%
1	0	4	40	60	20	10.67%
2	0	4	40	100	60	25.74%
3	1	5	40	50	10	5.74%
4	1	5	40	120	80	31.61%
5	2	5	40	30	(10)	neg.
<b>Total</b>			<b>200</b>	<b>360</b>	<b>160</b>	<b>12.47%</b>

What is the total carried interest to the GP?

- A. \$30 million
- B. \$32 million
- C. \$34 million

7. A hedge fund has the following fee structure:

- Annual management fee based on year-end AUM: 2%
- Incentive fee: 20%
- Hurdle rate before incentive fee collection starts: 4%
- Current high-water mark: \$610 million

The fund has a value of \$583.1 million at the beginning of the year. After one year, it has a value of \$642 million before fees. The net percentage return to an investor for this year is closest to:

- A. 6.72%.
- B. 6.80%.
- C. 7.64%.

## SOLUTIONS

1. A is correct. Although the gross return of Rotunda results in a \$360 million gross NAV, the deduction of the \$7.2 million management fee brings NAV to \$352.8 million, which is below the prior high-water mark. Rotunda earns a management fee of \$7.20 million but does not earn an incentive fee because the year-end fund value net of management fee does not exceed the prior high-water mark of \$357 million. Since Rotunda is still also below the prior-year high-water mark, the hurdle rate of return is also basically irrelevant in this fee calculation.

The specifics of this calculation are as follows:

End-of-year AUM = Prior year-end AUM  $\times$  (1 + Fund return) = \$288 million  $\times$  1.25 = \$360 million.

\$360 million  $\times$  2% = \$7.20 million management fee.

\$360 million – \$7.2 million = \$352.8 million AUM net of management fee.

The year-end AUM net of fees do not exceed the \$357 million high-water mark. Therefore, no incentive fee is earned.

2. B is correct.

Using Equation 2,

$$r_L = \text{Leveraged portfolio return/Cash position} = [r \times (V_b + V_c) - (V_b \times r_b)] / V_c.$$

Or, after re-arranging the formula,

$$r = \frac{(V_c \times r_L) + (V_b \times r_b)}{(V_c + V_b)}.$$

Substituting  $r_L = 0.15$ ,  $V_b = 200$ ,  $V_c = 500$ ,  $r_b = 0.045$ ,

$$r = (500 \times 0.15) + (200 \times 0.045) / (200 + 500) = 12\%.$$

Since leverage magnifies return when the borrowing cost is lower than asset returns, the unleveraged asset return must be lower than 15%.

3. B is correct. Accounting rules require that investments be recorded at their fair value for financial reporting purposes. The fair value of private equity that owns illiquid assets requires certain estimates, rather than observable transaction prices, to be factored into valuation. A model that relies on Level 3 inputs may reflect an imperfect theoretical valuation and not a true liquidation value. The lack of new market information over time may anchor the interim valuation at or near initial cost. The relatively stable accounting valuations may give investors a false sense that they are less volatile. At the same time, there is a potential conflict of interest for the GP to overstate interim return because of the implication for carried interest. As a result, returns may be smoothed or overstated and the volatility of returns understated.
4. C is correct. Despite investing for the shortest period of time in the fund (i.e., two years), Investor C has earned the highest net return compared to the other two investors. The following table illustrates the calculations based on the “2 and 20” fee structure and the high-water marks facing each investor in any particular year; in each case, the incentive fee is calculated using Equation 8:

$$R_{GP(\text{High-Water Mark})} = (P_2 \times r_m) + \max[0, (P_2 - P_{HWM}) \times p].$$

The net return is calculated using Equation 9:

$$r_i = (P_2 - P_1 - R_{GP})/P_1.$$

Year	Annual Gross Return	Investor A's Investment	Year-End AUM (Before Fee)	High-Water Mark	Management Fee (2%)	Incentive Fee (20%)	Year-End AUM (After Fee)
(Before Fee)							
0	—	100.00	100.00	—	—	—	100.00
1	20%	—	120.00	100.00	2.40	3.52	114.08
2	-15%	—	96.97	114.08	1.94	—	95.03
3	15%	—	109.28	114.08	2.19	—	107.10
4	10%	—	117.81	114.08	2.36	0.27	115.18
<b>Investor A's net return</b>							<b>15.18%</b>

Year	Annual Gross Return	Investor B's Investment	Year-End AUM (Before Fee)	High-Water Mark	Management Fee (2%)	Incentive Fee (20%)	Year-End AUM (Before Fee)
0	—	—	—	—	—	—	—
1	20%	—	—	—	—	—	—
2	-15%	100.00	85.00	100.00	1.70	—	83.30
3	15%	—	95.80	100.00	1.92	—	93.88
4	10%	—	103.27	100.00	2.07	0.24	100.96
<b>Investor B's net return</b>							<b>0.96%</b>

Year	Annual Gross Return	Investor C's Investment	Year-End AUM (Before Fee)	High-Water Mark	Management Fee (2%)	Incentive Fee (20%)	Year-End AUM (Before Fee)
0	—	—	—	—	—	—	—
1	20%	—	—	—	—	—	—
2	-15%	—	—	—	—	—	—
3	15%	100.00	115.00	100.00	2.30	2.54	110.16
4	10%	—	121.18	110.16	2.42	1.72	117.03
<b>Investor C's Net Return</b>							<b>17.03%</b>

5. A is correct. The distribution of profit of each investment is as follows:

Investment No.	Year		Amount (\$ m)		Profit			
	Invested	Sold	Invested	Sold	\$ m	%	LPs at 80%	GP at 20%
1	0	4	40	60	20	10.67%	16	4
2	0	4	40	100	60	25.74%	48	12
3	1	5	40	50	10	5.74%	10	0
4	1	5	40	120	80	31.61%	64	16

Investment No.	Year		Amount (\$ m)		Profit			
	Invested	Sold	Invested	Sold	\$ m	%	LPs at 80%	GP at 20%
5	2	5	40	30	(10)	neg.	(8)	(2)
<b>Total</b>			<b>200</b>	<b>360</b>	<b>160</b>	<b>12.47%</b>	<b>130</b>	<b>30</b>

Since the preferred return of the LP is 8%, Investments 1, 2, and 4 all meet the criterion and the profit is split 80/20 between the LPs and GP. Investment 3 does not earn the GP any carry because it fails to meet the preferred return; neither does Investment 5, whose profit is negative. Because of the clawback clause, the GP's carry is reduced by \$2 million (or 20% of the loss on Investment 5). Therefore, the total carried interest adds up to \$30 million, or  $30/160 = 18.75\%$  of the total profit made by the fund.

6. B is correct. A European waterfall occurs at the aggregate fund level. As long as the fund exit IRR exceeds 8% after five years, the GP will be eligible for the full 20% carried interest on the profit made from all investments—that is, \$160 million  $\times 20\% = \$32$  million.

Investment No.	Year		Amount (\$ m)		Profit			
	Invested	Sold	Invested	Sold	\$ m	%	LPs at 80%	GP at 20%
1	0	4	40	60	20	10.67%		
2	0	4	40	100	60	25.74%		
3	1	5	40	50	10	5.74%		
4	1	5	40	120	80	31.61%		
5	2	5	40	30	(10)	neg.		
<b>Total</b>			<b>200</b>	<b>360</b>	<b>160</b>	<b>12.47%</b>	<b>128</b>	<b>32</b>

7. C is correct. The management fee for the year is  $\$642 \text{ million} \times 0.02 = \$12.84$  million.

Because the ending gross value of the fund of \$642 million exceeds the high-water mark of \$610 million, the hedge fund can collect an incentive fee on gains above this high-water mark but net of the hurdle rate of return. The incentive fee calculation becomes

$$\{\$642 - [\$610 \times (1 + 0.04)]\} \times 0.20 = \$1.52 \text{ million.}$$

The net return to the investor for the year is

$$[(\$642 - \$12.84 - \$1.52)/\$583.1] - 1 = 0.07638 \approx 7.64\%.$$

## LEARNING MODULE

## 3

## Investments in Private Capital: Equity and Debt

### LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain features of private equity and its investment characteristics
<input type="checkbox"/>	explain features of private debt and its investment characteristics
<input type="checkbox"/>	describe the diversification benefits that private capital can provide

## INTRODUCTION

1

This Learning Module and the subsequent four Learning Modules explain the investment characteristics of specific alternative asset types, starting with private equity and private debt. The subsequent Learning Modules focus on real assets, natural resources, hedge funds, and digital assets. Each Learning Module introduces core characteristics, distinguishing features, and risk–return characteristics for the specific asset class. Alternative assets differ from the traditional asset classes—debt and equity—due to their unique return, risk, and information profiles and historically show low levels of correlation with debt and equity. Moreover, alternative assets often require highly specialized knowledge to select, manage, and divest these assets. Since these alternative assets are generally considered to be less liquid than traditional asset classes, understanding the valuation and return characteristics is a specialized skill.

### LEARNING MODULE OVERVIEW



- Private equity is a form of private capital funding sourced from outside public markets through non-traditional sources, such as venture capital and leveraged buyout firms. It can be injected at various stages of business development, from initial idea to final transition to public company status.
- The duration of a private equity investment also varies, with funds conducting their exits typically by the strategies of trade sales to strategic buyers or public listings through IPOs or special acquisition

companies (SPACs). Other strategies include recapitalizations, secondary sales, and liquidations, with all the strategies having their unique advantages and drawbacks.

- Compared to traditional investments, private equity can offer better returns combined with higher risks. This contrast is a function of private equity's distinct choice set, greater management control, and greater leverage. Data ambiguities make it challenging to reliably measure the benefits of private equity investing.
- Private debt primarily refers to the various forms of debt provided by investors directly to private entities. Its four major categories are direct lending, mezzanine loans, venture debt, and distressed debt, and it also includes unitranche debt of blended loans and other specialty loans.
- As in private equity investment, private debt can be arranged on a direct or indirect basis, with funds deployed over the corporate life cycle straight from an investor or intermediated through a fund. Investors receive interest payments and the return of principal after a designated term, with debt typically secured and having protections/covenants.
- Private debt has potentially higher returns and risks than traditional fixed income, with its investors needing specialized knowledge to adjust exposures for differences across company funding stages, debt structures, and underlying assets.
- Private debt and equity are distinct in terms of risks and performance from their public counterparts due to illiquidity and concentration risk and to the often-greater uncertainties of both their underlying businesses and the means to hedge away their risks. And a fundamental timing characteristic for private capital is its vintage year, with the valuation and economic environment at the origin of a private equity fund having a potentially substantial effect on realized results over the fund's set lifespan.
- To offset the potentially adverse performance effects of an ill-timed fund launch at an unfavorable stage of the business cycle, investors can diversify exposure across fund vintage years.
- Investments in private capital vary in terms of risk and return across the corporate capital structure hierarchy, with a diversified mix of private equity and debt investments potentially balancing private capital risks and returns. And when combined with public stocks and bonds, investments in private capital funds can add a moderate diversification benefit with opportunities for excess returns due to private capital's additional leverage, market, and liquidity risks.

#### SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. At the conclusion of a public company's leveraged buyout, the amount of its market-traded stock is substantially:

A. reduced.

- B. increased.
- C. unaffected.

**Solution:**

A is correct. After the transaction, the target company becomes or remains a privately owned company. Leveraged buyouts are sometimes called “going-private” transactions because after the acquisition of a publicly traded company, the target company’s equity is substantially no longer publicly traded.

2. Which of the following financing tools would *most likely* be used at the later stage of venture capital investment?

- A. Common stock
- B. Preferred stock
- C. Convertible debt

**Solution:**

B is correct. Preferred stock can be deployed as late into a company’s maturity as later-stage venture capital, when preferred stock can offer more protection to venture investors as a company transitions toward an IPO. A and C are incorrect because these instruments are more typically used in the earlier pre-seed and seed stages.

3. Which of the following transaction features is associated with mezzanine debt?

- A. Warrants
- B. Lines of credit
- C. Fixed payment schedules

**Solution:**

A is correct. Mezzanine debt often comes with additional features, such as warrants or conversion rights. These provide equity participation to lenders/investors. B is incorrect because lines of credit are associated with venture debt, which entrepreneurs may seek to obtain additional financing without further diluting shareholder ownership. C is incorrect because fixed payment schedules are associated with direct lending, in which, as with typical bank loans, payments are usually received on a fixed schedule.

4. In using private debt for a syndicated leveraged mortgage portfolio, the financial ratio of loan to value (LTV) is important at:

- A. origination and to the real estate fund sponsor.
- B. syndication and to the private debt fund lender.
- C. both transaction phases and to each of the parties.

**Solution:**

C is correct. LTV plays a significant role in both legs of this transaction. For a sponsor to be able to borrow and for a lender to be able syndicate the loans, the aggregate LTV ratio cannot be breached, and any deviation from LTV on an individual property level needs to be cured. As the loan amortizes, its outstanding principal declines, increasing LTV. However, if the value of the real estate were to drop, then the sponsor will be required to raise additional collateral to maintain the LTV level.

5. Vintage diversification is an advisable policy for implementation by private capital:

- A. funds.
- B. investors.
- C. users, such as company managers.

**Solution:**

B is correct. The vintage year, the time when fund deployment begins, is important for comparing PE and VC investments with other funds in the same year. Because of changing business and valuation environments, funds of a certain vintage have a relative advantage based on their start-up timing. That is why investors are encouraged to pursue vintage diversification by investing in multiple vintage years. A is incorrect because once capital commitments from a fund begin, all subsequent transactions are classified as part of the same vintage year. C is incorrect because the terms and conditions of capital use are more a function of the circumstances of their company than of the origin point of their fund source.

6. The potential diversification benefits from private capital investment are *most likely* related to its:

- A. wide range of exit strategies.
- B. various types of fee structures.
- C. lower correlation with public asset returns.

**Solution:**

C is correct. Investments in private capital funds can add a moderate diversification benefit to a portfolio of publicly traded stocks and bonds. Correlations with public market indexes vary from 0.63 to 0.83. A is incorrect because different exit strategies can offer funds the opportunity to maximize returns but do not necessarily reduce the volatility of returns over time. B is incorrect because while different fee structures may more effectively align the interests of funds and their investors, they do not necessarily change the risks of the underlying investments.

## 2

## PRIVATE EQUITY INVESTMENT CHARACTERISTICS



explain features of private equity and its investment characteristics

**Private capital** is the broad term for funding provided to companies that is not sourced from the public markets, such as from the sale of equities, bonds, and other securities on exchanges, or from traditional institutional providers, such as a government or bank. Capital raised from sources other than public markets and traditional institutions and in the form of an equity investment is called **private equity**. Comparably sourced capital extended to companies through a loan or other form of debt is referred to as **private debt**. Private capital relates to the entire capital structure, comprising private equity and private debt.

Private equity strategies include **leveraged buyout (LBO)**, venture capital (VC), and growth capital. **Leveraged buyouts**, or highly leveraged transactions, arise when private equity firms establish buyout funds (or LBO funds) to acquire public companies



or established private companies, with a significant percentage of the purchase price financed through debt. The target company's assets typically serve as collateral for the debt, and the target company's cash flows are expected to be sufficient to service the debt. The debt becomes part of the target company's capital structure after the buyout occurs. After the transaction, the target company becomes or remains a privately owned company. LBOs are sometimes called "going private" transactions because after the acquisition of a publicly traded company, the target company's equity is substantially no longer publicly traded.

The LBO may also be of a specific type. In a **management buyout** (MBO), the current management team participates in the acquisition, and in a **management buy-in** (MBI), the current management team is replaced with the acquiring team involved in managing the company. LBO managers seek to add value by improving company operations, boosting revenue, and ultimately increasing profits and cash flows. Cash flow growth, in order of contribution, comes from organic revenue growth, cost reductions and restructuring, acquisitions, and then all other sources. The financial returns in this category, however, depend greatly on the use of leverage. If debt financing is unavailable or costly, LBOs become less attractive and are less likely to take place. As business conditions and the availability of financing change, private equity managers may change focus.

A manager may manage many private equity funds, each composed of several investments, and the companies owned are called **portfolio companies** because they will be part of a private equity fund portfolio.

There are certain similarities and differences between private equity and public equity. Both types of equity represent direct ownership and control of the corporation. Owners are shareholders and as such have voting rights at the annual general meeting of shareholders electing the board, setting strategy, and making impactful decisions for the future of the company. Additionally, all owners have a direct and proportional claim to residual cash flow rights in the form of dividends. Ultimately, because of significant shareholdings, private equity ownership allows more direct control over decisions than public equity. Because of this, managing a direct private investment exposure requires specialized knowledge specific to the industry and sector the firm is in. Capital gains are typically the largest driver of returns, either through price appreciation or from free cash flow generated by the holdings.

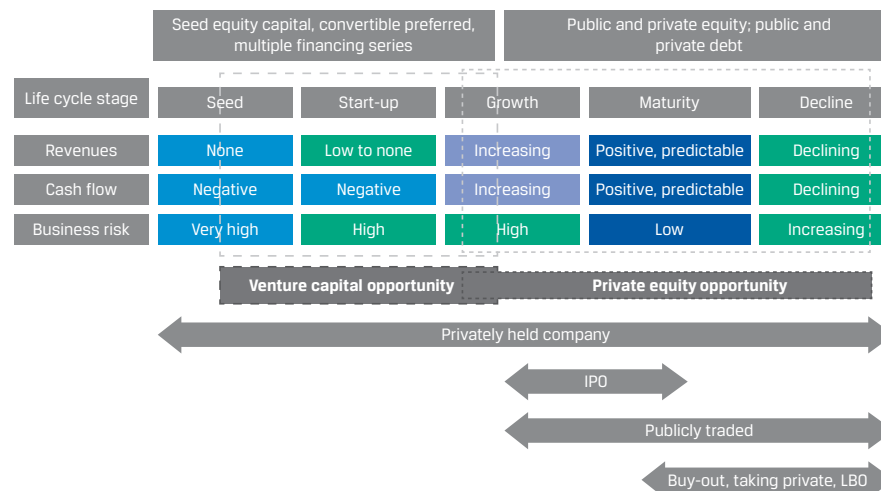
Equity investment, whether in public or private companies, has historically exhibited higher returns at the expense of greater risk compared with debt. The potential pay off for equity investors is unlimited upside with the downside limited to the amount invested. However, there are some differences as well.

## Private Equity Investment Categories

Private equity investments can be direct, through a single private equity fund, or indirect, through a fund-of-funds vehicle with stakes in various other private funds. With a direct private equity investment, the investment is made in a single, specific asset, but there may also be co-investments where the investor will participate alongside a lead sponsor who sources, structures, and executes the transaction.

Non-public equity is often categorized by the investor's *entry* point in a company's life cycle (venture capital versus private equity), as Exhibit 1 shows.

### Exhibit 1: Private and Venture Capital across the Corporate Life Cycle Stages



**Venture capital** entails investing in or providing financing to private companies with high growth potential. Typically, these are start-ups or young companies, but venture capital can be injected at various stages, ranging from concept creation for a company or near the point of a company's IPO (initial public offering) launch or its acquisition. The investment return required varies with the company's stage of development. Investors in early-stage companies will demand higher expected returns relative to later-stage investors because the earlier the stage of development, the higher the risk.

Venture capitalists, like all private equity managers, are active investors directly involved with their portfolio companies.

VC funds typically invest in companies and receive an equity interest but may also provide financing in the form of debt (commonly, convertible debt). Exhibit 2 summarizes the financing stages at different phases of the venture.

### Exhibit 2: Stages of Venture Capital

Stage	Pre-seed	Seed	Early stage   Later stage
Investors	<ul style="list-style-type: none"> <li>• Founders</li> <li>• Friends and family</li> <li>• Angel investors</li> </ul>	<ul style="list-style-type: none"> <li>• Seed funds</li> <li>• Angel investors</li> </ul>	<ul style="list-style-type: none"> <li>• Venture capital funds</li> <li>• Corporate venture funds</li> <li>• Private equity investors</li> <li>• Strategic investors</li> </ul>
Typical investment amount	USD5k–USD500k	USD25k–USD5mil	USD5mil+
Source of capital	Mainly individuals	Funds	<ul style="list-style-type: none"> <li>• Institutional investors</li> <li>• Family offices</li> <li>• Strategic investors</li> </ul>

- *Pre-seed capital*, or *angel investing*, is capital provided at the idea stage. Funds may be used to develop a business plan and to assess market potential. The amount of financing here is typically small and sourced from individuals, often friends and family, rather than by VC funds.

- *Seed-stage financing*, or *seed capital*, generally supports product development and marketing efforts, including market research. This is the first stage at which VC funds usually invest.
- *Early-stage financing* (early-stage VC), or *start-up stage financing*, goes to companies moving toward operation but prior to commercial production or sales, in both of which early-stage financing may be injected to initiate.
- *Later-stage financing* (expansion VC) comes after commercial production and sales have begun but before an IPO. Funds may be used to support initial growth, a major expansion (such as a physical plant upgrade), product improvements, or a major marketing campaign.

Later-stage financing generally involves management selling control of the company to the VC investor; financing is provided through equity and debt, although the fund may also use convertible bonds or convertible preferred shares. The VC fund offers debt financing for reasons of recovery and the control of assets in a bankruptcy situation, not to generate income. Simply put, debt financing affords the VC fund more protection than equity does.

**Mezzanine-stage financing** (mezzanine venture capital) prepares a company to go public as it continues to expand capacity and enhance its growth trajectory. It represents the bridge financing needed to fund a private firm until it can execute an IPO or be sold. The term *mezzanine-stage financing* is used because it is infused between private and public company status, principally distinguished by the timing of the financing rather than its method. While the terms sound quite similar, mezzanine financing is different from mezzanine-stage financing. Mezzanine financing relates to the use of equity–debt hybrid instruments, such as convertible debt or convertible preferred. Mezzanine-stage financing can use mezzanine financing, but typically at this stage the main financing is either equity-like (to capture potential gains from the planned IPO) or short-term debt.

Convertible preferred shares are often used in start-ups to raise private capital from venture capital funds. Since convertible preferred shares include an option for the holder to convert the preferred shares into a fixed number of common shares after a predetermined date and in some cases at a predetermined price, they provide incentive alignment between the entrepreneurs in the start-up and the investor. The conversion value is ultimately based on the valuation of the start-up. Investing in a start-up is risky because the investor is financing a new and unproven business. Preferred convertible equity provides investors with additional protections that are not available to common shareholders. Most importantly, in the event of a liquidation, preferred convertible shareholders have seniority over common shareholders and are entitled to recover the entire value of their investment before common shareholders receive any of the proceeds.

When investing, the manager of the venture capital fund is largely confident that the portfolio company's management team is competent and armed with a solid business plan showing strong prospects for growth. Because these companies are immature businesses without years of operational and financial performance history, estimating company valuations and their future prospects is highly subjective.

Once venture capital sees an exit opportunity, private equity can take over. Private equity firms specialized in the later-stage life cycle phase seek to generate returns by directly influencing management and implementing strategy changes, particularly for both publicly traded and privately held companies that are underperforming their peers. Here, private equity focuses on increasing the value of the core business by adaptively changing the overall business strategy by eliminating poor-performing business lines or businesses that do not generate sufficiently high returns. These

strategic changes can generate value that can be extracted either from the assets in place (selling underperforming assets) or by creating a more sustainable strategy and selling the company at a higher valuation.

Established companies in a transition phase may seek minority equity interest to expand, restructure, or acquire other companies. A private equity firm that engages in minority equity investing, also known as growth equity or growth capital, takes a less-than-controlling interest in more mature companies looking for capital to expand or restructure operations, enter new markets, or finance major acquisitions. Many times, minority equity investing is initiated and sought by the management of the investee company. The management's motive is to realize earnings from selling a portion of its shares before the company can go public but still retain control and participation in the success of the company. Although this scenario occurs most commonly with private companies, publicly quoted companies can seek private equity capital through PIPEs (private investments in public equities).

#### EXAMPLE 1

##### Tenderledge and Arguston—Buyout

A private equity firm, Tenderledge Opportunity Fund LLC, is evaluating Arguston Inc. Arguston is a mid-sized manufacturing firm operating in a mature industry and has seen both its revenues and earnings drop. Arguston does not have the needed capital to make the necessary long-term investments in technological upgrades to maintain its competitiveness or to recapture and extend its dwindling market share. The company is closely held, but the current owners and its management do not have the financial capacity to make the necessary investments and are willing to be bought out by Tenderledge.

To make the investment in Arguston, Tenderledge would not only provide needed capital but also restructure Arguston's operations to create efficiencies by reducing unnecessary redundancies and streamlining production, operations, and administration. It would likely replace management and install its own board members.

Tenderledge is likely to reduce overhead costs and cut employment levels. Once the restructuring and technological investments bear fruit, Tenderledge may also opt to consider acquiring some of Arguston's competitors to create additional growth through economies of scale and scope. Ultimately, the objective is to increase the value of Arguston and exit from the investment at a much higher price.

A **PIPE (private investment in public equity)** transaction is a private offering to select investors with fewer disclosures and lower transaction costs that allows the issuer to raise capital more quickly and cost effectively than with other means that may be more regulated, expensive, and lengthy. In a traditional PIPE transaction, either newly issued common stock or shares sold by existing stockholders—or a combination of both—in an already-publicly traded company are made available to certain investors. These investors, typically investment firms, mutual funds, or other institutional investors, enter into a definitive purchase agreement with the issuer and commit to purchase securities at a fixed price. These transactions are common in work-out or rescue situations where there is a material difference in the market price and valuations. A special case of PIPE is capital raised through convertible debt or convertible preferred stock. PIPE transactions are also dilutive to existing shareholders, and the new investors require a discount to market on the purchase price, which can introduce incentive conflicts between existing shareholders and new shareholders.

**EXPEDIA AND PIPE DURING COVID-19**

Private investment in public equity often signals confidence in a company's prospects and offers companies an accelerated approach to raise capital by selling large chunks of its equity to investors who often take board seats to directly get involved in setting business strategy. But these transactions can be costly.

In March 2020, the early stages of the COVID-19 pandemic led to a severe decline in travel, creating crisis conditions for the industry. Expedia, an online business and retail travel platform operating several brands, sold USD1.2 billion of preferred shares to two private equity managers, gave each firm a seat on its board, and issued USD2 billion of additional debt.

The preferred shares pay a fixed rate of 9.5% and warrants to purchase common stock. All in all, Expedia accumulated over USD4 billion in liquidity to weather the business disruption caused by the pandemic.

Expedia may redeem these preferred shares at preset prices during certain time periods and at preset periods.

Time period	Redemption price
Up to the first year of issuance	105.0%
Between first and second year of issuance	103.0%
Between second and third year of issuance	102.0%
Between third and fourth year of issuance	101.0%
After the fourth year of issuance	100.0%

When redeeming these shares, Expedia also has to pay any unpaid and accrued dividends. Additionally, Expedia also issued warrants to purchase 8.4 million shares of common equity at an exercise price of USD72.00 per share; these warrants have an expiration of 10 years.

Finally, Expedia also issued debt with an aggregate principal amount up to USD855 million. Its euro-denominated debt paid up to a 2.35% annual interest rate until 31 December 2021 and after that date paid up to 1.75%.

Assuming the adverse business conditions recovered fairly quickly to enable Expedia redemption of the financing at the midpoint (2 years) of the term sheet and it had not paid any preferred dividends beforehand, Expedia's cost would be  $[(1.02) + (0.095 \times 2)](\text{USD1.2 billion}) = \text{USD1.45 billion}$ , or USD250 million in excess of the preferred fund raise, plus the potential dilutive effects on earnings of 8.4 million extra common shares if their market price exceeded the USD72 warrant exercise price. As a benchmark comparison, the dividend yield of the largest US market preferred ETF, iShares Preferred and Income Securities, was approximately 5.6% in the summer of 2020, a substantially lower yield.

**Private Equity Exit Strategies**

Private equity firms seek to improve new or underperforming businesses and then exit them at higher valuations, buying and holding companies for an average of five years. Holding time, however, can range from less than six months to more than 10 years. Before deciding on an exit strategy, private equity managers assess the dynamics of the industry in which the portfolio company competes, the overall economic cycle, interest rates, and company performance.

Instead of a long-term buy-and-hold strategy that managers and owners of publicly held equity funds espouse, private equity seeks to aggressively maximize investment return by selling or even auctioning holdings in private assets to the highest bidder.

The added value that private equity investors can realize by strategically connecting firms and entities can further increase the value of the transaction and the returns to the private equity investors.

Typically, a private equity fund has an investment period of approximately five years and a subsequent harvesting period when exit occurs and the valuation environment becomes more relevant. Moreover, private equity fund investments are not made in one single payment but are spread over time using committed capital over several years, which provides managers a great deal of flexibility to optimize when their entry and exit points occur.

There are two main exit strategies: trade sale and public listing, which can take the form of IPOs, direct listings, or SPACs

### **Trade Sale**

There are two main exit strategies: trade sale and public listing. In a **trade sale**, a portion or a division of the private company is sold either via direct sale or auction to a strategic buyer interested in increasing the scale and scope of the existing business. Because the transaction may have an impact on the competitive environment, it may face regulatory scrutiny and approval or management or employee resistance.

A key advantage of a trade sale is that a strategic investor will be willing to pay a premium as they price in potential synergies with their existing business. Other advantages of a trade sale include the relatively fast and simple execution of the transaction, which compared with an IPO may be shorter and incur lower transaction costs. Since these are privately negotiated transactions, there is a higher confidentiality because there are just a few parties involved in the transaction or the auction. Additionally, the strategic buyers can better evaluate the fit with their existing business without external scrutiny, which potentially makes strategic buyers willing to pay more from anticipating synergies with their own business.

The disadvantages of trade sales include potential resistance from existing members of management, who may fear their job security and may wish to avoid ownership by a competitor. Similarly, management and employees may resist a private transaction because a public listing would monetize the shares and potentially attain a higher sale price. Finally, the universe of trade buyers may be limited, which can potentially raise regulatory scrutiny and reduce prices.

## **EXAMPLE 2**

### **Tenderledge and Arguston—Exit**

After Tenderledge Opportunity Fund LLC, bought out Arguston Inc., the mid-sized manufacturing firm operating in a mature industry, Tenderledge restructured Arguston's operations and acquired some of Arguston's direct competitors to increase its economies of scale. Additionally, it acquired one of Arguston's smaller, strategic suppliers to create a strategically resilient and efficient entity. To realize the efficiencies from streamlining production, operations, distributions, and administration, Tenderledge is now considering exiting from this investment.

Trade sale may be an alternative Tenderledge considers, particularly because there still are some competitors that may benefit from a strategic alliance with a more cost-efficient peer. Tenderledge has turned to BridgeRock LLC, another private equity firm, which owns one of Arguston's main competitors, Tetrawolf Inc., about merging the two companies before exiting from the investment by listing the companies on the market.

### Public Listing

Public listing on an exchange can take place either as an initial public offering (IPO), a direct listing, or a special acquisition company (SPAC). IPOs are the most common means of raising capital in public equity markets using financial intermediaries to underwrite the offering. When a private equity firm or company founder takes a company public, the portfolio company sells its shares, including some or all of those held by the private equity firm, to public investors.

There are several benefits of an IPO as an exit strategy. An IPO may potentially realize the highest price for the company, may increase the visibility of the company, and would continue to provide an upside for the private equity company because it retains a share in the new public entity. Moreover, the success of an IPO builds on management support and approval, and it is likely that management will be retained, which provides job security.

There are several disadvantages with an IPO and public listing. Apart from the high transaction fees to investment banks and lawyers, the time to complete the transaction may be long, and it requires onerous disclosure. The public equity market introduces stock market volatility, and the potential lockup period (mandating the private equity firm to retain an equity position for a specified period post-IPO) may limit a quick realization of value. Moreover, not all companies are suitable for an IPO; smaller companies and those operating in out-of-favor industries, ones with unclear strategic priorities and unstable financial position, or those that have limited operating histories may not be ideal candidates for an IPO. A less commonly used approach to exit from a private equity position is **direct listing**, where the equity of the entity is floated on the public markets directly, without underwriters, reducing the complexity and cost of the transaction.

A **special purpose acquisition company** (SPAC) is a technique also used for a public exit. Such a “blank check” company exists solely for the purpose of acquiring an unspecified private company within a predetermined period; otherwise, it must return capital to investors. Companies suitable for an IPO would be appropriate SPAC candidates, but the two strategies have different valuation methods: here a single counterparty sets SPAC terms, which reduces the uncertainty around the valuation.

There are several advantages of an SPAC exit; these transactions provide an extended time for public disclosure on company prospects to build investor interest, flexibility of transaction structure to best suit the company’s context, and association with potentially high-profile and seasoned sponsors and their extensive investor network. Moreover, the valuation of the entity is fixed in advance and does not change, which reduces both the volatility and the uncertainty of share pricing. Finally, SPACs are allowed to provide more forward formal guidance on a company’s prospects than is allowed under an IPO.

There are some noteworthy shortcomings of using SPACs for market access and exit. First, SPAC transactions increase the cost of capital because the various capital instruments, such as warrants, have dilutive effects. Second, there is a valuation spread between the value of the SPAC equity and the equity purchased by the SPAC. This can be further complicated by possible dilution effects: The shares and warrants of a SPAC can be issued and then traded separately. Third, there may also be specific deal risk associated with the successful execution of the definite purchase and merger agreement. Fourth, regulatory authorities, such as the US SEC, are reconsidering the classification of SPACs under long established rules that could impose more stringent standards on their operations. Finally, there may be significant trading in the SPAC equity in the months after the purchase transaction is announced that can lead to **stockholder overhang**, the downward pressure on the share price as large blocks of shares are being sold on the open market.

**EXAMPLE 3****Tenderledge and BridgeRock—SPAC Transaction**

After Tenderledge Opportunity Fund LLC and BridgeRock merged their two companies Arguston Inc. and Tetrawolf Inc., they continued to operate the merged company, Aurora Inc., each holding a 50% share in the merged entity. As both private equity firms are considering exiting from the investment at the same time, they were evaluating various exit alternatives.

For private equity, having an exit strategy is critical. Tenderledge and BridgeRock are evaluating various alternatives. Although IPOs and direct listings are popular exit strategies, SPACs have the ability to raise capital as companies such as Aurora enter the public market. Given the future uncertainty about valuations, the private equity firms chose to go the SPAC route and enter into a merger with a SPAC to bring Aurora to market a few months faster and at a higher price than the companies would have realized had they chosen an IPO or a direct listing.

**Exhibit 3: Pros and Cons of Common Private Equity Exit Strategies**

Strategy	Advantage	Disadvantage
<b>Trade Sale</b>	<ol style="list-style-type: none"> <li>1. Immediate cash exit</li> <li>2. Higher price from synergy-seeking strategic buyers</li> <li>3. Fast and simple execution</li> <li>4. Streamlined process on transaction cost, disclosure, and confidentiality from dealing with only one party</li> </ol>	<ol style="list-style-type: none"> <li>1. Potential management opposition</li> <li>2. Limited set of buyers</li> <li>3. Reduced financial appeal to employees due to forgone monetization of ownership stakes/options</li> </ol>
<b>IPO</b>	<ol style="list-style-type: none"> <li>1. Highest potential share price</li> <li>2. Likeliest management approval</li> <li>3. Notoriety to private equity sponsor'</li> <li>4. Sharing in potential share price appreciation from ongoing ownership stake</li> </ol>	<ol style="list-style-type: none"> <li>1. High transaction costs</li> <li>2. Long lead time</li> <li>3. Stock market volatility creating value uncertainty</li> <li>4. Onerous disclosure</li> <li>5. Potential lockup period freezing capital committed to deal</li> <li>6. Suitable mainly for large and fast-growing companies</li> </ol>
<b>SPAC</b>	<ol style="list-style-type: none"> <li>1. Extended disclosure time and ability to provide forward guidance to develop investor interest</li> <li>2. Fixed valuation with lower share price volatility</li> <li>3. Transaction structure flexibility</li> <li>4. Involvement of high-profile, seasoned sponsors and their investor networks</li> </ol>	<ol style="list-style-type: none"> <li>1. Potential higher capital costs of dilution, warrants, and fees</li> <li>2. Divergence between announced and true equity value due to dilution</li> <li>3. Deal and capital risk of potential redemptions</li> <li>4. Prolonged post-merger stockholder overhang and churn</li> </ol>



### Other Exit Strategies

In addition to the previously discussed exit strategies, other exit strategies include **recapitalization**, **secondary sale**, and **write-off/liquidation**.

- *Recapitalization.* Recapitalization via private equity describes the steps a firm takes to increase or introduce leverage to its portfolio company and pay itself a dividend out of the new capital structure. A recapitalization is not a true exit strategy, because the private equity firm typically maintains control; however, it does allow the private equity investor to extract money from the company to pay its investors and improve its internal rate of return (IRR).
- *Secondary sale.* This approach represents a sale of the company to another private equity firm or group of financial buyers. With the considerable amount of funds raised by global PE, there has been an increase in the proportion of secondary sale exits.
- *Write-off/liquidation.* A write-off occurs when a transaction has not gone well, and the investment is likely to lose value. The private equity firm then revises the value of its investment downward or liquidates the portfolio company before moving on to other projects.

The exit strategies we have discussed may be pursued individually or in combination with others or may be used for a partial exit strategy, such as divesting a self-contained product or business line. For example, private equity funds may sell a portion of a portfolio company to a competitor via a trade sale and then complete a secondary sale to another private equity firm for the remaining portion. Company shares may also be distributed directly to the investors (LPs) of the private equity fund, although such a move is unusual.

### Risk–Return from Private Equity Investments

Private equity investors expect ownership capital returns—cash flows from dividends and proceeds from exit—subject to underlying market conditions of the industry. However, private equity investments have distinct entry and exit points between which managers exercise greater direct control and apply specialized knowledge to add value over a specific life cycle phase. While both involve investment selection (stock selection using filings and financial statement analysis for public companies, target selection for private equity), vintage year is important for private equity for comparative purposes.

The higher-return opportunities that private equity funds may provide relative to traditional investments are due to their ability to invest in private companies, their influence on portfolio companies' management and operations, and their use of leverage. Investing in private equity, including venture capital, is riskier than investing in common stocks and requires a higher return for accepting its higher risk, including illiquidity and leverage risks.

Published private equity indexes may be an unreliable measure of performance. Measuring historical private equity performance is challenging; as with hedge funds, which will be discussed later, private equity return indexes typically rely on self-reporting and are subject to survivorship, backfill, and other biases. This typically leads to an overstatement of returns. Moreover, prior to the global financial crisis of 2008–2009, in the absence of a liquidity event, private equity firms did not necessarily mark their investments to market. Failure to mark to market combined with the lag to mark investments due to inherent illiquidity will understate measures of volatility and correlations with other investments. Thus, data adjustments are required to more reliably measure the benefits of private equity investing. As a result, many investors

expect companies to be marked on a quarterly or annual basis, preferably by an independent party. Investors should require a higher return for accepting a higher risk, including illiquidity and leverage risks.

### QUESTION SET



1. Determine the correct answers to fill in the blanks:

\_\_\_\_\_ is the broad term for funding provided to companies that is not sourced from the public markets, with its two primary sub-categories being \_\_\_\_\_ and \_\_\_\_\_.

**Solution:**

*Private capital* is the broad term for funding provided to companies that is not sourced from the public markets, with its two primary sub-categories being *private equity* and *private debt*.

2. Identify the following statement as true or false: Both public and private equity represent direct ownership and control of the corporation. Additionally, all owners have a direct and proportional claim to residual cash flow rights in the form of dividends.

**Solution:**

True. These are two of the similarities shared by public and private equity.

3. Describe a funding situation to which a PIPE transaction is well suited.

**Solution:**

A PIPE (private investment in public equity) transaction is a private offering to select investors with fewer disclosures and lower transaction costs that allows the issuer to raise capital more quickly and cost effectively than other means that may be more regulated, expensive, and lengthy. These transactions are common in work-out or rescue situations where there is a material difference in the market price and valuations.

4. Match each form of private capital investment with the combination of corporate life cycle characteristics *most appropriate* for it.

Investment Form	Corporate Life Cycle Characteristics
1. Private Equity	i. Negative cash flow, high business risk
2. Venture Capital	ii. Increasing cash flow, high business risk
3. Both Private Equity and Venture Capital	iii. Declining cash flow, increasing business risk

**Solution:**

Investment Form	Corporate Life Cycle Characteristics
1. Private Equity	iii. Declining cash flow, increasing business risk
2. Venture Capital	i. Negative cash flow, high business risk
3. Both Private Equity and Venture Capital	ii. Increasing cash flow, high business risk

As shown in Exhibit 1, the “iii” combination of characteristics is typical of the decline stage, when private equity would have the best fit. The “i” combi-

nation of characteristics is typical of the start-up stage of the corporate life cycle, when venture capital is most appropriate. Finally, the “ii” combination is typical of the growth stage, when the opportunities for the forms of investment overlap and either type would be suitable.

## PRIVATE DEBT INVESTMENT CHARACTERISTICS

### 3



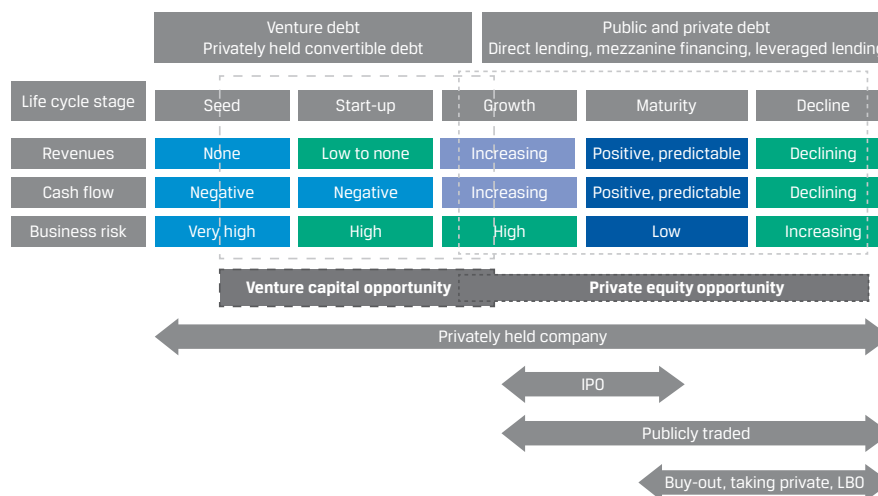
explain features of private debt and its investment characteristics

Private debt primarily refers to the various forms of debt provided by investors directly to private entities. In the past decade, the expansion of the private debt market has been largely driven by private lending funds filling the gap between borrowing demand and reduced lending supply from traditional lenders in the face of tightened regulations following the 2008 financial crisis.

We can organize the primary methods of private debt investing into four categories: direct lending, mezzanine loans, venture debt, and distressed debt. The broad array of debt strategies offers not only diversification benefits but also exposure to other investment spheres, such as real estate and infrastructure.

### Private Debt Categories

Analogous to private equity investment, an investor wanting to include private debt in a portfolio has various alternatives along a comparable direct versus indirect distinction. In direct private debt investment, the investor makes a loan directly to a specific operating company. In the indirect approach, the investor takes an intermediated path, purchasing an interest in a fund that pools contributions typically on behalf of multiple participants to buy into the debt from a set of operating companies. For both approaches, in exchange for the debt, the investors receive interest payments and the return of principal after a designated term. The debt is typically secured and has various protections/covenants in place. Exhibit 4 depicts the role of private debt in the corporate life cycle.

**Exhibit 4: Private Debt and Corporate Life Cycle**

**Venture debt** is private debt funding that provides venture capital backing to start-up or early-stage companies that may be generating little or negative cash flow. Entrepreneurs may seek venture debt, often in the form of a line of credit or term loan, to obtain additional financing without further diluting shareholder ownership. Venture debt can complement existing equity financing, allowing current shareholders to maintain ownership and control for a longer period. Similar to mezzanine debt, venture debt may carry additional features that compensate the investor/lender for the increased risk of default or for the start-up and early-stage companies that lack substantial assets for debt collateral. One such feature could grant the lender rights to purchase equity in the borrowing company under certain circumstances.

Private debt investors get involved in **direct lending** by providing capital directly to borrowers and subsequently receiving interest, the original principal, and possibly other payments in exchange for their investment. As with typical bank loans, payments are usually received on a fixed schedule. The debt itself typically is senior and secured and has covenants in place to protect the lender/investor. It is provided by a small number of investors to private and sometimes public entities and differs from traditional debt instruments, such as bonds, which can be issued to many participants and be publicly traded.

Direct lending primarily involves private debt firms (or private equity firms with private debt arms) establishing funds with money raised from investors desiring higher-yielding debt. Fund managers will then seek financing opportunities, such as providing a loan to a mid-market corporation or extending debt to another private equity fund that is seeking funds for acquisitions. In general, private debt funds provide debt, at higher interest rates, to entities needing capital but lacking good alternatives to traditional bank lenders, which themselves may be uninterested or unable to transact with these borrowers. As in private equity, private debt fund managers conduct thorough due diligence before investing.

In direct lending, many firms may also provide debt in the form of a **leveraged loan**, a loan that is itself levered. Private debt firms that invest in leveraged loans first borrow money to finance the debt and then extend it to another borrower. By using leverage, a private debt firm can enhance the return on its loan portfolio.

**EXAMPLE 4****Peterburgh Real Estate Fund, LLC—Syndicated Leveraged Mortgages**

The Peterburgh Real Estate Fund, LLC, has acquired a portfolio consisting of six commercial properties for a total of GBP100 million. From its investors, the fund secured GBP30 million in equity financing, and it turned to BridgeRock Credit Opportunities LLP, a private debt fund that underwrites commercial mortgages before syndicating them.

In this specific transaction, properties valued at GBP100 million serve as collateral for a total of GBP75 million mortgages, giving an aggregate loan to value (LTV) of 0.75, which provides GBP25 million excess collateral to BridgeRock. The terms of the mortgage are MRR + 150 bps, with a maturity of 15 years, GBP5 million in annual amortization, and all mortgages being secured by the first lien on the property. The lender, BridgeRock, then sells or syndicates the mortgages to other banks or institutional investors. The LTV plays a significant role in both legs of this transaction. For Peterburgh to be able to borrow and for BridgeRock to be able to syndicate the loans, the aggregate LTV ratio cannot be breached, and any deviation from the LTV on an individual property level needs to be cured. As Peterburgh amortizes the loan, the outstanding principal of the mortgages declines, which increases the LTV value. However, if the value of the real estate were to drop, then Peterburgh will be required to raise additional collateral to maintain the LTV.

In private debt, **mezzanine debt** refers to private credit subordinated to senior secured debt but senior to equity in the borrower's capital structure. Mezzanine debt is a pool of additional capital available to borrowers beyond senior secured debt, often used to finance LBOs, recapitalizations, corporate acquisitions, and similar transactions. Because of its typically junior ranking and its usually unsecured status, mezzanine debt is riskier than senior secured debt. To compensate investors for this heightened risk, investors commonly demand higher interest rates and may require options for equity participation. Mezzanine debt often comes with additional features, such as warrants or conversion rights. These provide equity participation to lenders/investors, conveying the option to convert their debt into equity or purchasing the equity of the underlying borrower under certain circumstances.

Involvement in **distressed debt** typically entails buying the debt of mature companies in financial difficulty. These companies may be in bankruptcy, have defaulted on debt, or seem likely to default on debt. Some investors identify companies with a temporary cash flow problem but a good business plan to help the company survive and ultimately flourish. These investors buy the company's debt expecting both the company and its debt to increase in value. Turnaround investors buy debt with an aim to be more active in distressed company management and direction, seeking to restructure and revive the company. Overall, investors concentrating on distressed debt need to develop specialized knowledge related to assessing the likelihood of default and the possible recovery rates because distressed debt is priced to the expected recovery rate, but occasionally recovery rates mistake the underlying risk of the exposures and have long time horizons. Bankruptcy procedures can be lengthy, complex, and capital intensive. Similarly, distressed debt investors need to understand how to restructure companies and restructure debt. Several distressed debt funds focus on debtor-in-possession (DIP) financing, which provides operating funds for firms already in bankruptcy.

### HERTZ AND DIP FINANCING

Hertz entered Chapter 11 reorganization in May 2020 after filing for bankruptcy due to COVID-19's near total shutdown of the global travel industry. In October 2020, it negotiated USD1.65 billion in operating funds secured through debtor-in-possession financing. That committed Hertz to pay creditors up to a market reference rate of (MRR) + 725 bps, permitting drawdowns from the facility in individual tranches of at least USD250 million. Up to USD1 billion was allowed for vehicle acquisition, and as much as USD800 million was allowed to fund working capital and general corporate needs.

The DIP financing was projected to provide Hertz with liquidity to support its continued operations throughout 2021, when the loan matured. Private capital firms Apollo Global Management, Diameter Capital Partners, and Silver Point Capital provided this funding. DIP financing is often a signal that lenders are confident in the company's ability to reorganize and pay the debt back in time.

Another type of debt that could be directly extended to borrowers is **unitranche debt**. Unitranche debt consists of a hybrid or blended loan structure combining different tranches of secured and unsecured debt into a single loan with a single, blended interest rate. Since unitranche debt is a blend of secured and unsecured debt, its interest rate will generally fall in between the interest rates often demanded on secured and unsecured debt. The unitranche loan will usually be structured between senior and subordinated debt in priority ranking.

Private debt firms may also provide *specialty loans*, extended to niche borrowers in specific situations. For example, in litigation finance, a specialist funding company provides debt to clients, usually plaintiffs in litigation, for their legal fees and expenses in exchange for a share of judgements.

### Risk–Return of Private Debt

Private debt investments may provide higher-yielding opportunities to fixed-income investors seeking increased returns relative to traditional bonds. Private debt funds may generate higher returns by taking opportunistic positions based on market inefficiencies. Private lending funds filled the financing gap left by traditional lenders following the 2008 financial crisis. Investors in private debt could realize higher returns from the illiquidity premium, which is the excess return investors require to compensate for lack of liquidity. Investors also benefited from increased portfolio diversification by owning these securities.

The interest rate of private debt is often expressed relative to a reference rate—for example, the Secured Overnight Financing Rate (SOFR) + 375 bps. As a result, the coupon varies in line with changes to the reference rate brought about by changes to the interest rate environment.

Differences between public and private debt include the distinct entry and exit points with lenders, which offer borrowers greater flexibility in arranging financing. Specialized knowledge for private debt financing is needed in order to add value for the investor. First, the financing and return on debt depends on the specific period of a company's phase of life cycle. Earlier debt financing typically carries higher risks and provides higher returns. Second, the structure of the debt is also of importance; for instance, CLOs (collateralized loan obligations) with the market reference rate, or MRR, as the base rate require specialized knowledge. Finally, the investor needs to have special knowledge about underlying assets, particularly for secured lending, such as real estate.

The potential for higher returns is connected to higher levels of risk. Private debt investments vary in risk and return, with senior private debt providing a steadier yield and moderate risk and mezzanine private debt carrying higher growth potential, equity upside, and higher risk than senior private debt. Overall, investing in private debt is riskier than investing in traditional bonds. Investors should be aware of these risks, including illiquidity and heightened default risk when loans are extended to riskier entities or borrowers in riskier situations. Modeling private equity or debt returns is not straightforward, due to a lack of good-quality data and artificially smooth returns.

**QUESTION SET**

1. Identify two categories of private debt that would typically be relied on in the growth stage or a later stage of the corporate life cycle.

**Solution:**

As shown in Exhibit 4, three private debt categories are featured in the later stages of the corporate life cycle:

1. Direct lending
2. Mezzanine financing
3. Leveraged lending

2. Determine the correct answers to fill in the blanks: Similar to mezzanine debt, \_\_\_\_\_ may carry additional features that compensate the investor/lender for \_\_\_\_\_ or for the start-up and early-stage companies that \_\_\_\_\_.

**Solution:**

Similar to mezzanine debt, *venture debt* may carry additional features that compensate the investor/lender for *the increased risk of default* or for the start-up and early-stage companies that *lack substantial assets for debt collateral*.

3. Describe the borrowing cost of unitranche debt.

**Solution:**

Unitranche debt consists of a hybrid or blended loan structure combining different tranches of secured and unsecured debt into a single loan with a single, blended interest rate. Since unitranche debt is a blend of secured and unsecured debt, its interest rate will generally fall in between the interest rates often demanded on secured and unsecured debt.

4. Identify the following statement as true or false: Modeling private debt returns is fairly straightforward because they are a function of a benchmark public debt return.

**Solution:**

False. While private debt and public debt share a reference point in being marked up from a benchmark return, modeling private equity or debt returns is not straightforward, due to a lack of good-quality data, more security-specific risk between assets, and artificially smooth returns.

## 4

## DIVERSIFICATION BENEFITS OF PRIVATE CAPITAL



describe the diversification benefits that private capital can provide

Since the performance of private debt and private equity greatly depend on the specific phase of a company's life cycle, performance, and risk, comparison of public debt and equity may not be appropriate. First, investing in a start-up carries greater risk than investing in a well-established firm. Second, investing in a company in a declining or disintermediated industry is unlikely to offer positive return over longer time horizons. Moreover, performance risk of a continuous investment in public equity and debt can easily be hedged away.

The vintage year is important for comparing private equity and VC investments with other funds in the same year. Each private equity fund carries a **vintage year**, typically defined as the year in which the fund makes its first investment. Typically, a private equity fund operates over a 10- to 12-year period, which is often segmented into an initial investment period and a subsequent harvesting period. The investment period usually is the first five years during which the capital is sourced from the limited partners and invested in various companies. The harvesting period is the remaining years of the fund, when the fund looks to exit its existing investments and to return capital to limited partners.

Because of changing business and valuation environments, funds of a certain vintage have the advantage of starting in a low-valuation, low-risk appetite, economic recovery phase and benefit from riding the wave of an economic recovery. Other vintages may be less fortunate and invest the bulk of their capital in a high-valuation environment preceding a market crash or a period of prolonged economic contraction. That is why investors are encouraged to pursue vintage diversification by investing in multiple vintage years.

The performance realized by the fund is greatly determined by the vintage year and the phase of the business cycle in which the vintage year occurred, as Exhibit 5 shows. Funds seeded during the expanding phase of the business cycle tend to earn excess returns if they fund early-stage companies. Funds seeded during the contracting phase of the business cycle tend to earn excess returns if they fund distressed companies.

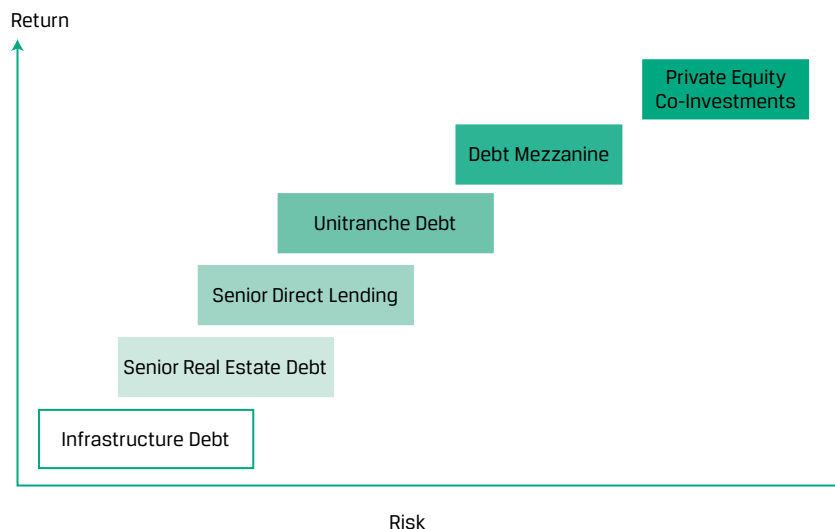


**Exhibit 5: US Private Equity Index Vintage Year Returns, Net Fund-Level Performance**


Vintage Year	2007	2011	2012	2013	2014	2015	2016	2017	2018
2020 Return	10.3%	14.7%	24.8%	29.6%	29.0%	33.9%	40.4%	29.7%	43.5%

Source: Cambridge Associates, data as of December 31, 2020. <https://www.cambridgeassociates.com/benchmarks/us-pe-vc-benchmark-commentary-calendar-year-2020/>

Investments in private capital vary in terms of risk and return across the corporate capital structure hierarchy. Typically, private equity, as the riskiest alternative, offers the highest returns, with private debt returns declining on a continuum down to the safest, most secured form of debt—infrastructure debt. Exhibit 6 outlines various private equity and private debt categories by their risk and return levels (mirroring the risk–return pathway for traditional equity and debt investing, note the trade-off as investors select between junior and senior debt and between equity and debt).

**Exhibit 6: Private Capital Risk and Return Levels by Category**


Source: Based on a graph from Leon Sinclair, “The Rise of Private Debt,” IHS Markit (7 August 2017).

Investments in private capital funds can add a moderate diversification benefit to a portfolio of publicly traded stocks and bonds. Correlations with public market indexes vary from 0.63 to 0.83, as shown in Exhibit 7. And if investors identify skillful fund managers, benefits from excess returns given the additional leverage, market, and liquidity risks are possible.

**Exhibit 7: Private Capital's Average Correlations with Public Market Indexes, March 2008–June 2021**

	<b>S&amp;P 500 Total Return</b>	<b>Russell 2000 Total Return</b>	<b>MSCI World Total Return</b>
Preqin, Private Equity	0.80	0.76	0.81
Preqin, Venture All Stage	0.65	0.67	0.63
Preqin, Buyout	0.82	0.76	0.83
Preqin, Private Debt	0.82	0.77	0.86

*Source:* Annualized quarterly returns of Private Capital Quarterly Index rebased to 31 December 2007, provided by Preqin.

#### QUESTION SET



1. State a private debt and equity investment factor that can make performance/risk comparisons with public debt and equity inappropriate.

**Solution:**

There are several private debt and equity investment factors that can invalidate such comparisons:

- Start-up investments carry greater risks than those of established firms.
- Investments in declining industries are unlikely to sustain gains over the long term.
- Ongoing performance risk in private investments can't be easily hedged.

2. Describe the life cycle segments of a private equity fund.

**Solution:**

The year in which a private equity fund makes its first investments is called its vintage year. Typically, operations span a 10- to 12-year period, which is often segmented into an initial investment period and a subsequent harvesting period. The investment period usually is the first five years during which the capital is sourced from the limited partners and invested in various companies. The harvesting period is the remaining years of the fund, when the fund looks to exit its existing investments and to return capital to limited partners.

3. Identify the following statement as true or false:

A private equity fund whose vintage dates to a high-valuation environment *most likely* starts with an advantage in having rich prices for its assets.

**Solution:**

False. Funds starting in a low-valuation, low-risk appetite, economic recovery phase benefit from riding the wave of an economic recovery and have an advantage over other vintages investing the bulk of their capital in a high-valuation environment preceding a market crash or a period of prolonged economic contraction.

4. Match the form of private capital investment with its *most likely* position on the risk/return continuum.

Investment Form	Risk/return combination
1. Mezzanine Financing	i. Lowest risk and return
2. Private Equity	ii. Intermediate risk and return
3. Senior Direct Lending	iii. Highest risk and return

**Solution:**

Investing in private debt and equity mirrors the risk–return pathway for traditional equity and debt investing. As investors select between senior and junior debt and between equity and debt, the potential risks and returns increase. Therefore, the lowest risk and return come from senior direct lending, and the highest risk and return are from private equity, while mezzanine financing falls in between the two.

Investment Form	Risk/return combination
1. Mezzanine Financing	ii. Intermediate risk and return
2. Private Equity	iii. Highest risk and return
3. Senior Direct Lending	i. Lowest risk and return

## PRACTICE PROBLEMS

1. Which of the following is *most likely* to participate in early-stage financing?
  - A. Founders
  - B. Angel investors
  - C. Strategic investors
2. Private equity funds are *most likely* to use:
  - A. leveraged buyouts.
  - B. option-based strategies.
  - C. merger arbitrage strategies.
3. A shared advantage of IPO and SPAC exit strategies for a private equity fund is their:
  - A. fast and simple execution.
  - B. ability to win market attention.
  - C. transaction structure flexibility.
4. Which of the following combinations of financial characteristics *most likely* would be associated with a venture debt transaction?
  - A. Positive revenues and cash flow
  - B. Declining revenues and cash flow
  - C. Low revenues and negative cash flow
5. In contrast to direct lending and distressed debt, mezzanine debt *most likely* requires higher interest rates for its investors due to its:
  - A. reliance on leverage.
  - B. overall ranking and status in company debt structure.
  - C. need for management involvement over an extended time.
6. A feature that private debt and public debt share in the setting of their investment returns is their:
  - A. relationship to benchmark interest rates.
  - B. need for specialized investment knowledge.
  - C. opportunity for illiquidity premiums in market crises.
7. The *most likely* effect on a portfolio's diversification when exposure to direct lending commences is that it:
  - A. decreases.

- B. remains the same.
  - C. increases.
- 8. Private equity funds whose vintage year occurs in the expanding phase of the business cycle tend to earn excess returns by investing in companies that are:
  - A. mature.
  - B. distressed.
  - C. early stage.
- 9. The private capital category *most likely* to offer the highest diversification benefit for portfolios holding public stock and bonds is:
  - A. private debt.
  - B. private equity.
  - C. venture capital.

## SOLUTIONS

1. C is correct. Strategic investors typically join in at early-stage to later-stage financing. A is incorrect because founders are typically part of the investor group at the earliest, pre-seed stage. B is incorrect because angel investors are typically involved in the beginning stages, either pre-seed or seed.
2. A is correct. Many private equity transactions involve leveraged buyouts to acquire public companies or established private companies. B and C are incorrect because these are strategies implemented by other types of alternative investors, such as hedge funds.
3. B is correct. IPOs can gain public attention for sponsors by high-profile business launches, and SPACs often have high-profile, seasoned sponsors and their investor networks as participants. A is incorrect because this is an advantage of a trade sale exit strategy. C is incorrect because it is a benefit specific to SPACs, with IPOs having fairly rigid and defined features, such as onerous disclosure requirements and lock-up periods.
4. C is correct. These financial characteristics are typical of the start-up stage in the corporate life cycle, when it is suitable to rely on venture debt. A is incorrect because these are features of the maturity stage, when direct lending, mezzanine financing, and leveraged lending are customary. B is incorrect because these are features of the maturity stage, when direct lending, mezzanine financing, and leveraged lending are customary.
5. B is correct. Because of its typically junior ranking and its usually unsecured status, mezzanine debt is riskier than senior secured debt. To compensate investors for this heightened risk, investors commonly demand higher interest rates. A is incorrect because leverage is typically a feature of a form of direct lending in which firms may also provide debt in the form of a leveraged loan, a loan that is itself levered with borrowing costs figuring into returns. C is incorrect because operational involvement over a possibly lengthy time with unpredictable risks describes the role of turnaround investors involved with distressed debt.
6. A is correct. For both public debt and private debt, return on debt capital tends to follow and change with the benchmark interest rate environment. B is incorrect because there is a need for specialized knowledge for private debt financing to add value to the investor through consideration of such factors as the debt's life cycle timing, its place in the financial structure, and the quality of underlying assets. C is incorrect because in market disruptions, such as the 2008 financial crisis, private debt exclusively benefited from an illiquidity premium when private lending funds filled the financing gap left by traditional lenders because traditional lenders were reluctant to underwrite public debt.
7. A is correct. While private capital can have overall positive contributions to diversification, direct lending can involve a large capital commitment to a single borrower, with increased concentration risk and reduced diversification. Investors attempt to protect against the risk of direct lending by having the debt itself classified as senior and secured with protective covenants in place to benefit from the associated higher interest rates while reducing non-diversifiable specific risk associated with a single borrower.
8. C is correct. Fund performance is greatly determined by the vintage year and the coinciding phase of the business cycle. Funds seeded during the expanding phase

tend to earn excess returns investing in early-stage companies. Funds seeded during the contracting phase tend to do best with distressed companies. Results may be intermediate with mature, stable companies.

9. C is correct. Research on the correlations between portfolios holding these public assets shows that venture capital has the lowest correlations across all major market indexes. This relationship implies a higher diversification benefit for venture capital.





## LEARNING MODULE

## 4

## Real Estate and Infrastructure

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain features and characteristics of real estate
<input type="checkbox"/>	explain the investment characteristics of real estate investments
<input type="checkbox"/>	explain features and characteristics of infrastructure
<input type="checkbox"/>	explain the investment characteristics of infrastructure investments

## INTRODUCTION

1

Broadly defined, real estate comprises land and buildings. Real estate investments involve developed land, including commercial and industrial real estate and residential real estate. Real estate has some unique features, including heterogeneity (no two properties are identical), long lives, and fixed geographical or physical location.

Raw land and less developed land used in agriculture and forestry are categorized as *natural resource* investments, while *infrastructure* involves land, buildings, and other fixed assets developed by public entities or public–private partnerships for economic use. Investments in real estate and infrastructure are included in many portfolios because they tend to exhibit low correlations with traditional asset classes and provide risk and return combinations across a broad spectrum. Here, as in other alternative assets, investors need specialized knowledge in selecting, acquiring, managing, and divesting these assets.

## LEARNING MODULE OVERVIEW



- Real estate includes two major sectors: residential and commercial. Residential real estate is the largest sector, totaling 75% of the global market. Commercial real estate includes office buildings, shopping centers, and warehouses.
- Real estate investing has some unique features, including heterogeneity (no two properties are identical), fragmentation, price discovery challenges, and costly and time-consuming transactions.
- Real estate investments can be direct or indirect, in the public market (e.g., REITs) or private transactions, and in equity.

- The return on real estate investments comes from income or asset appreciation or a combination of both. More than half of the returns commercial real estate investors earn are derived from income, and throughout an economic market cycle, real estate income is a more consistent source of return than capital appreciation.
- Investing in real estate can generate either lower-risk, bond-like cash flows from leases or higher-risk, equity-like speculative returns from realizing value from development projects or price appreciation.
- Real estate offers diversification benefits to portfolios. However, during certain market conditions, equity REIT correlations with market benchmarks increase, particularly during steep market downturns.
- Infrastructure consists of assets that are capital intensive and long lived and that are intended to provide essential services for public use.
- Infrastructure investments can take many forms, both direct and indirect. They can be broadly categorized as either economic (e.g., transportation, utility, and energy assets) or social (e.g., educational assets). They can also be categorized based on the underlying asset's stage of development as greenfield, secondary stage, or brownfield. The greenfield investment life cycle common among public–private partnerships is called the build-operate-transfer (BOT) life cycle.
- Most infrastructure assets are financed, owned, and operated by governments, and infrastructure is increasingly being financed privately through public–private partnerships by local, regional, and national governments. Investments in construction and development of new infrastructure are made with expectations to generate cash from either income or capital appreciation.
- Of the three stages of infrastructure investments, greenfield investments offer the highest expected return and have the highest expected risk, and secondary stage investments offer the lowest expected return and have the lowest expected risk.
- Infrastructure investors primarily expect the assets to generate stable long-term cash flows that also adjust for economic growth and inflation and secondarily expect capital appreciation, depending on the type and timing of their investment.
- Infrastructure investments provide an income stream, increase portfolio diversification by adding an asset class with typically low correlation with other public investments, provide some protection for changes in GDP growth, and offer some protection against inflation.

#### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. The two categories of real property are:

- A. residential and commercial.
- B. privately held and publicly traded.

C. individual market and institutional market.

**Solution:**

The correct answer is A. The two categories of real property are residential and commercial.

2. The preferred investment vehicles for public investors to own income-producing real estate are:

- A. real estate funds.
- B. mortgage-backed securities.
- C. real estate investment trusts.

**Solution:**

The correct answer is C. Real estate investment trusts (REITs) are the preferred investment vehicles for owning income-producing real estate for both private and public investors.

3. Which of the following entails the least risk?

- A. Value-add real estate
- B. Investment-grade commercial mortgage-backed securities
- C. Residential real estate with long-term leases and many lessors

**Solution:**

The correct answer is B. Of these three, investment-grade commercial mortgage-backed securities (CMBS) entail the least risk, and value-add real estate investments entail the most.

4. Which of the following entails the most risk?

- A. Mezzanine debt
- B. Core-plus real estate strategies
- C. Redevelopment of an existing property

**Solution:**

The correct answer is A. Of these three, mezzanine debt entails the most risk, and core-plus strategies entail the least.

5. The first stage of development of an infrastructure asset is typically called:

- A. bluesky.
- B. greenfield.
- C. early stage.

**Solution:**

The correct answer is B. The first stage of development of an infrastructure asset is typically called greenfield. Greenfield investing involves developing new assets and new infrastructure with the intention either to lease or sell the assets to the government after construction or to hold and operate the assets. Greenfield investors typically invest alongside strategic investors or developers that specialize in developing the underlying assets. The subsequent stages of development of infrastructure assets are typically called secondary stage and brownfield.

6. Direct infrastructure investment involves assets that are:

- A. illiquid.

**B.** securitized.

**C.** exchange traded.

**Solution:**

The correct answer is A. Like real estate, direct investment in existing infrastructure involves acquiring unique, illiquid assets with distinct location, features, and uses. Investors concerned about liquidity and diversification may invest indirectly using publicly traded infrastructure securities.

7. Which of the following types of infrastructure investments has the highest expected return?

**A.** Greenfield

**B.** Brownfield

**C.** Secondary stage

**Solution:**

The correct answer is A. Greenfield investments offer the highest expected return of the three. They also entail the highest expected risk. Secondary stage offers the lowest expected return and the lowest expected risk.

8. Which of the following tends to make the largest allocations to the infrastructure asset class?

**A.** Pension funds

**B.** Sovereign wealth funds

**C.** Life insurance companies

**Solution:**

The correct answer is B. Sovereign wealth funds tend to make the largest allocations to the infrastructure asset class—around 5%–6% of total AUM, according to Preqin.

## 2

## REAL ESTATE FEATURES



explain features and characteristics of real estate

Both individuals and institutions invest in real property: either in residential or commercial real estate. Residential real estate, or the housing market, consists of individual single-family detached homes and multi-family attached units, which share at least one wall with another unit, such as condominiums, cooperatives, townhouses, or terraced housing. Commercial real estate includes primarily office buildings, retail shopping centers, commercial and residential rental properties, and warehouses. In contrast to the owner-occupied market, rental properties are leased to tenants.

Residential real estate is by far the largest market sector by value and size. Savills World Research estimated in July 2018 that residential real estate accounted for more than 75% of global real estate values. Although the average value of a home is less than the average value of an office building, the aggregate space required to house people is much larger than that needed to accommodate office use and retail shopping.

## Real Estate Investments

Real estate investments exhibit general similarities to and differences from traditional equity and debt classes. Real estate can be held privately or traded publicly through real estate investment trusts (REITs). Equity investment involves direct or indirect ownership with claims to residual cash flows from the property. Depending on the property investment, these cash flows can be variable or fixed. Debt investment typically involves direct mortgage lending from financial intermediaries, part of which then can be securitized and then traded through various types of mortgage-backed securities (MBS). The main features of residential and commercial real estate are shown in Exhibit 1.

**Exhibit 1: Main Features of Residential and Commercial Real Estate**

	Residential real estate	Commercial real estate
Typical property	Owner-occupied, single residences; single-family residential property	Residential properties owned for lease or rental Office, retail, industrial, warehouse, hospitality, and mixed-use properties
Source of equity	Owners	Privately held by owners Publicly held through investors
Source of debt	Directly: Lenders (banks) through residential mortgages Indirectly: Investors in MBS that package residential mortgages	Directly: Lenders (banks) through commercial mortgages Indirectly: Investors in MBS that package commercial mortgages
Source of return to investors	Enjoyment of the property Price, or capital, appreciation	Income, or cash flow, generated by the property Price, or capital, appreciation

Real estate is uniquely different from other asset classes in several ways:

- The initial investment is typically large.
- Real estate is unique and distinct because there aren't two identical properties; each piece of real estate is heterogeneous and is uniquely characterized in terms of location, age, tenant credit mix, lease term, and market demographics.
- There are multiple types of real estate investment alternatives available: direct and indirect investment options spanning the spectrum from relatively liquid investments in stable, income-producing properties to illiquid investments over a long development life cycle across the purchase, construction/upgrade, occupancy, and sales phases.
- Diversification across all different types of real estate investment alternatives may be difficult to attain.
- Private market indexes replicating the performance of real estate are not directly investable.

Additionally, the price discovery process in the private real estate markets is opaque, for multiple reasons:

- Historical prices may not reflect prevailing market conditions.

- Transaction costs are typically high. Buying and selling real estate can be a time-consuming process, involving real estate professionals, banks, lawyers, and others needed to facilitate these transactions.
- Transaction activity may be limited in certain markets due to either supply or demand conditions.

Because of distinct and unique features—geographic location and potential uses—real estate markets are typically fragmented, with the local demand and supply conditions determining the value of the property. Consequently, the heterogeneity unique to real estate demands specialized skills. Selecting, valuing, acquiring, managing, and divesting a real estate portfolio is often more complicated than managing a portfolio made up of listed corporate debt and equity.

## Real Estate Investment Structures

Real estate investments take on a variety of public and private forms across equity and debt capital alternatives, as summarized in Exhibit 2.

**Exhibit 2: Selected Forms of Real Estate Investment**

	Debt	Equity
<b>Private</b>	Mortgage debt	<b>Direct ownership</b> Sole ownership Joint ventures Limited partnerships
	Construction loans	
	Mezzanine debt	<b>Indirect ownership</b> Real estate funds Private REITs
<b>Public</b>	MBS/CMBS/CMOs	<b>Publicly traded shares</b> Construction Operating Development Public REITs
	Covered bonds	
	Mortgage REITs	
	Mortgage ETFs	UCITS/Mutual funds/ETFs

### **Direct Real Estate Investment**

Direct private investing involves purchasing a property and originating debt for one's own account. Ownership can be free and clear, whereby the property title is transferred to the owner(s) unencumbered by any financing liens, such as from outstanding mortgages. Initial purchase expenses associated with direct ownership may include legal expenses, survey costs, engineering/environmental studies, and valuation (appraisal) fees. There are distinct advantages to owning real estate directly for property investors:

- *Control.* Only the owner can decide when to buy or sell, when and how much to spend on capital projects, whom to select as tenants based on credit quality preference and tenant mix, and what types of lease terms to

offer. Owners generate cash flow returns from the use and enjoyment of the property, the receipt of lease payments, and the potential for capital appreciation.

- *Tax benefits.* The owners can reduce their taxable income using non-cash property depreciation expenses and tax-deductible interest expenses.
- *Diversification.* Historically, real estate has exhibited low correlation with other asset classes, and adding real estate to a portfolio has been demonstrated to increase portfolio diversification and reduce portfolio risk.

There are also disadvantages to investing directly in property:

- *Complexity.* The owners need to dedicate time to manage the property. Making the purchase itself is more complicated as well, with requirements including property selection, negotiating terms, performing due diligence, title search, contract review, and property inspection.
- *Need for specialized knowledge.* The owners need to understand both general and local market characteristics, which requires local market knowledge.
- *Significant capital needs.* The owners need to have access to a potentially significant amount of debt and equity capital because of the large initial capital outlay needed for real estate investments.
- *Concentration risk.* Owners, particularly smaller investors, cannot create a well-diversified real estate portfolio through direct investment.
- *Lack of liquidity.* It is typically difficult to quickly buy or sell direct investments in real estate, and transaction costs are typically high.

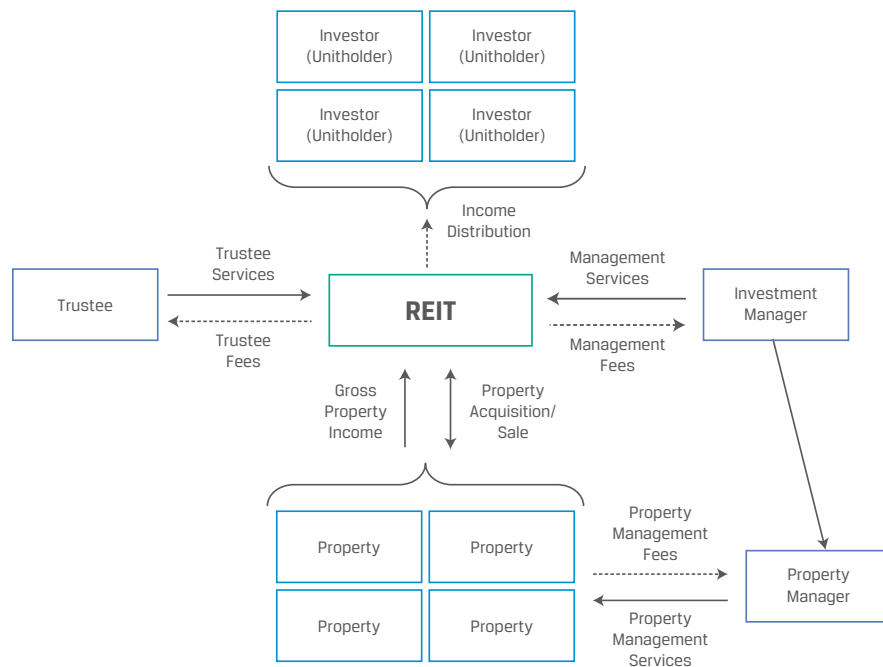
Real estate investors may choose to handle all aspects of investing in and operating the property internally. However, commercial real estate investors often hire advisers to identify investments, negotiate acquisition and lease terms, perform due diligence, conduct real estate operations, and assist with the eventual disposal. Institutional investors that hire advisers or managers to manage their direct real estate investment can also use a separate account structure that allows the investor to control the timing and value of acquisitions and dispositions.

Owners can also borrow from mortgage lenders to fund the acquisition. Additional debt closing costs are incurred when owners take out loans to fund their investments.

### ***Indirect Real Estate Investment***

Indirect investment in real estate pools assets from different investors to acquire one or several properties; here the exposure is indirect through a variety of investment vehicles. These can be public or private, such as limited partnerships, mutual funds, equities, REITs, and exchange-traded funds (ETFs). Sometimes investors form joint ventures with other investors to invest in real estate. Joint ventures are especially common when one party can uniquely contribute something of value, such as land, capital, development expertise, debt due diligence, or entrepreneurial talent.

Tax-advantaged trusts that own, operate, and sometimes develop income-producing real estate property are known as real estate investment trusts. Their structure is shown in Exhibit 3. There are three main forms of REIT: equity REITs, which invest in properties outright or through partnerships and joint ventures; mortgage REITs, which underwrite loans to real estate (mortgages) or invest in MBS; and hybrid REITs, which invest in both these types.

**Exhibit 3: Real Estate Investment Trust Structure**

REITs are the preferred investment vehicles for owning income-producing real estate for both private and public investors. The main appeal of the REIT structure is the elimination of double corporate taxation. Corporations pay taxes on their income, from which they make dividend distribution to their owners from after-tax earnings. The shareholders, in turn, are taxed at their personal tax rate. REITs can avoid corporate income taxation by distributing dividends equal to 90%–100% of taxable net rental income.

The business strategy for equity REITs is simple: Maximize property occupancy rates and rents while minimizing ongoing operating and maintenance expenses to maximize cash income and dividends. REITs are a popular investment vehicle both in the United States, where they originated, and in other countries. REITs and REIT-like structures have similar legal and taxation structures and provide unique tax advantages to investors and the corporate entity.

Equity REITs, like other public companies, must report earnings per share based on net income as defined by generally accepted accounting principles (GAAP) or International Financial Reporting Standards (IFRS). Many report non-traditional measures, such as net asset value or variations of gross cash flow, such as funds from operations (FFO), which makes adjustments for depreciation, distributions, and preferred dividends, to better estimate future dividends, because non-cash depreciation expenses can be high for asset-intensive businesses.

These publicly traded REITs address many of the disadvantages related to private real estate investing. These REITs provide investors with greater transparency. Additionally, a REIT investor only needs to buy or sell REIT shares instead of buying or selling real estate directly. The REIT is not forced to sell the company's underlying real estate like open-end funds experiencing mass redemptions. Finally, REITs have the know-how to manage the properties in order to align the interests of the REIT with those of its investors. However, a disadvantage of REITs is their higher correlation with the public equity markets when compared to private real estate.



**EXAMPLE 1****Luxury Dreams REIT**

Luxury Dreams Real Estate Investment Trust acquires high-quality, income-oriented, high-end real estate around the world and offers investment alternatives for investors looking for long-term, stable, predictable cash flows, price appreciation in the underlying properties from the inclusion of properties, and active management of the properties, at generally lower volatility because of its focus on high-end real estate.

The REIT invests at least 75% of assets in high-end residential properties and up to 25% of assets in privately held, real estate-related securities, mortgage debt, and cash. The REIT is capped at USD5 billion. Because this REIT is not traded publicly, shares in the trust can be redeemed at the end of each month at a price equal to the prior month's net asset value (NAV) per share in increments of USD10,000. NAV per share is determined at the end of each month and is released to holders within 15 business days after the end of the month. Each shareholder is limited to transact 2% of the total NAV per month and 5% of NAV per quarter.

REITs and other private real estate funds are structured as infinite-life, open-end funds and allow investors to contribute or redeem capital throughout the life of the fund in a fashion parallel to mutual fund structures. Open-end funds generally offer exposure to well-leased, high-quality commercial and residential real estate in the best markets, also called **core real estate strategies**. Investors expect core real estate to deliver stable returns, primarily from income from the property.

Investors seeking higher returns may also accept additional risks from development, redevelopment, repositioning, and leasing. For such opportunistic investment preferences, finite-life, closed-end funds are more commonly used. Investors may focus on **core-plus real estate strategies**, value-add investments that require modest redevelopment or upgrades to lease any vacant space together with possible alternative use of the underlying properties. To earn higher returns, investors may engage in **value-add real estate strategies**, such as larger-scale redevelopment and repositioning of existing assets. The most **opportunistic real estate strategies** include major redevelopment, repurposing of assets, taking on large vacancies, or speculating on significant improvement in market conditions.

Mortgage REITs and hybrid REITs invest in real estate debt, typically MBS (covered elsewhere in the curriculum). These debt REITs can be both private and publicly traded funds.

**QUESTION SET**

1. True or false: The distinct and unique features of real estate make managing a real estate portfolio less complex than managing a portfolio of listed corporate bonds.

**Solution:**

False. Selecting, valuing, acquiring, managing, and divesting a real estate portfolio is often more complicated than managing a portfolio made up of listed corporate debt. The heterogeneity unique to real estate demands specialized skills, and real estate markets are typically fragmented, with the local demand and supply conditions determining the value of the property.

2. The largest sector of the real estate market is:

- A. residential real estate.
- B. real estate investment trusts.
- C. publicly traded mortgage-backed securities.

**Solution:**

The correct answer is A. Residential real estate is by far the largest market sector by value and size. Savills World Research estimated in July 2018 that residential real estate accounted for more than 75% of global real estate values.

3. Describe the business strategy for equity REITs.

**Solution:**

The business strategy for equity REITs is to maximize property occupancy rates and rents while minimizing ongoing operating and maintenance expenses to maximize cash income and dividends.

4. When a property title that is transferred to a new owner is unencumbered by any financing liens, such as from outstanding mortgages, the new ownership is considered:

- A. privately held.
- B. free and clear.
- C. direct equity ownership.

**Solution:**

The correct answer is B. “Free and clear” refers to the lack of any financing liens on a purchased property. If a direct private investor purchases a property and receives a title that is unencumbered by any financing liens, that ownership is considered free and clear.

### 3

## REAL ESTATE INVESTMENT CHARACTERISTICS



explain the investment characteristics of real estate investments

Investments in real estate provide competitive long-term total returns from income generation combined with potential price appreciation. Many commercial real estate companies offer multiple-year leases with fixed rents; hence, the income earned from these leases is typically both predictable and stable. Moreover, real estate tends to provide inflation protection because the lease payments are regularly adjusted, which allows for a clear valuation and pricing of the property. Finally, real estate historically has low correlations with other asset classes, and adding real estate to an investment portfolio provides diversification benefits at relatively lower levels of risk. There are a variety of indexes globally designed to measure total and component real estate returns for listed securities and non-listed investment vehicles.

## Source of Returns

The return on real estate investments comes from income or asset appreciation or a combination of both. Income-producing real estate generates income primarily from the collection of rental or lease payments, including lease renewals. Expenses include direct and indirect management expenses, such as maintenance and improvement costs. Income-producing real estate investment is mostly low-risk direct investment that depends on the timely receipt of lease payments. Investors relying on capital appreciation typically expect that their longer-term property development projects are successful and the exit price will exceed the aggregate investments and any cash flow that they can generate in the meantime.

The risk and return spectrum for real estate investments, depicted in Exhibit 4, includes strategies across both debt and equity investment. Investing in real estate can generate either lower-risk, bond-like cash flows from leases or higher-risk, equity-like speculative returns from realizing value from development projects or price appreciation. Moreover, most real estate strategies are financed by a combination of debt and equity, with the proportion depending on the type of real estate.

**Exhibit 4: Real Estate Risk–Return Spectrum**



The low-risk/low-return part of the spectrum, starting in the lower left section of the exhibit, is made up by relatively low-risk, senior debt, such as first mortgages and investment-grade commercial mortgage-backed securities (CMBS). Since the underlying assets are bonds, both the risks and returns are bond-like.

The second part of the spectrum is made up of the core, stable income-producing REITs, investing in properties that generate stable cash flows either from properties with long-term leases and with many lessors (typically residential real estate) or from sale-leaseback transactions. Because in a sale-leaseback transaction the owner of the property sells the property to an investor and leases the property for continued use, the risk of default is low, providing investors a stable and relatively safe return. Additionally, real estate investors often use sale leaseback structures to secure financing in a cost-efficient way or to reduce their leverage. Overall, the returns are relatively higher compared to low-risk senior debt and are generally bond-like because the main source of return comes from long-term lease payments.

The third and fourth parts of the spectrum offer higher, riskier, and less predictable potential returns; the main source of return moves away from the predictable, bond-like, contractual cash flows and is replaced by more speculative sources from potential price appreciation. The main source of return for core-plus holdings comes from leases, but the cost of acquiring these leases and maintaining and updating the underlying properties may become significant, particularly when the property needs refurbishment, renovation, and redevelopment. For value-add real estate, the returns are increasingly equity-like, with the price appreciation component becoming progressively meaningful.

The final, fifth phase of the spectrum provides the greatest return potential at the expense of the highest levels of risk from opportunistic real estate. For instance, investments in distressed properties and in property development are subject to greater risks than investments in properties in sound financial condition or with stable operations, such as core real estate. The increasing equity-like return reflects that property development is subject to special risks, including regulatory issues, construction delays, and cost overruns. Environmental regulation is one regulatory hurdle, as is the failure to receive zoning, occupancy, and other approvals and permits. Because the lifecycle of such projects can be very lengthy, economic conditions may change. All these issues may increase construction time or delay successful leases, which increases construction costs and reduces the level of rents relative to initial expectations. This could result in a reduction of IRR versus expectation—leaving the investor uncompensated for the higher risk and illiquidity of real estate investment.

In sum, real estate investments offer a wide range of investment opportunities along the risk/return spectrum. Also, the amount of leverage deployed in each strategy shifts the risk/return trade-off by magnifying both the gains and the losses. Particularly for the more speculative real estate investors, this can increase their default risk, especially when there are unexpected changes in the level of interest rates, access to financing, or government land-use regulations. Therefore, the performance of real estate investments can vary substantially depending on the measurement period being considered.

## EXAMPLE 2

### Peterburgh Real Estate Fund, LLC—Change to LTV

The Peterburgh Real Estate Fund, LLC, has acquired a portfolio consisting of six commercial properties for a total of GBP100 million and secured GBP75 million in mortgages from BridgeRock Credit Opportunities LLP, a private debt fund that syndicated them. The terms of the mortgage are MRR + 150 bps, with a maturity of 15 years, GBP5 million in annual amortization, and all mortgages being secured by the first lien on the property. The loan agreements stipulated that the loan to value (LTV) should remain at 0.75. But after one year, the economic environment worsened substantively and the assessed value of some of the commercial properties owned by Peterburgh declined by GBP8 million to GBP92 million, breaching the LTV of 0.75. More specifically, the outstanding mortgage balance was reduced through amortization by GBP5 million to GBP70 million and the value of the property is GBP92 million, which gives an LTV of 0.76. BridgeRock and the other members of the syndicate would now require that Peterburgh fix the breach by adding GBP1.3 million in collateral to bring the LTV down to 0.75.

Returns to both debt and equity investors in real estate depend to a large extent on the ability of the owners or their agents to successfully operate the underlying properties. Property values vary with global, national, and local conditions. Real estate tends to provide inflation protection if leases provide regular contractual rent step-ups or can be frequently marked to market. Research suggests that inflation-hedging

ability varies significantly by geographic location, market segment, and time period. In particular, the inflation-hedging potential of real estate may be more difficult to detect if the high-inflation period of the late 1970s and early 1980s is excluded from the period of study. In addition to these macro and micro conditions, the prevailing level of interest rates influence valuation, returns, and risk.

## Real Estate Investment Diversification Benefits

Investors in real estate—whether direct or indirect, public or private, debt or equity—seek high, stable, and steady returns. Real estate investments generate bond-like cash flows from their medium- to long-term property leases and offer bond-like returns. The variability of these returns can be reduced by longer-term leases, better credit quality of tenants, and the possibility of rent increases.

Real estate investments offer the opportunity for capital appreciation from the underlying assets that can bolster returns. In fact, more than half of the returns commercial real estate investors earn are derived from income, with the rest coming from long-term price appreciation. Additionally, throughout an economic market cycle, the real estate income is a more consistent source of return than capital appreciation. This reduces the risk from investing for real estate investors and provides a source of diversification: stock market returns mostly derive from long-term capital appreciation. Effectively, investment in real estate is somewhat like a convertible bond but with several advantages: steady cash flows, the possibility of capital appreciation, and low correlation between real estate price appreciation and equity market price appreciation.

Different investors have differing views on how to approach investments in REITs. Some institutional investors consider it a separate alternative asset class, some consider it a sub-sector of the broader real estate market, some consider it a fixed-income/equity hybrid, and others consider it a fixed-income asset with an incremental yield advantage over investment-grade corporate bonds.

Whether listed real estate behaves like stocks or private real estate is a matter of ongoing debate. The market prices listed REITs continuously, whereas private real estate is appraised perhaps once a year. This mismatch in appraisal timing leads to correlation numbers that are artificially low. Additionally, equity investors in public real estate discount future cash flows, while appraisers of private real estate place heavy emphasis on current market conditions and recent trends. The various real estate strategies provide different diversification benefits, as Exhibit 5 shows.

**Exhibit 5: Historical Correlation between Real Estate Investment Strategies and Market Returns, March 2008–June 2021**

Correlation	S&P 500 Total Return	MSCI US REIT Total Return	MSCI World Total Return
Preqin, real estate	0.51	0.49	0.46
Preqin, real estate debt	0.40	0.48	0.38

*Source:* Annualized quarterly returns of Private Capital Quarterly Index rebased to 31 December 2007, provided by Preqin.

Moreover, during certain market conditions, equity REIT correlations with market benchmarks increase, particularly during steep market downturns, such as the 2007–08 financial crisis. As these high correlations remained high during the post-crisis recovery, they lifted the value of most asset classes. Similar patterns in financial asset prices and real estate prices appear to have been observed during the COVID-19 recovery.

All in all, there appears to be a consensus that real estate offers diversification benefits to portfolios. Real estate markets around the world can be highly idiosyncratic and often have low correlations with traditional asset classes. For example, Oxford Economics studied the expected performance of listed European real estate as an asset class, comparing it with equities, fixed income, and commodities.<sup>1</sup> “A substantial allocation to listed real estate,” the authors concluded, “does enhance the risk–return characteristics of a multi-asset portfolio.” They went on to recommend larger allocations to real estate for European investors.

### QUESTION SET



1. True or false: Real estate historically has high correlations with other asset classes.

**Solution:**

False. Real estate historically has low correlations with other asset classes, and adding real estate to an investment portfolio provides diversification benefits at relatively lower levels of risk.

2. The sources of long-term return for real estate investments are \_\_\_\_\_ and \_\_\_\_\_.

**Solution:**

*income generation, potential price appreciation*

3. In what ways are real estate investments similar to bond investments?

**Solution:**

Real estate investments can be similar to bond investments in that they are stable, predictable, lower-risk cash flows from leases that are similar to bond coupon payments

4. In what ways are real estate investments similar to equity investments?

**Solution:**

Real estate investments can be similar to equity investments in that they are speculative returns that can be realized from price appreciation of the real estate asset

## 4

### INFRASTRUCTURE INVESTMENT FEATURES



explain features and characteristics of infrastructure

Infrastructure investments have a societal purpose; facilitate broad economic, technological, and social development purposes; and usually combine land, buildings, and other long-lived fixed assets. Infrastructure supports public transportation, airports, utilities (water, gas, and electricity), and more recently, information (telecommunication,

<sup>1</sup> Listed Real Estate in A Multi-Asset Portfolio: A European Perspective, pg. 2, Oxford Economics, EPRA September 2019.

cable, and wireless networks). Early infrastructure investments were typically financed by private corporations to realize profits. Subsequently, many governments took on a larger proportion of infrastructure investment and by the second half of the 20th century became the main source of investments. The late 20th century saw waves of privatizations of public transportation, and the scope of private infrastructure investment expanded to assets that governments had historically financed, owned, and operated themselves. The market for privately funded infrastructure is sizeable.

## Infrastructure Investments

Infrastructure investments are real, capital-intensive, and long-lived assets intended for public use and provide essential services, such as airports, health care facilities, and sewage treatment plants. These investments have similarities to and differences from common equity and debt. Investments include equity, with its usual claims to residual cash flows, and debt, to finance and maintain such investments. Like real estate, investment in existing infrastructure involves acquiring unique, illiquid assets with distinct locations, features, and uses. Investments in construction and development of new infrastructure are made with expectations to generate cash either from income or from capital appreciation.

Infrastructure investments often involve a consortium that combines one or several strategic partners that have specialized operational or technical skills with the financial investors. Rather than leases or rentals from commercial or residential tenants, infrastructure cash flows in most cases arise from contractual payments, such as the following:

- *Availability payments*, which are payments received to make the facility available
- *Usage-based payments*, such as tolls and fees for using the facilities
- *“Take-or-pay” arrangements*, which obligate buyers to pay a minimum purchase price to sellers for a pre-agreed volume.

Allocations to infrastructure investments are driven both by the increased demand for infrastructure and by many governments’ search for alternative funding sources for infrastructure investments, such as investors interested and experienced in building, managing, and running infrastructure. Investors have an interest in financing infrastructure investments, demand-side growth. As governments provide more opportunities by expanding infrastructure assets, they also continue to privatize government services.

Most infrastructure assets are financed, owned, and operated by governments, and a substantive proportion of these investments comes from public sources in the developing world. However, increasingly infrastructure is being financed privately through public–private partnerships (PPPs) by local, regional, and national governments. A **public–private partnership** is typically defined as a long-term contractual relationship between the public and private sectors for the purpose of having the private sector deliver a project or service traditionally provided by the public sector. Infrastructure investors may intend to lease the assets back to the government, to sell newly constructed assets to the government, or to hold and operate the assets until they reach operational maturity or perhaps for even longer.

Infrastructure investments are also made in partnership with development finance institutions, which are specialized financial intermediaries that provide risk capital for economic development projects on a non-commercial basis. There are global, international, national, and local development finance institutions. For instance, the European Bank for Reconstruction and Development (EBRD) invests to improve municipal services, including infrastructure, and regularly taps global financial markets to finance infrastructure and other types of investments.

### Categories of Infrastructure Investments

To categorize infrastructure investments, investors frequently rely on the underlying assets, with the broadest categorization distinguishing between economic and social infrastructure assets, as shown in Exhibit 6.

**Exhibit 6: Categorizing Infrastructure Investments**

Economic Infrastructure Investments			
Transportation Assets	Information and Communication Technology Assets	Utility and Energy Assets	Social Infrastructure Investments
<ul style="list-style-type: none"> <li>▫ roads</li> <li>▫ bridges</li> <li>▫ tunnels</li> <li>▫ airports</li> <li>▫ seaports</li> <li>▫ railway systems</li> </ul>	<ul style="list-style-type: none"> <li>▫ telecommunication towers</li> <li>▫ data centers</li> </ul>	<ul style="list-style-type: none"> <li>▫ electrical grid</li> <li>▫ power generation, transmission, distribution</li> <li>▫ potable water production</li> <li>▫ gas storage and distribution</li> <li>▫ liquefied natural gas terminals</li> <li>▫ oil and gas infrastructure</li> <li>▫ solid waste treatment</li> </ul>	<ul style="list-style-type: none"> <li>▫ educational assets</li> <li>▫ health care assets</li> <li>▫ social housing</li> <li>▫ correctional facilities</li> <li>▫ government/municipal buildings</li> </ul>

**Economic infrastructure investments** support economic activity through transportation assets, information and communication technology (ICT) assets, and utility and energy assets:

- *Transportation assets* include roads, bridges, tunnels, airports, seaports, and heavy and light/urban railway systems. Income will usually be linked to demand based on traffic, airport and seaport charges, tolls, and rail fares and hence is deemed to carry market risk.
- *ICT assets* include infrastructure that stores, broadcasts, and transmits information or data, such as telecommunication towers and data centers.
- *Utility and energy assets* generate power and produce potable water; transmit, store, and distribute gas, water, and electricity; and treat solid waste.

Utility investments encompass environmentally sustainable development, with an increasing focus on renewable technologies, including solar, wind, and waste-to-energy power generation. Other energy assets may encompass downstream oil and gas infrastructure, the electrical grid, and liquefied natural gas terminals. The income earned from utility assets may also carry demand risk because buyers' energy and natural resources needs fluctuate. Alternatively, utilities can institute "take-or-pay" arrangements, locking buyers into minimum purchases whether supply is needed or not. Buyers usually have recourse if the utility falls short on performance or delivers supplies that are late or of inferior quality.

**Social infrastructure investments** are directed toward human activities and include such assets as educational, health care, social housing, and correctional facilities, with the focus on providing, operating, and maintaining the asset infrastructure. The relevant services administered through those facilities are usually provided separately by the public authority or by a private service provider contracted



by the public authority. In some countries, this model has been extended to other public infrastructure, including government and municipal buildings. Income from social infrastructure is typically derived from a type of lease payment that depends on availability payments and on managing and maintaining the asset according to predefined standards.

### EXAMPLE 3

#### Clarkswood Infrastructure Fund LP

Clarkswood Infrastructure Fund LP invests its assets in equity and equity-like securities and debt issued by issuers that own or operate infrastructure assets in developed and developing countries. Infrastructure lays the foundation of basic services, facilities, and institutions on which the growth and development of a nation and country directly depend. More specifically, in several developed countries, such as the United States, the United Kingdom, and Australia, many infrastructure assets have been privatized as national, state, and local governments spend less on building, investing, maintaining, and operating infrastructure. The private sector has stepped in to provide equity financing to fund infrastructure.

For instance, the fund invests in an operator of Australian toll roads listed on the Australian Stock Exchange, as well as the operator of the Auckland Airport (New Zealand's largest airport), also listed on a stock exchange. It also invests in several corporations that provide infrastructure services in both the developing world and the developed world, including China and India, that see significant demand for infrastructure products and services and seek private sector investments. The fund invests up to 50% of its assets in US assets, up to 30% in infrastructure assets located in OECD (Organisation for Economic Co-operation and Development) countries, up to 30% in assets in non-OECD countries, and no more than 10% in any country with no more than 5% in any infrastructure provider.

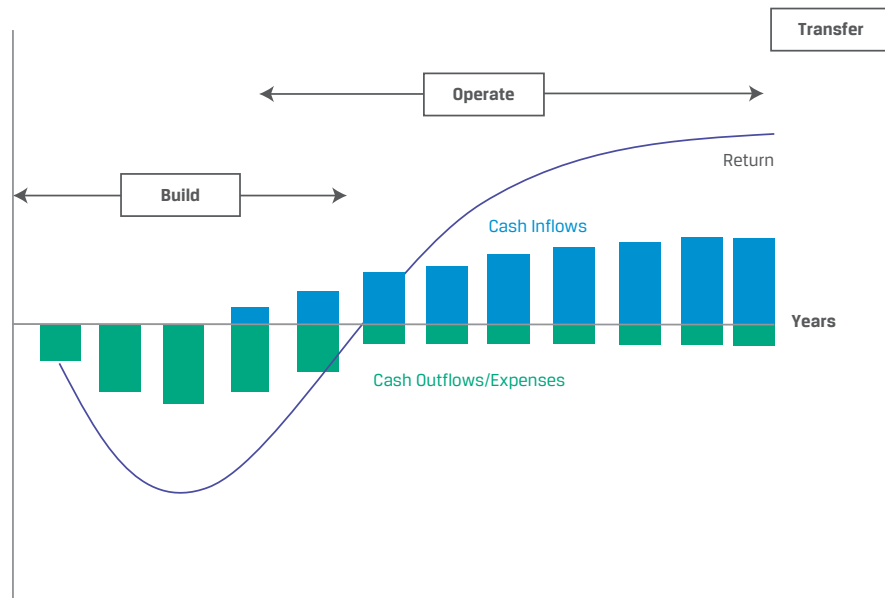
#### *Stages of Infrastructure Development*

Infrastructure investments can also be categorized by the underlying assets' stage of development. Typically, we distinguish among greenfield investments, secondary-stage investments, and brownfield investments.

**Greenfield investments**, developing new assets and new infrastructure, are opportunistic investments. The intent may be to lease or sell the assets to the government after construction or to hold and operate the assets. If they are held, it can be over the long term or for a shorter period until operational maturity, with subsequent sale to new investors, thus ensuring capital appreciation to reflect the construction and commissioning risk. Greenfield investors typically invest alongside strategic investors or developers that specialize in developing the underlying assets.

The greenfield investment life cycle common among public-private partnerships is called the build-operate-transfer (BOT) life cycle and is shown in Exhibit 7. The build phase is often characterized by an initial longer approval and construction phase with negative cash flows. The subsequent operate phase is governed by a concession agreement in which the private investor generates income based on pre-agreed parameters. In the final transfer phase, the investment is transferred to a government entity based on pre-determined parameters, sold to a third party, or decommissioned.

### Exhibit 7: Illustrative Build-Operate-Transfer Model for Greenfield Infrastructure



**Brownfield investments** expand existing facilities and may involve privatization of public assets or a sale leaseback of completed greenfield projects. They are characterized by a shorter investment period with immediate cash flows and an operating history. Typically, some of the assets' financial and operating history is available, and so brownfield investments may be sought after by both strategic investors specializing in operating the assets and—particularly with privatizations—financial investors focused on long-term, stable returns.

**Secondary-stage investments** invest in existing infrastructure facilities or fully operational assets that do not require further investment or development over the investment horizon. These assets generate immediate cash flow and returns expected over the investment period. Some assets never reach this stage because they continuously require additional capital and development.

#### Forms of Infrastructure Investment

As with real estate investments, infrastructure investments come in a variety of forms. The choice affects liquidity, cash flow, and income streams. Infrastructure investments can be direct or indirect. The vast majority of investors focus on equity investments, with some interest in pure debt (infrastructure bonds) and convertible debt.

- *Direct investment in the underlying infrastructure* provides control and the opportunity to capture full value. It requires a large investment and results in both concentration and liquidity risks while the assets are managed and operated. Because of this risk and the typical long-term horizon, direct infrastructure investment usually takes place with a group or consortium of strategic investors that share the financial risk and/or assume a specific role in building, operating, or managing the assets. Such strategic partners, such as large pension funds or sovereign wealth funds, are frequent direct investors because they are better placed to manage certain risks to limit individual concentration risk. Frequently these funds invest under specific mandates in infrastructure projects and specifically prioritize domestic infrastructure needs.

**EXAMPLE 4****Public–Private Partnership, Iguá Saneamento S.A.**

In March 2021, the Canada Pension Plan Investment Board paid CAD270 million (BRL1,178 million) for a 45% stake in Iguá Saneamento S.A. (Iguá), a water and sewage service company operating 18 concessions and contracts across five Brazilian states. The company serves more than 6 million people, provides sanitation services, and is the third-largest Brazilian private water and sewage treatment service provider.

The objective of the transaction is to support Iguá's growth objectives and to maintain operating funds to support its operations. Iguá is considering acquiring additional water and sanitation concessions in Brazil, including the formation of public–private partnerships there. Other institutional investors include Alberta Investment Management Corporation (39%), BNDES Participações S.A. (11%), and IG4 Capital Group (11%).

• *Indirect investments* include infrastructure funds (similar in structure to private equity funds and either closed end or open end), infrastructure ETFs, and holding equity in publicly traded infrastructure providers, or master limited partnerships (MLPs). Investors concerned about liquidity and diversification may choose publicly traded infrastructure securities. Publicly traded infrastructure securities benefit from liquidity, reasonable fees, transparent governance, observable market prices, and transparent pricing, in addition to diversification among underlying assets. An investor should be aware, however, that publicly traded infrastructure securities represent a small segment of infrastructure investment: S&P Dow Jones Indices reported a total global market cap of USD2.23 trillion as of 31 December 2021.<sup>2</sup> Publicly traded infrastructure investments also tend to be clustered in certain asset categories.

Master limited partnerships (MLPs) trade on exchanges, are pass-through entities like REITs, and share the income pass-through structure taxation rules that minimize double taxation for investors. MLPs are most commonly used in energy transportation, processing, or storage; generate relatively stable cash flows from fee-based income; and distribute larger parts of their free cash flow to their investors.

Debt financing for infrastructure projects can be both private debt and publicly traded debt. Normally, the terms are flexible to accommodate periods of zero cash flow and long development or investment horizons. Publicly issued debt, such as the Airport Authority of Hong Kong perpetual bonds and Indonesian Infrastructure Fund US dollar bonds highlighted in earlier fixed-income modules are other approaches to financing infrastructure projects.

**QUESTION SET**

1. Infrastructure cash flows primarily arise from:

- A. dividends.
- B. commercial tenants.
- C. contractual payments.

**Solution:**

The correct answer is C. Rather than leases or rentals from commercial or residential tenants, infrastructure cash flows arise from contractual payments, such as: availability payments (payments are received to make the

<sup>2</sup> S&P Dow Jones Indices, "Approaches to Benchmarking Listed Infrastructure" (June 2022, p. 3). [www.spglobal.com/spdji/en/documents/research/research-approaches-to-benchmarking-listed-infrastructure.pdf](https://www.spglobal.com/spdji/en/documents/research/research-approaches-to-benchmarking-listed-infrastructure.pdf).

facility available), usage-based payments (e.g., tolls and fees for using the facilities), and “take-or-pay” arrangements (which obligate buyers to pay a minimum purchase price to sellers for a pre-agreed volume).

2. Most infrastructure assets are financed, owned, and operated by:

- A. governments.
- B. public–private partnerships.
- C. development finance institutions.

**Solution:**

The correct answer is A. Most infrastructure assets are financed, owned, and operated by governments, and a substantive proportion of these investments comes from public sources in the developing world. However, increasingly infrastructure is being financed privately through public–private partnerships by local, regional, and national governments. Infrastructure investments are also made in partnership with development finance institutions, which are specialized financial intermediaries that provide risk capital for economic development projects on a non-commercial basis.

3. Which of these statements about infrastructure investing is true? Infrastructure investments:

- A. can generate cash income.
- B. are intended to be non-profit.
- C. do not have capital appreciation.

**Solution:**

The correct answer is A. Investments in construction and development of new infrastructure are made with expectations to generate cash either from income or from capital appreciation.

4. Which of the following is a characteristic of direct investment in infrastructure?

- A. High liquidity
- B. Concentration risk
- C. Short-term horizon

**Solution:**

The correct answer is B. Direct investment in infrastructure requires a large investment and results in both concentration and liquidity risks while the assets are managed and operated. Because of these risks and the typical long-term horizon, direct infrastructure investment usually takes place with a group or consortium of strategic investors that share the financial risk and/or assume a specific role in building, operating, or managing the assets.

## 5

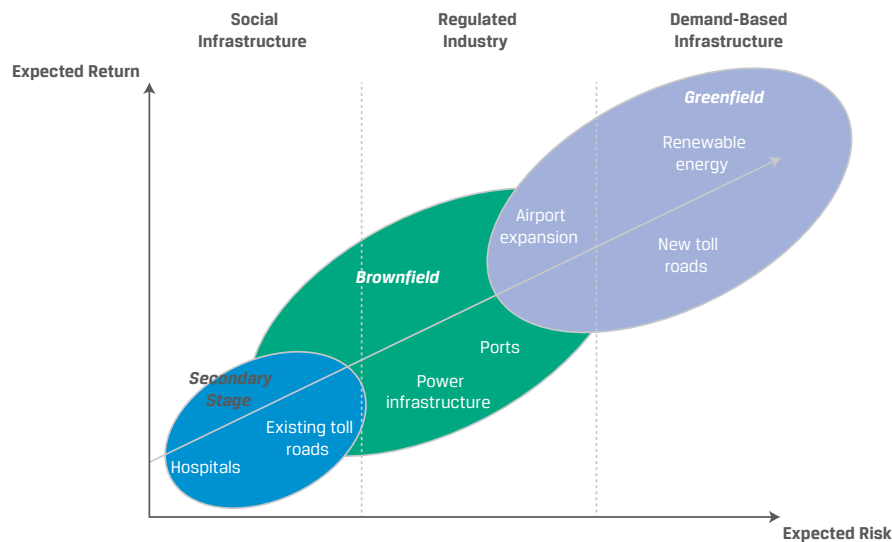
## INFRASTRUCTURE INVESTMENT CHARACTERISTICS



explain the investment characteristics of infrastructure investments

The type of the underlying infrastructure investment, its stage of development, its geographic location, and the way the investment is structured define the expected risk and returns, as Exhibit 8 depicts.

**Exhibit 8: Infrastructure Investment Characteristics**



In the three groupings of the infrastructure development cycle—greenfield, operational secondary-stage, and brownfield—the operational secondary-stage assets with an existing track record of generating steady, bond-like cash flows possess the lowest risk and offer the lowest return to the investors. Brownfield investments, redevelopment of existing infrastructure, are incrementally riskier, and greenfield projects are the riskiest. Exhibit 9 shows the historical risk and return statistics for infrastructure assets compared to other real asset categories.

**Exhibit 9: Historical Annualized Return and Standard Deviation of Return for Infrastructure and Real Assets, March 2008–June 2021**

Annualized	Average return	Standard deviation	Coefficient of variation
Preqin, infrastructure	8.57%	0.07	0.82
Preqin, natural resources	3.29%	0.09	2.66
Preqin, real estate	3.43%	0.10	2.86
S&P Infrastructure Total Return	3.52%	0.19	5.52
MSCI US REIT Total Return	7.77%	0.26	3.38

Source: Annualized quarterly returns of Private Capital Quarterly Index rebased to 31 December 2007, provided by Preqin.

Additionally, the type of infrastructure investment is material in determining the risk and return. While investments in basic social services infrastructure or existing regulated industries typically involve less risk and lower expected return, demand-based infrastructure projects often build on projections of future economic growth and increased usage demands and are riskier.

In developing market economies, where infrastructure investments are needed to underpin economic, social, and societal growth and benefit from increased wealth created by the economy, risks are considerable, but the returns—over long time horizons—are considerable. Greenfield infrastructure projects in developing countries offer exceptional return opportunities over very long time horizons. Exhibit 10 shows the risk profiles of various infrastructure projects.

**Exhibit 10: Private Infrastructure Fund Illustrative Target Returns**

High-Risk Profile	Medium-Risk Profile	Low-Risk Profile
Greenfield projects without guarantees of demand upon completion—e.g., variable electricity prices, uncertain traffic on roads and through ports	Mostly brownfield assets (with some capital expenditure requirements) and some greenfield assets (with limited construction and demand risk)	Brownfield assets with mitigated risks—e.g., fully constructed with contracted/regulated revenues
Located in OECD countries and emerging markets	Located primarily in OECD countries	Located in the most stable OECD countries
High weighting to capital appreciation	Mix of yield and capital appreciation	High weighting to current yield
<b>Target equity returns of 14%+</b>	<b>Target equity returns of 10%–12%</b>	<b>Target equity returns of 6%–8%</b>

*Note:* Target equity returns are net of fees.

*Source:* Cambridge Associates, “Digging In: Assessing the Private Infrastructure Opportunity Today,” Research Note (April 2017).

Most infrastructure funds gravitate toward the medium- and low-risk profiles, generating an average long-term annual return around 10%. As for other alternative investments, less liquid forms of direct equity ownership investments tend to offer the highest expected return with the greatest risk, while publicly traded forms of debt offer the lowest potential returns. Assets backed by stable long-term concession arrangements provide the most stable returns.

## Infrastructure Diversification Benefits

Infrastructure investors expect the assets to primarily generate stable long-term cash flows that also adjust for economic growth and inflation and secondarily expect capital appreciation, depending on the type and timing of their investment. Because infrastructure investments typically support services that face inelastic demand and/or benefit from high barriers to entry, generate steady cash returns, and have a longer life cycle, equity investments in infrastructure offer lower correlation with public market equities and the broader economy. Effectively, infrastructure investments provide an income stream, increase portfolio diversification by adding an asset class with typically low correlation with other public investments, provide some protection for changes in GDP growth, and offer some protection against inflation. Exhibit 11 summarizes the diversification benefits of infrastructure investments and shows the low correlation between infrastructure and other market returns. It is notable that public and private infrastructure returns exhibit low correlations.

**Exhibit 11: Historical Annualized Correlations for Infrastructure Assets,  
March 2008–June 2021**

Correlation	S&P 500 Total Return	S&P Infrastructure Total Return	MSCI US REIT Total Return	MSCI World Total Return
Preqin, infrastructure	0.12	0.14	0.33	0.08
Preqin, natural resources	0.68	0.67	0.61	0.68
Preqin, real estate	0.51	0.39	0.49	0.46

*Source:* Annualized quarterly returns of Private Capital Quarterly Index rebased to 31 December 2007, provided by Preqin.

Most institutional investors consider infrastructure investments to balance public equity holdings because infrastructure has proven to be relatively resilient to swings in the equity markets. Given the stable underlying nature of cash flows, infrastructure debt tends to experience lower default rates and higher recovery than similar fixed-income instruments, and it is less subject to fluctuation over the economic cycle.

Additionally, infrastructure may better match the longer-term liability structure of certain investors, such as pension funds, superannuation schemes, and life insurance companies. It also suits the longer-term horizon of sovereign wealth funds, which tend to make the largest allocations to this asset class—around 5%–6% of total assets under management, according to Preqin. Long-term correlation benefits also result from the fact that most infrastructure assets have a link to inflation through regulation, concession agreements, or other fee contracts whose rates rise to or above the rate of inflation.

**QUESTION SET**


1. True or false: Greenfield infrastructure projects in developing countries offer exceptional return opportunities over very long time horizons.

**Solution:**

True. In developing market economies, where infrastructure investments are needed to underpin economic, social, and societal growth and benefit from increased wealth created by the economy, risks are considerable but the returns—over long time horizons—are also considerable.

2. Which of these types of infrastructure investment has the highest expected risk?

- A. Regulated industry
- B. Social infrastructure
- C. Demand-based infrastructure

**Solution:**

The correct answer is C. Demand-based infrastructure investments have the highest expected risk of the three. Social infrastructure has the lowest.

3. Which of the following has the highest weighting to capital appreciation?

- A. Greenfield assets with limited construction and demand risk
- B. Fully constructed brownfield assets with contracted revenues

**C.** Greenfield assets without guarantees of demand upon completion

**Solution:**

The correct answer is C. Greenfield projects without guarantees of demand upon completion—for example, variable electricity prices, uncertain traffic on roads and through ports—have a high weighting to capital appreciation. Greenfield assets with limited construction and demand risk have a mix of yield and capital appreciation. Brownfield assets with mitigated risks (e.g., fully constructed with contracted/regulated revenues) that are located in the most stable OECD countries have a high weighting to current yield.

4. Which of the following is true regarding infrastructure investments?

- A.** Infrastructure investments typically generate volatile cash returns.
- B.** Infrastructure investments typically support services that face inelastic demand and/or benefit from high barriers to entry.
- C.** While public infrastructure returns have low correlation with market returns, private infrastructure returns have high correlation with market returns.

**Solution:**

The correct answer is B. Because infrastructure investments typically support services that face inelastic demand and/or benefit from high barriers to entry, generate steady cash returns, and have a longer life cycle, equity investments in infrastructure offer lower correlation to public market equities and the broader economy. It is notable that public and private infrastructure returns exhibit low correlations.



## PRACTICE PROBLEMS

1. A REIT is considered to be “hybrid” if it invests in both:
  - A. equity REITs and mortgage REITs.
  - B. sole ownership and joint ventures.
  - C. direct and indirect property ownership.
2. A direct private real estate investor can reduce taxable income using:
  - A. double taxation.
  - B. cash depreciation expenses.
  - C. tax-deductible interest expenses.
3. The main appeal of the REIT structure is the elimination of:
  - A. dividend distributions.
  - B. double corporate taxation.
  - C. the requirement to report earnings per share.
4. An advantage of REITs is:
  - A. low non-cash depreciation expenses.
  - B. more transparency than private real estate markets.
  - C. lower correlation with the public equity markets compared to private real estate.
5. Which of the following tends *not* to characterize real estate investments?
  - A. Predictability
  - B. Protection from inflation
  - C. High correlations with other asset classes
6. Akasaka Investment Company established a portfolio of warehouse properties with a total market value of THB3.60 billion. It secured mortgage financing of THB2.61 billion. The terms of the mortgage required Akasaka to maintain a loan-to-value ratio of 0.725.

After 18 months, the portfolio value had dropped to THB2.23 billion and the mortgage liability was THB2.35 billion.

By how much must Akasaka reduce its mortgage liability to return its LTV back to the required level?

  - A. THB6.00 million
  - B. THB8.25 million
  - C. THB9.19 million

7. Unlike appraisers, equity investors tend to place heavy emphasis on:
  - A. recent trends.
  - B. discounted cash flows.
  - C. current market conditions.
8. Infrastructure investments can be categorized based on the nature of their underlying assets as:
  - A. economic or social.
  - B. local, regional, or national.
  - C. usage-based or take-or-pay.
9. Which of these would most likely appeal to an investor who wants long-term, stable returns?
  - A. Greenfield investments
  - B. Brownfield investments
  - C. Secondary-stage investments
10. Which of the following is true regarding infrastructure investments?
  - A. They cannot be made using publicly traded securities.
  - B. Master limited partnerships minimize double taxation for investors.
  - C. Large pension funds are frequent members of direct investment consortiums because of their preference for a short-term horizon.
11. Which of the following types of infrastructure investments entails the lowest expected risk?
  - A. Greenfield
  - B. Brownfield
  - C. Secondary stage
12. Which of the following has the highest weighting to current yield?
  - A. Greenfield assets with limited construction and demand risk
  - B. Fully constructed brownfield assets with contracted revenues
  - C. Greenfield assets without guarantees of demand upon completion
13. Compared with similar fixed-income instruments, infrastructure debt:
  - A. has higher recovery rates.
  - B. experiences higher default rates.
  - C. has more fluctuation over the economic cycle.

## SOLUTIONS

1. The correct answer is A. There are three main forms of REIT: (1) equity REITs that invest in properties outright or through partnerships and joint ventures, (2) mortgage REITs that underwrite loans to real estate (mortgages) or invest in MBS, and (3) hybrid REITs that invest in both of these types.
2. The correct answer is C. Direct private real estate owners can reduce their taxable income using *non*-cash property depreciation expenses and tax-deductible interest expenses.
3. The correct answer is B. The main appeal of the REIT structure is the elimination of double corporate taxation. Corporations pay taxes on their income, from which they make dividend distributions to their owners from after-tax earnings. The shareholders, in turn, are taxed at their personal tax rate. Equity REITs, like other public companies, must report earnings per share based on net income as defined by generally accepted accounting principles or International Financial Reporting Standards.
4. The correct answer is B. An advantage of REITs is more transparency than private real estate markets. As an asset-intensive business, REITs can have high non-cash depreciation expenses. REITs have higher correlation with the public equity markets compared to private real estate, and this is a disadvantage of REITs.
5. The correct answer is C. Real estate has historically low correlations with other asset classes, and adding real estate to an investment portfolio provides diversification benefits at relatively lower levels of risk. Many commercial real estate companies offer multiple-year leases with fixed rents; hence, the income earned from these leases is typically both predictable and stable. Moreover, real estate tends to provide inflation protection because the lease payments are regularly adjusted.
6. The correct answer is B.  

$$\text{LTV} = \text{Mortgage liability} / \text{Portfolio value}.$$

$$\text{Mortgage liability} = \text{LTV} \times \text{Portfolio value}.$$

$$\text{Required mortgage liability} = \text{Required LTV} \times \text{Portfolio value}.$$

$$\text{Required reduction in mortgage liability} = \text{Mortgage liability} - \text{Required mortgage liability}.$$

$$\text{Required reduction in mortgage liability} = \text{Mortgage liability} - (\text{Required LTV} \times \text{Portfolio value}).$$

$$8.25 = 235 - (0.725 \times 323).$$
7. The correct answer is B. Equity investors in public real estate discount future cash flows, while appraisers of private real estate place heavy emphasis on current market conditions and recent trends.
8. The correct answer is A. To categorize infrastructure investments, investors frequently rely on the underlying assets, with the broadest categorization distinguishing between economic and social infrastructure assets.
9. The correct answer is B. With brownfield investments, typically some of the financial and operating history is available. Therefore, brownfield investments may be sought after by both strategic investors specializing in operating the assets

and—particularly with privatizations—financial investors focused on long-term, stable returns.

10. The correct answer is B. Master limited partnerships trade on exchanges, are pass-through entities like REITs, and may also share with REITs taxation rules that minimize double taxation for investors.  
Investors concerned about liquidity and diversification may choose publicly traded infrastructure securities.  
Because of the concentration and liquidity risks and the typical long-term horizon, direct infrastructure investment usually takes place with a group or consortium of strategic investors that share the financial risk and/or assume a specific role in building, operating, or managing the assets. Such strategic partners, such as large pension funds or sovereign wealth funds, are frequent direct investors because they are better placed to manage certain risks to limit individual concentration risk.
11. The correct answer is C. In the three groupings of the infrastructure development cycle—greenfield, operational secondary stage, and brownfield—the operational secondary-stage assets with an existing track record of generating steady, bond-like cash flows possess the lowest risk and offer the lowest return to the investors. Brownfield investments, redevelopment of existing infrastructure, are incrementally riskier, and greenfield projects are the riskiest.
12. The correct answer is B. Brownfield assets with mitigated risks (e.g., fully constructed with contracted/regulated revenues) that are located in the most stable OECD countries have a high weighting to current yield. Greenfield assets with limited construction and demand risk have a mix of yield and capital appreciation. Greenfield projects without guarantees of demand upon completion—e.g., variable electricity prices, uncertain traffic on roads and through ports—have a high weighting to capital appreciation.
13. The correct answer is A. Given the stable underlying nature of cash flows, infrastructure debt tends to experience lower default rates and higher recovery than similar fixed-income instruments, and it is less subject to fluctuation over the economic cycle.

## LEARNING MODULE

## 5

## Natural Resources

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain features of raw land, timberland, and farmland and their investment characteristics
<input type="checkbox"/>	describe features of commodities and their investment characteristics
<input type="checkbox"/>	analyze sources of risk, return, and diversification among natural resource investments

## INTRODUCTION

1

Natural resources comprise commodities and raw land used for agricultural purposes, specifically farming and timber. Managing this asset class requires specialized knowledge of the features of natural resources. An increasing number of portfolios include natural resources, which justifies a separate examination of the sector. There has been rapid development in offering indirect investment in natural resources through exchange-traded funds (ETFs), limited partnerships, REITs, swaps, and futures.

Commodities, such as crude oil, soybeans, copper, and gold, are seen as investments. Investments in commodities can be either “hard” (those mined, such as copper, or extracted, such as oil) or “soft” (those grown over a period of time, such as livestock, grains, and cash crops, such as coffee).

Timberland investment involves ownership of raw land and the harvesting of its trees for lumber, thus generating an income stream and the potential for capital gain, and timberland has been included in large institutional portfolios for decades. Farmland as an investment is a more recent phenomenon, with only a few dedicated funds involved. With population growth, weather, and water management becoming more topical, however, investors may turn to these assets to actively address sustainability.

## LEARNING MODULE OVERVIEW



- Investments in farmland and timberland are similar in certain respects to real estate investments but also exhibit several important differences.

- While raw land's investment returns occur strictly from price changes of the asset, both farmland and timberland generate returns from the assets' income stream in addition to price changes of the assets.
- Timberland's income stream differs from that of farmland in that the harvest time of timber can be chosen while crops from farmland are harvested on a regular cycle.
- Commodity investments are typically entered into via derivative markets, although some investors may find value in investment in physical commodities directly or through specialized funds.
- The prices available on commodities through derivative markets must be related to the prices on the same commodities in physical markets to prevent arbitrage opportunities.
- The forward price of a commodity will be greater than the spot price on the same commodity only if the carrying costs of owning the physical commodity are greater than the non-cash benefits of owning the physical commodity.
- Investing in commodities is motivated by its potential for high expected return, its potential for diversifying a portfolio of traditional assets, and inflation protection. Commodities exhibit high correlation with inflation over the last 30 years, suggesting that commodities are an effective inflation hedge.
- Farmland and timberland investments trade infrequently and in non-public markets. As a result, they are likely to appear as less volatile than commodities and other publicly traded risky assets (such as stocks), despite the fact that both asset classes face significant risks, such as weather-related threats.
- Farmland and timberland provide diversification potential to portfolios consisting primarily of traditional assets (i.e., stocks and bonds). Historical correlations between these asset classes and traditional assets have been close to zero.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which of the following asset characteristics is shared by both farmland and real estate investments?
  - A. Both are liquid investments.
  - B. Both are illiquid investments.
  - C. Physical improvements are a primary focus of the investment value for both.

#### **Solution:**

B is correct. Farmland and real estate share a feature of illiquidity: It is costly to find a buyer when sale of the investment is desired. A is incorrect given that both are illiquid investments. C is incorrect because physical improvements are a focus of value only for real estate investments, not for farmland investments.

2. Which of the following natural resource investments is least likely to use the real estate investment trust (REIT) ownership structure?

- A. Farmland
- B. Raw land
- C. Timberland

**Solution:**

B is correct. Raw land is typically acquired through direct ownership or a partnership structure. Also, raw land has no inherent income stream and returns accrue purely from price appreciation, making the income pass-through REIT structure less relevant. Both A and C are incorrect because both farmland and timberland investments are included in REIT structures, as well as other ownership forms.

3. Which type of investor is likely to prefer investing in commodities using exchange-traded products?

- A. Those seeking simplified trading through a brokerage account
- B. Those seeking to gain access to dynamic commodity trading strategies
- C. Those seeking expertise in a specific commodity sector

**Solution:**

A is correct. Exchange-traded products allow investors to gain commodity exposure through a simple exchange-traded instrument that can be accessed via a brokerage account. B is incorrect because it describes investors who choose to use commodity trading advisors. C is incorrect because this type of investor will choose a specialized commodity fund for its expertise.

4. Which of the following describes a non-cash benefit of holding a physical commodity rather than a derivative contract on the same commodity?

- A. Interest
- B. Convenience yield
- C. Storage

**Solution:**

B is correct. In market environments in which physical inventories of a commodity become low, investors in that commodity will prefer to hold the physical asset rather than a derivative contract with the asset as an underlying. The premium on the spot price resulting from this preference is called the convenience yield. A and C are both incorrect because interest and storage reflect costs associated with owning the physical commodity.

5. Which of the following statements most correctly reflects commodity supply and demand fundamentals?

- A. Supply of commodities adjusts equally to demand for commodities.
- B. Supply of commodities adjusts more rapidly than does demand for commodities.
- C. Supply of commodities adjusts more slowly than does demand for commodities.

**Solution:**

C is correct. Commodity supply adjusts slowly to demand because of long production times; for example, agricultural crops require a growing cycle.

6. Which of the following measures is best used to assess the potential for portfolio diversification when adding farmland or timberland to a portfolio of traditional assets?

- A. Returns of other asset classes
- B. Volatility of other asset classes
- C. Correlation between other asset classes

**Solution:**

C is correct. Correlation between asset classes best reflects the potential for portfolio diversification. An asset class that exhibits lower (i.e., closer to zero) correlation with traditional asset classes (such as stocks and bonds) has better diversification potential compared to an asset class exhibiting higher (i.e., closer to one) correlation. A and B are incorrect because returns and volatility strictly reflect reward and risk for the asset class without consideration as to how the asset class performs compared to other asset classes.

## 2

### NATURAL RESOURCES INVESTMENT FEATURES



explain features of raw land, timberland, and farmland and their investment characteristics

Natural resources comprise different production inputs that are basic to the economy and everyday life: plants and animals (i.e., soft commodities); energy and minerals (hard commodities); and metals and industrial goods used to manufacture goods and produce services. A notable proportion of natural resource investments are directly through farmland, raw land with exploration and mining rights, and timberland. Direct ownership spans a broad spectrum: from farmers producing grain to institutional investors building solar farms.

Many large institutional investors create exposure to natural resources outright by purchasing land with rights to farm agricultural commodities; to extract oil and gas; to build facilities for alternative energy generation, such as solar and wind farms; or to mine commodities, such as iron, coal, and other industrial metals. By investing in these assets, institutional investors often seek to fulfil their environmental, social, and governance (ESG) objectives, such as sustainability, water conservation, and other environmental goals.

#### Land Investments vs. Real Estate

Farmland, timberland, and raw land are similar to real estate investments in that they are unique, illiquid assets with distinct geographic location and features, where the latter two characteristics have an influence on the value of the resource itself. They involve forms of ownership capital (claims to residual cash flows). In the case of developed real estate and farmland, there may also be steady cash flow streams (leases).

Less developed land includes farmland, timberland, and raw land, as well as associated mineral or drilling rights. Sources of return include expected price appreciation over time and cash flows, such as farm lease payments (for an owner), farm operating income (owner-operator), farm timberland income, and mineral and drilling royalties.



Estimates suggest nearly half of private investable timberland globally is in the United States. The next biggest timberland regions are Central and Eastern Europe, Latin America, Australia, and New Zealand.

There are several differences between real estate investments and raw land, farmland, and timberland investments. The first difference is that unlike real estate, there is limited or no focus on the physical improvements to the land. It is not the value of buildings, construction, and development that matters but, rather, the quality of the soil, climate features (farmland, timberland), or geology (mineral rights). In contrast, it is the actual, potential, and planned improvements that determine the value of the property in real estate investments. The location of land is also important; the closer it is to transportation and markets, the higher the price. While the proximity to transportation is also a factor for real estate, transportation expenses can be a significant component of the price of the products paid by the end-user of timberland and farmland.

To make investments in raw land, timberland, or farmland, investors need specialized knowledge and understanding of the specifics of the natural resource. Investors investing directly in timberland need forest investment expertise to manage a forest over its life cycle. Many large institutional investors that do not have this expertise rely on **timberland investment management organizations** (TIMOs), entities that support institutional investors by managing their investments in timberland by analyzing and acquiring suitable timberland holdings.

Both commercial and residential real estate offer a wide variety of financing alternatives; however, there are fewer alternatives for farmland, timberland, and raw land. Often these investments are financed through bank loans or direct, private debt investment. Finally, these are illiquid assets that have a limited number of potential buyers and sellers due to the specialized knowledge and capital needed for these transactions. These features are outlined in Exhibit 1.

**Exhibit 1: Raw Land, Farmland and Timberland**

	Raw land	Farmland	Timberland
Return drivers	Price of land	Harvest quantities Commodity prices Price of land	Biological growth Harvest quantities Lumber prices Price of land
Source of direct revenue	Price appreciation Lease revenue	Sale of crops and other agricultural products Price appreciation Lease revenue	Sale of trees, wood, and other timber products Price appreciation Lease revenue
Value	Physical location	Physical location Growth cycle Soil quality	Physical location Quality of timber Phase in timber production
Main risks	Best alternative use	Weather factors and climate change Biological factors, diseases	
Owners	Mostly institutional, some individual	Mostly individuals, some institutional	Mostly institutional, some individual
Ownership structure	Direct ownership, partnership	Direct ownership, partnership, REIT	Direct ownership, partnership, REIT, TIMO

## Features and Forms of Farmland and Timberland Investment

Sustained interest in farmland and timberland investments stems from their common nature (everyone eats and requires shelter), the recurring income from crops, inflation protection from holding land, and their degree of insulation from financial market volatility. US farmland, for example, enjoyed positive returns both during periods when US GDP declined significantly (1973–1975 and 2007–2009) and when the United States experienced higher-than-normal inflation (1915–1920, 1940–1951, and 1967–1981).

Timberland has been part of institutional and ultra-high-net-worth portfolios for decades, typically trading in large units of land (several thousands of acres or hectares). One of the main challenges of these investments is their long market cycle, particularly in new-growth forest and crops that are picked, such as fruit. In contrast, farmland can be found in much smaller sizes—perhaps tens of or a few hundred acres or hectares. Many farms are still family owned, as is 98% of US farmland. Globally, farmland remains a main source of family wealth.

Investments in farmland and timberland—owned directly, owned indirectly, or leased—generate returns from selling crops and timber. Farmland consists mainly of row crops that are planted and harvested (more than one round of planting and harvesting can occur in a year depending on the crop and the climate) and permanent crops that grow on trees or vines. Farmland may also be used as pastureland for livestock. Farm products must be harvested when ripe, with little flexibility in production. By contrast, timberland serves as both a factory and a warehouse. Timber (trees) can be grown (i.e., timberland's factory characteristic) and easily stored by simply not harvesting the trees (i.e., timberland's warehouse characteristic). This characteristic offers the flexibility of harvesting when timber prices are up and delaying harvests when prices are down.

As part of the returns generated by selling the output from the land, both farmland and timberland generate returns from price changes in their output. The market prices for agricultural products and timber may fluctuate considerably over time, and these prices combined with harvest quantities dictate the revenue generated by the sale of the land's output. Finally, the value of the land may change over time for both farmland and timberland, and these land price changes also contribute to the return on farmland and timberland investments. The return drivers are summarized in the first row of Exhibit 1.

For centuries, direct farmland and timberland ownership has been the initial dominant form, with a focus on long-term tax-exempt investors, such as pension funds, foundations, and endowments. That is why the primary investment vehicles of smaller investors for timber and farmland are investment funds, whether offered on the public markets, such as real estate investment trusts in the United States, or administered privately through limited partnerships. Direct timberland investors use TIMOs to select, manage, and sell assets in accordance with investor objectives. TIMOs are often used in conjunction with indirect investment alternatives, such as limited partnerships, limited liability corporations (LLC), and private REITs.

Larger investors can consider direct investments for assets with appeal. For example, Middle Eastern sovereign wealth funds have made investments in farmland in Africa and Southeast Asia. Increasingly, farmland funds and limited partnerships and publicly traded farmland REITs have been launched.

These indirect investment vehicles usually involve separately managed accounts that distinguish between owner and owner-operator models; in the former case, owners rent land used for row crops (i.e., grains, etc.), while in the latter, they retain some operating control in the case of permanent crop properties (orchards, vineyards, etc.). Cash flows are typically fixed in the former case and variable in the latter, with investors taking on some operating risk.

Owning physical farmland opens the door to a wider variety of foodstuffs: spices, nuts, fruits, and vegetables—a much broader array than the corn, soy, and wheat offered by futures investment. However, there is limited price transparency or information to guide investment decisions without the assistance of sector specialists. The illiquidity of direct farmland and timberland investments is also limiting.

In terms of risk, farmland is highly sensitive to unexpected weather changes and climate developments that can easily destroy crops and eradicate revenue. The impact of weather spans the entire growing season, making agricultural volumes and prices difficult to predict. That is why agricultural commodity futures contracts can be combined with farmland holdings to generate an overall hedged return. A farm has an inherent long position in its crop and, therefore, will sell futures for delivery at the time of the harvest. The following discussion provides a more detailed case study of climate risks associated with timberland investing.

There is an indirect benefit from farmland and timberland investments: These natural resources consume carbon as part of the plant life cycle and their value comes not just from the harvest but also from the carbon offset to human activity. Water rights are also part of the direct and implied value of these properties; conservation easements may create value by supporting traditions and nature conservation. Demand for arable land may rise as interest in investments that adhere to ESG considerations grows.

## CASE STUDIES

### Investing Responsibly in Timberland Assets: A Climate-Conscious Case Study



Campbell Global (CG) is a global investment manager focused on forest and natural resources investments. Based in Portland, Oregon, with offices in 14 US states and New Zealand, the firm has nearly four decades of experience in sustainable value creation. CG is committed to managing its forests in a manner that promotes the best long-term interests of its clients, while also striving to address economic and ESG considerations. In addition to their economic value, forests serve as vast carbon sinks, with trees removing atmospheric CO<sub>2</sub> and providing carbon storage. In one year, a single Douglas fir tree, a common commercial timber species in the US Pacific Northwest, stores the CO<sub>2</sub> equivalent of driving 400 miles in a standard automobile. Globally, the Earth's forests are estimated to absorb as much as 30% of human-induced CO<sub>2</sub> emissions.

Sustainably harvested wood products and materials also store atmospheric CO<sub>2</sub> long after removal from a forest, with one cubic meter of wood capturing nearly a metric ton of CO<sub>2</sub>. In addition to carbon sequestration, forests provide benefits of clean water and wildlife habitat, recreational opportunities, and a source of living-wage jobs in rural communities. These attributes agree with the UN's Sustainable Development Goals and contribute to advancing the UN's mission for a sustainable future globally. With these effects, there is increasing awareness that well-managed forests are a critical component of any global climate change strategy.

CG uses scenario analyses to identify climate-related risks beginning at a broad country-level scale, narrowing down to a specific property, and then testing the impact of various risks to a site's present and future suitability. Factors analyzed to gauge climate risks include precipitation patterns, temperature fluctuations, severity of weather events, presence of pests or disease, and annual average growth rates for commercial tree species. While many climate-related risks in forestry are mitigated through active management, during this iterative process, CG analyzes the potential positive and negative impacts associated with

these risks to assess potential changes in net asset value. The following table illustrates climate risks evaluated, their impact on the forest, and the ability to mitigate the risks.

Climate Risk	Implication	CG Mitigants
Change in temperature	Increased fire danger	Property-specific fire plans; re-evaluate target regions/countries for investment
Change in precipitation patterns	Changes in tree species range; increased drought and related fire risk	Vegetation suitability modeling and genetic tree improvement; re-evaluate target regions/countries for investment
Frequency of extreme weather events	Loss of standing timber from wind events	Re-evaluate target regions; property-specific response plans; geographically diverse portfolio construction
Presence of pests or disease	Early onset and increased frequency of individual tree mortality	Immediate treatment, which may include removal of affected trees to prevent further spread of pests or disease in the forest
Change in growth	Increased or decreased growth rates	Effects will vary by region, may influence planting stock decisions; re-evaluate forest growth model assumptions

Climate change opportunities and challenges highlighted in the CG investment process include the following:

- Identifying afforestation (establishment of new forest) opportunities that mitigate climate change by sequestering CO<sub>2</sub> emissions from the atmosphere in trees and soil, while offering many important co-benefits for communities, biodiversity, and soil and water quality.
- Protecting existing carbon stocks by minimizing impacts to carbon stored on the forest floor through tailored forest management practices.
- Enhancing forest carbon sequestration by replanting areas as soon as possible so the new forest will quickly begin removing CO<sub>2</sub> from the atmosphere.

The ability to quantify, evaluate, and report the year-over-year changes in the carbon footprint of a forest can influence the impacts an organization has on the environment, leading to increased transparency and more-informed business decisions. Incorporating climate change factors in its investment process not only mitigates climate-related risks; it also promotes and enhances the natural solutions forests provide. Understanding and measuring the comprehensive carbon stores of forests may lead to business decisions improving carbon sequestration, critical for addressing climate change.

## QUESTION SET



1. Identify the three primary return drivers of investing in timberland.

### Solution:

The three primary return drivers of investing in timberland are (1) the biological growth of the timber to be harvested in the future, (2) the price of lumber, and (3) changes in the price of the land.

2. Which of the following statements provides the most accurate description of timberland investment management organizations?
- A. TIMOs are entities that use their forest investment expertise to analyze and acquire suitable timberland holdings on behalf of institutional investors.
  - B. TIMOs are investment funds that raise money from individual investors to buy timberland.
  - C. TIMOs are entities that only facilitate direct ownership of timberland by institutional investors.

**Solution:**

A is the correct response. Timberland requires asset-specific expertise, and TIMOs use their expertise to analyze and acquire timberland holdings either directly or indirectly for institutional investors. B is incorrect because TIMOs are not investment funds. C is incorrect because TIMOs can be used by institutional investors in conjunction with indirect investing in timberland.

3. Describe one important similarity and one important difference between investing in timberland versus investing in real estate.

**Solution:**

Similarities between timberland and real estate include the fact that both asset classes involve investing in unique assets with distinct geography and the fact that both asset classes have a high degree of illiquidity.

An important difference between the two asset classes is the degree to which value reflects physical improvements to the land. Specifically, real estate investing values actual and potential improvements while timberland investing does not.

4. Describe a significant difference in the income component of farmland investing versus timberland investing.

**Solution:**

Timberland provides flexibility in the timing of harvesting trees. Unlike timberland, farm products must be harvested when ripe, with little flexibility in production.

## COMMODITY INVESTMENT FORMS

# 3



describe features of commodities and their investment characteristics

## Commodity Investment Features

Commodities themselves do not generate cash flows but usually incur costs (*cost of carry* introduced in derivatives learning modules), such as those for transportation, storage, and insurance for physical commodities. Investors seek to benefit from commodity price appreciation (in excess of carry cost) based on their future economic value rather than actual use of the underlying asset.

Moreover, governments have realized the importance of controlling natural resources and taken an increasingly important role in natural resource markets. For instance, many governments provide food price subsidies to customers and price support to farmers.

Governments often control extractable natural rights, such as energy resources and mining. In many emerging markets, governments or government-owned enterprises control strategic energy production or mining resources. For instance, SOCAR, the State Oil Company of Azerbaijan Republic, is the fully state-owned national oil and gas company of the country and extracts oil and natural gas from onshore and offshore fields of the Caspian Sea. Moreover, it operates Azerbaijan's only oil refinery and operates several oil and gas export pipelines. It is a major source of income for the country. In other countries, owners of land may only be able to cultivate the soil and may extract only certain minerals. Often the government owns and manages subsurface rights and has the right to extract certain resources, such as oil, gas, coal, gold, and silver.

Environmental factors play a direct role in natural resource investments, because governments are increasingly implementing environmental safeguards to meet climate objectives and control activities with climate impact, such as mining, agriculture, and energy extraction and production. More specifically, global climate change policies seek to reduce reliance on fossil fuels (coal, oil) and increase renewable energy use. To do so, countries are in the process of adopting national programs that intend to increase renewable energy (wind, solar, biomass) and reduce the reliance on fossil fuels.

The policy objective of reducing the reliance on fossil fuels has shifted focus to electric vehicles and advances in battery technology. A potential impact of the reliance on low-carbon energy technologies is the higher demand for many minerals and metals, such as lithium, cobalt, and nickel. As mining activities for these critical metals will increase, there will be significant impacts on local water systems, ecosystems, and communities.

Finally, there is growing interest from ESG investors seeking to promote sustainable farming practices or use timberland investments for carbon offsets.

## Distinguishing Characteristics of Commodity Investments

Commodity sectors include precious and base (i.e., industrial) metals, energy products, and agricultural products. Exhibit 2 offers examples of each type. The relative importance, amount, and price of individual commodities evolve with society's preferences and needs. Increasing industrialization of emerging markets has driven strong global demand for commodities. These markets need increasing amounts of oil, steel, and other materials to support manufacturing, infrastructure development, and the consumption demands of their populations. Emerging technologies, such as advanced cell phones and electric vehicles, create demand for new materials and destroy demand for old resources as markets for specific commodities evolve over time.

**Exhibit 2: Examples of Commodities**

<b>Sector</b>	<b>Sample Commodities</b>
Energy	Oil, natural gas, electricity, coal
Base metals	Copper, aluminum, zinc, lead, tin, nickel
Precious metals	Gold, silver, platinum
Agriculture	Grains, livestock, coffee
Other	Carbon credits, freight, forest products

Commodities may be further classified by physical location and grade or quality. For example, there are many grades and delivery locations for crude oil and wheat. Commodity derivative contracts thus specify quantity, quality, maturity date, and delivery location.

The majority of commodity investing is implemented through derivatives. Physical commodities often generate unwelcome tax obligations and costs arising from storage, insurance, brokerage, and transportation. Additionally, physical commodity markets lack price transparency. As such, commodity investments are usually made through financial derivative instruments, most frequently commodity futures and forwards and occasionally options on futures. Using derivatives to establish exposures to natural resources has several benefits: Because these instruments are traded on organized exchanges, they are very liquid and provide opportunities for price discovery.

Futures contracts are obligations to buy or sell a specific amount of a given commodity at a fixed price, location, and date in the future. Futures contracts are exchange traded, are marked to market daily, and may or may not be settled on delivery or receipt of the physical commodity at the end of the contract. This delivery obligation becomes dramatically important during stressful periods. For example, with oil during the global financial crisis in 2008 and during the COVID-19 pandemic in 2020, as demand collapsed, oil producers could not find buyers and global storage filled suddenly. Even commodity-related ETFs were affected, forcing some to close and impose large losses on investors.

For futures contracts, counterparty risk is managed through the settlement process between the clearinghouse/exchange and clearing brokers. Commodity exposure can be achieved through means other than direct investment in commodities or commodity derivatives, including the following:

- *Exchange-traded products.* ETPs, either funds (ETFs) or notes (ETNs), may be suitable for investors restricted to equity shares or seeking simplified trading through a standard brokerage account. ETPs may invest in commodities or commodity futures. For example, the SPDR Gold Shares ETF seeks to track the price of physical gold by holding bullion in vaults. It owned just under USD53 billion in gold bullion as of December 2022. ETPs may use leverage and may replicate the pay-offs from a long or short position, with the latter form considered inverse or “bearish.” Similar to mutual funds or unit trusts, ETPs charge fees included in their expense ratios.
- *Investing with commodity trading advisers.* CTAs are another way to gain commodity exposure. CTAs are managed futures funds that make directional investments primarily in futures markets based on technical and fundamental strategies. A commodity-focused CTA might concentrate on a specific commodity (such as grains) or be broadly diversified across commodities. However, one would need to find a fund focused solely on the desired commodity, because modern CTAs often invest in a variety of futures, including commodities, equities, fixed income, and foreign exchange. Individual investors may establish accounts that are managed in

accordance with their specific investment preferences and risk tolerance called **separately managed accounts** (SMAs). These types of individual accounts are common for commodity investments. More details on CTAs and managed futures are covered in the learning module on hedge funds.

- *Specialized funds investing in specific commodity sectors.* An example of specialized funds is private energy partnerships, which are similar in structure to private equity funds and enable institutional exposure to the energy sector. Management fees can range from 1% to 3% of committed capital, with a typical life span of 10 years and extensions of 1- and 2-year periods. Publicly available energy mutual funds and unit trusts typically focus on the oil and gas sector, often acting as fixed-income investments to pay dividends from rents or capital gains. They may focus on upstream (drilling), midstream (refineries), or downstream (chemicals). Their management fees are comparable with those of other public equity managers and range from 0.4% to 1%.

Commodity investments are typically direct underwriting of the acquisition, management, and extraction of the natural resource itself. These investments usually involve direct or indirect claims to *physical* assets themselves and not claims on residual (equity) or fixed (debt) cash flows.

This indirect approach, however, comes with the added risks from financing and operations and, when privately held, the impact of additional illiquidity.

## Basics of Commodity Pricing

Investors seeking to benefit from direct commodity price exposure typically use derivative instruments, such as exchange-traded futures and options and forwards, with individual commodities or an index as the underlying asset. Different commodity indexes are composed of different commodities and have materially different index weights, which determines varying exposures to not only specific commodities but also commodity sectors. However, as we will see later in this module, the correlation between commodities and traditional asset classes is typically low, which means improved portfolio diversification is possible regardless of the index chosen.

Since commodities trade in both physical and financial markets, there is a direct relationship between their prices in both the physical cash markets and the financial derivative markets. Such no-arbitrage conditions, which we have encountered earlier in the derivatives readings, dictate that the difference in prices between the cash or spot markets and derivative markets is equal to the *cost of carry*, which is the opportunity cost of holding these assets, and mirrors the risk-free rate and the cost of storing, transporting, and insuring the commodity. The holder of the physical commodity should expect to be compensated via a higher forward price,  $F_0^+(T)$ , than the prevailing cash price.

However, there may be non-cash benefits from holding the physical commodity instead of gaining exposure using a derivative. Such *convenience yield* may arise under conditions when the market participants prefer to hold the physical commodity; for instance, the owners want to ensure that they have continuous access to this commodity. Usually, the convenience yield is related inversely to inventory levels of the underlying commodity. Because convenience yield is a benefit and *accrues* to the owner, it *reduces* the forward price.

The pricing relationship between cash,  $S_0$ , and derivative markets can be expressed under continuous compounding as

$$F_0(T) = S_0 e^{(r+c-i)T}, \quad (1)$$



where  $c$  is the cost of carry,  $i$  is the convenience yield,  $r$  is the risk-free rate, and  $T$  is the time to the expiration of the forward contract. Effectively, the relationship between commodity forward and spot prices over time depends on the relative relationship between the cost of carry and the convenience yield.

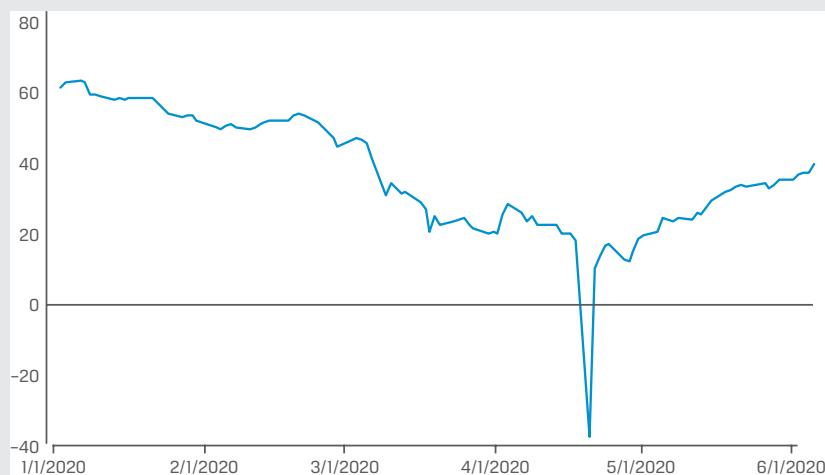
Looking at the relationship between the convenience yield and the cost of carry, there are two relationships. When the spot price is above the forward prices, there is **backwardation**, a downward-sloping, or inverted, forward curve. This can occur for physically settled contracts when the convenience yield is positive and the benefit of holding the commodity outright exceeds the cost of carry. When the spot price is below the forward prices, there is **contango** because the cost of ownership exceeds the benefit of a convenience yield and the forward price will be above the underlying spot asset price. As a rule of thumb, a contango scenario generally lowers the return of the long-only investor, and a backwardation scenario enhances it. The following example shows how market changes can lead to significant shifts in the shape of the commodity forward curve:

### EXAMPLE 1

#### Crude Oil Going from Backwardation to Contango

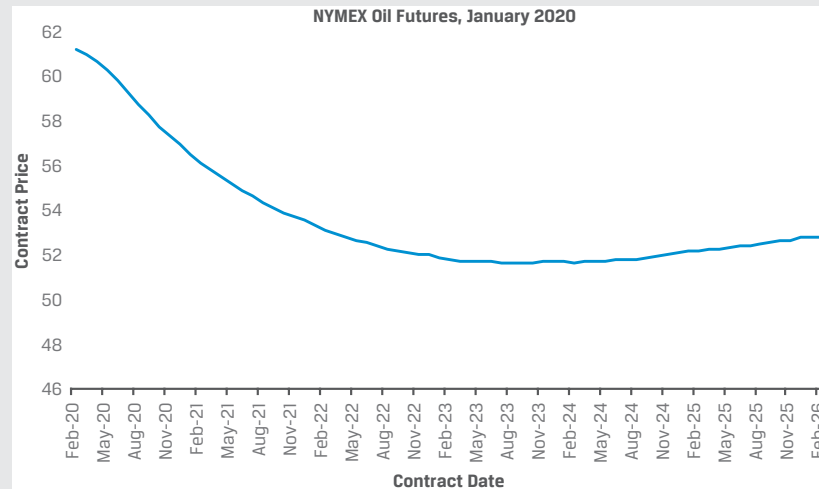
In April 2020, the price of crude oil futures on the New York Mercantile Exchange (NYMEX) fell below zero for the first time ever; sellers paid buyers to take on an exposure to oil. This situation is highly unusual and was caused by the lockdowns in the wake of the COVID-19 pandemic that eroded the demand for oil. Producers could not cut crude oil production quickly enough, and storage facilities were overflowing with oil. This caused oil inventories to skyrocket. Exhibit 3 shows the NYMEX oil futures price from January to June 2020.

**Exhibit 3: NYMEX Oil Futures Price (US dollars per contract), January–June 2020**

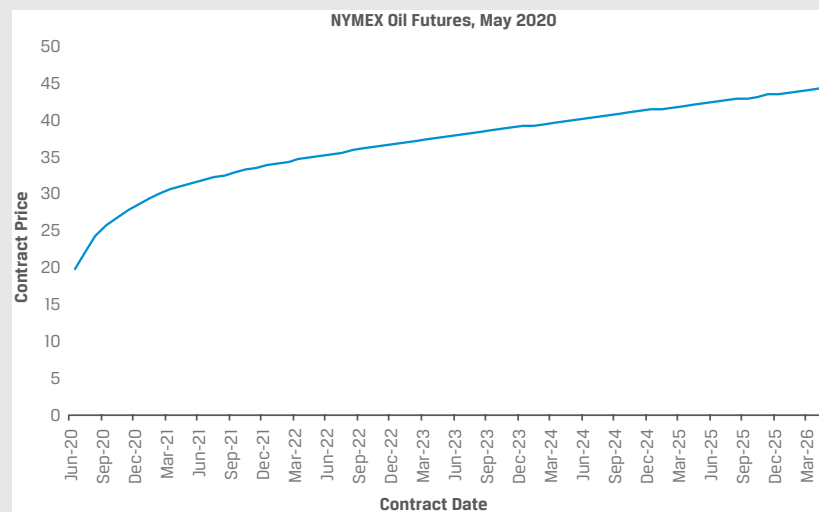


Source: Bloomberg.

The sudden disappearance of the demand for oil impacted the shape of the oil futures curve, the series of oil futures contract prices with different forward settlement dates. Prior to the outbreak of the COVID-19 pandemic in January 2020, the US oil markets had the lowest inventory levels in over a year and expected a continued healthy demand from a vibrant economy. Because of low inventory levels, the oil futures curve was in backwardation at the beginning of January 2020 because the benefit of holding oil outright exceeded the cost of carry of having oil in inventory (see Exhibit 4).

**Exhibit 4: NYMEX Oil Futures Curve, January 2020**

As lockdowns continued into mid-2020, US oil market inventory approached near all-time high levels but demand remained muted. Carry costs then outweighed any benefit of storing oil for production purposes (and there was hardly any open storage capacity available), and the oil futures curve was in *contango* at the beginning of May 2020 (see Exhibit 5).

**Exhibit 5: NYMEX Oil Futures Curve, May 2020****QUESTION SET**

1. Compare farmland investment to commodity investment with respect to sources of value.

**Solution:**

A similarity between the two is that both forms of investment rely on changes in the price of underlying. A difference between the two is that commodity investments, which typically rely on derivative contracts, do not generate cash flows while farmland investment generates cash flows through the income generated from harvesting the agricultural crops.

2. Which of the following statements is most correct about investors seeking commodity exposure through a commodity trading adviser (CTA)?

- A. The investor is seeking to make direct investment in commodities.
- B. The investor is seeking to benefit from an income stream from commodities.
- C. The investor is seeking to profit from specific directional trends in commodity futures contracts.

**Solution:**

C is correct. Commodity trading advisers devise trading strategies using derivative contracts on commodities that are focused on predicting upcoming bull or bear trends. A is incorrect because CTAs do not advise on physical commodity transactions. B is incorrect for a similar reason, because the income stream from most commodities would require direct or indirect land ownership, which produces commodities.

3. Discuss how the relationship between costs of carry and benefits of owning a commodity outright affect the relationship between the forward price and spot price of the commodity.

**Solution:**

A simple framework for expressing the relationship between forward price and spot price in terms of costs and benefits of owning the underlying commodity is as follows:

$$\text{Forward price} = \text{Spot price} + \text{Costs of carry} - \text{Benefits of ownership.}$$

Thus, if costs of carry exceed benefits of ownership, then forward price is greater than spot price for the commodity. In contrast, forward price is less than spot price if benefits exceed costs.

4. Explain why low inventories of a commodity may result in backwardation for the commodity.

**Solution:**

Backwardation reflects a downward-sloping term structure of prices for a commodity. In the near term, backwardation implies forward price below spot price. Low inventories of a commodity cause investors to prefer to hold the physical commodity over derivative contracts (i.e., forwards). This preference will cause price for the physical asset to be bid higher, and if this non-cash benefit of owning the physical commodity (also known as convenience yield) exceeds costs of ownership of the commodity (such as interest and storage), then the spot price of the commodity exceeds its forward price.

## NATURAL RESOURCE INVESTMENT RISK, RETURN, AND DIVERSIFICATION

# 4



analyze sources of risk, return, and diversification among natural resource investments

Commodity prices have risk and return drivers that are often related to but do not always directly coincide with the timing of the economic cycle affecting the prices of common equity and debt securities. Commodities, farmland, and timberland have different return drivers and cycles. Commodities are priced on a second-by-second basis on public exchanges, whereas land generally has an infrequent pricing mechanism and may include imprecise estimates, as opposed to actual transactions. Keeping these market structure differences in mind helps investors consider their relative benefits and challenges.

## Commodities

Physical commodity supply dynamics are determined by production (in the case of hard commodities), seasonal crop yields (for soft commodities), and inventory levels in the short term, while commodity end user/consumer use of these basic inputs drives ultimate demand. Supplies of physical commodities are determined by production and inventory levels and secondarily by the actions of non-hedging investors. Demand for commodities is determined by the needs of end users and secondarily by the actions of non-hedging investors. Investor actions can both dampen and stimulate commodity price movements, at least in the short term. Gold, a physical commodity and a precious metal, is often a preferred safe haven given its historical use as a store of value among investors and as a non-currency based reserve among central banks.

Producers cannot alter commodity supply levels quickly because extended lead times are often needed to affect production levels. For example, agricultural output may be altered by planting more crops and changing farming techniques, but at least one growing cycle must pass before there are results. And at least one factor beyond the producer's control—the weather—will significantly affect output. Building the necessary infrastructure for increased oil and mining production may take many years, involving both developing the mine itself and the necessary transportation and smelting components. For commodities, suppliers' inability to quickly respond to changes in demand may result in supply too low in times of economic growth and too high when the economy slows. And despite advancing technology, the cost of new supply may grow over time.

Investing in commodities is motivated by its potential for returns, portfolio diversification, and inflation protection. Investors may choose commodities if they believe prices will increase in the short or intermediate term. Commodity futures contracts may offer investors a liquidity premium or other trading opportunities, creating the prospect for a positive real return. In the 30-year period referenced in Exhibit 6, commodity investments outperformed global stocks and global bonds but with much higher volatility.

**Exhibit 6: Historical Returns of Commodities, Q3 1992–Q2 2022 (quarterly data)**

	Global Stocks	Global Bonds	Commodities
Annualized return			
1992:Q3–2022:Q2	6.89%	4.39%	7.81%
Annualized standard deviation			
1992:Q3–2022:Q2	16.76%	6.14%	24.39%
Worst calendar year	–43.54%	–5.17%	–42.80%
	(2008)	(1999)	(2008)

	Global Stocks	Global Bonds	Commodities
Best calendar year	31.62%	19.66%	50.30%
	(2003)	(1995)	(2009)

Sources: Global stocks, MSCI ACWI; global bonds, Bloomberg Barclays Global Aggregate Index; commodities, S&P GSCI Total Return.

Exhibit 6 shows a summary of investment performance and volatility of global stocks, global bonds, and commodities over a 30-year time horizon from Q3 1992 through Q2 2022. Commodities exhibit the highest average return and the highest volatility among the three asset classes. The worst performance for both global stocks and commodities occurred during 2008, coinciding with the middle of the global financial crisis of 2007–2009. As global economies began recovering in 2009, commodities exhibited their best calendar-year performance. While the data may imply that commodities behave similarly to global stocks, we directly address the correlation between these two asset classes later in this lesson. Overall demand levels are influenced by global manufacturing dynamics and economic growth. When demand levels and investors' orders to buy and sell during a given period change quickly, the resulting mismatch of supply and demand may lead to price volatility.

## Farmland and Timberland

Farmland and timberland, in contrast, are far less frequently traded and derive their value from different sources. In the case of farmland, it is multiple growing seasons over time that generate the return. In the case of timberland, it is the longer forest/tree growth cycle and the demand for lumber that determine returns once the lumber has been cut down. The size of the global investable farmland market is estimated at approximately USD1 trillion, with a relatively small proportion, less than 5%, held by institutional investors. Institutional investors hold about one-quarter of the global investable timberland base, valued at roughly USD285 billion. A large majority of timberland held by these investors is located in the United States, Australia, and New Zealand.

Turning to land, Exhibit 7 provides a comparison of returns on US timber and farmland. The National Council of Real Estate Investment Fiduciaries (NCREIF) constructs a variety of appraisal-based indexes for property, timberland, and farmland. Over the 30-year time period from Q3 1992 to Q2 2022, farmland had the higher annualized return and timber had the higher standard deviation.

### Exhibit 7: Historical Returns of US Real Estate Indexes, Q3 1992–Q2 2022 (quarterly data)

	NCREIF Data	
	Timberland	Farmland
Annualized return	8.69%	10.95%
Annualized standard deviation	6.76%	5.88%
Worst calendar year	–5.30%	2.02%
	(2001)	(2001)
Best calendar year	22.36%	33.90%
	(1993)	(2005)

Although the data in Exhibit 7 make farmland appear to be a very attractive investment, it has definite risks. Liquidity is very low, the risk of negative cash flow is high because fixed costs are relatively high (land requires care and crops need fertilizer, seed, and so on), and revenue is highly variable based on weather. The risks of timberland and farmland are similar to those of real estate investments in raw land, but weather is a unique and more exogenous risk for these assets and does not have the same impact on traditional commercial and residential real estate properties. Drought and flooding can dramatically decrease the harvest yields for crops and thus the expected income stream.

In contrast to the local nature of real estate, farmland and timberland are exposed to more global risks given that these investments generate commodities that are globally traded and consumed. For example, there have been interruptions in world trade, and growing agricultural competition has resulted in declining grain prices. Therefore, it seems difficult to repeat these returns over the next 30 years. Timberland and farmland investments should consider the international context as a major risk factor.

Finally, investment in vacant or raw land that has not been developed or prepared for construction generally involves greater risk than farmland or timberland.

## Inflation Hedging and Diversification Benefits of Natural Resource Investments

Investors often consider commodity investments as a hedge against inflation and as a source of portfolio diversification relative to a portfolio of traditional assets (i.e., stocks and bonds).

### *Hedge against Inflation*

The argument for commodities as a hedge against inflation derives from some commodity prices being components of inflation calculations. Commodities, especially energy and food, affect consumers' cost of living. The volatility of commodity prices, especially energy and food, is much higher than that of reported consumer inflation. Consumer inflation is computed from many products, including housing, whose prices change more slowly than commodity prices, and inflation calculations use statistical smoothing techniques and behavioral assumptions. Exhibit 8 shows a summary of calendar year returns on global stocks, global bonds, commodities, farmland, and timberland segmented by whether the US CPI (i.e., inflation) is above or below its median of 2.26%.

**Exhibit 8: Historical Asset Class Returns Divided by Median US CPI, 1993–2021 (annual returns)**

	Global Stocks	Global Bonds	Commodities	Farmland	Timberland
Higher inflation	+9.40%	+5.66%	+22.87%	12.78%	10.44%
Lower inflation	+5.43%	+4.18%	−9.26%	9.85%	5.70%

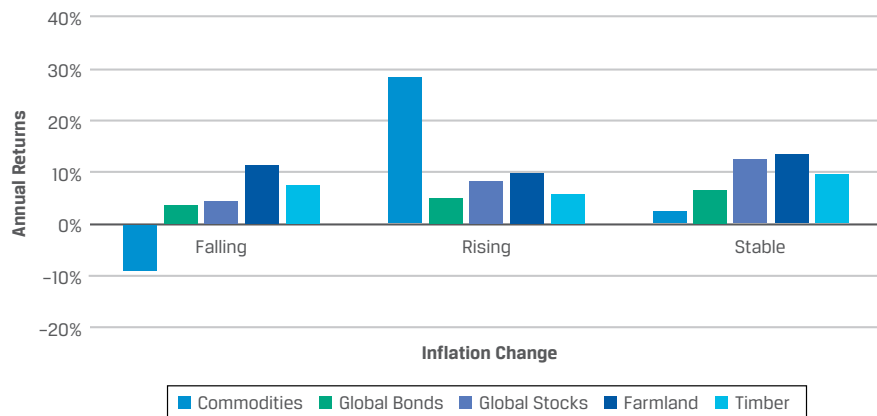
*Sources:* Global stocks, MSCI ACWI; global bonds, Bloomberg Barclays Global Aggregate Index; commodities, S&P GSCI Total Return; Farmland and Timberland, NCREIF

The data from Exhibit 8 covering 29 full calendar years are consistent with the idea that investments in commodities perform well when inflation is higher and perform poorly when inflation is lower. This difference also holds if the 29 calendar years are split into low, middle, and high inflation rates. In fact, the performance distinctions

on commodities are even larger using the three inflation categories instead of only two. However, there is less evidence that investment performance of farmland or timberland differs significantly in differing inflation environments.

As an alternative to presenting asset class returns during higher or lower inflation, Exhibit 9 shows average return data for commodities, global bonds, and global stocks when inflation has moved lower, higher, or very little. Here, stable inflation is defined as a less than a 10 bp move from year to year, so falling and rising inflation regimes are those years with larger year-to-year changes in inflation.

**Exhibit 9: Annual Asset Class Returns in Falling, Rising, and Stable Inflation Environments, 1993–2021**



Sources: Global stocks, MSCI ACWI; global bonds, Bloomberg Barclays Global Aggregate Index; commodities, S&P GSCI Total Return Farmland and Timber, NCREIF

Consistent with the data shown in Exhibit 8, the returns presented in Exhibit 9 highlight that commodities provide greater returns in rising inflation environments at a cost of negative returns as inflation rates decline. In stable inflation environments, commodity returns have tended to be low but positive. None of the other asset classes, including farmland and timberland, demonstrate such large contrasts in performance based on changing inflation.

### Portfolio Diversification

Farmland, timberland, and commodities exhibit potential for portfolio diversification. Historically, all three of these asset classes have low correlations with investment returns from traditional assets (i.e., stocks and bonds) during the business cycle. Exhibit 10 shows the quarterly correlation between farmland, timberland, selected commodities, global equity, and global bond indexes from Q3 1992 through Q2 2022. All three of the alternative asset classes show correlations of approximately zero with global bonds. Timberland shows the lowest correlation of the alternative asset classes with global stocks, at approximately 0.02. Farmland's correlation with global stocks is still low, at approximately 0.12, while commodities demonstrate the least diversification potential with global stocks, given the correlation of 0.41. The correlations of stocks, bonds, and commodities are expected to be positive because all the assets have some exposure to the global business cycle. The commodity index, the S&P GSCI (Goldman Sachs Commodity Index), is heavily weighted toward the energy sector, with each underlying commodity possibly exhibiting unique behavior.

The data in Exhibit 10 also complement the earlier discussion of the potential for asset classes to serve as inflation hedges. The correlation between commodities and inflation of 0.54 is much higher than that of other asset classes with inflation and is consistent with the idea of commodities as an inflation hedge. Neither farmland (−0.17) nor timberland (0.02) shows a high degree of correlation with inflation, implying that these asset classes are less effective as inflation hedges. However, the prior section suggests that the lack of correlation has more to do with their investment performance not suffering during periods of low or declining inflation. Commodity returns, in contrast, are negative during low or declining inflation, as shown earlier.

**Exhibit 10: Historical Commodity Return Correlations, 1990–Q1 2020 (quarterly data)**

	Farmland	Timberland	Commodities	Global Bonds	Global Stocks	Inflation
Farmland	1					
Timberland	0.4352	1				
Commodities	−0.1276	−0.1180	1			
Global Bonds	−0.0209	−0.0047	0.0006	1		
Global Stocks	0.1221	0.0215	0.4106	0.1465	1	
Inflation	−0.1724	0.0166	0.5404	−0.2180	0.0954	1

*Sources:* Global stocks, MSCI ACWI; global bonds, Bloomberg Barclays Global Aggregate Index; commodities, S&P GSCI Total Return; Farmland and Timberland, NCREIF

Overall, investments in natural resource assets exhibit significant portfolio diversification potential because of low correlations with traditional asset classes. Farmland and timberland display somewhat high correlation with each other (approximately 0.44), but each of these asset classes has slightly negative correlations with commodities.

Finally, commodity prices are often more sensitive to geopolitical factors and natural phenomena, such as weather conditions. Also, commodity investments frequently employ leverage and seek to capitalize on expected price changes based on more complex strategies, such as futures contract delivery timing and location, which will be addressed in later lessons.

#### QUESTION SET



1. Contrast the pricing of commodities with the pricing of farmland/timberland investments.

##### **Solution:**

Commodities are priced on public exchanges, and their pricing reflects the latest information. Land investments, such as farmland and timberland, are traded only when actual transactions occur; thus pricing is infrequent and relies on imprecise estimates.

2. Explain why commodity supply is usually slow to adjust to changes in demand for the commodity.

##### **Solution:**

Commodity supply adjusts slowly to changes in commodity demand because producers are unable to alter supply quickly due to the need for extended lead times to affect production levels. For example, increased



demand for a specific agricultural crop requires at least one growing cycle to produce more supply.

3. Explain one rationale as to why consumer price inflation is likely to be less volatile than commodity prices.

**Solution:**

- One possible rationale is that consumer price inflation reflects many additional products beyond commodities (such as housing), which dampens the effect of commodity prices on inflation.
- Another possible rationale is that statistical techniques used to compute consumer price inflation cause smoothing in the data while commodity prices reflect real-time changes on public exchanges.

4. Analyze the results in Exhibit 9 as to how different asset classes perform as inflation changes.

**Solution:**

Commodities clearly perform well in periods of rising inflation and clearly exhibit negative performance as inflation declines. Global stocks and bonds do not exhibit such clear differences in performance related to inflation. Both global stocks and bonds perform best during periods of stable inflation.

## PRACTICE PROBLEMS

1. Which of the following possible value drivers does *not* apply to both raw land and farmland investments?
  - A. Lease revenue
  - B. Sale of agricultural products
  - C. Price appreciation of property
2. Which of the following most correctly describes a significant difference between farmland and timberland investment?
  - A. Timberland provides environmental benefits because of the ability of trees to absorb carbon, while farmland does not.
  - B. Farmland provides a resource necessary for human existence, while timberland does not.
  - C. Farmland is commonly family owned, while timberland is commonly owned by institutional investors.
3. Which of the following best reflects an advantage of institutional ownership of physical farmland as opposed to buying exposure to crops through futures contracts?
  - A. Liquidity of physical farmland
  - B. Price transparency of farmland
  - C. Flexibility to gain price exposure to a wider variety of agricultural products
4. Which of the following is most consistent with backwardation in a commodity market?
  - A. The convenience yield is negative.
  - B. The benefit of holding the physical commodity exceeds the cost of carry.
  - C. The forward price is above the spot price.
5. Which of the following futures market price conditions would be most expected in a period of low commodity inventories?
  - A. Backwardation
  - B. Falling prices
  - C. Contango
6. Which of the following characteristics is *not* a benefit of derivative instruments (compared to ownership of the physical commodity) as a means of gaining commodity exposure?
  - A. Price transparency
  - B. Non-cash benefits

**C. Liquidity**

7. Which of the following statements most correctly describes why commodity investments are thought to provide a hedge against inflation?
  - A. The returns on commodity investing are driven by commodity price changes, and inflation partially reflects these changes.
  - B. Commodity prices increase after inflation rates increase.
  - C. Expectations of higher inflation cause commodity prices to increase.
8. Which of the following statements is most correct about commodity investments?
  - A. Commodity investments are expected to perform worse in inflationary environments.
  - B. Commodity investments exhibit high risk because of high leverage.
  - C. Commodity investments provide weak portfolio diversification because of their high risk.
9. Which of the following statements is most correct if you observe that the correlation between farmland and inflation is significantly lower than the correlation between commodities and inflation (and that both correlations are positive)?
  - A. Commodities are expected to provide a better inflation hedge than farmland.
  - B. Farmland is expected to provide a better inflation hedge than commodities.
  - C. Differences in correlation do not provide any information that is useful in assessing whether an asset class is an inflation hedge.

## SOLUTIONS

1. B is correct. Raw land's value is derived solely from lease revenue and price appreciation. Farmland, in contrast, also generates value from the production and sale of agricultural products.
2. C is correct. Timberland tracts typically consist of thousands (or more) of acres of land, while farmland is quite frequently owned in smaller tracts of tens or hundreds of acres. As such, farmland is much more suited to family ownership, while timberland is more commonly owned by institutions. A is incorrect because carbon offset is capable in both trees (i.e., timberland) and crops (i.e., farmland). B is incorrect because the lumber from timberland provides the raw material for housing, which is a basic human need.
3. C is correct. Futures contracts are available on a very limited number of common crops (i.e., wheat, corn, etc.). Ownership of physical farmland opens up the possibilities of growing crops not traded on futures exchanges, thus providing a larger universe of agricultural product price exposures. A and B are incorrect because futures contracts provide greater liquidity and price transparency than available in physical farmland, which is not traded on public exchanges.
4. B is correct. Backwardation in a commodity market implies that forward prices are lower than spot prices. This can occur only if the total benefits of physical ownership of the commodity exceed the total costs. A is incorrect because convenience yield cannot be negative. C is incorrect because this statement implies a contango market.
5. A is correct. Low inventories of a specific commodity create incentives for market participants to own the physical commodity rather than a derivative contract. This incentive drives up spot prices relative to forward prices and can lead to spot prices being greater than forward prices (i.e., backwardation). B is incorrect because low inventories of a commodity indicate scarcity and would be more likely to contribute to a rising price for the commodity. C is incorrect because a contango market is the opposite of a market in backwardation.
6. B is correct. Non-cash benefits refer to the convenience yield, which causes a preference for owning the physical commodity rather than a derivative contract. A is incorrect in that derivatives provide price transparency while the market for physical commodities is much less transparent. C is incorrect in that the more frequent trading of derivatives on organized exchanges enhances liquidity while physical commodity markets are characterized by infrequent trading and thus poor liquidity.
7. A is correct. Commodity prices are a significant portion of consumer prices because commodities include aspects of everyday life, such as food and energy, and thus consumer price inflation will incorporate the effects of commodity price changes. By investing in commodities, an investor is, at least partially, hedged against the inflation that occurs with rising commodity prices. B is incorrect because inflation and commodity prices do not move together, but instead, changes in the inflation rate lag behind changes in commodity prices. C is incorrect because in this case, commodity price increases occur before inflation changes.
8. B is correct. Commodity investments are typically entered into through derivative contracts, which are highly leveraged financial instruments. As a result, observed returns are highly volatile. A is incorrect because commodity investments

are expected to perform better during inflationary environments. C is incorrect because commodity investments tend to exhibit low correlations with traditional assets and thus are typically used as portfolio diversifiers.

9. A is correct. An effective hedge exhibits relatively high correlation relative to the risk exposure being hedged. Thus, the higher correlation between commodities and inflation implies that commodities provide a better hedge against inflation compared to farmland.



## LEARNING MODULE

## 6

## Hedge Funds

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	explain investment features of hedge funds and contrast them with other asset classes
<input type="checkbox"/>	describe investment forms and vehicles used in hedge fund investments
<input type="checkbox"/>	analyze sources of risk, return, and diversification among hedge fund investments

## INTRODUCTION

1

Hedge funds originally started as an equity investment vehicle in which offsetting short and long positions protected the overall portfolio against major stock market moves. Today, the name *hedge funds* is a misnomer. They are not restricted to equities or just hedging strategies. Hedge funds are private pooled investment vehicles that can invest in a wide variety of products, including equities, fixed income, derivatives, foreign exchange, private capital, and real assets. It is the investment *approach* rather than the underlying investments that distinguish hedge funds. Many hedge funds operate in all kinds of financial markets by using leverage, short selling, or using financial instruments that are not often used by other similar commingled funds, such as mutual funds. This may result in a very different risk and return profile than owning underlying assets themselves.

The hedge fund industry is in a state of constant change as several hundred new funds are launched each year, with a similar number of funds exiting or being liquidated. While several jurisdictions around the world regulate hedge funds, often they are lightly regulated compared with other investment vehicles.

## LEARNING MODULE OVERVIEW



- Hedge funds are private investment vehicles with pooled funds from institutions and high-net-worth (HNW) investors. Hedge funds typically have more flexible investment strategies than other options, such as mutual funds and ETFs.

- Hedge funds are not an asset class but are a variety of investment vehicles driven by a set of disparate investment strategies. Most hedge funds utilize some form of leverage to enhance potential returns.
- Hedge funds are typically classified by strategy. A variety of classifications are possible, which helps in the selection of appropriate investment strategies and appropriate performance benchmarks and in reviewing aggregate performance.
- Most hedge funds are set up as limited partnerships, with the portfolio manager acting as a general partner (GP) and the institutional investors acting as limited partners (LPs). This is the direct form of hedge fund setup. For smaller and retail investors, indirect forms, such as funds of funds, help obtain a hedge fund exposure.
- The legal and contractual relationship between the GPs and LPs is governed by the fund offering documents. In addition, a manager could draft a “side letter” applicable to some investors only, with different legal, regulatory, tax, operational, or reporting requirements.
- Hedge funds use several strategies, such as market-neutral, relative value, and event-driven strategies, to obtain diversification benefits and to attempt to outperform equity markets on a risk-adjusted basis.
- Hedge fund strategies are classified by a combination of the instruments in which they are invested, the trading philosophy followed, and the types of risks assumed.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. Which statement about hedge funds is most accurate?

- A. Hedge funds are investment products offered to the public and are traded daily on the OTC market.
- B. Hedge funds are benchmarked to an index or industry/sector, and managers use complex strategies to mimic the index or industry/sector.
- C. Hedge funds are private pooled funds, applying strategies with a goal of maximizing returns while reducing risk.

**Solution:**

C is correct. A hedge fund is a pooled investment vehicle that uses complex trading (using leverage, short selling, using derivatives, etc.) and risk management techniques to enhance performance for a private group of accredited investors.

A is incorrect. Mutual funds, not hedge funds, are regulated investment products offered to the public and available for daily trading.

B is incorrect. Exchange-traded funds (ETFs) are normally benchmarked to an index or industry/sector and typically track a specific industry or index. Hedge funds are benchmarked to either a hedge fund index or performance measured in absolute returns.



2. Which of the following statements about relative value strategies is *least accurate*?

- A. Relative value strategies seek to profit from a price or return discrepancy between securities based on a short-term relationship.
- B. Relative value funds are inherently structured to minimize net market risk and credit risks.
- C. The investments made under a relative value strategy are all within a single asset class or sector, using assets with a sufficient price differential to arbitrage their movements to equilibrium prices.

**Solution:**

C is correct because it is the least accurate statement. Relative value strategies often involve investments in different asset classes. A and B are true.

3. Which of the following statements is *least accurate* about hedge funds?

- A. Merger arbitrage strategies generally assume that an acquirer will be overpaying for the target.
- B. Event-driven hedge funds flourish in a stable market environment, where minor deviations in asset prices quickly converge to equilibrium.
- C. An activist strategy expects to realize higher returns due to the manager being more effective in driving the corporate policies or strategic direction of the investment.

**Solution:**

B is correct because it is the least accurate statement. Event-driven hedge funds thrive in a rising market environment with a high level of corporate activity in a strong economy. These are the times that accelerate merger and acquisition activity. A and C are accurate statements.

4. Which of the following is *not* a characteristic of hedge funds?

- A. Hedge funds are mostly illiquid, with little trading possibilities.
- B. Hedge fund managers use leverage; however, the overall risk is lower.
- C. Hedge funds are a different asset class, with a distinct risk/reward profile.
- D. Managers demand higher remuneration and have more discretionary freedom in the choice of investments.

**Solution:**

C is correct. Hedge funds invest in traditional asset classes but use a specific investment strategy. They are not a distinct asset class.

5. In January, HedgeAway, a new hedge fund, started operations with an initial amount of USD100 million. The fund charges a management fee of 1.6% based on end-of-year value and a performance fee of 18% on gross returns payable on the excess over a hurdle rate of 8% after fees. The fund ended the year with assets under management (AUM) of USD120 million. What was the investors' return during the year?

- A. 16.38%
- B. 18.08%

**C.** 18.19%

**Solution:**

A is correct.

Management fee = 1.6% of 120 million = 1.92 million.

Growth during the year = 20 million, excess over the hurdle

= 20 million – (100 million × 0.08) – 1.92 million = 10.08 million.

Performance fee = 10.08 million × 0.18 = 1.81 million.

Total fees = 1.92 million + 1.81 million = 3.72 million.

Return to the investors = 20 million – 3.72 million = 16.38 million.

Investors' return = 16.38%.

## 2

## HEDGE FUND INVESTMENT FEATURES



explain investment features of hedge funds and contrast them with other asset classes

As private investment vehicles, hedge funds are distinguished by their investment approach rather than the underlying investments. Hedge funds combine traditional debt and equity instruments with leverage, derivatives, short selling, and other strategies to generate and enhance their returns. The objective of a hedge fund is to generate high returns, either in an absolute sense or on a risk-adjusted basis relative to its portfolio-level volatility. The strategies hedge funds use can make benchmarking their performance relative to traditional index performance benchmarks difficult. Thus, many hedge funds evaluate their performance using an absolute return standard instead of tracking a benchmark. Hedge funds are attractive for their diversification effects because their returns typically demonstrate low correlation with traditional asset investing.

Hedge funds normally apply common principles that seemingly increase portfolio risk, such as borrowing money to invest, using leverage (derivatives), and short selling. On their own, they do not hedge risky positions against a market move; on the contrary, they seem to amplify the risks. It may seem like the name *hedge funds* is a misnomer. However, the investment strategy splits a portfolio such that each component helps hedge the risks from the other. Thus, by internally neutralizing market risk and by managing the portfolio components, the hedge fund manager can obtain enhanced risk-adjusted returns.

While mutual funds and hedge funds seem similar, in that they both invest clients' money to achieve a better risk/reward profile, there are some major differences. Mutual funds managers are paid a fixed compensation and may not necessarily invest in the funds they manage. Hedge fund managers are paid a performance-based fee, and many require the managers to invest in the hedge fund. Some hedge funds incorporate a high-water mark, in which the manager will get a performance fee only when the returns exceed the previous highest value of the fund.

Normally, hedge fund managers have a great deal of freedom to make trading decisions and to decide how to allocate client funds. Mutual funds are highly regulated since they are available to public investors. Hedge funds are available only to institutional and accredited investors.

Hedge funds are different from other fund types, such as mutual funds, ETFs, bond funds, and REITs, in that a hedge fund is privately owned. And unlike many of these funds, hedge funds are lightly regulated. They are different from private equity funds, in that hedge funds typically have a shorter time horizon and invest in more liquid asset classes.

Hedge funds select investments from one or more asset classes (equities, credit, fixed income, commodities, futures, foreign exchange, loans, and sometimes even hard assets, such as real estate). Some hedge funds implement strategies focused on one specific asset class, while others combine multiple asset classes. Hedge funds can also be geographically focused or agnostic and implement their strategies across different geographic regions.

Leverage—through short selling, borrowing, or derivatives and occasionally combining all three—is often used by hedge funds to enhance returns. Since leverage is often a core component of the strategy, hedge funds continuously need to monitor the value of their exposures. This is particularly important when a hedge fund takes both long and short positions (when possible) solely using derivatives.

Hedge funds are typically classified by strategy. One such classification includes five broad categories of strategies:

- equity hedge funds,
- event-driven hedge funds,
- relative value hedge funds,
- opportunistic hedge funds, and
- multi-manager hedge funds.

Many hedge funds trade sovereign and corporate debt, commodities, futures contracts, options, derivatives, and even real estate investments. However, not all hedge funds maintain short positions or use leverage. Instead, many simply exploit niche areas of expertise in a sophisticated manner; hedging and leverage may or may not be involved. Finally, there are funds of hedge funds that create a diversified portfolio of hedge funds. These vehicles are attractive to smaller investors without the resources to select individual hedge funds and build a portfolio of them.

Hedge fund categorization allows investors to review aggregate performance data, select strategies with which to build a portfolio of funds, and select or construct appropriate performance benchmarks. Exhibit 1 shows examples of the five broad strategy hedge fund categories mentioned above.

**Exhibit 1: Hedge Fund Strategies**

Equity	Event Driven	Relative Value	Opportunistic	Multi-Manager
Long/Short Equity	Merger Arbitrage	Convertible Bond Arbitrage	Global Macro	Fund of Funds
Short Biased	Distressed	Fixed-Income Arbitrage	Managed Futures	
Market Neutral	Special Situations Activist	Multi-Strategy		

## Equity Hedge Fund Strategies

Hedging long positions through short selling can be considered the original hedge fund category. Long/short equity funds focus on public equity markets and take long and short positions in equity and equity derivative securities. Most equity hedge strategies use a “bottom-up” security-specific approach—company-level analysis, followed by overall industry analysis, followed by overall market analysis—with relatively balanced long and short exposures. A contrasting “top-down” approach entails global macro analysis, followed by sector/regional analysis, followed by individual company analysis or any market-timing approach. Some equity long/short strategies may use index-based short hedges to reduce market risk or single-name shorts for portfolio alpha and added absolute return. The following are examples of equity hedge strategies.

- **Fundamental long/short:** In this strategy, the hedge fund takes long positions in companies that are trading at inexpensive levels compared to their potential intrinsic value and shorts those that trade in the other direction, with the intention of reversing this trade to obtain alpha.  
In all cases, the strategy takes a long position in those securities (buys stocks or call options) whose valuations are underestimated/undervalued by the market or have a potential for growth that the market has not yet identified. The strategy also concurrently shorts stocks or an index to reduce the risk. The manager typically maintains a net long exposure but may adjust the amount of net market risk depending on his or her market forecast.  
Most hedge funds that use a long/short strategy have a long bias, which differentiates this strategy from the short bias strategy.
- **Fundamental growth:** These strategies use fundamental analysis to identify companies expected to exhibit high growth and capital appreciation. The hedge fund will take a long position in these stocks. The fund will short companies with business models that are under downward pressure and expected to exhibit low or negative growth and suffer capital depreciation. Effectively, the spread between growth and value expectations drives the investment strategy and portfolio performance. Most of these portfolios tend to end up long biased; hence, they may not be market neutral and their returns may exhibit a non-zero beta.
- **Fundamental value:** These strategies use fundamental analysis to identify undervalued and unloved companies for which there is the possibility that a corporate turnaround, with future revenue and cash flow growth, will result in higher valuations. The hedge fund takes long positions in these companies to capture expected future stock price rises. Effectively, it is the spread between value and growth expectations that drives portfolio performance.
- **Short biased:** These strategies use quantitative, technical, and fundamental analysis to short the overvalued equity securities with limited or no long-side exposures. Managers are often forensic in their fundamental analysis and sometimes try to expose previously unrecognized accounting or business flaws. The expectation is that company share price will fall and thus improve the profitability of the fund's portfolio. These funds vary their short exposure over time. Short-biased managers tend to be contrarian; they are shorting shares in otherwise successful companies. These funds can be useful additions to larger portfolios during periods of market stress. Short-biased managers, however, have had a difficult time overall posting meaningful long-term returns during the past 30 years of generally positive market conditions.

- **Market neutral:** These strategies use quantitative, fundamental, and technical analysis to identify under- and overvalued equity securities. The hedge fund takes long positions in undervalued securities and short positions in overvalued securities, while seeking to maintain a market-neutral net position. Ideally, the manager achieves an overall beta relative to the market close to zero.

The intent is to profit from the movements of individual securities, undervalued ones rising and overvalued ones falling, while avoiding movements in the overall market. To achieve a meaningful return, market-neutral portfolios may require the application of leverage. These portfolios generally seek stable, single-digit returns that are independent of the market (market neutral), but because leverage is used to amplify the returns, these funds may experience higher risk during periods of unexpected volatility unless they reduce their leverage.

**EXAMPLE 1****Tenderledge Investments LLC—Equity Strategies**

Tenderledge Investments, a fund-of-funds hedge fund, is benchmarking the performance of various equity strategies as measured by monthly hedge fund returns over a 10-year period, shown in the following tables.

<b>Equity Strategies: Monthly Returns</b>	<b>Market Index</b>	<b>Fundamental Value</b>	<b>Market Neutral</b>	<b>Fundamental Growth</b>	<b>Short Bias</b>
Average	0.30%	0.03%	−0.04%	0.03%	−0.5%
Standard deviation	2.0%	1.0%	0.5%	1.5%	2.3%
Coefficient of variation	14.8%	2.6%	−7.4%	1.9%	−20.1%
Max.	5.2%	2.2%	1.0%	3.5%	7.2%
Min.	−8.1%	−4.0%	−2.7%	−10.4%	−11.3%

<b>Equity Strategies: Correlation of Monthly Returns</b>	<b>Market Index</b>	<b>Fundamental Value</b>	<b>Market Neutral</b>	<b>Fundamental Growth</b>	<b>Short Bias</b>
Market Index	1.00				
Fundamental Value	0.80	1.00			
Market Neutral	0.22	0.24	1.00		
Fundamental Growth	0.67	0.64	0.38	1.00	
Short Bias	−0.72	−0.53	−0.23	−0.60	1.00

The market index, measured by the S&P 500 Index, outperformed each strategy both in absolute terms as measured by the average monthly returns and in relative terms as measured by the coefficient of variation of returns. Moreover, analyzing the correlations between these strategies shows that fundamental value and growth have exhibited high correlation with the market index.

Market-neutral funds have the lowest positive correlation with the market index, reflecting the fact that these strategies intend to generate returns without taking on market-level exposure. As expected, the short-bias funds have a negative correlation with the other strategies. Note that over short time frames, short-bias strategies are negatively correlated with the market index–based strategies. However, in the long term, similar correlations may not hold.

## Event-Driven Strategies

These bottom-up strategies seek to profit from defined events that are expected to change valuations, typically involving changes in corporate structure, such as an acquisition or restructuring. Event-driven strategies may include long and short positions in common and preferred stocks, debt securities, and options. Further subdivisions of this category by Hedge Fund Research (HFR) include the following:

- **Merger arbitrage:** Generally, these strategies involve going long (buying) the stock of the company being acquired at a discount to its announced takeover price and going short (selling) the stock of the acquiring company when the merger or acquisition is announced. The manager may expect to profit once this initial deal spread (the price between the buyer and the object of the acquisition) narrows to the closing value of the transaction after it is fully consummated. This spread exists because of timing and uncertainty over the closure of the deal due to legal and regulatory hurdles, or the acquirer may decide to step away. Shorting the acquirer is also a way to express the risk of merger overpayment. The primary risk in merger arbitrage is that the announced combination fails to occur and the value of the fund holdings are negatively impacted before it can unwind its position. Since the expected risk and return on a merger arbitrage strategy stems from the modest spread in prices, leverage is regularly used to amplify returns but also increases losses when the strategy fails.
- **Distressed/restructuring:** These strategies focus on securities of companies either in or perceived to be near bankruptcy. In one approach, hedge funds simply purchase fixed-income securities trading at a significant discount to par but that are still senior enough to be backed by sufficient corporate assets. The expectation is that these securities should be valued at par or at least at a significant premium to the current bond purchase price in a bankruptcy reorganization or liquidation. Alternatively, a fund may purchase a debt instrument that is expected to be converted into new equity upon restructuring or bankruptcy, typically called a *fulcrum* security, and then either hold onto the equity or exit.
- **Special situations:** These strategies focus on opportunities to buy equity of companies engaged in security issuance or repurchase, special capital distributions, rescue finance, asset sales/spin-offs, or other catalyst-oriented situations.
- **Activist:** The term “activist” is short for “activist shareholder.” Here, managers secure sufficient equity holdings to allow them to seek a position on the company board and influence corporate policies or direction. They seek to create business changes that move the investment towards a desired outcome. For example, an activist hedge fund may advocate for divestitures, restructuring, capital distributions to shareholders, or changes in management and company strategy affecting their equity holdings. Such hedge funds are distinct from private equity because they operate primarily in the public equity market.

Event-driven strategies tend to be long biased, with merger arbitrage having the least bias; the time to complete a merger transaction typically takes somewhere between 6 and 24 months, while a bankruptcy or a reorganization can take years to complete.

**EXAMPLE 2****Activism of Carl Icahn and Hertz**

Carl Icahn, an activist hedge fund manager, has been financially successful in several cases where he took a controlling interest in companies to direct their operations for an increase in enterprise value. Hertz, a car rental company, is an interesting example. In 2014, Icahn started investing in Hertz and in 2020 held more than 55 million, or 39%, of its common shares. During May 2020, Hertz sought bankruptcy protection because of the impact of COVID-19 and the collapse of the global travel industry. Before Hertz filed for bankruptcy protection, Icahn's holdings were worth around USD2.3 billion. After Hertz filed, he sold his shares at a loss of USD2 billion, even though he acquired 11 million shares only a few weeks earlier.

Originally, Icahn acquired his stake in Hertz starting in 2014 based on his notion that Hertz had a strong brand and operational foundation but was lacking discipline and management. And Icahn accumulated a more than USD1.1 billion position in the company by the end of that year. When Icahn entered Hertz, the company was emerging from years of operational, strategic, and financial problems that eroded its financial strength. Icahn put new board members and a new management in place, which in the ensuing years instilled fiscal and operational discipline. The company restated previous years' earnings, finished the integration of various enterprise-wide systems, and secured a qualified management team. At the same time, the car rental industry was impacted by two strategic shifts: One was the increased preference for SUVs, and the other was the emergence of ride-hailing apps. While the company showed higher revenue and increasingly consistent profits, the cost of restoring the company eroded its financial and liquidity position. After Hertz came out of bankruptcy proceedings in July 2021, Icahn has been closely following the company.

**Relative Value Strategies**

Relative value strategies seek to profit from a pricing discrepancy between related securities based on an unusual short-term relationship. The expectation is the short-term discrepancy will be resolved over time. Examples of relative value strategies include the following:

- *Convertible bond arbitrage.* This conceptually market-neutral investment strategy seeks to exploit a perceived mispricing between a convertible bond and its component parts: the underlying bond and the embedded call option. There may be relative mispricing between equity and the convertible bond. The strategy typically involves buying convertible debt securities and simultaneously selling a certain amount of the same issuer's common stock based on the delta of the embedded call option. This strategy can be sensitive to bankruptcy risks; however, they may be hedged away using either equity put options or credit default swap derivatives on the issuer.
- *Fixed income (general).* These strategies focus on the relative value within the fixed-income markets, with an emphasis on sovereign debt (relative value rates) and sometimes the relative pricing of investment-grade corporate debt (relative value credit). Strategies may incorporate long–short trades between two different issuers, between corporate and government issuers, between different parts of the same issuer's capital structure, or

between different parts of an issuer's yield curve. Here, spread and currency dynamics together with considerations around the shape of government yield-curve considerations drive investment choices and returns.

- *Fixed income (asset backed, mortgage backed, and high yield).* These strategies focus on the relative value of various higher-yielding securities, such as asset-backed securities, mortgage-backed securities, high-yield loans and bonds, and their derivatives. Hedge funds seek to generate an attractive and highly secured coupon return and to exploit relative security and quality mispricings.
- *Multi-strategy.* These strategies trade relative value within and across asset classes or instruments. Rather than focusing on one type of trade (e.g., convertible arbitrage), a single basis for a trade (e.g., merger arbitrage), or a particular asset class (e.g., fixed income), this strategy instead looks for any available investment opportunities, often with different pods of managers executing unique market approaches. The goal of a multi-strategy manager is to initially deploy (and later redeploy) capital efficiently and quickly across various strategy areas as conditions change.

## Opportunistic Strategies

There are also funds that focus on macro events and commodity trading. These strategies may often use index ETF securities or derivatives in addition to individual securities.

- *Macro strategies* emphasize a top-down approach to identify economic trends. Trades are made on the basis of expected movements in economic variables. Generally, these funds trade opportunistically in fixed-income, equity, currency, and commodity markets. Macro hedge funds use long and short positions to profit from a view on overall market direction as it is influenced by major economic trends and events. Because these funds generally benefit most from periods of higher volatility, the active moves by national authorities, such as central banks, to smooth out economic shocks likely shrink their investment sphere.
- *Managed futures* funds are actively managed funds making diversified directional investments primarily in the futures markets on the basis of technical and fundamental strategies. Managed futures funds are also known as commodity trading advisers (CTAs) because they historically focused on commodity futures. However, CTAs may include investments in a variety of futures, including commodities, equities, fixed income, and foreign exchange. CTAs generally use models that measure trends and momentum over different time horizons. CTA investments can be useful for portfolio diversification, particularly in times of strong trending market conditions and especially during periods of extended market stress when other fundamental strategies may be expected to perform poorly. CTAs can be relied on to profit from having purchased short positions in falling markets. However, mean-reverting markets, which may cause false momentum breakout signals, can lead to extended drawdown periods before strong trends emerge for the CTA. To the extent that many CTAs have migrated to trade more and more financial products (such as stock index futures and bond futures), the reliability of CTA diversification benefits has diminished.



Commodity-focused managed futures funds are unique (versus global macro) because there is a constant price tension between suppliers and consumers: High prices cripple demand (tending to lower prices), and low prices shut in supply (and thus raise prices). This situation creates a unique balance absent in traditional stocks and bonds.

## Distinguishing Characteristics of Hedge Fund Investments

The key characteristics distinguishing hedge funds and their strategies from traditional investments include the following:

1. Less legal and regulatory constraints
2. Flexible mandates permitting the use of shorting and derivatives
3. A larger investment universe on which to focus
4. Aggressive investment styles that allow concentrated positions in securities offering exposure to credit, volatility, and liquidity risk premiums
5. Relatively liberal use of leverage
6. Liquidity constraints that include lockups and liquidity gates
7. Relatively high fee structures involving management and incentive fees

While hedge funds frequently invest in publicly traded equity or debt instruments and often use financial statement analysis techniques to value securities, their return and risk characteristics generally differ from those of exchange-traded funds and mutual funds. The typical relative value hedge fund generates returns using a combination of long and short positions in equities, increases its asset base using borrowed funds, and implements opportunistic positions in special situations, seeking to earn very different risk–return profiles than those of common long-only funds.

Hedge funds are also subject to much lighter regulatory, compliance, and transparency requirements. With more flexibility in portfolio construction, hedge funds enjoy leeway to invest in situations in which time may be needed to generate an expected return and thus are unsuitable for a mutual fund offering daily liquidity. Investors in modern hedge funds are subject to extended holding periods (known as *lockup periods*) and subsequent *notice periods* before an investment redemption is possible. Some hedge funds partially limit fund redemptions through a liquidity gate provision so that assets can be liquidated over a longer time period. Compared to, for example, many mutual funds allowing easier redemption and guaranteed liquidity on one day's notice, such lengthy mandatory lockup and notice periods allow hedge funds more flexibility than mutual funds or other types of investments.

A redemption fee may be charged, typically payable to the fund itself (rather than the manager) to protect remaining investors in the fund, particularly in circumstances where the redemption takes place during the lockup period. This characteristic is called a *soft lockup*, and it offers a path (albeit an expensive one) to redeem early.

With reduced operating constraints, hedge funds may avail themselves of less liquid and unnoticed opportunities, the true valuation of which may at times be opaque. For instance, several hedge fund strategies—long/short, activist, distressed, and arbitrage—build on highly concentrated, long-term, leveraged holdings in equities, debt, and derivatives. Building up such an exposure may take a longer time to execute and unwinding such a position can be complex and quite time consuming, particularly when financial markets are under transitory stress. That is why restrictions on *redemptions* are typically imposed.

There is also reduced transparency for more complex hedge fund investments and asymmetric information between managers and investors. This leads to incentive-based performance fees to bridge the gap between the manager's and the investor's interests. Investors look to earn high returns, and sharing some proportion of these high returns with the managers incentivizes managers to perform in the interest of the

shareholders. Hedge funds are generally deemed riskier from an oversight (fraud risk) point of view, but some hedge funds take less absolute market risk in their portfolio construction than is taken by registered products available to retail investors. A hedge fund's true market risk and its distinction between regulatory risk and illiquidity risk can thus often be confused.

Hedge funds generally utilize active management by experienced managers and an integrated risk management approach. A variety of strategies are possible, which makes each hedge fund uniquely different from others. They can be customized for a specific investor preference. They provide an ability to generate alpha and uncorrelated risk-adjusted returns. Some of the strategies also implement guardrails to insure against significant losses. Such attributes help an investor build a diversified portfolio that stabilizes market volatility within a multi-asset class portfolio.

Investors should consider several issues when choosing a hedge fund. Broadly, they should review the limited partner capabilities, including their operational framework, risk management practices, and hands-on monitoring of portfolio performance. As for the GP performance, the investor should consider their fiduciary management guidelines, manager experience, and alignment of interests towards the fund's strategies.

There have been quite a few reported incidents of fraud in the hedge fund space, including setting up a fund as a Ponzi scheme and reporting false performance data. Some hedge funds had complex strategies that failed during market turmoil. The reduced regulatory oversight, along with the large investments that are typically required to participate in a hedge fund, opens the door for some unscrupulous fund managers to take advantage of investors' trust in them.

While evaluating the investment, the investor should take into account its strategy, transparency, liquidity, and reporting practices. In addition, the investor should thoroughly evaluate the fund manager's past performance and be aware of how the fund compensates the fund managers and calculates the fees charged to investors.

#### QUESTION SET



1. Both hedge funds and private equity invest in equity stock of public or private enterprises, and there are many commonalities between them. The following statements are some of the commonalities, and one of these statements is false. Choose the *false* statement.
  - A. Both are structured as partnerships of investors with private pooling of funds and are primarily intended for high-net-worth individuals.
  - B. Both utilize leverage to invest in a variety of marketable securities.
  - C. Both are less liquid than mutual funds or ETFs.
  - D. Both are less regulated, and the transparency/reporting requirements are not strict.

#### Solution:

B is false. Hedge funds normally invest in public securities (including debt, equity, and derivatives), while private equity funds invest directly in private operating companies, which are not marketable securities, nor do private equity firms apply leverage to their fund.

2. Both hedge funds and private equity invest in equity shares of public or private enterprises. However, there are many differences between them. The

following statements lists some of their differences. Choose the statement that is *false*.

- A. Private equity funds invest for the long term, while hedge funds invest in equities for the shorter term.
- B. Typical hedge funds are transaction oriented; they make several offsetting trades. Private equity funds make stable, long-term investments in few companies.
- C. An investor normally funds the hedge fund at the start of the investment, while private equity funds are committed at the start and funded over time, upon demand.
- D. Private equity is redeemable on a periodic basis, while hedge funds require a longer-term commitment.

**Solution:**

D is the correct answer choice because the statement is false. Hedge funds are redeemable on a periodic basis, and private equity funds require a longer-term commitment. The other statements are correct.

3. Hedge funds versus mutual funds: Choose the *false* statement.

- A. Mutual funds are open to any investor and are generally more liquid instruments with minimal constraints on redemptions.
- B. Hedge fund fees are typically negotiable by an investor, while mutual funds operate with the same fees for all investors.
- C. Hedge funds are more restricted in what they can trade compared to mutual funds.

**Solution:**

C is the false statement. Hedge funds are less restricted than mutual funds as to the securities they can trade. They are also subject to less disclosure and transparency requirements. A and B are true statements.

4. How many of the following statements comparing hedge funds and ETFs are true?

- 1. ETFs are exchange-traded public securities, while hedge funds are private partnership funds.
  - 2. Any investor can invest in an ETF, while specific restrictions apply to who can invest in a hedge fund.
  - 3. ETFs have very low fees and expense ratios compared to hedge funds.
  - 4. ETFs are highly regulated, with specific reporting requirements, while hedge funds are lightly regulated.
- A. One of the statements is true.
  - B. Two of the statements are true.
  - C. Three of the statements are true.
  - D. Four of the statements are true.

**Solution:**

D is correct. All the statements are true.

5. Select the statements that are true:

A. The primary drivers of returns from stocks are growth projections, dividends, and retained earnings.

B. The primary drivers of returns from a bond are interest rates, credit risk, and coupon payments.

C. The primary drivers of return from hedge funds are market volatility and market inefficiency.

**Solution:**

A, B, and C are all true.

### 3

## HEDGE FUND INVESTMENT FORMS



describe investment forms and vehicles used in hedge fund investments

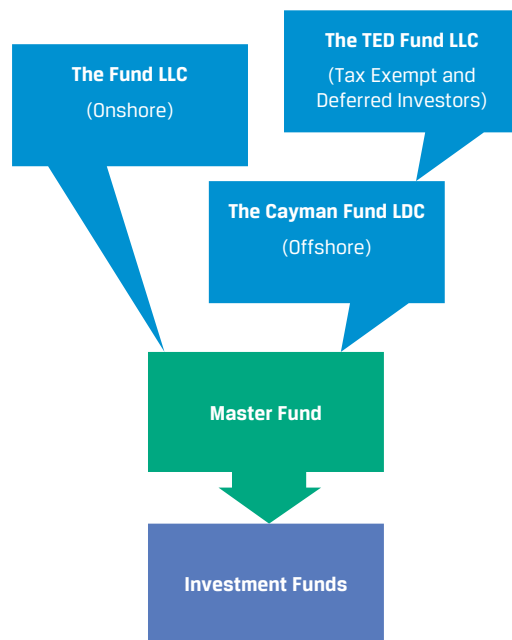
A common structural characteristic of a hedge fund is that it is set up as a private investment partnership either onshore or in a tax-advantaged offshore location. Under certain legal restrictions (which vary by jurisdiction), the offering can be open only to a limited number of investors meeting certain income and net worth guidelines.

Hedge funds, like private equity funds, are legally typically incorporated and organized as private limited partnerships or limited liability companies with a general partner or managing member who is the hedge fund manager. The partnership or the managing member receives a management fee, and the general partner receives compensation based on fund performance. Hedge fund investors purchase a share of the fund or partnership and receive in return a fixed percentage of the fund returns, minus applicable fees.

The fund documents—private placement memorandum, the partnership agreement, or the articles of incorporation—lay out the legal and contractual relationship between the fund manager and the fund investor and create the operational framework for the fund. Normally, the fund structures are incorporated as perpetual legal entities, but in reality, as the hedge funds close, they are liquidated on a regular basis.

### Direct Hedge Fund Investment Forms

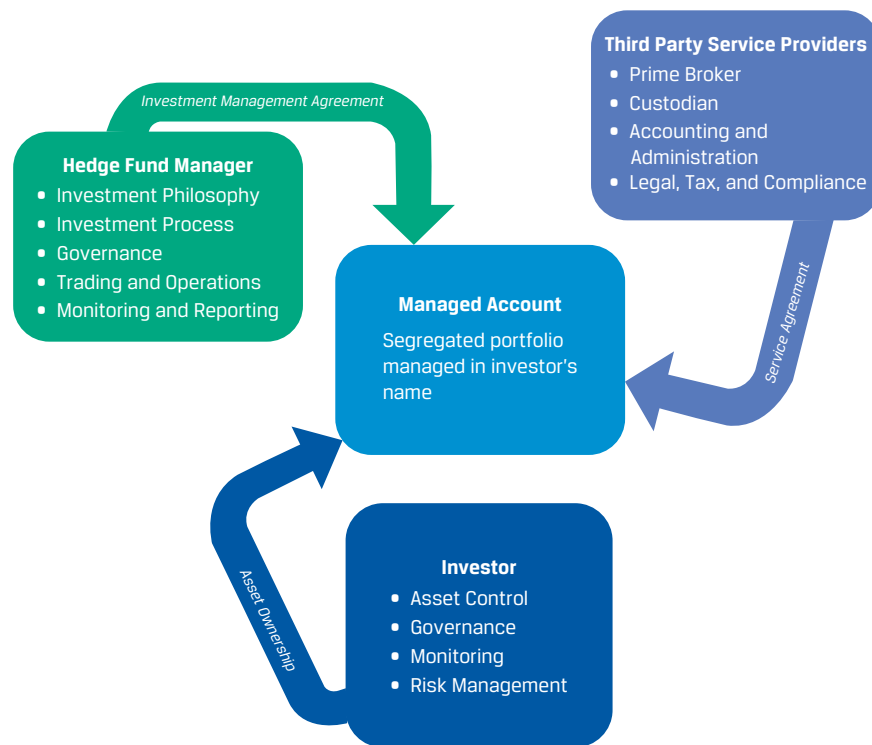
A common hedge fund form is a master feeder structure. The master feeder structure is set up for optimum tax efficiency and consists of an offshore feeder fund and an onshore feeder fund—both feeding into a master fund that invests the capital based on its contractual partnership agreements. This structure is depicted in Exhibit 2.

**Exhibit 2: Sample Master-feeder Structure**

A common hedge fund fee structure is known as “two and twenty.” In this setup, the hedge fund partnership entity that operates and administers the fund receives a 2% management fee. The general partner, who manages the fund investments, receives 20% of the fund’s net profits. The investment returns, after management and performance fees, flow back to the feeder funds to the investors. Recently, the fee structure has been changing due to pressure from investors. Some of the newer funds are offered with a fee of 1 or 30, where the manager receives the greater of a 1% management fee or an incentive fee of 30% of the fund’s alpha or outperformance against a benchmark, instead of a performance fee based simply on total profits.

In addition to the partnership agreement that delineates responsibilities, hedge funds often use side letters to address the specific legal, regulatory, tax, operational, and reporting requirements of an investor. Such side letters complement and can occasionally supersede the terms of the fund’s documents and are typically used when a hedge fund investor requires concessions without changing the private placement memorandum, the partnership agreement, or the articles. Occasionally, specific rights are conferred to a particular investor, such as enhanced information rights.

For larger investors, the hedge fund structure could be a fund of one or a separately managed account (SMA). These are separate investment accounts over which the investor retains more influence. In the case of a fund of one structure, the hedge fund is created for one investor, and in the case of an SMA, the investor creates his or her own investment vehicle and the underlying assets are held and registered in the name of the investor. However, the day-to-day management of the account is delegated to the hedge fund manager. These structures may require additional agreements and service providers to operate efficiently and seamlessly. Exhibit 3 shows a typical structure.

**Exhibit 3: Separately Managed Hedge Fund Investment Account**

Source: KPMG. <https://assets.kpmg/content/dam/kpmg/pdf/2015/09/hedge-fund-managed-accounts.pdf>.

An SMA structure allows for a customizable portfolio, with investor-specific investment mandates, better transparency, efficient capital allocation, and higher liquidity, over which the investor can exercise enhanced control while keeping the fees lower. However, SMAs are operationally more complex and also demand greater governance oversight. That is why these accounts are more appropriate for larger, institutional investors.

An SMA structure has some downsides as well. Unlike a commingled fund, the managers do not have a stake in the fund investments. The investors negotiate lower fees and fund expenses but in return, may receive allocations only to the fund manager's most liquid investment trades. Hence, the overall motivation of the managers for investment performance could be reduced. Structuring in favor of incentive fees can mitigate this problem.

### Indirect Hedge Fund Investment Forms

Indirect investment in hedge funds aims to make hedge fund exposures more accessible to smaller institutional and larger retail investors or to those who may lack specialized skills in managing certain asset types or want to create multiple and concurrent exposures to different strategies. The choice of indirect exposure is often motivated by reducing management costs, increasing performance transparency, and improving liquidity. Often the choice is the fund-of-hedge-funds approach, which is a managed portfolio of hedge funds.

Funds of hedge funds pool funds from investors and invest the proceeds in a diversified portfolio of hedge fund investments across a variety of hedge funds. This approach provides direct diversification benefits across fund strategies, investment regions, and management styles. These funds offer generally lower investment minimums, reduced lockup periods, and typically better exit liquidity.

For the investor, all this comes with a higher fee structure—often an additional 1%—because the manager of the fund of funds adds its own fees on top of the hedge fund management fees. Fund-of-funds investors often face a 10% incentive fee in addition to those fees charged by underlying hedge funds. The incentive fee is typically calculated on profits net of management fees at both the hedge fund level and the fund-of-funds level. This raises the cost for the investors, but at the same time, they gain access to hedge funds in which they otherwise would not have been able to invest in a diversified pool of funds. Fee layering reduces the end investor's initial gross investment returns and may result in an investor paying fees more than once for management of the same assets. Despite the additional fees, investors choose funds of funds since they provide an opportunity to invest in an underlying hedge fund that might be closed to new investors otherwise. Greater liquidity in funds of funds may result in weaker performance due to fund redemptions in times of market turmoil.

The fund-of-hedge-fund managers must have expertise in conducting hedge fund due diligence, must monitor both absolute and relative performance, and are often able to negotiate better redemption or fee terms than individual investors can.

### EXAMPLE 3

#### Tenderledge Investments LLC—Comparing strategies

Tenderledge Investments, a fund-of-funds hedge fund, is benchmarking the performance of various hedge fund strategies as measured by the annual strategy returns over a 10-year period. Focusing on the correlation between various hedge fund strategies, the highest correlation between fund-of-funds returns is with fundamental growth, and the lowest is with short-bias funds.

Correlation between Annual Returns	Fund of Funds
Fundamental Growth	0.91
Convertible Arbitrage	0.89
Distressed/Restructuring	0.84
Multi-Strategy	0.83
Credit	0.74
Equity Market Neutral	−0.02
Macro/CTA	−0.16
Short Bias	−0.84

An increasing number of exchange-traded products, such as ETFs, seek to replicate hedge fund investment *styles* without directly investing in hedge funds themselves. Exhibit 4 shows where these investments are on the liquidity spectrum.

**Exhibit 4: Liquidity Spectrum of Various Investment Alternatives**

These funds can deliver returns similar to many popular hedge fund strategies, such as long/short equity, market neutral, and event driven, through indexing or active management using liquid assets. Effectively, hedge fund replication ETFs seek to generate returns with high correlations with actual hedge fund returns. By relying on quantitative tools, they imitate a broad spectrum of hedge fund returns or a specific style return. However, the returns from these strategies often fall short compared to pure hedge fund strategies because these instruments are publicly traded, are subject to a much heavier regulatory burden, do not impose restrictions on redemptions, and cannot use leverage to the same level. Yet, these investments benefit from greater liquidity, lower fees, and increased transparency than comparable hedge fund or fund-of-funds strategies and seek to match the monthly returns of hedge fund indexes, as in the following example.

**EXAMPLE 4****Tenderledge Investments LLC—Creating a Hedge Fund ETF**

Tenderledge is in the process of developing a hedge fund ETF that would track, before fees and expenses, the performance of a fund-of-hedge-funds strategy index. The strategy index seeks to replicate the risk-adjusted return characteristics of fund of hedge funds and underlying multiple hedge fund investment styles—more specifically, long/short equity, market neutral, event driven, fixed-income arbitrage, and distressed situations.

Although Tenderledge is a fund of hedge funds, the ETF will not invest in any hedge funds but, rather, incorporates Tenderledge's experience and knowledge in selecting hedge fund investments for the incorporation of its flagship fund of funds. Tenderledge will use quantitative tools to select traded debt and equity instruments, as well as certain derivatives, to replicate the performance of these five hedge fund strategies.

This ETF intends to achieve portfolio diversification by tracking the performance of the hedge fund universe and the five styles it seeks to replicate, generating returns with high correlation with these strategies over the long-term. Moreover, by combining these distinct strategies, the ETF will reduce its exposures to traditional sources of risk, such as interest rate volatility and equity market risk factors, such as beta.



## QUESTION SET



1. Choose the correct statements regarding benefits of a master feeder structure.

- A. It allows investors in taxable jurisdictions to invest in an offshore hedge fund without any tax liability.
- B. Pooling funds from offshore and onshore funds creates economies of scale.
- C. This structure allows hedge funds to accept funding from global investors with relative ease.
- D. Many regional regulatory requirements can be avoided by such a structure.

**Solution:**

B, C, and D are correct. A is incorrect because investors in taxable jurisdictions do not avoid taxes by investing in offshore funds.

2. Which of the following statements about SMAs is *least* accurate?

- A. SMAs are a preferred choice of high-net-worth investors with specific investment mandates because they are highly customizable.
- B. SMAs provide better transparency for the investor than other fund structures.
- C. SMAs are characterized by simpler fee structures compared to mutual funds.
- D. The potential for conflicts of interest exists for SMAs since managers are not personally invested in the funds and the regulation requirements are light.

**Solution:**

C is correct because it is the least accurate statement. Mutual fund fees are clearly disclosed in the fund prospectus. The SMA fees are negotiated with the manager for each account and require sufficient care to structure in a way to incentivize the manager. A, B, and D are accurate statements.

3. Which of the following is *least* likely an investor objective that would help inform the choice of a specific fund structure?

- A. Reduce tax leakage and enhance returns by efficient tax planning
- B. Ensure an appropriate amount of investor control on companies and strategies
- C. Maximize the net returns on the investments
- D. Reduce the regulatory and compliance requirements

**Solution:**

C is correct because it is the least likely investor objective. The objective of the investment manager is to maximize the returns, while the objective of

the structure is to ensure that the other options (Choices A, B, and D) are taken care of.

4. Identify the investment structure most appropriate for the investor.

A. Managed futures

B. SMA

C. Fund of funds

1. A high-net-worth investor who requires tax-efficient investment channels with a high degree of control over allocation decisions	
2. An institutional investor intending to invest in commodity markets	
3. A small investor who would like hedge fund exposure at a lower risk	

**Solution:**

1. A high-net-worth investor who requires tax-efficient investment channels with a high degree of control over allocation decisions	B. SMA
2. An institutional investor intending to invest in commodity markets	A. Managed futures
3. A small investor who would like hedge fund exposure at a lower risk	C. Fund of funds

**Choice A:** Managed futures funds are also known as commodity trading advisers because they historically focused on commodity futures. However, CTAs may include investments in a variety of futures, including commodities, equities, fixed income, and foreign exchange.

**Choice B:** An SMA structure allows a customizable portfolio, with investor-specific investment mandates, better transparency, efficient capital allocation, and higher liquidity, over which the investor can exercise enhanced control while keeping fees lower. However, SMAs are operationally more complex and also demand greater governance oversight. That is why these accounts are more appropriate for larger investors.

**Choice C:** Funds of funds allow smaller investors to make investments in hedge funds for which the minimum investment and research requirements would otherwise be overly burdensome. Volatility is generally low in these offerings because the fund-of-funds manager invests in many different hedge funds.

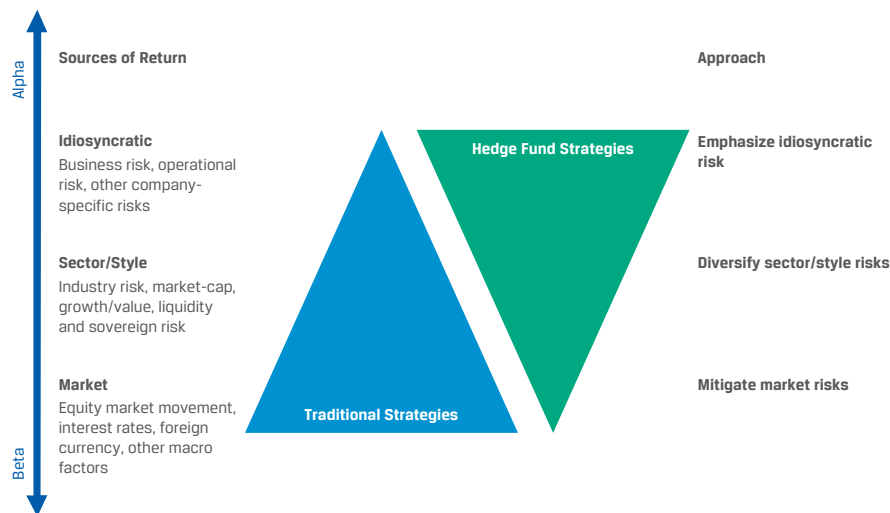
## HEDGE FUND INVESTMENT RISK, RETURN, AND DIVERSIFICATION

4

- analyze sources of risk, return, and diversification among hedge fund investments

Hedge fund portfolios approach return generation differently from traditional portfolios, as Exhibit 5 shows. The most significant difference is that hedge funds seek to limit market exposure and returns from beta and primarily focus on generating idiosyncratic returns by identifying sources of unique return, or alpha. The primary source of hedge fund excess return is market inefficiencies (which may be short lived) and the skills of the manager in leveraging them.

**Exhibit 5: Comparing Traditional and Hedge Fund Portfolio Sources of Return**



Some specific sources of alpha are the manager skills in specific stock selection and utilizing higher-return strategies that minimize risks. The performance of hedge funds can be attributed to three distinct sources:

- Market beta—the broad market beta that can be realized using market index-based funds/ETFs
- Strategy beta—the beta attributed to the investment strategy of the hedge fund applied across the broad market
- Alpha—the manager-specific returns, due to the selection of specific positions

The managers can realize the strategy beta and alpha returns due to their skills in identifying mis-priced securities and sectors, correctly timing the market, and utilizing operational control on the company business model, as well as using leverage to amplify the results.

This does not suggest that the traditional asset pricing models are incorrect. They operate within a set of assumptions—not the least of which is that the markets are efficient. The returns from hedge funds are due to systematic and idiosyncratic alternative risk factors not considered by the model.

Typically, investors do not realize the full returns from the hedge fund. Hedge funds are characterized by higher fees, which reduce the alpha generated by them. Some underperforming hedge funds that close out for performance issues face another problem. The capital redeemed from liquidated positions may result in a lower payout, effectively diminishing the total return from the fund.

To compare the risk-adjusted return characteristics of individual hedge funds and aggregate hedge fund strategies, we use hedge fund indexes that are created using publicly available hedge fund performance data. There are several vendors that create such indexes, and each index has its own features, but there are some general considerations around these indexes that distinguish them from other indexes used for performance comparison. Most hedge fund indexes are based on information reported by the hedge fund managers and others who receive performance information with the right to share that information. It should also be noted that the reporting by any hedge fund is voluntary. This situation introduces several sources of bias and suggests that hedge fund performance is likely overestimated.

Selection bias can be an issue. Individual funds are allocated to strategy peer groups in an inconsistent manner: sometimes based on the prospectus, sometimes based on historical style analysis, sometimes using a combination of approaches, each of which adds and compounds bias. Indexes may also have inconsistent sources of the underlying data. A handful of these indexes operate with transparent and constant inclusion, selection, and exclusion of individual managers and/or their funds. Some index providers impose requirements on AUM, vintage, and whether the hedge fund is open to new investors.

Survivorship bias can also be an issue. When funds that have stopped reporting are removed from the index, the index will likely show better performance. This effect can be mitigated by including the returns of funds of funds that are active and those that have stopped reporting. For instance, a hedge fund manager consistently underperforming a peer group may have limited incentives to continue to highlight underperformance, particularly if the manager is actively sourcing new investors or launching new funds. Hence, the manager may discontinue reporting poor performance, which overestimates reported performance.

Additionally, hedge funds that are closed to new investors or closed because of underperformance are treated equally. Their performance is excluded from the index value. Additionally, the hedge fund performance data are published with a delay, normally a time lag of four weeks or one month. Since these indexes are non-investable and illiquid, replicating their performance may be difficult.

In addition, backfill bias can be an issue. When a successful fund starts reporting performance for the first time, it is very likely that its past performance was stellar, since typically only hedge funds with favorable returns are reported. When such funds are added to an index, the benchmarks will overstate the actual performance. This is a variation of the survivorship bias that results when a new hedge fund is included in a given index and its past performance is “backfilled” into the index’s database. Typically, larger indexes have less backfill bias.

A large majority of hedge fund indexes are not weighted by assets under management. In these hedge fund indexes, each hedge fund receives an equal index weighting in the performance peer group. As a result, comparison of large and small funds can be skewed compared to the performance of a size-weighted index.

## Hedge Fund Investment Risks and Returns

Many traditional investment funds, such as long-only mutual funds and index ETFs, diversify away much of the idiosyncratic risks in their holdings by investing in a large number of stocks and achieve most of their returns primarily by bearing the systematic market risk (beta). Unlike in traditional funds, hedge funds use a variety of instruments across asset classes and techniques in seeking to generate absolute returns in all market environments.

The structure of a hedge fund complicates the benchmarking of the returns on a frequent basis. Due to the high degree of flexibility the managers have over the investments and the minimal level of disclosure (as well as the frequency of disclosure) they offer, it is hard to conduct any performance attribution analysis. Additionally, the relative illiquidity of investments held by the funds makes marking to market a problematic and potentially futile process.

Hence, attributing the sources of returns and risks is a composite process that is further complicated by the complexity of the strategies and compounded using various sources of leverage. That is why risk and return comparisons are typically made to fund-of-funds composite indexes to minimize return distortions. That approach controls for the effects of self-reporting and selection biases. Furthermore, it ensures that the fund-of-funds benchmark index is investable.

Nonetheless, over the long term (since 1990), hedge funds have enjoyed higher returns (at least prior to fees and expenses) than either stocks or bonds and a standard deviation nearly identical to that of bonds. Specifically, Exhibit 6 compares the returns, risk, and performance measures for the HFRI Fund of Funds Composite Index, the MSCI ACWI Index, and the Bloomberg Barclays Global Aggregate Index. The HFRI Fund of Funds Composite Index is an equally weighted performance index of funds of hedge funds included in the HFR Database.

As mentioned earlier, hedge fund indexes typically build on data that each hedge fund reports. Because these data are self-reported, there is a degree of survivorship bias that the HFRI Fund of Funds Composite Index mitigates by reflecting the actual performance of portfolios of hedge funds. The measures shown here may reflect a lower reported return because of the added layer of fees, but they likely represent a fairer, more conservative, and more accurate estimate of average hedge fund performance than HFR's composite index of individual funds. The returns are likely biased toward equity long/short funds since these are frequently a substantial portion of funds of funds' allocation mix. There are also other hedge fund indexes, allowing a more thorough analysis of the source of returns.

As shown in Exhibit 6, over the 25-year period between 1990 and 2014, hedge funds had higher returns than either stocks or bonds and a standard deviation almost identical to that of bonds. This, however, depends on the benchmarks used, which typically are broad market benchmarks. In the 1990-2014 period, hedge funds had modest overall correlation with global stock returns (0.56) and negligible correlation with global bond returns (0.07) over this 25-year period. Hedge funds certainly offered added value to institutional investors as a portfolio diversification agent in this time period.

### Exhibit 6: Historical Risk–Return Characteristics of Hedge Funds and Other Investments, 1990–2019

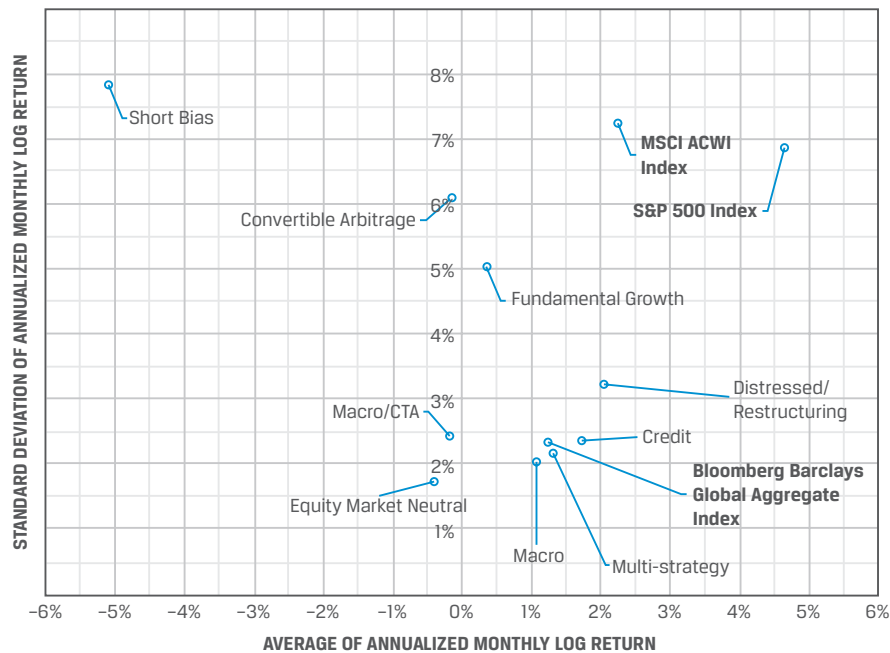
	FoF	Global Stocks	Global Bonds
Annualized return			
1990–2014	7.2%	6.9%	6.3%

	FoF	Global Stocks	Global Bonds
2015–2019	2.48%	10.41%	2.42%
Annualized volatility			
1990–2014	6.0%	16.5%	5.8%
2015–2019	1.08%	11.42%	1.33%
FoF correlation (avg. monthly)			
1990–2014		0.56	0.07
2015–2019		0.86	0.03
Percentage of positive months	69.3%	61.3%	62.7%
Best month	6.8%	11.9%	6.2%
Worst month	–7.5%	–19.8%	–3.8%
Worst drawdown	–22.2%	–54.6%	–10.1%

*Sources:* Fund-of-funds (FoF) data are from the HFRI Fund of Funds Composite Index; global stock data are from the MSCI ACWI Index; global bond data are from the Bloomberg Barclays Global Aggregate Index.

Notably, Exhibit 6 shows that for the subsequent five-year period between 2015 and 2019, the absolute return of funds of funds relative, in particular, to global equities has declined, while their performance correlation with equity markets actually increased. This trend has made hedge fund allocations arguably less useful and also somewhat less popular. But some allocators have continued to find value to maintain or actually increase their allocation to a mix of hedge funds as a bond market substitute in their overall portfolio building.

Looking at the performance of various hedge fund strategies over time, an approach is to examine the relationship between hedge fund returns and the risk measured by the standard deviation of returns. Exhibit 7 shows the log returns and standard deviation of returns for selected hedge fund strategies on an annualized monthly basis as reported by Hedge Fund Research. The data cover the period from January 2008 to November 2021, a period of almost 14 years.

**Exhibit 7: Return and Risk of Selected Hedge Fund Strategies, 2008–2021**

Source: Hedge Fund Research (HFR), annualized monthly log returns.

Some of the hedge fund strategies generated negative returns on average. Short-bias strategies underperformed all the other strategies in terms of both return and risk measured by standard deviation. For comparison, the S&P 500, MSCI ACWI, and Bloomberg Barclays Global Aggregate Index returns are also plotted to show that many strategies have widely varying risk/return characteristics compared to common equity and fixed income benchmarks.

Another approach to analyze the benefits from investing in hedge funds looks at the risk/return trade-off measured by the coefficient of variation of annual hedge fund returns. The coefficient of variation can be thought as the price of return in terms of risk or the relative return adjusted for risk: A higher coefficient of variation provides greater return for the same amount of risk.

## Diversification Benefits of Hedge Fund Investments

The original hedge fund strategy was a market-neutral strategy in which the long and short positions made the portfolio beta neutral and generated a positive alpha. Stated differently, hedge funds initially kept their portfolios' net market sensitivity exposure to the market benchmark index at zero. By holding long and short positions, they sought to create value for their investors with minimal exposure to the market.

Over time, hedge fund investments have evolved from these equity hedge strategies with little or no equity market correlation to complex strategies across various asset classes. While hedge fund strategies vary widely, market-neutral, relative value, and event-driven strategies tend to outperform equity markets during market downturns and when individual stock correlations fall and tend to have weaker performance when correlations are high (reducing relative value opportunities) and equities move uniformly higher.

The diversification benefits of hedge funds first came to prominence with the dot-com bubble unwinding in 2000–2002, when they generally performed well compared with traditional long-only investment products. Starting with funds of funds, institutional investors increased their hedge fund exposure and then expanded it through direct allocations following the 2008 financial crash. Seeking better diversification and risk mitigation, despite high hedge fund fees, the investors sought absolute and uncorrelated risk-adjusted returns rather than outsized upside performance. In the early 2000s, when hedge funds came to prominence, the primary drivers were the absolute excess returns, regardless of market direction. The verification efforts from investors on hedge fund operations were minimal. The 2008 crisis (including Lehman Brothers' bankruptcy and the Bernie Madoff fraud) brought to light the need for additional due diligence. The correction after the 2008 crisis brought several changes. The inflow from high-quality institutional investors and pension funds increased. Such investors demanded additional transparency and control over the fund's investment process, fund operations, leverage applied, and managers' performance. This change has benefited only the highest-quality funds, and most other hedge funds failed to keep pace with the positive equity and bond market advances of 2009–2019. They continue to have a place in institutional asset allocations because of their risk-diversification properties.

Exhibit 8 shows return relationships between hedge funds and US stock and bond indexes from 2001 to 2021. Although diversification is typically used to reduce idiosyncratic risk, diversification using hedge fund investments seeks to add idiosyncratic outperformance.

While hedge fund risk diversification benefits can merit investigation, experience also reflects that their prospective advantage can vary over time. Investors need to be thorough in conducting due diligence when selecting a hedge fund manager. Hedge fund performance has a very low correlation with that of traditional asset classes, such as investment bonds and currencies/cash. For traditional investors in these asset classes, hedge funds provide an enhanced opportunity for portfolio diversification and for generating consistent returns over time. Adding hedge funds to a traditional 60/40 portfolio typically decreases the total portfolio standard deviation and increases the Sharpe ratio, thus enhancing portfolio diversification and risk-adjusted return.

#### Exhibit 8: Historical Hedge Fund Monthly Return Correlations, 2001–2021

	Hedge Funds	S&P 500	Investment-Grade Bonds
Hedge Funds	1	0.82	0.10
S&P 500		1	–0.06
Investment-Grade Bonds			1

Notes: Hedge funds = Dow Jones Credit Suisse Hedge Fund Index. Investment-grade bonds = Bloomberg Barclays US Aggregate Bond Index.

As Exhibit 8 shows, hedge funds have a fairly high correlation with equities and a low correlation with investment-grade bonds.

#### QUESTION SET



1. Selecting investment trades with high idiosyncratic risk is *most likely* associated with the management of which type of fund?
  - A. Equity index ETF



**B.** Long-only equity mutual fund

**C.** Long/short equity hedge fund

**Solution:**

C is correct. Long/short equity hedge funds are the most likely type of funds to seek out trades with high idiosyncratic risk, so their performance is driven primarily by stock selection and less by other factors, such as sector and market exposures. B is incorrect because long-only mutual funds are more likely to seek returns from market risk and sector risks with less emphasis on undiversifiable, security-specific risks. A is incorrect because equity index funds do not seek out idiosyncratic risk and simply try to mimic the performance of an index.

2. Hedge funds are not obligated to report their performance to the public. Many do, however, to attract additional investors. Hedge fund benchmarks and indexes are created using the reported data. Which of the following is *least likely* a bias that hedge fund investors should be aware of when evaluating the performance of hedge fund benchmarks and indexes?

**A.** Survivorship bias: Unsuccessful and liquidated hedge funds are removed from an index, resulting in their underperformance not being accounted for in the index.

**B.** Self-reporting bias: Hedge funds have the freedom to not report their performance, and only the ones that report are included in an index.

**C.** Societal bias: This relates to the perception of hedge funds at a societal level, where hedge funds that are perceived to be high performing are included in the index.

**Solution:**

C is correct. Societal bias is not one of the biases that hedge fund investors need to be aware of when evaluating hedge fund performance. A and B are biases that distort the performance of hedge fund benchmarks and indexes.

3. An investor wants to invest in a diversified hedge fund that minimizes the return correlation with the traditional asset classes but would prefer the fund to be more liquid and transparent while minimizing the leverage obtained by borrowing or shorting. What would be the *most* appropriate hedge fund the investor can choose?

**A.** Fundamental value

**B.** Managed futures

**C.** Multi-strategy

**D.** Fund of funds

**Solution:**

B is correct. Managed futures have, historically, exhibited low correlation with traditional assets and invest in active futures in liquid commodities and foreign exchange markets. They are also able to increase exposure without resorting to borrowing or shorting.

## PRACTICE PROBLEMS

1. Choose the *false* statement about hedge funds:
  - A. While traditional fund managers charge mainly a management fee, hedge fund managers charge both a management fee and a performance-based incentive fee.
  - B. Redemption of funds from a hedge fund prior to its liquidation is very hard, due to the initial lockup period and specific terms of redemption.
  - C. Individual retail investors cannot invest in hedge funds, and only institutional investors can.
2. A fundamental long/short hedge fund manager is evaluating specific securities to build a portfolio's positions. Which of the following is the strategy the manager would *least likely* adopt?
  - A. Long securities that have an upside potential relative to current price
  - B. Short sectors with macro trends negatively impacting the company
  - C. Long securities that trade at a significant discount, expecting an increased valuation in case of a bankruptcy
3. Which of the following fund structures is most likely to be suitable for an institutional hedge fund investor that seeks a highly customizable offering with negotiable fees?
  - A. Separately managed hedge funds
  - B. Commingled hedge funds (master feeder funds)
  - C. Mutual funds
  - D. Funds of funds
4. Which of the following parties is responsible for the portfolio management of a fund of hedge funds?
  - A. Accredited investor
  - B. General partner
  - C. Limited partner
5. Which of the following best explains why it is unlikely a poor-performing hedge fund would be added to an index?
  - A. Survivorship bias
  - B. Backfill bias
  - C. Selection bias
6. The strategy that identifies opportunities for future merger, bankruptcy, or

spin-offs and seeks profit from pricing inefficiencies is known as:

- A. event driven.
- B. relative value.
- C. opportunistic.

7. A money manager was reviewing the automobile sector and identified that the stock of General Motors (GM) is relatively overvalued compared to Ford. A money manager purchases 100 shares of Ford and shorts 150 shares of GM. It turned out that the manager's perception was right. At the end of the quarter, she unwinds both positions, making a profit on both positions. What is this strategy called?

- A. Equity long/short
- B. Event driven
- C. Relative value

## SOLUTIONS

1. C is correct because it is a false statement. Accredited retail investors can invest in various types of hedge funds. A and B are true.
2. C is correct. Participating in a potential bankruptcy situation would be characteristic of an event-driven hedge fund manager and not a fundamental long/short manager. B is incorrect because a fundamental long/short manager would invest in securities expected to exhibit high growth and capital appreciation. C is incorrect because a fundamental long/short manager would short securities in sectors that project negative growth.
3. A is correct. A hedge fund SMA can be tailored to a single investor seeking a tailored portfolio with negotiated fees. The other fund structures do not generally offer customization or negotiable fees.
4. B is correct. The general partner is responsible for choosing the hedge funds in a fund of hedge funds. A and C are incorrect because they refer to customers or investors in the fund.
5. C is correct. Selection bias refers to when the benchmark inclusion criteria cover only those funds that have good performance and hence report their performance to attract new investors. A is incorrect; survivorship bias is when the benchmark stops including funds that have ceased operations, most likely due to poor performance, and hence does not fully represent the hedge fund universe. B is incorrect; backfill bias occurs when an index retroactively includes the performance of a fund before it is added to the index.
6. A is correct. Event-driven strategies include mergers, bankruptcies, and spin-offs. Relative value strategies (Choice B) seek to profit from a price or return discrepancy between securities based on a short-term relationship. Opportunistic strategies (Choice C) use managed futures and macro strategies.
7. A is correct. Equity long/short strategies purchase undervalued stocks and sell short overvalued stocks. Event-driven managers (Choice B) participate in such events as mergers and spin-offs. Relative value managers (Choice C) seek to arbitrage values between related securities.

## LEARNING MODULE

## 7

## Introduction to Digital Assets

## LEARNING OUTCOMES

<i>Mastery</i>	<i>The candidate should be able to:</i>
<input type="checkbox"/>	describe financial applications of distributed ledger technology
<input type="checkbox"/>	explain investment features of digital assets and contrast them with other asset classes
<input type="checkbox"/>	describe investment forms and vehicles used in digital asset investments
<input type="checkbox"/>	analyze sources of risk, return, and diversification among digital asset investments

## INTRODUCTION

## 1

As introduced in Alternatives Learning Module 1, digital assets are a relatively new investment class that covers assets that can be created, stored, and transmitted electronically and have associated ownership or use rights. This class includes a wide variety of digital assets, including cryptocurrencies, tokens, and digital collectables. Based on the innovative distributed ledger technology (DLT), or blockchain technology, digital assets utilize advanced encryption techniques that assure the authenticity of digital assets. While cryptocurrencies have their own blockchains, crypto-tokens are built on an existing blockchain.

As with other types of alternative investments, digital assets have characteristics distinct from traditional investments. Since the advent of Bitcoin in 2009, when it was a niche concept in the technology world, digital assets have gradually become more mainstream alternative investments for investors worldwide. We will give an overview of the common forms of digital asset investments and discuss the key concepts and pitfalls in this space.

Despite their special features and technological characteristics, digital assets offer investors diversification while providing higher expected returns than traditional investments provide. However, their risks are also higher. We will discuss the sources of risks in digital assets to provide a better understanding of this evolving asset class.

**LEARNING MODULE OVERVIEW**

- Blockchain and distributed ledger technology might offer a new way to store, record, and track digital assets on a secure, distributed basis. Additionally, DLT could bring efficiencies to post-trade and compliance processes through automation, smart contracts, and identity verification.
- DLT can take the form of either permissionless or permissioned networks.
- A consensus protocol is a set of rules that govern how blocks are cryptographically chained together in a blockchain network for the verification of the complete and immutable history of transaction records. Two broad types of consensus protocols are “proof of work” (PoW) and “proof of stake” (PoS).
- Digital assets are frequently seen as an alternative asset class. As digital assets become more developed, institutional investors may continue to seek some exposure to these assets for their higher returns and possible diversification benefits.
- Digital assets differ from traditional financial assets in terms of their inherent value, transaction validation approach, uses as a legal medium of exchange, and legal and regulatory protection.
- The most common digital assets are cryptocurrencies, including Bitcoin and altcoins (including stablecoins and meme coins). There are also digitalized tokens that include non-fungible tokens, security tokens, utility tokens, and governance tokens.
- Many cryptocurrencies are designed with self-imposed limits on the total supply through complex computer algorithms. Such limits could help maintain a value from a technical perspective, yet there is no economic consensus on how they should be valued. Cryptocurrency exchanges are classified into centralized exchanges and decentralized exchanges. Both centralized and decentralized exchanges face problems with fraud and manipulation because they are not subject to rigorous oversight and are generally not regulated as financial exchanges.
- Investment in digital assets can take the form of direct ownership of cryptocurrencies and other digital assets on the blockchains or indirect investment in exchange-traded products, hedge funds, trusts, futures, and thematic stocks.
- Asset-backed tokens are digital claims on physical assets, financial assets, or financial instruments and are collateralized by these underlying assets.
- The push for financial decentralized applications based on open-source codes and smart contracts has grown into a movement known as decentralized finance, or DeFi. DeFi seeks to design, combine, and develop decentralized financial applications as building blocks for sophisticated financial products and services.
- The price of Bitcoin and other digital assets are driven by expectations on future asset appreciation rather than any underlying cash flow. The market demand for the limited supply of cryptocurrencies is a significant driver of prices.

- The performance of the first widely traded digital asset, Bitcoin, has been characterized by high return, high volatility, and low correlations with traditional asset classes.
- Due to the historically low correlations with other asset classes, digital assets offer potential diversification benefits to a well-diversified portfolio. But the correlations are observed to have risen, especially during periods of high market uncertainty.

### LEARNING MODULE SELF-ASSESSMENT



These initial questions are intended to help you gauge your current level of understanding of this learning module.

1. The consensus protocol on a distributed ledger technology network refers to:
  - A. the standardized approach that governs how digital assets generated from a blockchain network should be valued.
  - B. programs embedded in electronic transaction records that are coded to self-execute according to predetermined conditions.
  - C. the set of rules governing how blocks are cryptographically linked to the chain to become immutable on the distributed ledger network.

**Solution:**

The correct answer is C. When transactions enter a node of the distributed ledger, they are bundled into “blocks” and cryptographically “chained” together to facilitate verification of the prior history. How blocks are chained together is determined by the consensus protocol, a set of rules governing how blocks can join the chain and become the immutable “truth.” The consensus protocols are designed to resist attempts at malicious manipulation up to a certain level of security.

2. Cryptocurrencies are a common form of digital asset that:
  - A. can be mined only through “proof of work” on blockchain networks.
  - B. exist both in physical forms and electronic records with rights to use, buy, or sell by owners.
  - C. can be used to transfer or store value, which allows time-efficient transactions between parties without the need for an intermediary.

**Solution:**

The correct answer is C. Cryptocurrencies are used to transfer or store value, which allows near-real-time transactions between parties without the need for an intermediary. As electronic mediums of exchange, cryptocurrencies lack physical form and exist only as electronic records on distributed ledgers. Depending on the particular consensus protocols used, cryptocurrencies can be generated either by “proof of work” or “proof of stake” by miners or validators on networks.

3. Digital assets differ from traditional financial securities in that:
  - A. digital assets are subject to a broader legal protection framework.
  - B. digital assets are not valued based on their expected future cash flow.

**C.** transaction records of digital assets require a centralized intermediary.

**Solution:**

The correct answer is B. Most digital assets do not have a fundamental value based on underlying assets or on the potential cash flow or earnings they are expected to generate. To date, the legal and regulatory frameworks for digital assets are still evolving, and there is generally less legal protection compared to traditional financial securities. Digital asset transactions are recorded on a distributed ledger, and no centralized intermediary is required in the process.

4. A special type of cryptocurrency that is backed by and pegged to a fiat currency is called:

- A.** altcoin.
- B.** stablecoin.
- C.** meme coin.

**Solution:**

The correct answer is B. A stablecoin is designed to maintain a stable value by pegging its value to another asset and is collateralized by a basket of assets, typically a fiat currency, precious metals, or other cryptocurrencies.

5. An indirect investment in digital assets can be made through:

- A.** entering into cryptocurrency futures.
- B.** purchasing Bitcoins on cryptocurrency wallets.
- C.** participating in the initial coin offering of a new digital token.

**Solution:**

The correct answer is A. Investors interested in an indirect investment in digital assets can trade cryptocurrency futures on established exchanges, such as the Chicago Mercantile Exchange. However, the purchase of Bitcoins and buying new tokens through an initial coin offering are direct forms of investment.

6. A cryptocurrency ETF seeks to:

- A.** gain exposure to cryptocurrencies through cash and cryptocurrency derivatives.
- B.** replicate digital asset investment returns by investing directly in cryptocurrencies.
- C.** gain exposure to the cryptocurrency theme by investing in public equities related to the digital asset sector.

**Solution:**

The correct answer is A. A cryptocurrency ETF seeks to replicate digital asset investment returns by cash and cryptocurrency derivatives.

7. Bitcoin as an alternative investment has historically exhibited:

- A.** low return, low risk, and high correlations with traditional assets.
- B.** high return, high risk, and low correlations with traditional assets.



C. high return, high risk, and high correlations with traditional assets.

**Solution:**

The correct answer is B. Despite no future cash flow generation, Bitcoin as an alternative investment has been characterized by high return, high volatility, and low correlations with traditional asset classes.

8. Early investors in cryptocurrencies have enjoyed significant price appreciation but many later-stage investors suffered huge losses most likely because:

- A. blockchain technology favors early-stage investors.
- B. there are substantial price fluctuations in cryptocurrencies.
- C. cryptocurrencies have increased their appeal to institutional investors.

**Solution:**

The correct answer is B. Because cryptocurrencies are a relatively new innovation, their market is subject to rapid price swings, changes, and uncertainty. Their prices have been extremely volatile over the years, and price drawdowns were often much more substantial than for traditional asset classes. As a result, later-stage investors may suffer huge losses depending on their points of entry and exit.

## DISTRIBUTED LEDGER TECHNOLOGY

## 2



describe financial applications of distributed ledger technology

**Distributed ledger technology (DLT)** based on a distributed ledger (defined later) represents a technological development and offers potential improvements to delivering financing services and financial record keeping. DLT networks are being considered as a means to create, exchange, and track ownership of financial assets on a peer-to-peer (P2P) basis. Potential benefits of using this technology include greater accuracy, transparency, and security in record keeping; faster transfer of ownership; and P2P interactions. However, the technology is not fully secure, and breaches in privacy and data protection are possible. Additionally, the computational processes underlying DLT generally require massive amounts of energy to verify transaction activity.

A **distributed ledger** is a type of database that can be shared among potentially infinite numbers of entities in a network. In a distributed ledger, entries are recorded, stored, and distributed across a network of participants so that each participating entity has a matching copy of the digital database, making each copy of the database a verified record of all current and previous transactions. Basic elements of a DLT network include a digital ledger, a consensus mechanism used to confirm new entries, and a participant network.

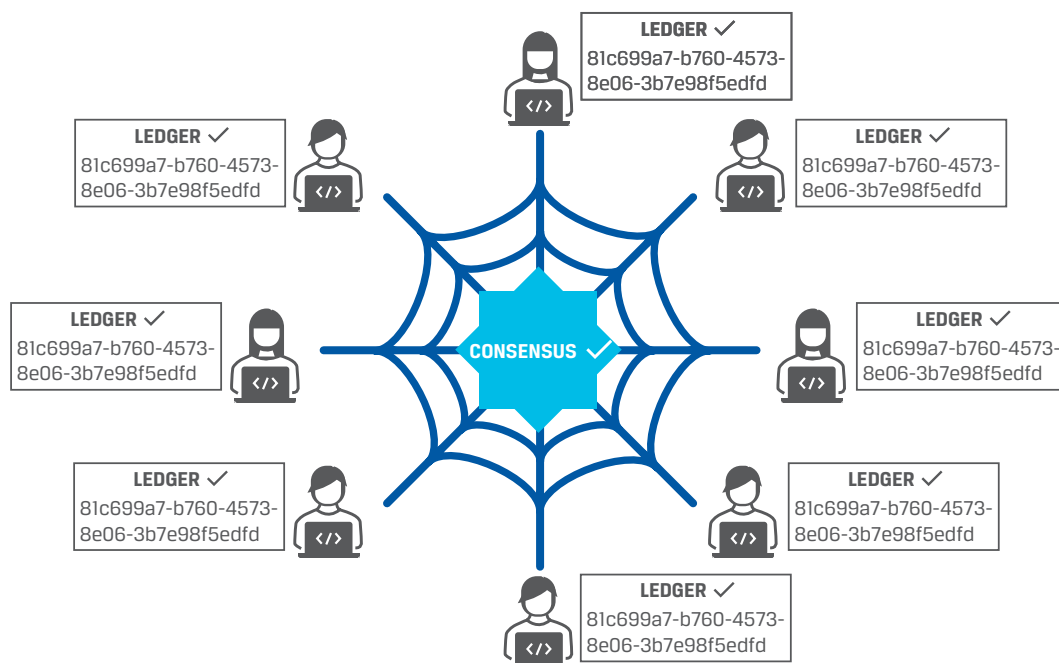
The consensus mechanism is the process by which the computer entities (or nodes) in a network agree on a common state of the ledger. Consensus generally involves two steps: transaction validation and agreement on ledger update by network parties. These features enable the creation of records that are, for the most part, considered immutable, or unchangeable, yet they are transparent and accessible to network participants on a near-real-time basis. There are various approaches to establishing consensus.

Features of DLT include the use of **cryptography**—an algorithmic process to encrypt data, making the data unusable if received by unauthorized parties—which enables a high level of network security and database integrity. For example, DLT uses cryptographic methods of proof to verify network participant identity and for data encryption.

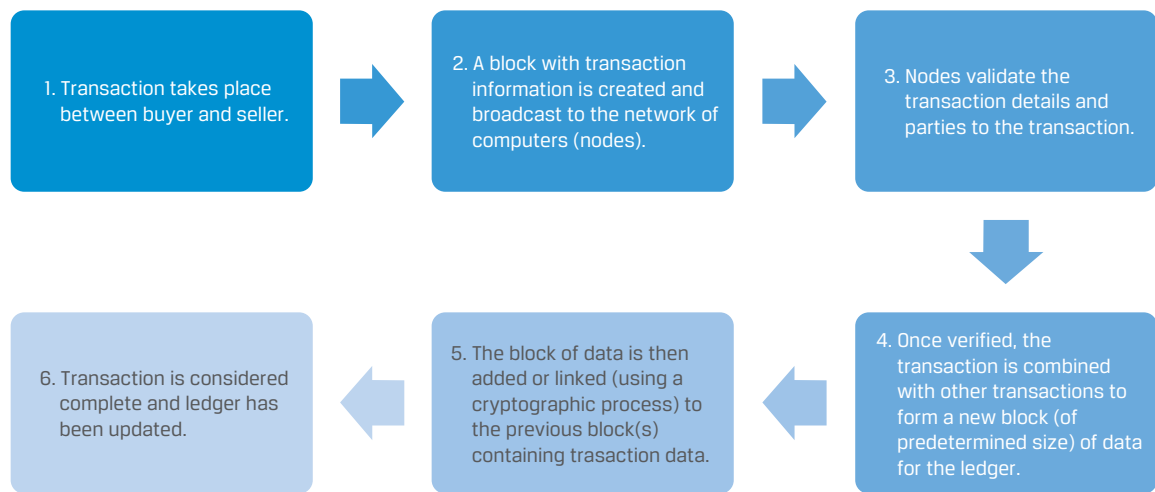
DLT has the potential to accommodate “**smart contracts**,” which are computer programs that self-execute on the basis of pre-specified terms and conditions agreed to by the parties to a contract. Examples of smart contract use are the automatic execution of contingent claims for derivatives and the instantaneous transfer of collateral in the event of default.

Exhibit 1 illustrates a distributed ledger network in which all participants (nodes) are connected to one another, each having a copy of the distributed ledger. The term “consensus” is shown in the center of the network and represents the consensus mechanism in which the nodes agree on new transactions and ledger updates.

**Exhibit 1: Distributed Ledger Network Setup**



**Blockchain** is a type of digital ledger in which information, such as changes in ownership, is recorded sequentially within blocks that are then linked or “chained” together and secured using cryptographic methods. The steps outlining adding a new transaction to the network are outlined in Exhibit 2. There are several key steps a transaction must go through before it is added to the blockchain.

**Exhibit 2: Adding a Transaction to the Blockchain**

Each block contains a grouping of transactions (or entries) and a secure link (known as a hash) to the previous block. New transactions are inserted into the chain only after validation via a consensus mechanism in which authorized members agree on the transaction and the preceding order, or history, in which previous transactions have occurred.

### Proof of Work vs. Proof of Stake

Fundamentally, blockchains are software protocols that enable many parties to interact under common assumptions and knowledge without having to trust each other. When transactions enter into a node of the blockchain, they are bundled into “blocks” and cryptographically “chained” together to facilitate verification of the prior history. How blocks are chained together is determined by the **consensus protocol**, a set of rules governing how blocks can join the chain and become the immutable “truth.” The consensus protocols are designed to resist attempts at malicious manipulation up to a certain level of security. We distinguish between two main types of protocols.

#### *The Proof of Work (PoW) Protocol*

The proof of work protocol determines which specific block to add through a computationally costly lottery. The PoW consensus mechanism used to verify a transaction involves a cryptographic problem that must be solved by some computers on the network (known as miners) each time a transaction takes place. **Miners** use powerful computers and significant amounts of energy to solve complex algorithm puzzles to validate and lock blocks of transactions into the blockchain, earning cryptocurrency for themselves in the process. The “proof of work” consensus process to update the blockchain can require substantial amounts of computing power, making it very difficult and extremely expensive for an individual third party to manipulate historical data. To manipulate historical data, an individual or entity would have to control most nodes in the network. The success of the network, therefore, relies on broad network participation.

Participants on the network agree that the *longest chain* of blocks is the only truthful representation of all previous transactions. If a malicious attacker wanted to make a longer chain including fraudulent transactions, it must outperform the entire network’s computational power, which theoretically requires most of the network’s

available computational power. This is the famous 51% attack threshold and serves as the boundary of the PoW protocol's security standards. Amassing this much network power on the most widely used blockchains behind cryptocurrencies, such as Bitcoin or Ethereum, is extremely difficult for any individual, organization, or group of coordinated actors. Even if most of the network's computational power can be controlled momentarily, the length of block history that can be altered is limited by the duration of this control. This has been the most widely used consensus protocol for digital assets.

### *The Proof of Stake (PoS) Protocol*

This protocol requires selected participants on the networks, the validators, to pledge capital to vouch for the block's validity. This stake signals to the network that a validator is available to verify the veracity of a transaction and propose a block. A majority of the other validators, who similarly staked a digital asset to the network, must then attest to the validity of proposed block. Validators benefit from both proposing and attesting to the validity of blocks that have been proposed by other participants in a similar staking process. The boundary of the PoS protocol's security standards builds on a group of stakers (and their pledged stake) controlling the network's computational power and protecting access from malicious parties gaining a majority.

Under both consensus protocols, the validation of the transactions comes with rewards: mining. Successful miners that validate the transaction obtain a new digital asset, whether it is a cryptocurrency or a token. The rate of mining and the speed at which new transactions on the network are validated generally control the amount of a new digital asset that can be created on a specific digital ledger network.

## Permissioned and Permissionless Networks

DLT can take the form of either permissionless or permissioned networks. **Permissionless networks** are open to any user who wishes to make a transaction, and all users within the network can see all transactions that exist on the blockchain. In a permissionless, or open, DLT system, any network participant can perform all network functions.

The main benefit of a permissionless network is that it does not depend on a centralized authority to confirm or deny the validity of transactions, because this takes place through the chosen consensus mechanism. This means no single point of failure exists because all transactions are recorded on a single distributed database and every node stores a copy of the database. Once a transaction has been added to the blockchain, it cannot be changed, barring manipulation; the distributed ledger becomes a permanent and immutable record of all previous transactions. In a permissionless network, trust between transacting parties is not a requirement.

**Bitcoin** is a well-known use of an open permissionless network. Bitcoin was created in 2009 to serve as the public ledger for all transactions occurring on its virtual currency. Since the introduction of Bitcoin, many more cryptocurrencies, or digital currencies, which use permissionless DLT networks, have been created.

In **permissioned networks**, network members might be restricted from participating in certain network activities. Controls, or permissions, can be used to allow varying levels of access to the ledger, from adding transactions (e.g., a participant) to viewing transactions only (e.g., a regulator) to viewing selective details of the transactions but not the full record. Exhibit 3 compares the salient features of permissioned and permissionless blockchains.

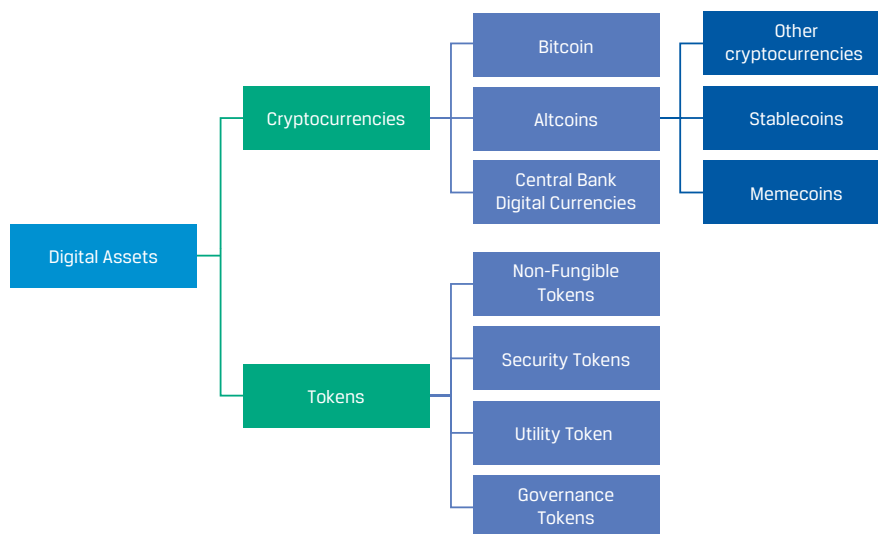
**Exhibit 3: Features of Permissioned and Permissionless Blockchains**

	Permissioned blockchain	Permissionless blockchain
<b>Speed</b>	Faster as only a limited number of members participate or are authorized to validate transactions	Slower as a large number of members have to reach consensus, which decreases network speed and scalability
<b>Cost</b>	Cost-effective as few members are required to validate each transaction	Not cost-effective as many members are required to validate each transaction
<b>Decentralization</b>	Partially decentralized as there are a limited number of members in the chain	Decentralized as all members can access the network
<b>Access</b>	Membership is limited.	Membership is unlimited.
<b>Governance</b>	The governance is determined by a centralized organization.	The governance is decentralized and is maintained by the members.

Based on their decentralized nature, permissionless distributed ledgers are fully open to all, but that high level of decentralization usually requires more processing power and bandwidth, making them less cost-effective. The opposite holds true for permissioned blockchains. Governance and transaction verification are two other aspects that are specific to the type of blockchain.

## Types of Digital Assets

Potential applications of DLT to financial services and investment management include the creation of digital assets, such as cryptocurrencies, and the process to deliver financial services more efficiently, such as processes tokenization, post-trade clearing and settlement, and compliance. Exhibit 4 summarizes different digital assets.

**Exhibit 4: Different Digital Assets**

### **Cryptocurrencies**

Digital assets are assets that exist only as an electronic record with rights to use, buy, or sell. They can be securities, currencies, properties, or commodities. The most common digital assets are units used to transfer or store value, referred to as **cryptocurrency** and digital currency, which allow near-real-time transactions between parties without the need for an intermediary.

As electronic mediums of exchange, cryptocurrencies lack physical form and are issued privately by individuals, companies, and other organizations. Hence, they do not benefit from the backing of a central bank or a monetary authority. Most issued cryptocurrencies utilize open DLT systems in which a decentralized distributed ledger is used to record and verify all digital currency transactions. Several different types of cryptocurrency exist, such as altcoins and stablecoins.

Many cryptocurrencies have a self-imposed limit on the total amount of currency they may issue. Although such limits could help maintain their store of value, it is important to note that many cryptocurrencies have experienced high levels of price volatility. There is no consensus on how to value cryptocurrencies, and the apparent lack of clear fundamentals underlying these currencies has contributed to their volatility.

Cryptocurrencies are not government backed or regulated. Central banks around the world, however, are recognizing potential benefits and examining use cases for their own cryptocurrency versions as an alternative to physical currency. Such **central bank digital currencies (CBDCs)** are typically designed as a tokenized version of the currency issued by the central bank (“fiat currency”)—essentially a digital bank note or coin.

### **Tokens**

Transactions involving physical assets, such as real estate, luxury goods, and commodities, often require substantial efforts in ownership verification and examination each time a transfer in ownership takes place. Through **tokenization**, the process of representing ownership rights to physical assets on a blockchain or distributed ledger, DLT has the potential to streamline this process by creating a single, digital record of ownership with which to verify ownership title and authenticity, including all historical activity.

Another form of digital assets is the **non-fungible token (NFT)**. An NFT links digital assets to certificates of authenticity using blockchain technology. NFTs differ from “fungible” tokens, such as cryptocurrencies, because each token and the authenticated object it represents is unique. As such, they uniquely “stamp” assets and can represent digital assets in a virtual world. The most common use for NFTs is trading in digital artwork.

**Security tokens** digitize the ownership rights associated with publicly traded securities. The custody of these security tokens can be stored on a blockchain, which increases the efficiency of post-trade processing, settlement, record keeping, and custody. Such a single ledger solution would eliminate the need for reconciliation and validation of transactions and allow participants to perform various transactions more easily and with greater transparency.

An example of security tokens is **initial coin offering (ICO)**, an unregulated process whereby companies sell their crypto-tokens to investors in exchange for money or for another agreed upon cryptocurrency. An ICO is typically structured to issue digital tokens to investors that can be used to purchase future products or services being developed by the issuer. ICOs provide an alternative to traditional, regulated capital-raising processes, such as IPOs. Compared to the regulated IPO market, ICOs might have lower associated issuance costs and shorter capital-raising time frames. However, most ICOs do not typically have attached voting rights. Regulation for ICOs is under consideration in several jurisdictions, and numerous instances of investor loss have resulted from fraudulent schemes.

There are also **utility tokens** that provide services within a network, such as pay for services and network fees. While security tokens may pay out dividends, utility tokens only compensate for activities on the network.

**Governance tokens** are important in permissionless networks. They serve as votes to determine how particular networks are run. For instance, when there is a technical problem on a permissionless blockchain network, it is the owners of governance tokens who can vote on a solution or any major changes to maintain the stability and integrity of the network.

### QUESTION SET



1. Which of the following is *not* a potential benefit of distributed ledger technology?

- A. Facilitation of smart contracts
- B. Energy-efficient way of record keeping
- C. Immutable and secure transaction records

**Solution:**

The correct answer is B. Distributed ledger technology provides greater accuracy, transparency, and security in record keeping; enables faster transfer of ownership; and enables peer-to-peer interactions. A DLT network relies on certain consensus mechanisms in which all nodes that are connected on the network agree on new transactions and ledger updates. Once verified by all the nodes of the network, the transaction ledger is immutable and is kept by each of the nodes. However, the transaction validation process requires material computational power of all the miners on the network, especially in the case of proof-of-work consensus protocol.

2. Identify the following statement as true or false: Prices of cryptocurrencies can be volatile because there are finite limits on the total amount of currencies that may be issued.

**Solution:**

False. Prices of cryptocurrencies have been volatile because there is no consensus on how to value cryptocurrencies. The apparent lack of clear fundamentals underlying these currencies has contributed to their volatility.

3. The process where a node on a blockchain network pledges its digital asset to verify a new block's validity is called:

- A. tokenization.
- B. proof of work.
- C. proof of stake.

**Solution:**

The correct answer is C. This proof-of-stake protocol requires selected participants on a blockchain network, the validators, to pledge digital assets to vouch for the block's validity. This stake signals to the network that a validator is available to verify the veracity of a transaction and propose a block. Other validators who stake a digital asset to the network must then attest to the validity of proposed block. Validators benefit from both proposing and attesting to the validity of blocks that have been proposed by other participants in a similar staking process in the form of new digital assets.

4. Identify the various types of digital assets according to their general classifications:

A. Cryptocurrencies	1. ICO
B. Tokens	2. NFT
	3. CBDC
	4. Ether
	5. Stablecoin
	6. Utility token

**Solution:**

A. Cryptocurrencies	3. CBDC
	4. Ether
	5. Stablecoin
B. Tokens	1. ICO
	2. NFT
	6. Utility token

### 3

## DIGITAL ASSET INVESTMENT FEATURES



explain investment features of digital assets and contrast them with other asset classes

Digital assets have a growing presence in the financial services industry and have become a more prevalent asset class for investors. Most digital assets are cryptocurrencies, and their growth has been spectacular. While there were around 70 cryptocurrencies recorded in 2013, by early 2022, there were close to 10,000 different cryptocurrencies issued by corporations, organizations, and in many cases, individuals.

Cryptocurrency investment is typically viewed as an alternative asset. As digital assets increase in prominence, some institutional investors selectively seek exposure to these assets for their potential higher returns and possible diversification benefits. When institutional investors become more comfortable with investing in digital assets, it may be a very early indication that the market may be close to reaching a tipping point. As more institutional investors are becoming comfortable with the idea of owning digital assets outright, various financial service providers, such as digital exchanges and custodians, have expanded their infrastructure in anticipation to support potential investment in these digital assets.



## Distinguishing Characteristics of Digital Assets

There are notable differences between digital assets and traditional financial assets. The main similarity between these asset types is the emergence of indirect investment vehicles such as exchange-traded funds and hedge funds that invest both in traditional financial assets and in digital assets. The main differences between these asset types are as follows:

- *Differences in inherent value.* Unlike financial assets, most digital assets do not have an inherent value based on underlying assets or on the potential cash flow—interest and dividends—they can or are expected to generate. With no expected earnings to inform today's value and indicate the likely direction for tomorrow's prices, digital assets do not have a fundamental value. They derive their price solely from an anticipated asset appreciation from a perceived scarcity value (due to limitations on the total supply of currency) and the potential ability to transfer value in the future (due to unique features in the underlying algorithms that may facilitate certain types of financial transactions).
- *Differences in validating transactions.* One key difference between digital assets and traditional financial instruments is that traditional assets are generally recorded in private ledgers maintained by central intermediaries. The ownership and exchange of digital assets are generally recorded on a decentralized digital ledger using cryptography and enhanced algorithms. Whether a digital asset transaction is validated in a permissionless or permissioned network using either a PoS or PoW standard impacts its perceived value.
- *Differences in the uses as a medium of exchange.* Financial assets are traditionally priced and traded in widely accepted currencies; they can be readily transacted and exchanged into fiat currencies. Fiat currencies form the foundation of modern financial systems as the mediums of exchange in any country. In contrast, some digital assets—for example, cryptocurrencies, such as Bitcoins—are used as direct substitutes of real-world fiat currencies in certain circumstances—especially for online transactions in the emerging Web3 ecosystem. Web3 is a concept that refers to the third iteration of the internet built on blockchain technologies, decentralization, and token-based economies.
- While many digital assets, such as digital art and NFTs, are transacted and traded using cryptocurrencies, there is only a very limited acceptance of digital assets as a medium of exchange in the mainstream financial system to date. A reason has been the relative cost of the transactions, which can be prohibitively high to meaningfully process a large volume of transactions. While cryptocurrencies are being used to pay for certain goods and services and as investments, they are not legal tender in most jurisdictions and cannot usually be used to legally settle and extinguish debt in the same way as fiat currencies. Finally, the use of cryptocurrencies for payment and investment purposes is severely restricted in multiple countries; some countries have banned its usage outright and digital asset ownership may result in criminal conviction. Some governments are exploring the concept of CBDC as a tokenized version of fiat currencies, but the jury is still out how the idea will evolve.

**EXAMPLE 1****Tesla's Acceptance of Bitcoin and Dogecoin**

A high-profile supporter of cryptocurrencies and tokens has been Elon Musk, the CEO and major shareholder of Tesla Inc., the electric vehicle automaker. In February 2021, Tesla announced in a public filing that it has invested USD1.5 billion in Bitcoin to diversify its cash balance (of over USD19 billion at the end of 2020). It also indicated it would start accepting Bitcoin as a payment method for its vehicles subject to applicable laws and on a limited basis initially. But in May 2021, the company announced that it had suspended vehicle purchases using Bitcoin, citing concerns about increasing use of fossil fuels for Bitcoin mining and transactions. In July 2021, Musk indicated that Tesla would most likely restart accepting Bitcoin as payments once it had conducted due diligence on the amount of renewable energy used in Bitcoin mining. It resumed accepting Bitcoin; in January 2022, Musk announced that Tesla would start accepting Dogecoin as a payment method for selected brand merchandise. It is now the only cryptocurrency that Tesla accepts for online payment for certain Dogecoin-eligible products.

In July 2022, Tesla announced that it had sold off 75% of its Bitcoin holding as the price had plummeted, but none of its Dogecoin.

- *Differences in legal and regulatory protection.* The rules governing financial instruments and the trading of financial instruments are clear, predictable, and well defined across most jurisdictions. Legal and regulatory standards for traditional financial instruments and assets and trading in these are well developed and are often comparable across national borders; this is not the case with digital assets. Comprehensive rules that are specific to digital assets—their formation, trading, and legal standards—are still being developed. The unclear, ambiguous framework puts digital assets at a disadvantage. US regulators consider certain digital currencies as commodities, and other regulators consider them a non-financial asset. With no comprehensive and clear legal recognition—and protection—for digital assets, their value is highly speculative. Finally, the various exchanges at which digital currencies are traded are typically not regulated as a traditional financial exchange. Therefore, certain market behaviors, such as price manipulation and outright fraud, that are considered to be illegal activities on traditional financial transactions are not necessarily prohibited on a digital exchange. Exhibit 5 shows differences between digital and traditional financial assets.

**Exhibit 5: Digital Assets vs. Traditional Financial Assets**

	Digital Assets	Traditional Financial Assets
Inherent value	<ul style="list-style-type: none"> <li>▪ No fundamental value or future cash flow generation</li> <li>▪ Price driven by certain features on the blockchain</li> </ul>	<ul style="list-style-type: none"> <li>▪ Value determined by future cash flow generated from the assets</li> </ul>
Transaction validation	<ul style="list-style-type: none"> <li>▪ Usually recorded on decentralized digital ledgers using cryptography and algorithms for permissionless blockchain networks</li> </ul>	<ul style="list-style-type: none"> <li>▪ Recorded in private ledgers maintained by central intermediaries</li> </ul>

	Digital Assets	Traditional Financial Assets
Uses as a medium of exchange	<ul style="list-style-type: none"> <li>Very few digital assets are used as a direct medium of exchange, mainly targeting large-scale commercially viable acceptance</li> </ul>	<ul style="list-style-type: none"> <li>Not used directly as a medium of exchange but can be readily transacted and exchanged into traditional fiat currencies that are widely used in the real world</li> </ul>
Legal and regulatory protection	<ul style="list-style-type: none"> <li>Ambiguous, often contradictory, evolving framework; generally unregulated, with minimal legal protections</li> <li>Use can be illegal or criminal in some countries</li> </ul>	<ul style="list-style-type: none"> <li>Well-established, tested, and proven legal, regulatory, and commercial standards that are clear, predictable, and well defined across all jurisdictions</li> </ul>

## Investible Digital Assets

The rapid growth of digital assets has been remarkable. That several digital assets and cryptocurrencies exist reflects that their underlying blockchains are designed and optimized for different purposes. The most popular cryptocurrencies are Bitcoin and Ether. Bitcoin and Ether together make up over 80% of total cryptocurrency market value as of July 2022.

Bitcoin (BTC or XBT) was launched to secure payments in a P2P network. It was designed as an alternative to traditional currencies: as a medium of exchange and store of value. It is the best-known and most widely traded cryptocurrency and its fundamental design continues to influence the development of new types of digital assets. Thousands of other cryptocurrencies exist based on technology similar to Bitcoin: These are called **altcoins**.

### Altcoins

The most prominent altcoin is **Ether**, launched in 2015 on its own Ethereum network. Ether involves the added feature of a programmable blockchain that allows users to construct applications using the blockchain to validate or secure transactions or payments for markets or other uses. Ether and other programmable coins therefore have the possibility to be more widely used than simply as a store of value. Such programmable altcoins are also called smart coins or smart contracts, a self-executing contract with the salient terms of the contract directly written into the lines of code, which allows for execution of the contract using the blockchain network. That execution provides a trackable and irreversible record, subject to immutable verification by the nodes of the network. Other altcoins include stablecoins and meme coins.

### Stablecoins

**Stablecoins** are designed to maintain a stable value by linking their value to another asset and are collateralized by a basket of assets, typically legal tender, precious metals, or other cryptocurrencies. The reserve basket protects the holders from price volatility and minimizes the risk to the holders of the stablecoin should the cryptocurrency face transaction problems, including failure. Some stablecoins—smart stablecoins or algorithmic stablecoins—are designed to use algorithms to control the available supply of the asset, such as minting additional assets when there is increased demand for the coin. It is important to note that stablecoins cannot be exchanged for fiat money and do not have any legal or regulatory backing.

Stablecoins may potentially facilitate settlement and streamline cross-border trading, investing, and payments. In recent years, several competing solutions have been developed to facilitate transactions across various physical and tokenized financial assets and instruments using the stablecoin concept. A special example is the **asset-backed token**, which maintains price parity with some target asset (the

US dollar or gold) through tokenization and will be covered later in this module. For an investor holding assets in a cryptographic wallet, asset-backed tokens provide exposure both to assets that are on the chain (cryptocurrencies and digital assets) and off the chain (financial instruments).

## EXAMPLE 2

### Market Shakeout of Stablecoins

Stablecoins are designed to track the value of real-world conventional fiat currencies and to provide immediate access to liquidity for traders to redeem their digital stablecoins at par with the value of fiat currencies. Some of these stablecoins are collateralized by financial assets, such as a basket of fiat currencies, gold, Treasuries, and other liquid or illiquid assets. Two of the largest stablecoins in terms of collateral, Tether and USD Coins, use this approach and can either be traded freely on the open market or be redeemed for their dollar value from the issuers.

Another popular stablecoin, Terra, was often called an “algorithmic stablecoin.” It was backed not by collateral of financial and physical assets but through an algorithm that sought to keep its value stable.

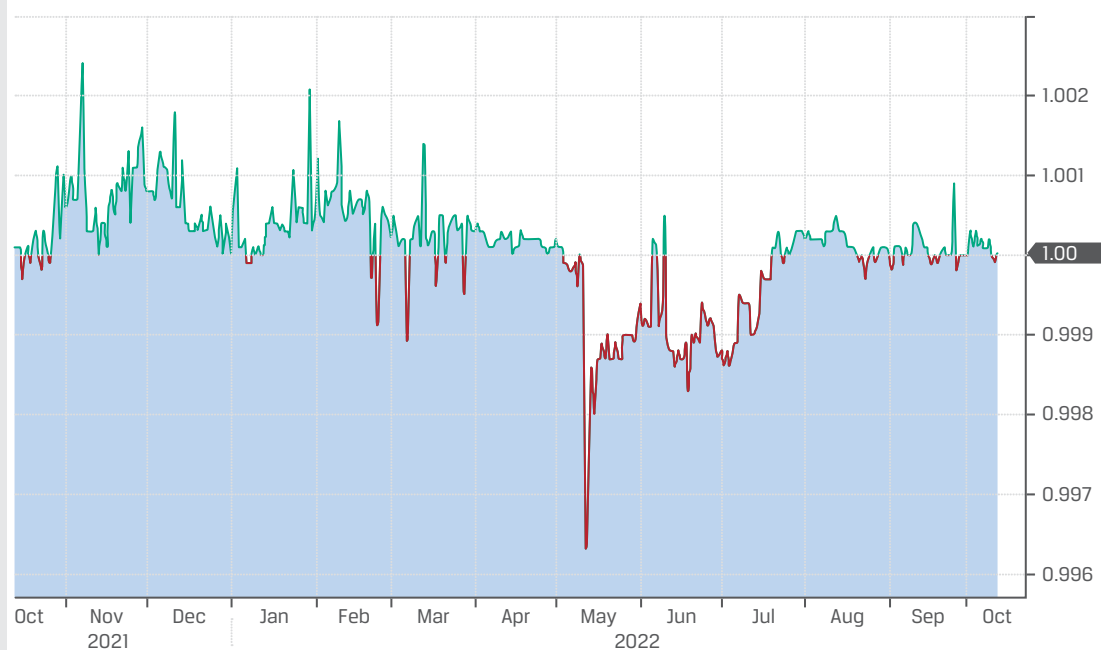
For instance, TerraUSD was Terra’s stablecoin pegged to the US dollar “algorithmically” instead of the value of the underlying collateral in US dollar reserves. The computer algorithm developed by the issuer of Terra, Terraform Labs, used Luna, a governance token also issued by Terraform Labs, to maintain the stability of Terra’s value. The relationship between Terra and Luna was designed so that the holders of Terra could always redeem it for a stable value of newly minted Luna coins. If the pricing relationship between Terra and Luna was not in equilibrium, new Luna coins could be issued or existing Luna coins would be destroyed to maintain the value relationship and this algorithmic peg between Terra and Luna. Through financial arbitrage, the stability of Terra’s value by trading between Terra and Luna would be held. Effectively, Terra was simply backed by Luna, both bearing a US dollar value.

In May 2022, this relationship was tested. On 9 May, the price of Luna first began to slide, putting pressure on the Terra’s peg. Many Terra owners rushed to redeem their coins, leading to a sudden surge in supply of newly issued Luna: In just one week, the number of Luna increased from 350 million coins to 6.5 trillion. Consequently, the price of Luna collapsed, as did that of Terra.

TerraUSD was finally depegged from the US dollar on 9 May 2022. Though Terra’s co-founder tried to revive Terra in a reincarnated form of TerraClassicUSD, the stablecoin had since lost its credibility. With its appeal diminished, the coin was practically defunct.

The collapse of Terra sparked serious concern over the collateral backing of its largest stablecoin counterparts, Tether and USD Coins, because of the limited transparency of their reserves. A massive run and redemption by nervous holders of these stablecoins soon followed.

While the “securitized” stablecoins, such as Tether, may have survived market turbulence, other “algorithmic” stablecoins may not when their technical flaws are exploited. Likewise, the value of asset-backed tokens depends on both the underlying assets and the prices of the relevant tokens. An example of Tether is shown in Exhibit 6.

**Exhibit 6: Market Price of Tether**

Source: Bloomberg

**Meme Coins**

**Meme coins** are cryptocurrencies often inspired by a joke and are generally launched for entertainment purposes as they gain popularity in a short period of time, which allows early purchasers to sell their holdings at an often considerable profit.

**EXAMPLE 3****Meme Coins—The History of Dogecoin**

The original and probably most popular meme coin is Dogecoin, which is based on the popular “doge” internet meme. Dogecoin, featuring a Shiba Inu dog on its logo, was originally founded in 2013 as a mockery of the popularity of cryptocurrencies, mainly Bitcoin. Even the name “dog” was misspelled as “doge” as part of a light-hearted parody.

While some cryptocurrencies are scarce by design, Dogecoins were intentionally designed to have no limit in supply. But what started as a joke soon gained popularity among fervent followers as a cult symbol, especially after social media endorsements by some high-profile business figures. At its peak in May 2021, Dogecoin’s market value had jumped to over USD80 billion. However, the price and market cap of Dogecoins plummeted as investors and speculators dumped these coins because their value seemed hardly justifiable. By May 2022, its market dropped to USD11 billion.

**QUESTION SET**

1. The market value of a digital asset is primarily driven by:

- A. the future price expectation of speculators.
- B. the future earnings generated from the digital assets.
- C. the cryptographic algorithm of the blockchain network.

**Solution:**

The correct answer is A. Unlike financial assets, most digital assets do not have an inherent value based on underlying assets or on the expected cash flow that can be generated. In other words, digital assets do not have a fundamental economic value. Their prices depend solely on the expected asset appreciation due to the perceived scarcity value and the potential ability to transfer value in the future; however, that value reflects market expectations at the time the transaction takes place.

2. Identify the following statement as true or false: The legal protection regulations on securities and commodities are generally applicable to digital assets.

**Solution:**

False. Whereas legal and regulatory standards for traditional financial instruments are well developed, legally tested, and proven; widely and uniformly enforceable; and often comparable across different jurisdictions and legal systems, comprehensive rules that are specific to digital assets—their formation, trading, and legal standards—are being developed. Additionally, the various digital exchanges on which digital currencies are traded are typically not as heavily regulated as traditional financial markets. Consequently, certain irregular market behaviors, such as price manipulation and outright fraud, are not always prohibited on a digital exchange.

3. The ownership and exchange of digital assets on permissionless networks are usually recorded on a:

- A. centralized ledger maintained by a central intermediary.
- B. decentralized ledger maintained by a central intermediary.
- C. decentralized ledger without any central intermediary.

**Solution:**

The correct answer is C. The ownership and exchange of digital assets on permissionless networks are generally recorded on a distributed (i.e., decentralized) digital ledger using cryptography and enhanced algorithms without the use of any central intermediary.

4. Compare and contrast the main types of stablecoins.

**Solution:**

The traditional type of stablecoin, or “securitized stablecoin,” is backed by physical collateral of fiat currencies, precious metals, or other financial assets as reserves for every stablecoin issued. Each coin can be traded freely on the open market or redeemed for its dollar value from the issuer by liquidating the collateral. A special example of the stablecoin is the asset-backed

token, which maintains price parity with some target asset—for example, the US dollar or gold—through tokenization.

Another unique type of stablecoin is called a “smart stablecoin” or an “algorithmic stablecoin,” because there is no backing of physical assets for the cryptocurrency. Rather, its value is backed by another cryptocurrency or token and is linked to the US dollar “algorithmically” instead of using actual dollar reserves.

## DIGITAL ASSET INVESTMENT FORMS

# 4



describe investment forms and vehicles used in digital asset investments

Investment in digital assets can take the form of direct investment on the blockchain or indirect investments in exchange-traded products and hedge funds. Direct ownership of Bitcoin and other cryptocurrencies involves the use of a **cryptocurrency wallet**, which stores the (public and private) digital codes required to access the asset on a computer website or mobile device application. Two different types of cryptocurrency exchanges exist.

- *Centralized exchanges* are the most popular type of exchange. These privately held exchanges provide trading platforms for cryptocurrencies and offer volume, liquidity, and price transparency. Although the concept of centralized exchanges is incompatible with Bitcoin’s decentralized ideology, they are very popular. Trading is electronic and direct, without any intermediating broker or dealer, and is hosted on private servers, exposing the centralized exchanges and their clients to security vulnerabilities. Should the exchange’s servers become compromised, the entire system may become paralyzed, halting trade, and leaking vital user information—such as cryptographic keys accessing the custodial wallets leaked. Some exchanges are regulated, and depending on jurisdiction, these exchanges may be regulated as financial exchanges or other types of financial intermediaries.
- *Decentralized exchanges* emulate blockchain’s decentralized protocol and operate similarly to how Bitcoin operates. Decentralized exchanges lack a centralized control mechanism and operate on a distributed platform without central coordination or control. This comes with the benefit that should one of the computers on the network be attacked, the exchange remains operational since there are numerous other computers that continue to operate on the network. That is why attacking decentralized exchanges is substantially more difficult, rendering such attacks almost certain to fail. Decentralized exchanges are difficult to regulate because no single individual, organization, or group controls the system. This means that those trading on decentralized exchanges are generally free to transact without any regulatory scrutiny, allowing for potentially illegal activity.

Both centralized and decentralized exchanges may face problems with fraud and manipulation and raise investor-protection concerns because they are not subject to rigorous oversight. Unlike the exchanges for more traditional assets, such as equity securities and futures contracts, cryptocurrency and cryptocurrency trading venues

are largely unregulated, and individuals or groups may engage in fraud or market manipulation (including using social media to promote cryptocurrencies in a way that artificially increases the price of a cryptocurrency).

#### EXAMPLE 4

### Pump and Dump Schemes of Cryptocurrencies

In a typical pump and dump scheme, earlier investors of an asset talk it up to encourage others to buy, pushing the price up. When the price is high enough, the scheme orchestrators quickly realize profits and dump the asset in the market, leaving unknowing investors with often steep trading losses. Given the hype around cryptocurrencies and the unregulated nature of the digital assets, pump and dump schemes are widespread.

EthereumMax (EMAX) is a digital token created on the Ethereum blockchain in May 2021, but it has no other connection to Ethereum. Touted as a “culture coin” that “bridges the gap between community-driven coins and foundational coins,” it was endorsed and heavily promoted by pop celebrities and famous sports stars just a month after it came into being. The price of EMAX rose sharply, with frenzied buying likely fueled by receiving heavy social media exposure. Its ascent was rapid, but its descent was faster: After one month, it had lost almost all its value and the interest in trading in it disappeared. It was then alleged that EMAX was a classic example of the typical pump and dump scheme. Subsequent legal action against the celebrities endorsing and “pumping” EMAX with false and misleading statements alleged that this scheme allowed EMAX executives to dump their holdings of EMAX for substantial profit while leaving other investors with worthless tokens.

#### EXAMPLE 5

### FTX Bankruptcy

FTX, a Bahamas-based cryptocurrency exchange that also owns FTX.US (available to US investors), grew rapidly after its founding in 2019.

As a centralized exchange for spot cryptocurrency and stablecoins, as well as futures, FTX became one of the largest such exchanges. It also provided custodial and transaction services. In 2021, it had 1.2 million users and generated over USD 1 billion in revenue. Like many digital currency exchanges, FTX issues an exchange token (FTT), a form of utility token used on its platform by market participants. As FTT may be borrowed or lent—and grants holders a discount on FTX trading fees—it represents some value (shown in Exhibit 7).

FTX has been funded by several successive rounds of venture capital (VC) investments, raising USD 400 million from VC firms and a major pension fund. In January 2022, FTX was valued at USD 31.6 billion.

FTX was considered to be a relatively stable and trusted alternative among digital currency exchanges. In 2022, declining cryptocurrency prices and rising interest rates created liquidity pressures for FTX and other cryptocurrency market participants.

This growing liquidity stress was compounded by reports that FTX and Alameda Research, a quantitative trading firm controlled by FTX's founder, held significant collateral assets in illiquid investments, including FTX-related assets. On 2 November 2022, a *Coindesk* article cited close ties between FTX and Alameda Research and noted that Alameda Research held over a third of its



USD 14.6 billion balance sheet either directly in FTT or as FTT-based collateral. This revelation prompted an FTT selloff by investors and customer withdrawals, with the FTT price falling from over USD 25 to around USD 1 in just over a week, as Exhibit 7 shows. On 11 November 2022, FTX filed for bankruptcy.

**Exhibit 7: FTT Digital Token Price (USD), January–November 2022**



Source: Bloomberg.

In its bankruptcy filing, FTX listed USD 9 billion in liabilities, with liquid investments comprising just USD 900 million of total assets. The number of creditors exceeds 100,000.

## EXAMPLE 6

### Ponzi Scheme Allegation over Forsage

A Ponzi scheme is an investment scam that typically involves the distribution of high returns to existing investors not from any real investment but from funds contributed by new investors. A variant of the Ponzi scheme is the pyramid Ponzi scheme, in which each level of schemers lures new investors in by promising high returns from investment opportunities that are usually fabricated. The new funds then go in to pay back the “returns” one level up. In effect, the “returns” earned by the earlier investors at the top are paid for by the cash contribution from the different levels of new investors.

Launched in January 2020, Forsage ran a decentralized blockchain platform that purportedly enabled online businesses to transact through smart contracts based on Ethereum, Tron, and Binance networks. But in August 2022, the US Securities and Exchange Commission (SEC) laid charges against 11 individuals, including the four founders based in Russia, Georgia, and Indonesia and certain US promoters, for investment fraud. The SEC alleged that Forsage operated a typical fraudulent crypto pyramid and Ponzi scheme that raised more than USD300 million from retail investors worldwide, helped by heavy endorsement and marketing by promoters on social media.

## Direct Digital Asset Investment Forms

Direct investments in digital assets are made on various digital exchanges where the transaction is recorded on the blockchain. Once such a transaction is entered between the parties, it becomes validated on the blockchain and a permanent record of the transaction is created. Most cryptocurrency exchanges are open and available 24/7, which allows for continuous trading.

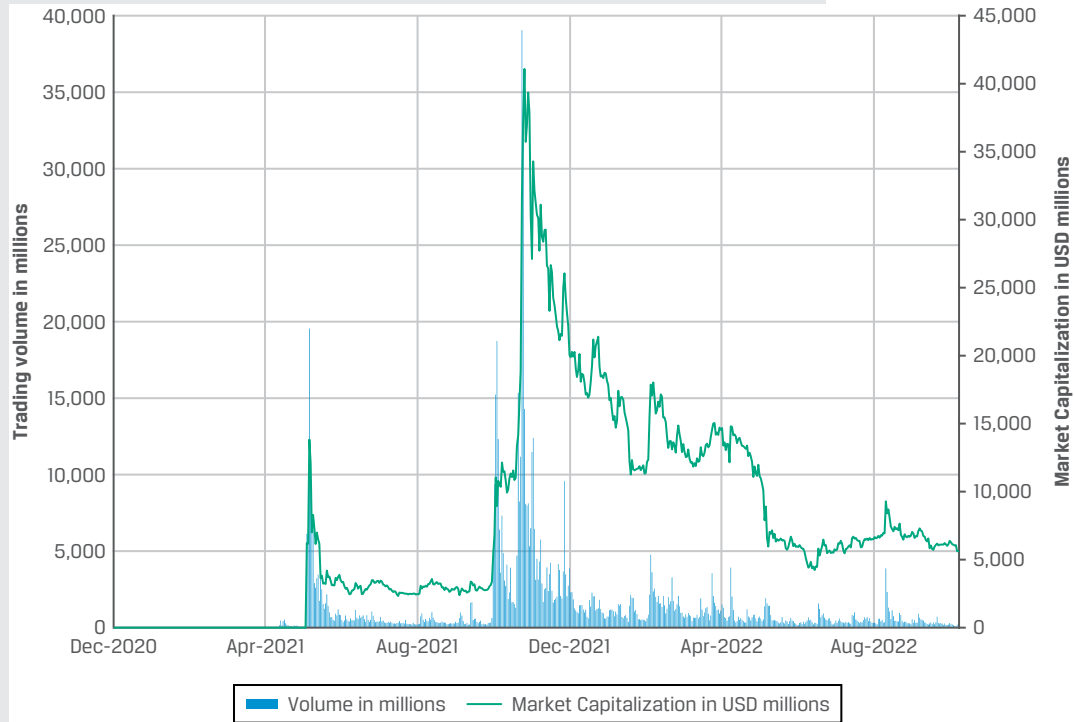
There are several risks with direct investment in cryptocurrencies. First, there is the risk for fraud, which has increased with the popularity of cryptocurrencies. Such fraud comes in many forms and includes scam ICOs, various pump and dump schemes, market manipulation, theft, and schemes that seek to gain access to credentials needed to access cryptocurrency wallet information. Second, since cryptocurrencies are usually held in a wallet that is accessible only using a unique passkey, losing access to the passkey makes the holdings in the wallet irretrievable. Around 20% of all Bitcoins are reportedly in lost or deserted wallets that their owners cannot access.

Moreover, many of the smaller cryptocurrencies may be held primarily by a small number of holders, sometimes referred to as “whales.” “Whales” is jargon used by the cryptocurrency community to mean individuals or entities that hold an amount of a cryptocurrency large enough to have the ability to manipulate the price.

### EXAMPLE 7

#### Cryptocurrency Whales

It was reported in mid-2022 that big investors on the Ethereum blockchain were accumulating the newly created meme coin, Shiba Inu (SHIB), a close competitor to Dogecoin. According to Whalestats, the crypto whale-watching platform, by August 2022, 41% of SHIB were “burned” (sent to “dead wallets” and effectively taken out of circulation). However, the next top 10 SHIB holders were estimated to be holding around 21%–23% of the total tokens (or 36%–39% of total coins in circulation). This might have allowed those “whales” to easily influence the price of SHIB. A glimpse into the trading volume might offer a hint about the effect on price action in a thin trading market environment, when the average daily trading volume was less than 10% of the total SHIB tokens. Exhibit 8 shows data on SHIB’s price and trading volume.

**Exhibit 8: Price Action and Trading Volume in Shiba Inu**

## Indirect Digital Asset Investment Forms

Several alternatives exist to gain indirect exposure to digital assets, including the following:

- *Cryptocurrency coin trusts* allow investors to trade shares in trusts holding large pools of a cryptocurrency and that trade over the counter (OTC) and behave like closed-end funds. For an investor in a coin trust, there is no need to create a digital wallet and use encryption keys to invest in cryptocurrencies. Additionally, the trusts may provide additional transparency into trading. The trusts charge substantial fees and expenses, in some cases in excess of 2%, and may trade at a premium or discount to their net asset values.
- *Cryptocurrency futures* contracts are agreements to buy or sell a specific quantity of Bitcoin or other cryptocurrency at a specified price on a particular future date. For instance, trading in Bitcoin futures on the Chicago Mercantile Exchange (CME) is based on the CME CF Bitcoin Reference Rate, which tracks the price of spot Bitcoin trading on cryptocurrency exchanges. Unlike physical commodities, these contracts are typically cash settled, with no actual cryptocurrency changing hands. Trading in futures is inherently leveraged. The market for cryptocurrency futures may be less developed and potentially less liquid and more volatile than more established futures markets.

- *Cryptocurrency exchange-traded funds:* An increasing number of exchange-traded products, such as ETFs, seek to replicate digital asset investment returns. These ETFs typically do not directly invest in cryptocurrencies and gain exposure to the value of cryptocurrencies using cash and cryptocurrency derivatives.

#### Baywhite Financial Cryptocurrency ETF (Fund) Term Sheet

<b>Fund description</b>	<p>The Fund is designed to provide investment results that generally correspond to the performance of major traded cryptocurrencies. It offers investors a convenient way to incorporate a rapidly growing digital asset into their portfolios as an alternative to traditional investments.</p> <p>The Fund does not invest directly in Bitcoin or other cryptocurrencies.</p>
<b>Fund investment strategy</b>	<p>The Fund seeks to provide capital appreciation primarily through managed exposure to cryptocurrency futures contracts. The Fund does not invest directly in cryptocurrencies. The value of cryptocurrencies is not backed by any government, corporation, or other identified body authorized to legally issue currency. Instead, its value is determined in part by the supply and demand in markets created to facilitate trading of cryptocurrencies. Ownership and transaction records for cryptocurrencies are protected through public-key cryptography. The supply of cryptocurrencies is typically determined by protocols that are unique to each specific cryptocurrency.</p> <p>The Fund may also invest in</p> <p><i>Cryptocurrency futures contracts</i>, standardized, cash-settled cryptocurrency futures contracts traded on exchanges</p> <p><i>Money market instruments</i>, such as short-term cash instruments with high-quality credit profiles, including short-term government debt and repurchase agreements</p>
<b>Fund trading</b>	The Fund is bought and sold through a brokerage account, eliminating the need for a cryptocurrency exchange account or wallet.
<b>Fund expenses</b>	Annual Fund operating expenses are 1.50% of NAV.
<b>Additional considerations</b>	<p>The Fund is non-diversified, with the ability to invest a relatively high percentage of its assets in financial instruments with a single counterparty or a few counterparties.</p> <p>Cryptocurrencies and cryptocurrency futures are relatively new investments. They are subject to unique and substantial risks and historically have been subject to significant price volatility. The value of an investment in the Fund could decline significantly and without warning, including to zero.</p>

- *Cryptocurrency stocks* provide indirect exposure due to their activity and relationship to digital assets. Examples include equity in publicly traded digital exchanges; payment providers accepting cryptocurrencies; corporations accepting cryptocurrencies as payments, investing in cryptocurrencies, or mining cryptocurrencies; and corporations developing and/or manufacturing products or services that are used for running blockchain networks, such as specialized computers used for mining.
- *Hedge funds investing in cryptocurrencies*, such as discretionary long, long/short, quantitative, and multi-strategy, have emerged as a major source of indirect digital asset investing. Several hedge funds actively mine for Bitcoin to generate further returns.

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**Sequester Capital LLC Crypto Hedge Fund ("Fund") Term Sheet**


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<b>Fund description</b>	The Fund is designed to provide investment results that generally correspond to the performance of major traded cryptocurrencies. The Fund invests directly in multiple cryptocurrencies.
<b>Fund investment strategy</b>	<p>The Fund uses a combination of common hedge fund strategies, including quantitatively driven trading strategies, discretionary long/short positions, and discretionary long-only positions in cryptocurrencies and other digital assets.</p> <p>The Fund invests in multiple cryptocurrencies, including Bitcoin, Ether, Litecoin, Cardano, Solana, and Polkadot. Additionally, the Fund trades in cryptocurrency derivatives and may from time to time take speculative short positions in certain cryptocurrencies that it believes to be overvalued.</p> <p>The Fund may engage in lending and borrowing cryptocurrencies.</p>

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## Digital Forms of Investment for Non-Digital Assets

These represent similar digital forms of investment with an underlying *non-digital* asset from which the investment derives its value.

A recent development is asset-backed tokens, digital claims on physical assets, financial assets, or financial instruments that are collateralized by the underlying asset and derive their value directly from the underlying asset. Asset-backed tokens are a digital representation of the ownership. Tokenized assets include gold, crude oil, real estate, and equities.

Asset-backed tokens could increase liquidity by allowing for fractional ownership of high-priced assets, such as houses, art, precious metals, and precious stones, which allows multiple investors to possess a fractional interest of the same asset. The digital representation of the ownership allows for an immutable record of ownership information and ownership transfer, which increases the transparency of these transactions and reduces transaction, intermediation, and record-keeping costs.

Financial regulators typically classify asset-backed tokens as securities, as the ownership of the token entitles the holder to an ownership interest in the underlying asset.

Asset-backed tokens are often issued on the Ethereum network or other smart contract platforms that allow for peer-to-peer interaction using interoperable, transparent smart contracts that persist for the duration of the chain. These decentralized applications, or dApps, allow for transactions to take place—and to be recorded on the blockchain—without a central coordinating mechanism.

The push for financial decentralized applications grew into a movement known as decentralized finance, or DeFi. DeFi seeks to design, combine, and develop various open-source financial applications as building blocks for sophisticated financial products and services. Effectively, DeFi is a marketplace of dApps that are designed to perform various core financial functions, including potentially acting as a medium of exchange, storage of value, tokenization of underlying assets, and immutable record keeping of ownership and transfer of ownership. Effectively, all kinds of smart contracts can be embedded in dApps to handle nearly all aspects of the traditional financial system, such as lending, trading, investment, settlement, payment, and transfer in a decentralized, authenticated and instantaneous manner. Some advocates of DeFi argue that the modern blockchain ecosystem may have more technical advantages over the traditional finance system—for example, time saving and risk reduction in settlement and asset transfer. However, the nascent idea of DeFi is yet to be fully developed. To date, most dApps have centered around extending leverage to investment and speculation in digital assets.

## QUESTION SET



1. Compare and contrast centralized cryptocurrency exchanges and decentralized cryptocurrency exchanges.

**Solution:**

- *Centralized exchanges* are privately held. They provide trading platforms for cryptocurrencies and offer volume, liquidity, and price transparency. Trading is electronic and direct, without any intermediating broker or dealer, and is hosted on private servers, exposing the centralized exchanges and their clients to security vulnerabilities. Should the exchange's servers be attacked, the entire system may be compromised, halting trade, and leaking vital user information. Depending on jurisdiction, some centralized exchanges are regulated as financial exchanges or other types of financial intermediaries.
- *Decentralized exchanges* emulate blockchain's decentralized protocol. Decentralized exchanges lack a centralized control mechanism and operate on a distributed platform without central coordination or control. The benefit is that should one of the computers on the network be attacked, the exchange remains operational since there are numerous other computers that continue to operate on the network. That is why decentralized exchanges are substantially less susceptible to computer hacks. Decentralized exchanges are difficult to regulate because no single individual, organization, or group controls the system. Therefore those trading on decentralized exchanges are generally free of any regulatory scrutiny, allowing for potentially illegal activity.

2. Identify the direct and indirect forms of investment in digital assets:

A. Direct investment	1. Cryptocurrency ETF
B. Indirect investment	2. Cryptocurrency coin trust
	3. Initial coin offering
	4. Hedge fund investing in digital tokens
	5. Buying a digital art NFT
	6. Trading cryptocurrency stocks
	7. Trading tokens on a cryptocurrency exchange
	8. Buying Bitcoin futures on a futures exchange

**Solution:**

A. Direct investment	3. Initial coin offering
	5. Buying a digital art NFT
	7. Trading tokens on a cryptocurrency exchange
B. Indirect investment	1. Cryptocurrency ETF
	2. Cryptocurrency coin trust
	4. Hedge fund investing in digital tokens
	6. Trading cryptocurrency stocks,
	8. Buying Bitcoin futures on a futures exchange.

3. Which of the following is *not* a risk in direct investment in digital assets?

- A. Market manipulation
- B. Pump and dump schemes
- C. Failure to validate asset transfers

**Solution:**

The correct answer is C. There are several risks with direct investment in cryptocurrencies, including the risk for fraud, such as scam ICOs, pump and dump schemes, market manipulation, theft, and schemes that seek to gain access to credentials needed to access cryptocurrency wallet information. However, transactions in digital assets such as cryptocurrencies and tokens are verified and authenticated by cryptographic algorithms and consensus protocols. Once a transaction is entered between the parties, it becomes validated on the blockchain and creates a permanent record of the transaction. Therefore, the DLT technology makes it very unlikely to fail validating asset transfers.

4. Determine the correct answers to fill in the blanks: DeFi is loosely defined as a marketplace for \_\_\_\_\_ that allows for financial transactions to take place and to be recorded on the \_\_\_\_\_ without a central coordinating mechanism. DeFi seeks to design, combine, and develop various \_\_\_\_\_ as building blocks for sophisticated financial products and services.

**Solution:**

DeFi is loosely defined as a marketplace for *dApps (or decentralized applications)* that allow for financial transactions to take place and to be recorded on the *blockchain* without a central coordinating mechanism. DeFi seeks to design, combine, and develop various *open-source financial applications* as building blocks for sophisticated financial products and services.

## DIGITAL ASSET INVESTMENT RISK, RETURN, AND DIVERSIFICATION

# 5



analyze sources of risk, return, and diversification among digital asset investments

Value appreciation of digital assets, such as Bitcoin and Ethereum, has been rapid since inception, most recently fueled by the introduction of more traditional indirect forms of investment. Because cryptocurrencies are a relatively new innovation, their market is subject to rapid price swings, changes, and uncertainty, with most investors considering them alternative investments.

## Digital Asset Investment Risks and Returns

Bitcoin and other cryptocurrency values are based solely on asset appreciation, with no underlying cash flows. The market demand for the limited supply of cryptocurrencies is a significant driver of prices. For instance, the supply of Bitcoin is limited to 21 million Bitcoins, by design. For this reason, Bitcoins are sometimes viewed by some investors as the digital version of gold.

Since its launch in 2009, Bitcoin's performance has been characterized by high return, high volatility, and low correlations with traditional asset classes, as seen in Exhibit 9 and Exhibit 10.

**Exhibit 9: Monthly Log>Returns for Bitcoin and Asset Class Benchmarks between January 2011 and January 2022**

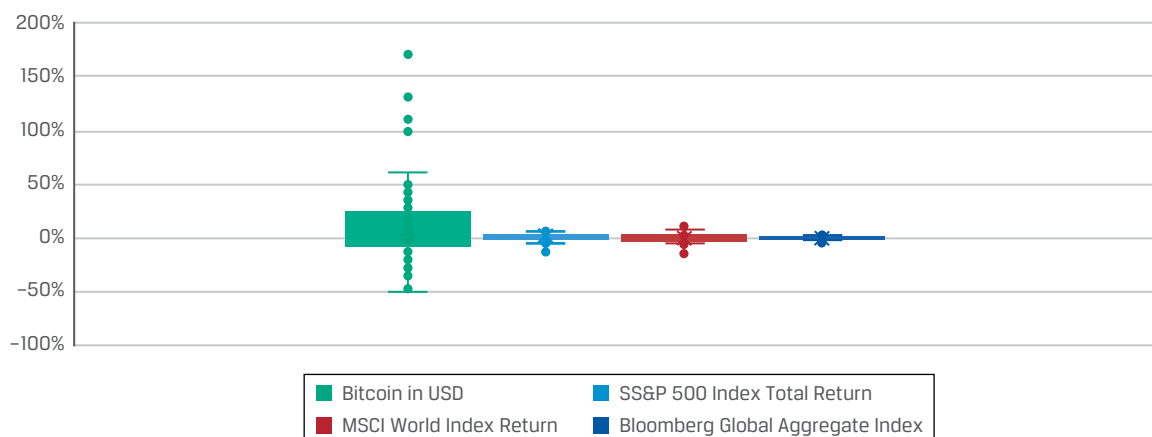
	Bitcoin in USD	S&P 500 Index Total Return	MSCI World Index Return	Bloomberg Global Aggregate Index
<b>Average</b>	8.84%	1.13%	0.66%	0.16%
<b>Standard deviation</b>	0.32	0.04	0.04	0.01
<b>Coefficient of variation</b>	3.66	3.43	6.09	8.16

*Notes:* Bitcoin price in US dollars, S&P 500 Total Return, MSCI World Index, and Bloomberg Global Aggregate Bond Index are log-returns for January 2011 to December 2022. All series are in US dollars.

From mid-2010, when publicly available market prices were available, Bitcoin price has risen from USD0.05 at inception to the historical peak of USD68,789 on 10 November 2021, before a subsequent crash took it down to around USD17,709 on 18 June 2022. Early investors in Bitcoins were hugely rewarded with phenomenal price appreciation.

Investing in cryptocurrencies therefore comes with material risks unique to digital assets. Cryptocurrencies have a relatively brief track record, yet their volatility has remained high, as Exhibit 10 shows. Although the volatility of Bitcoin has declined, it is still much higher than the volatility of traditional financial assets, such as the S&P 500 Index.

**Exhibit 10: Distribution of Monthly Log>Returns for Bitcoin and Asset Class Benchmarks between January 2011 and January 2022**





*Notes:* Bitcoin price in US dollars, S&P 500 Total Return, MSCI World Index, and Bloomberg Global Aggregate Bond Index are log-returns for January 2011 to January 2022. All series are in US dollars.

Bitcoin-like price and return patterns are often observed in other cryptocurrencies and may reflect a great deal of uncertainty about cryptocurrencies as an investable asset class. Given that regulation of cryptocurrencies is evolving, there still is no clear legal protection for using them as a medium of exchange. While they are regulated as a digital commodity in the United States and comprehensive rules in the EU remain pending, there is a great deal of uncertainty from an investor perspective about the quality of legal protection an investor can enjoy when investing in these assets. Considering that fraud and criminal activities are rampant among buyers, sellers, issuers, and marketers of digital assets, the continued legal and regulatory uncertainty is non-trivial. Multiple countries have placed extensive restrictions on trading and owning cryptocurrencies, such as China, which banned the asset in 2021.

## Diversification Benefits of Digital Asset Investments

Although purely speculative in nature, considered by some to have value drivers distinct from common equity and debt markets, cryptocurrencies have exhibited low correlations with traditional asset class returns. This may provide a hint that the long-term determinants of cryptocurrency prices differ from those of traditional investment assets. In practice, prices (or returns) of cryptocurrencies are driven more by market adoption, network effects, technological advancement, regulatory development, speculation, and general market risk appetite. Some of these factors are unique to this asset class. Exhibit 11 shows the monthly correlation of Bitcoin with some benchmarks.

**Exhibit 11: Correlations between Monthly Log-Returns for Cryptocurrencies and Selected Asset Classes (January 2011 to January 2022)**

	Bitcoin in US Dollars	S&P 500 Index Total Return	MSCI World Index Return	Bloomberg Global Aggregate Index
Bitcoin in US dollars	1			
S&P 500 Index Total Return	0.21	1		
MSCI World Index Return	0.22	0.97	1	
Bloomberg Global Aggregate Index	0.14	0.25	0.33	1

*Notes:* Bitcoin price in US dollars, S&P 500 Total Return, MSCI World Index, and Bloomberg Global Aggregate Bond Index are log-returns for January 2011 to January 2022. All series are in US dollars.

In this way, cryptocurrencies are potential portfolio diversifiers. The correlation of cryptocurrencies with traditional assets appears to increase, and the correlation and diversification benefits of cryptocurrencies remain to be seen.

**QUESTION SET**

1. More institutional investors are allocating capital to cryptocurrencies because of their:
  - A. low price volatility.
  - B. high expected cash flow.
  - C. low correlation with other traditional asset classes.

**Solution:**

The correct answer is C. The value of Bitcoin and other cryptocurrencies is based solely on asset appreciation, with no underlying cash flows. Historically, their price volatility has been extremely high. As such, cryptocurrencies are often considered a high-risk investment. Nevertheless, more institutional investors allocate their investment into cryptocurrencies because of the low historical correlation with other traditional asset classes, providing potential diversification benefits to a portfolio.

2. Explain the historical characteristics of cryptocurrencies' correlation with other traditional investments.

**Solution:**

Cryptocurrencies have historically exhibited low long-term correlations with traditional asset class returns. Prices of cryptocurrencies are driven more by market adoption, network effects, technological advancement, regulatory development, and general market risk appetite. Some of these factors are unique to this asset class. But as institutional participation increases in digital assets, the correlations with traditional assets have risen.

3. Identify the following statement as true or false: Regulatory uncertainty is a non-trivial risk for investors in cryptocurrencies.

**Solution:**

True. Regulatory uncertainty is a non-trivial risk for investors in cryptocurrencies because there is considerable risk of fraud and criminal activity among participants, and legal protection over these assets remains uncertain. Regulation of cryptocurrencies is evolving; some countries have adopted them, while some have banned them altogether.

## PRACTICE PROBLEMS

1. A benefit of distributed ledger technology (DLT) that increases its use by the investment industry is its:
  - A. scalability of underlying systems.
  - B. ease of integration with existing systems.
  - C. streamlining of current post-trade processes.
2. What is a DLT application suited for physical assets?
  - A. Tokenization
  - B. Cryptocurrencies
  - C. Permissioned networks
3. A cryptocurrency miner can earn new digital assets by:
  - A. solving complex algorithm puzzles to validate blocks of transactions onto a blockchain network based on a PoS protocol.
  - B. staking his own cryptocurrencies to validate blocks of transactions onto a blockchain network based on a PoW protocol.
  - C. validating and locking transactions onto a blockchain irrespective of the consensus protocol adopted by the particular network.
4. The perceived value of a cryptocurrency depends on the following factors *except*:
  - A. the maximum supply of the cryptocurrencies.
  - B. the underlying cash flow of the cryptocurrencies.
  - C. whether the relevant blockchain operates on a permissionless or permissioned network.
5. A key difference between digital assets and traditional financial assets is that:
  - A. digital assets are usually subject to a well-defined set of regulations.
  - B. transactions in digital assets are most likely kept on private centralized ledgers.
  - C. digital assets are valued not on their earnings but on the expected price appreciation given supply limitation.
6. Asset-backed tokens have the potential of improving the liquidity of the underlying assets because:
  - A. they allow for fractional ownership of high-value assets.
  - B. costs of validating the transactions are reduced on the blockchains.
  - C. they make it easier to trade the underlying assets on centralized exchanges.

7. Which of the following is the least vulnerable to a computer security hack?
  - A. Cryptocurrency wallets
  - B. Centralized cryptocurrency exchanges
  - C. Decentralized cryptocurrency exchanges
8. An investor who wants to replicate the return on Bitcoins without the use of a digital wallet can *best* do so by:
  - A. investing in a cryptocurrency coin trust that holds Bitcoins.
  - B. investing in a listed cryptocurrency exchange stock.
  - C. buying Bitcoins from a centralized cryptocurrency exchange.
9. Which of the following is a key building block of dApps?
  - A. Bitcoins
  - B. Smart contracts
  - C. Centralized exchanges
10. Compared to traditional equity investment, the historical return distribution of Bitcoins shows:
  - A. negative skewness.
  - B. high mean returns.
  - C. low standard deviation.
11. Despite higher volatility, cryptocurrencies may offer diversification benefits to an investment portfolio due to their:
  - A. scarcity value.
  - B. expected higher return.
  - C. lower correlation with other traditional investments.
12. Price volatility of cryptocurrencies is high most likely because they are:
  - A. subject to self-imposed limit on supply.
  - B. correlated with traditional risky asset classes.
  - C. highly speculative with no fundamental value or cash flow.

## SOLUTIONS

1. The correct answer is C. DLT has the potential to streamline the existing, often complex and labor-intensive post-trade processes in securities markets by providing close to real-time trade verification, reconciliation, and settlement, thereby reducing related complexity, time, and costs.
2. The correct answer is A. Through tokenization—the process of representing ownership rights to physical assets on a blockchain or distributed ledger—DLT has the potential to streamline this rights process by creating a single digital record of ownership with which to verify ownership title and authenticity, including all historical activity.
3. The correct answer is C. Under both PoW and PoS consensus protocols, the validation of the transactions, or “mining,” always comes with rewards. A successful miner that validates the transactions obtains new digital assets—either a cryptocurrency or a token. For blockchain networks based on the PoW protocol, the miner earns his digital assets by solving complex algorithm puzzles to validate blocks of transactions onto a blockchain network. For blockchain networks based on the PoS protocol, the miner earns his digital assets by staking his own to validate and attest to the new blocks of transactions.
4. The correct answer is B. A cryptocurrency derives its price solely from an anticipated asset appreciation given the perceived scarcity value arising from a self-imposed limit of supply. Whether a cryptocurrency transaction is validated in a permissionless or permissioned network using either PoS or PoW standard also impacts its perceived value. But most digital assets do not have an inherent value based on underlying assets or on the potential cash flow.
5. The correct answer is C. Unlike traditional financial assets, most digital assets do not have an inherent value based on underlying assets or the potential cash flow or earnings they are expected to generate. Digital asset transactions are usually kept on decentralized digital ledgers using cryptography and algorithms without any central intermediaries. Regulations around digital assets are also ambiguous and evolving, unlike most traditional financial assets, which are subject to a well-defined set of regulations.
6. The correct answer is A. Asset-backed tokens can increase liquidity by allowing for fractional ownership of high-priced assets, such as houses, art, precious metals, and precious stones, which allows multiple investors to possess a fractional interest of the same asset. B is incorrect because the validation of transactions on the blockchains takes up a substantial amount of computing power, which can be expensive.
7. The correct answer is C. Decentralized exchanges operate on a distributed platform without central coordination or control. The benefit is that should one of the computers on the network be attacked, the exchange remains operational since there are numerous other computers that continue to operate on the network. For this reason, decentralized exchanges are substantially more difficult to hack, rendering such attacks almost certain to fail. However, transactions on centralized cryptocurrency exchanges are hosted on private servers, exposing the centralized exchanges and their clients to security vulnerabilities. Should the exchange's servers become compromised, the entire system may become paralyzed, halting trade and leaking vital user information, such as cryptographic keys stored in the cryptocurrency wallets.

8. The correct answer is A. Investing in a cryptocurrency coin trust that holds Bitcoins is the best way to replicate the return on Bitcoins. Investing in a listed cryptocurrency exchange stock provides very indirect exposure that may not necessarily replicate the performance of Bitcoins because of a variety of company-specific factors, as well as the sensitivity to the general stock market environment. Buying Bitcoins from a centralized cryptocurrency exchange provides a direct exposure, but it would require a digital wallet.
9. The correct answer is B. Smart contracts are the building block of dApps. dApps are usually developed on the Ethereum blockchain network or other smart contract platforms that allow for peer-to-peer interaction using interoperable, transparent smart contracts that persist for the duration of the chain. These decentralized applications, or dApps, allow for transactions to take place—and to be recorded on the blockchain—without a central coordinating mechanism or centralized exchange.
10. The correct answer is B. Historical Bitcoin returns are characterized by high mean returns and high standard deviation. Despite the high volatility, the return distribution is positively skewed.
11. The correct answer is C. Although purely speculative in nature, cryptocurrencies have historically exhibited low correlations with traditional asset class returns and are considered to have value drivers distinct from common equity and debt markets. Therefore, cryptocurrencies are potential diversifiers of an investment portfolio.
12. The correct answer is C. Cryptocurrencies are purely speculative, with no fundamental value or future cash flow. There is no consensus on how to value them. Their markets are therefore subject to rapid price swings, changes, and uncertainty. But historically, their performance has relatively low correlations with traditional asset classes.